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A COMMENT ON UNSTEADY–PERIODIC FLOW FRICTION FACTOR: AN ANALYSIS ON EXPERIMENTAL DATA GATHERED IN PULSATILE PIPE FLOWS

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ABSTRACT

In 1940's, Schultz- Grunow proposed that time-average value of friction factor, $\lambda_{u,ta}$ was similar to its corresponding steady state value, λ for the presence of gradual and slow oscillations in pulsatile flows. A recent approach was available for low frequency pulsatile flows through narrow channels in transitional and turbulent regimes by Zhuang et al, in 2016 and 2017. In this analysis; extensive experimental data of $\lambda_{u,ta}$ in fully laminar and turbulent sinusoidal flow are processed in the measured time-average Reynolds number range of $1390 \leq Re_{ta} \leq 60000$ disregarding the transitional regime. The ranges of dimensionless frequency-Womersley number, $\sqrt{\omega'}$ and oscillation amplitude, A_1 are $2.72 \leq \sqrt{\omega'} \leq 28$ and $0.05 \leq A_1 \leq 0.96$ respectively. A multiplication element is defined as $Mel = Re_{ta} \times \sqrt{\omega'}$. A modified friction multiplier, λ_{Mel} which is similar to the conceptual parameter of Zhuang et al's friction factor ratio C ($\lambda_{Mel} = \frac{\lambda_{u,ta}}{\lambda}$) is also referred. The correlation of $\lambda_{Mel} = \lambda_{Mel}(Mel)$ is dependent on flow regime and the magnitude of Re_{ta} for the range of $\sqrt{\omega'} > 1.32$. The proposal of Schultz-Grunow is verified irrespective of the oscillations in turbulent regime since the magnitude of $\lambda_{Mel} = 1$ is observed for turbulent flow cases with $Re_{ta} \geq 35000$. In laminar regime the magnitude of Re_{ta} is governing the fact. The magnitude of λ_{Mel} varies in $0.589 \leq \lambda_{Mel} \leq 28.125$ for $Re_{ta} \leq 5000$ while $\lambda_{Mel} = 1$ is obtained for $Re_{ta} > 5000$. The graphical representation of $\lambda_{Mel} = \lambda_{Mel}(Mel)$ can be considered as a counterpart of Moody Diagram in pulsatile fields for a significant practice.

Keywords: *Time-Average Friction Factor, Time-Average Reynolds Number, Womersley Number, Multiplication Element, Modified Friction Multiplier = C*

INTRODUCTION

Hagen and Poiseuille [1] determined steady laminar flow through a circular pipe by means of their independent experiments in 1840 's. Reynolds determined the flow nature change from laminar to turbulent in 1883. Darcy-Weisbach equation is used for the calculation of frictional head loss through a steady pipe flow in terms of steady mean velocity, U steady flow friction factor, λ, and the pipe dimensions of length L_p and diameter D. In the case of smooth pipes, λ is only a function of steady flow Reynolds number, $Re = \frac{UD}{\nu}$. The plot of λ against Re on a log-log chart is known as Stanton Diagram. Blasius is the first correlating the smooth pipe friction factors in turbulent flow fields by experiments leading to the well known Blasius Formula in 1913 [2]. In reference to the Hagen-Poiseuille equation, smooth pipe friction factors inside laminar range can also be determined. Moody Diagram [3] which has the confirmation of Nikuradse through his controlled rough pipe experiments in 1940 [4] is still in use for the determination of frictional characteristics of steady pipe flows (Table 1) The validity of Moody Diagram for an unsteady- periodic pipe flow is the major topic of the discussion since Schultz-Grunow's historical proposal [5, 6] on the equivalence between λ and $\lambda_{u,ta}$ still needs verification.

A sinusoidal-pulsatile flow having a periodic time-dependence is considered as a special sub-class. The experimental data gathered on fully laminar and fully turbulent pulsatile fields through the research projects; MF 97-04 [7] and MF 09-09 [8] granted by Gaziantep University–Turkey are processed. The friction factors of sinusoidal flow are given in comparison with the steady flow basics by means of introduced correlation parameters for the purpose.

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Table 1. Comparison of steady and sinusoidal flow fields

Flow	Steady	Sinusoidal
Governing Equation	Force Balance (Linear Momentum Principle) $\frac{d}{dt} = 0$	$\rho \frac{d\bar{U}_m(t)}{dt} + \frac{4\bar{\tau}_w(t)}{D} = \frac{\Delta\bar{P}_{(t)}}{L}$ $\frac{d}{dt} \neq 0$
Friction Factor **Available Equation**	$\lambda = (2\, g\, h_f D) / (L_p U^2)$ h_f frictional head loss L_p pipe length $\lambda = 64/Re$ (Re<2000) $\lambda = 0.3164/Re^{0.25}$ (2000< Re< 100000)	$\lambda_{u,ta} = \frac{8}{\rho(\bar{U}_{m,ta})^3 T} \int_0^T \bar{\tau}_w(t)\bar{U}_m(t)dt$
Governing Parameters of Flow Dynamics	Re = UD/υ	$Re_{ta} = \frac{\bar{U}_{m,ta}D}{\nu}$ $\sqrt{\omega'}$ $A_1 = \frac{\lvert U_{m,os,1}\rvert}{U_{m,ta}}$

BASICS OF PULSALITE FLOW DYNAMICS

Although the details of the flow dynamics can be found in the previous publications of the author [9-12] based upon [13, 14] the following basics are essential to reveal the discussion:

- Sinusoidal flow is generated by a variety of frequency, f and amplitude of oscillation, A_1 resulting in a range of time-average Reynolds numbers Re_{ta} $\left(Re_{ta} = \frac{\bar{U}_{m,ta}D}{\nu}\right)$. Since the pipe size, D is a serious restriction on f, a non-dimensional frequency parameter, Womersley number ($\sqrt{\omega'} = R\sqrt{\frac{\omega}{\nu}}$, $\omega = 2\pi f$) is used as a common base. As first described by Ohmi et al [15, 16] and verified by the author's cited research above; quasi-steady region and inertia dominant region is coupled with $\sqrt{\omega'} \leq 1.32$ and $\sqrt{\omega'} > 27.72$ respectively. The so-called intermediate region of which $1.32 < \sqrt{\omega'} < 27.72$ indicates the flow range having the greatest departure from a steady one under the influence of oscillations.

- Pulsatile flow dynamics is governed by the following momentum-integral equation which is valid for both laminar and turbulent regime:

$$\rho \frac{d\bar{U}_m(t)}{dt} + \frac{4\bar{\tau}_w(t)}{D} = \frac{\Delta\bar{P}_{(t)}}{L} \tag{1}$$

The usual practice without an accuracy loss is to use an approximation by the first harmonics of the fitted FFT transformation as follows:

$$\bar{U}_m = \bar{U}_{m,ta} + \lvert\bar{U}_{m,os,1}\rvert sin\left(\omega t + \angle\bar{U}_{m,os,1}\right) \tag{2.a}$$

$$\Delta\bar{P}_{(t)} = \Delta\bar{P}_{ta} + \left|\Delta\bar{P}_{os,1}\right| sin\left(\omega t + \angle\Delta\bar{P}_{os,1}\right) \quad (2.b)$$

Table 2. The experimental data range with a brief outline on the measurement-data acquisition chain and calculated sensitivities [11, 17]

Data Source	***Re_ta*** **Range** (Sensitivity in Re_{ta},±) (Uncertainty in $\lambda_{u,ta}$)	***f*** **Range Hz** $\sqrt{\omega'}$ range (Sensitivity in $\sqrt{\omega'}$ ±)	**Velocity** **Pressure** Measurement Details (Sensitivity)	**Data Acquisition System** (Data Accumulation)
MF 97-04	2000-60000 (1.5%-17%) (7.1 %)	0.1-3 7- 28,0 (0.01% - 4% as a function of Re_{ta})	DANTEC CTA 56C01 (±0.15%) Difference, HBM-PD1 (±1%)	I/O board KEITHLEY,DAS 1602 100 kilo samples/s (30 phases of oscillation, 200 cycles at each phase 6000 data readings)
MF 09-09	1390-4817 (3.4%), (7.1 %)	0.1-14 2.72-28 (1.2%)	DANTEC CTA 56C01 (±0.15%) Local, WIKA-SL1 (±0.8%)	IO TECH Daq 3001 USB 100 Hz (5000 data readings)

Instantaneous wall shear stress, $\bar{\tau}_w(t)$ can be calculated using the direct measurements of instantaneous velocity and pressure drop in Equation 1. The calculation of $\bar{\tau}_w(t)$ is used to determine the non-dimensional time average friction factor, $\lambda_{u,ta}$ [15, 16] as:

$$\lambda_{u,ta} = \frac{8}{\rho(\bar{U}_{m,ta})^3 T}\int_0^T \bar{\tau}_w(t)\bar{U}_m(t)dt \quad (3)$$

- The comparison of a sinusoidal flow with a steady one is outlined in Table 1. There is a need for a widely accepted equation for the calculation of $\lambda_{u,ta}$ in terms of relevant parameters. Therefore following functional relationship can be estimated:

$$\lambda_{u,ta} = \lambda_{u,ta}\left(Re_{ta}, \sqrt{\omega'},\ A_1\right) \quad (4)$$

THE DETAILS OF EXPERIMENTAL RESEARCH

The range of the experimental data, the measurement and data acquisition-details, sensitivities and uncertainties of the basic parameters are given in Table 2. The test systems are open circuit ones through which air flow is generated by suction type and blowing type arrangements. The rigid and smooth PVC pipes of D = 50.4 mm and D = 26.6 mm are used in each system. The primary difference of the test systems come from the method of oscillation generation. In the first system [7, 11, 13] an oscillation generator in the form of a reciprocating piston driven by a scotch yoke mechanism is used. The amplitude of oscillation is controlled by using a variety of piston strokes while frequency control is by means of speed control of the scotch-yoke mechanism. In the second test system [8, 14, 17] an electronic mass flow controller unit, MFC [18] of Durst et al is used. The electro dynamic coil system of MFC is operated by analog voltage inputs (in 0-10 V range) to control amplitude and *f* of oscillation. The test systems generate an extensive range of pulsatile air field in the controlled magnitudes of *f* and amplitude.

The measurements of local instantaneous velocity wave forms and local instantaneous pressures are used to calculate $\lambda_{u,ta}$.

The sinusoidal nature of the flow is verified by the measured instantaneous local velocity and pressure wave forms and the calculated wall shear stress. The analysis on the frictional field characteristics of pulsatile flow can be used to determine the critical state in passage from laminar to turbulent regime. [19, 20, 21] Meanwhile the time-averaged cross-sectional velocity distribution of pulsatile flow resemble the well known Blasius and Prandtl 1/n th power laws for laminar and turbulent flow regime of steady flow respectively. The influence of $\sqrt{\omega'}$on the velocity field is such that $\sqrt{\omega'} = 8.61$ [20] is the estimated limit for which flow pattern is varying for the range of high and low magnitudes of $\sqrt{\omega'}$. However the magnitude of A_1 is only governing the transition process from laminar to turbulent flow separately in low and high Womersley number regions. Inside laminar and turbulent flow regimes A_1 does not have a strong influence on flow dynamics. The strong influence of A_1 is found in the generation of first turbulent bursts at the onset of transition to turbulence [8, 22, 23, 24]. Therefore disregarding the non-linear transition process, inside laminar and turbulent fields; the governing oscillation parameter is $\sqrt{\omega'}$.
The aim is to determine the validity of Schultz-Grunow's proposal and deduce on suitability of Moody Diagram for a sinusoidal flow field in laminar and turbulent regimes through a rigid smooth pipe according to Equation 4. The flow is laminar and turbulent for $1390 \leq Re_{ta} \leq 10000$ and $Re_{ta} \geq 35000$ respectively.

RESULTS AND DISCUSSION

Sinusoidal Flow Nature and Limits of The Experimental Data

The covered parameter ranges (Table 2) are $1390 \leq Re_{ta} \leq 60000$, $2.72 \leq \sqrt{\omega'} \leq 28$, $0.05 \leq A_1 \leq 0.96$. The generation and control of oscillation is by means of f and A_1 meanwhile the measurement of cross-sectional velocity distribution is coupled with the calculation of generated flow's Re_{ta}. The nature of sinusoidal flow is such that flow is fully laminar for $Re_{ta} \leq 17929$ and it is fully turbulent for $Re_{ta} \geq 23763$ as was proposed by Carpinlioğlu [19]. The utilized data belong to the following cases:

- Laminar flow with $Re_{ta} \leq 5000$ for 15 separate test cases
- Laminar flow with $Re_{ta} = 10000$ for 5 separate test cases
- Turbulent flow with $Re_{ta} = 35000$ and $Re_{ta} = 60000$ for 10 separate test cases

The turbulence level of flow, I is 1.5% in the close proximity of the pipe surface while a minimum magnitude of 0.5% prevails at the pipe centerline. The amount of uncertainty in the calculation of $\lambda_{u,ta}$ is found as ± 7,1% [8, 14, 20, 24] .This is in conformity with the accepted magnitude of experimental data scattering of λ as ± 5% in log-log plot of λ versus Re in Stanton Diagram for steady pipe flows [1].

The range of the experimental magnitudes of $\lambda_{u,ta}$ are given in Table 3 as a function of Re_{ta}. Inside laminar flow $\lambda_{u,ta}$ varies between 0.01 and 0.9 for $Re_{ta} \leq 5000$ while at $Re_{ta} = 10000$, $\lambda_{u,ta}$ varies between 6,4 x10^{-3} and 9.6x10^{-3}. Inside turbulent flow of $\lambda_{u,ta}$ varies between 0.02 and 0.03 for $Re_{ta} \geq 35000$. The measured minimum magnitudes of $\lambda_{u,ta}$ belong to the cases of laminar regime at Re_{ta} = 10000.
A correlation study is conducted to determine the influence of $\sqrt{\omega'}$, A_1 in laminar and turbulent regimes which are expressed by the determined Re_{ta} ranges.

Table 3. The range of the sinusoidal flow time average friction factors with the calculated range of modified friction factors

Re_{ta} Sinusoidal	Sinusoidal Flow Nature	Range of $\lambda_{u,ta}$ Experimental	λ (λ_{sL} , λ_{sT})
1473	Laminar	0.01 - 0.9	0.043
5000			0.0128
10000		(6.4 – 9.6) x10^{-3}	6.4 x10^{-3}
35000	Turbulent	0.02 - 0.03	0,032
60000			0.020

Ohmi and Iguchi [25] deduced that magnitude of instantaneous friction factor in pulsating turbulent pipe flows was regardless of oscillation in quasi steady region. However for intermediate region and inertia dominant region deviation from steady state value is a fact. Moreover $\lambda_{u,ta}$ increased with increase in A_1 and $\sqrt{\omega'}$ in terms of their correlation parameter through $\sqrt{\omega'}$ and $Re_{ta}{}^{3/4}$.

Although their analysis set the interactive influence of A_1 and $\sqrt{\omega'}$ which was also noted in the author's early paper [12] ; the significant influence of A_1 is restricted to the onset of transition in reference to [8, 14, 21, 22, 23, 24]. This interactive influence is discussed in a recent study of the author [26].

However for the analysis herein for the extensive range of A_1, $0.05 \leq A1 \leq 0.96$, the magnitudes of Re_{ta} and $\sqrt{\omega'}$ of each case reflect the interactive influence of A_1 for fully laminar and turbulent regime. Therefore Equation 4 can be given by Equation 5 as:

$$\lambda_{u,ta} = \lambda_{u,ta}\,(Flow\ Regime\ , Re_{ta}\ , \sqrt{\omega'}) \tag{5}$$

The deductions of Schultz-Grunow cited later [6] are such that the magnitude of $\lambda_{u,ta}$ is the same of its steady state magnitude, λ. for water flow through a converging-diverging passage for the presence of gradual and slow oscillations. However there is no information either on the magnitudes of $\sqrt{\omega'}$ and A_1 or on their interactive influence even with no comment on the flow regime.

As a first trial on Schultz-Grunow's proposal, a modified sinusoidal flow friction factor is defined expressing the influence of oscillations in terms of Re_{ta}. In this respect λ_{sL} and λ_{sT} are calculated using the assumption of $Re_{ta} = Re$ in steady flow formula of λ given in Table 1 for laminar and turbulent regimes . The calculated magnitudes of λ_{Sl} and λ_{sT} are given in the fourth column of Table 3. In fully turbulent flow regime the magnitudes of $\lambda_{u,ta}$ and λ_{sT} are almost the same. Contrary to this fact, in laminar flow; the magnitudes of $\lambda_{u,ta}$ are considerably different from those of λ_{sL}. The influence of oscillations in frictional field behaviour is apparently a flow regime–dependent fact. It is also important to note that sinusoidal flow at $Re_{ta} = 10000$ is of laminar nature while steady flow at $Re_{ta} = Re = 10000$ is of turbulent nature. The magnitudes of $\lambda_{u,ta}$ at $Re_{ta} = 10000$ which are considerably smaller than the ones of $\lambda_{u,ta}$ for $Re_{ta} \leq 5000$ need an explanation.

Definition of Dimensionless Parameters For Correlation of Time-Average Friction Factor Through Moody Diagram

In order to take into account the sole influence of $\sqrt{\omega'}$ on a specified Re_{ta}, in the covered ranges of Re_{ta} and $\sqrt{\omega'}$, a multiplication element *Mel*, is defined as follows:

$$Mel = Re_{ta} \times \sqrt{\omega'} \tag{6.a}$$

Since quasi-steady pulsatile flow can be described by steady flow basics this definition is valid for the following Womersley number range:

$$\sqrt{\omega'} > 1.32 \tag{6.b}$$

Therefore Equation 5 can be given as follows:

$$\lambda_{u,ta} = \lambda_{u,ta}\,(Mel) \tag{7.a}$$

Similar to Moody Diagram, a log-log plot of Equation 7.a is given for the laminar cases of $Re_{ta} \leq 5000$ and $Re_{ta} = 10000$ in Figure 1 and for turbulent cases of $Re_{ta} = 35000$ and $Re_{ta} = 60000$ in Figure 2.

As can be seen from Figure 1, $\lambda_{u,ta}$ is a function of *Mel* for $Re_{ta} \leq 5000$ such that an increase in *Mel* is resulted in an increase in $\lambda_{u,ta}$. Increase in *Mel* is either increase of Womersley number at a constant Re_{ta} or vice versa. However it is not possible to fit an equation to describe Equation 7.a due to the considerable data scattering. On the other hand at $Re_{ta} = 10000$ for $Mel < 200000$, $\lambda_{u,ta}$ is not a serious function of Womersley number thus Equation 7.a is expressed by:

$$\lambda_{u,ta} = cst \tag{7.b}$$

The magnitude of *cst* in Equation 7.b is 6.4x10^{-3}. A slight increase in $\lambda_{u,ta}$ is observed for $Mel \geq 200000$ in conformity with Equation 7.a. The data behaviour for turbulent flow (Figure 2) is similar of $Re_{ta} = 10000$ given by Equation 7.b with different magnitudes of $\lambda_{u,ta}$ and *Mel.* The validity range of Equation 7.b is for $178500 \leq Mel \leq 1680000$. An increase in Re_{ta} is coupled with a decrease in $\lambda_{u,ta}$ such that the magnitude of *cst* of Equation 7.b is 0.028 and 0.022 at $Re_{ta} = 35000$ and $Re_{ta} = 60000$ respectively. Therefore minor influence of Re_{ta} on $\lambda_{u,ta}$ can be estimated as follows:

$$\lambda_{u,ta} = \lambda_{u,ta}(Re_{ta}) \tag{7.c}$$

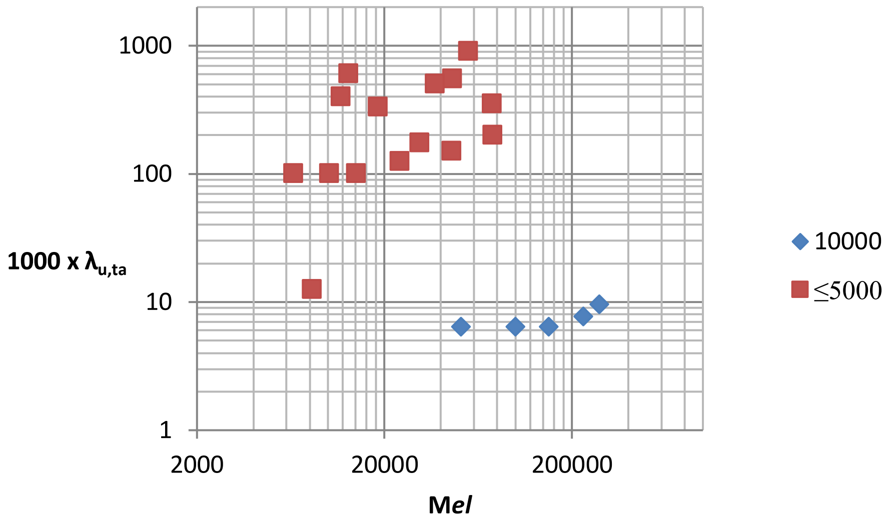

Figure 1. Variation of $\lambda_{u,ta}$ with M*el* inside laminar sinusoidal flow as a function of Re_{ta} for $2.72 \leq \sqrt{\omega'} \leq 28$ and $0.05 \leq A_1 \leq 0.96$

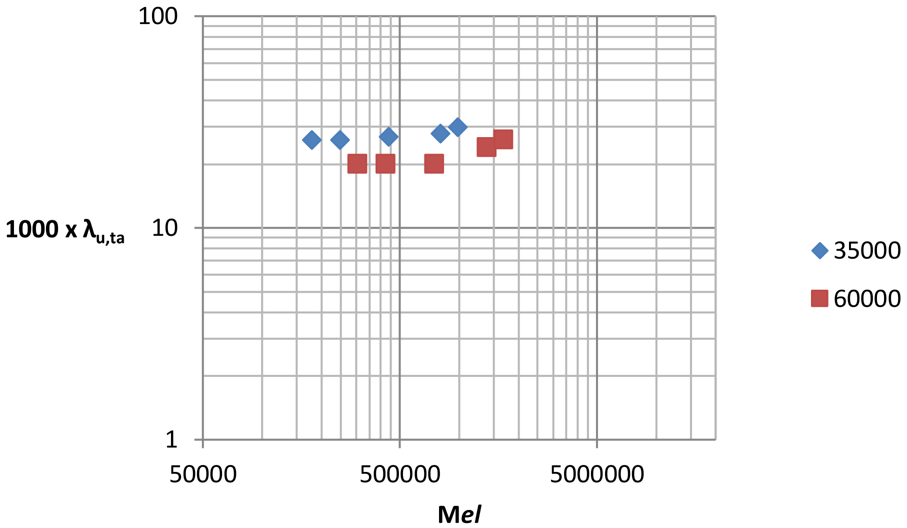

Figure 2. Variation of $\lambda_{u,ta}$ with M*el* inside turbulent sinusoidal flow as a function of Re_{ta} for $2.72 \leq \sqrt{\omega'} \leq 28$ and $0.05 \leq A_1 \leq 0.96$

Sinusoidal flow regime limits can be given by $8208 \leq Mel \leq 75644$ for laminar case at low Re_{ta} and $51000 \leq Mel \leq 280000$ at $Re_{ta} = 10000$. Meanwhile turbulent cases have the range of $178500 \leq Mel \leq 1680000$.

In order to complete the discussion with a general functional relationship valid for periodic flows, a modified friction multiplier λ_{Mel} which is the same of Zhuang et al's friction ratio C is defined with an emphasis on flow regime as follows:

$$\lambda_{Mel} = \frac{\lambda_{u,ta}}{\lambda_{sL}} \tag{8.a}$$

inside laminar flow for $Re_{ta} \leq 17929$,

$$\lambda_{Mel} = \frac{\lambda_{u,ta}}{\lambda_{sT}} \tag{8.b}$$

inside turbulent flow for $Re_{ta} \geq 23763$.

These definitions are conceptually similar to the one introduced by Zhuang et al [27, 28] for their analysis on transitional and turbulent low frequency pulsatile flows in narrow mini channels.

Thus Equation 7.a can be given as follows:

$$\lambda_{Mel} = \lambda_{Mel}(Mel) \tag{9.a}$$

A log-log plot of Equation 9.a is given in Figure 3. As can be seen from Figure 3 for $8208 \leq Mel \leq 75644$ corresponding to $Re_{ta} \leq 5000$ an increase of λ_{Mel} from 0.589 to 28.125 is observed. However no influence of oscillations is seen with $\lambda_{Mel} = 1$ both in turbulent flows for $178500 \leq Mel \leq 1680000$ and in laminar flows for $51000 \leq Mel \leq 200000$ at $Re_{ta} = 10000$.

Therefore Schultz-Grunow 's emphasis on gradual and slow oscillations should be shifted to the statement of low and high Re_{ta} flows taking into flow regime into account as follows:

- Presence of oscillations in turbulent flow is such that steady state empirical equations can be used for the calculation of $\lambda_{u,ta}$ with $Re = Re_{ta}$. It seems that neither Re_{ta} nor Womersley number is governing the field. The nature of turbulence overcome the oscillations since the magnitude of λ_{Mel} is almost around 1 in the range of $Mel > 200000$.
- In laminar flow at high Re_{ta} the minimum magnitudes of $\lambda_{u,ta}$ with the satisfaction of steady state empirical equation may be due to the weaking influence of oscillations towards the passage of turbulent flow. The magnitude of λ_{Mel} is almost around 1 in the range of $51000 \leq Mel \leq 200000$.
- In laminar flow at low Re_{ta}, the presence of oscillations cause a drastic departure from steady state empirical equations for the calculation of $\lambda_{u,ta}$ as a function of Mel. The strong influence of the oscillations in laminar regime sensed by a delayed transition to turbulence may be the reason of radical increase of λ_{Mel}. The delayed transition can be defined as a periodic re-laminarization which possibly causes an increase in $\lambda_{u,ta}$.

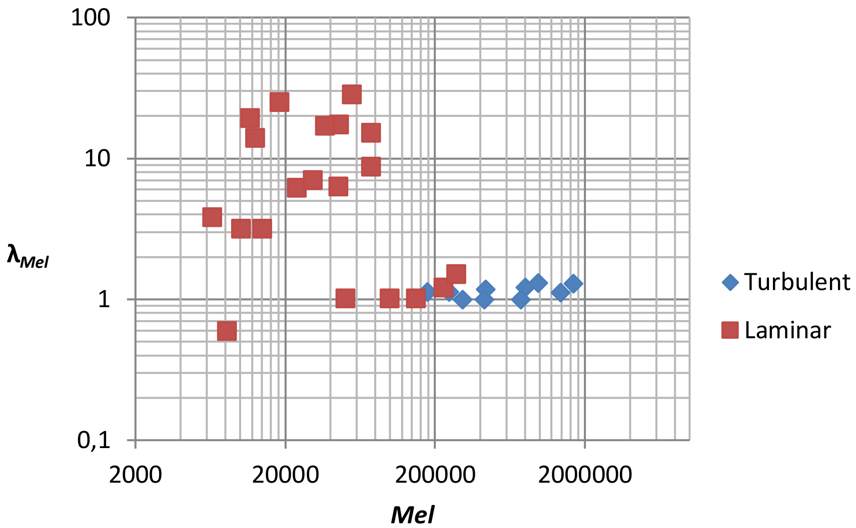

Figure 3. *Mel*-Moody Diagram

Discussion on Zhuang et al's Approach

An important discussion and comparison should be given as a concluding part in reference to the similar correlative approach applied recently by Zhuang et al [27]. In their experimental study transitional range of pulsatile flow for narrow channels was analysed. The range of Re_{ta} is $575 \leq Re_{ta} \leq 5583$, with a rather low frequency given by Womersley number $\sqrt{\omega'}$range of 0.52-2.34 and for an oscillation amplitude range A_1 ; $0.056 \leq A_1 \leq 0.988$. Although the same ranges of oscillation amplitudes are used ; their ranges of Re_{ta} and Womersley number are restricted in comparison with the utilized range of the present study due to their emphasis on transitional period. Another important difference is the size–shape of the channel and the flowing fluid. Furthermore their Womersley number range belongs to quasi-steady region. Although it is estimated that in quasi-steady region [23, 25] the influence of oscillations are ignorable. They denoted the interactive influence of Womersley number and oscillation amplitude in quasi steady range reversely as an important contribution. It seems that inside quasi- steady region low frequency oscillations with a great range of amplitudes have a serious effect on Re_{ta} . Therefore they introduced a dimensionless acceleration parameter and proposed a correlation for critical Re_{ta} in terms of both Womersley number and amplitude of oscillation. They found that increase in amplitude and frequency of oscillation (in their restricted quasi-steady range) results in an increase of critical Re_{ta}. Meanwhile their range of C corresponding to λ_{Mel} of the present study has a maximum magnitude in the order of 1.5. In our analysis a similar argument is observed for laminar pulsatile flow in the range of $51000 \leq Mel \leq 200000$.

In [28] they considered turbulent pulsatile flow characteristics to determine turbulence generation mechanism as a major function of aspect ratio of mini channels. They determined a variety of C magnitudes as maximum ones in the interactive influence of Womersley Number and oscillation amplitude A_1.

The following ranges are listed for the approximate maximum values of C;

- For $Re_{ta} = 8500$ in a rather low frequency given by Womersley number range of $\sqrt{\omega'}$; 1.56 - 3.12 and for an oscillation amplitude range A_1; $0.05 \leq A_1 \leq 0.5$ C has a maximum value in the order of 1.2.
- For $Re_{ta} = 7460$ and $Re_{ta} = 9600$ in a rather low frequency given by Womersley number range of $\sqrt{\omega'}$; 0.5 - 3 and for low oscillation amplitudes of $A_1 = 0.18$ and $A_1 = 0.11$ C has a maximum value in the order of 1.35.
- For low magnitudes of A_1 and $\sqrt{\omega'}$; 0.5 - 3 $A_1 = 0.14$ and $A_1 = 0.24$ with $\sqrt{\omega'} = 0.82$ and $\sqrt{\omega'} = 0.61$ increase in Re_{ta} causes a decrease in C. The used range of Re_{ta} is between 7000 - 9500 and the maximum value of C is 1.6 at low Re_{ta}..

In their detailed investigation the overall values of C can be treated in the order of 1. Furthermore their verbal conclusion as "*As a result superimposed unsteadiness has no (or slight) impact on frictional characteristics*" is confirming the suggestion of the presented deduction above as " … seems to be true for turbulent pulsatile flow λ_{Mel} (C) = 1 in the range of $Mel > 200000$". The discrepancies between the determined magnitudes of present analysis and Zhuang et al's are due to the differences between the experimental limitations coming from the size of the channels and the rather narrow range of their pulsation parameters particularly with Womersley number in the range of quasi-steady region. However the conformity in so- called turbulent flows is the justification of the physical fact determined separately.

Mel- Moody Diagram provides a considerable practice for the calculation of periodic flow friction factors in the range of Womersley number > 1.32 as listed below:

- For laminar periodic smooth pipe flows Re_{ta} and Womersley number are both governing the field and the following Equation 9. b can be used to determine the magnitude of $\lambda_{u,ta}$in the proposed validity range of $8208 \leq Mel \leq 75644$

$$\lambda_{Mel} = 4.5\, x10^{-4}\ Mel \tag{9.b}$$

Equation 9.b means that increase in oscillation frequency even at a constant flow speed, is coupled with a periodic flow friction increase as a contradiction to the case in steady flows sensed by an increase in flow speed is associated with a decrease in flow friction.

- For laminar and turbulent periodic smooth pipe flows in the suggested approximate range of $Mel \geq 178500$ the following Equation 9.c can be used.

$$\lambda_{Mel} = 1 \tag{9.c}$$

Equation 9.c means that oscillations which have the specified ranges of frequency do not have a significant influence on the flow frictional characteristics of a steady field. The proposed *Mel*-Moody diagram and Equations 9.b and 9.c in this paper are for the purpose.

- Zhuang et al's [27] mini channel (pointwise) transitional range study is confirming the change observed for laminar regime variation through proposed equations 9.b and 9.c. Zhuang et al's [28] study is also confirming the physical fact in turbulent regime.

The utilization of oscillations as a flow control tool requires further research [29] for the determination of interactive limits of frequency, amplitude and time-average velocity mainly. The deductions of Zhuang et al's in mini channels regarding influence of oscillations in quasi-steady region are also challenging.

In reference to a very recent study [30]; continuing efforts on frictional–resistance characteristics of pulsatile flows in different ranges of Re_{ta}, A_1 and $\sqrt{\omega'}$ for a variety of industrial applications are also confirming the importance of the presented analysis herein.

CONCLUSION

In reference to the extensive experimental research on sinusoidal pipe flows, it is found that presence of oscillations and their influence is strongly dependent on the flow regime sensed by different functional relationships of $\lambda_{u,ta} = \lambda_{u,ta}(Re_{ta})$. A multiplication element which is defined as $Mel = Re_{ta} \times \sqrt{\omega'}$ and a modified friction multiplier, $C = \lambda_{Mel}$ which is defined as $\lambda_{Mel} = \frac{\lambda_{u,ta}}{\lambda}$ are referred for $\sqrt{\omega'} > 1.32$ inside laminar and turbulent regimes. The critical magnitude of Re_{ta} inside laminar regime is estimated as 5000. The magnitude of $\lambda_{u,ta}$ increases with *Mel* for $Re_{ta} \leq 5000$. However at $Re_{ta} = 10000$, the magnitudes of $\lambda_{u,ta}$ which are less than the ones for $Re_{ta} \leq 5000$ are almost independent of $\sqrt{\omega'}$. Inside turbulent regime the magnitudes of $\lambda_{u,ta}$ are not varying with *Mel* . The increase in Re_{ta} from 35000 to 60000 is associated with a very slight decrease in $\lambda_{u,ta}$.

The variation of λ_{Mel} with *Mel* is given as a log - log plot. This plot is defined as *Mel* –Moody Diagram similar to the well-known Moody Diagram. It is possible to determine the interactive influence of Womersley number and Re_{ta}. since *Mel* takes the role of $Re_{ta} = Re$ in steady flows.

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NOMENCLATURE

A_{cross}	Cross sectional pipe area, m^2
A_1	Oscillation amplitude, $A_1 = \frac{\lvert U_{m,os,1} \rvert}{U_{m,ta}}$
C	Zhuang et al's friction ratio, (same conceptual definition of λ_{Mel})
D	Pipe inner diameter, m
f	Frequency of oscillation, Hz
g	Gravitational acceleration, m^2/s
h_f	Frictional head loss for steady flow, m

I	Turbulence intensity
L	Axial distance used for local static pressure gradient, m
L_p	Pipe length, m
Mel	Multiplication element of periodic flows, $Re_{ta} \times \sqrt{\omega'}$
N	Number of periods
R	Pipe radius, $R = \frac{D}{2}$, m
Re	Steady flow Reynolds number, $Re = \frac{UD}{\nu}$
Re_{ta}	Time-average Reynolds number, $Re_{ta} = \frac{\bar{U}_{m,ta}D}{\nu}$
t	Time coordinate, s
T	Period of oscillation, s
$\bar{U}$	Ensemble-averaged local axial instantaneous velocity, $\bar{U} = \frac{1}{N}(\sum_{i=0}^{N-1} u_i)$, m/s
$\bar{U}_m(U)$	Ensemble-averaged cross-sectional mean velocity, $\bar{U}_m = \frac{Q}{A_{cross}}$ (steady flow velocity), m/s
$\bar{U}_m(t)$	Instantaneous cross-sectional mean velocity, m/s
$\lvert U_{m,os,1} \rvert$	Oscillating component of cross sectional mean velocity for the fundamental first wave in FFT, m/s
$\bar{U}_{m,ta}$	Time-averaged component of cross-sectional mean velocity, m/s
Q	Volumetric flow rate, m^3/s

Greek Symbols

ε	Pipe surface roughness, m
$\Delta\bar{P}_{(t)}$	Instantaneous local pressure drop, Pa
λ	Steady flow Darcy friction factor
$\lambda = \frac{64}{Re}$	For laminar flow
$\lambda = \frac{0.3164}{(Re)^{0.25}}$	For turbulent flow
λ_{Mel}	Modified friction multiplier for periodic flows $\frac{\lambda_{u,ta}}{\lambda_{sL}}$ for laminar periodic flow $\frac{\lambda_{u,ta}}{\lambda_{sT}}$ for turbulent periodic flow
λ_{sL}	Modified laminar periodic flow friction factor, $\lambda_{sL} = \frac{64}{\left(\frac{\bar{U}_{m,ta}D}{\nu}\right)}$
λ_{sT}	Modified turbulent periodic flow friction factor, $\lambda_{sT} = \frac{0.3164}{\left(\frac{\bar{U}_{m,ta}D}{\nu}\right)^{1/4}}$
$\lambda_{u,ta}$	Periodic flow time average friction factor, $\lambda_{u,ta} = \frac{8}{\rho(\bar{U}_{m,ta})^3 T}\int_0^T \bar{\tau}_w(t)\bar{U}_m(t)dt$
ν	Kinematic viscosity, m^2/s
ρ	Fluid density, kg/m^3
$\bar{\tau}_w(t)$	Instantaneous wall shear stress, Pa
ω	Angular frequency of oscillation, $\omega = 2\pi f$, rad/s
ω'	Dimensionless frequency of oscillation, $\omega' = R^2\frac{\omega}{\nu}$
$\sqrt{\omega'}$	Womersley number, $\sqrt{\omega'} = R\sqrt{\frac{\omega}{\nu}}$

Other symbols

Cst	Constant in Equation 7.b
ta	Time-average (long-time averaged)
∠	Phase lag
‾	Ensemble-average (short –time averaged)

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