

Analysis and Simulation of Turbulence Effects on Gaussian Beam Propagation Based on Generalized Modified Atmospheric Spectrum

F. YILDIZ^{1,2,*}, H. KURT²

¹Roketsan Missiles Industries Inc., Ankara, 06780, Turkey

²Department of Electrical and Electronics Engineering, TOBB University of Economics and Technology, Ankara, 06560, Turkey

Abstract:

Atmospheric turbulence has been extensively studied for many years in physics and engineering disciplines. When the laser beam propagates throughout the atmosphere, it can be influenced by different optical phenomena including scattering, absorption, and turbulence. The turbulence effect of the atmosphere results from changes in the refractive index. The eddies with different size affect optical wave propagation through the atmosphere. These changes of refractive index cause different variations for the propagating laser beam such as beam wandering, beam spreading, and image jitter. All these effects can severely degrade the beam quality (M-squared) and decrease the performance efficiency of the system in some applications including free-space optical communication, LIDAR-LADAR applications, and directed energy weapons systems [1-5]. Traditionally, the turbulence is defined by the Kolmogorov model type. The Kolmogorov spectrum is with the power law value of 11/3 which is used to describe Gaussian distribution [6]. There are many spectra which have specific inner and outer scale like Tatarskii, von Karman, Kolmogorov, and generalized modified spectra [7]. In this study, the generalized modified atmospheric spectra model is applied. We numerically and analytically perform the propagation behavior of Gaussian laser beam at different propagation distances. Also, we examine the influence of some parameters on beam propagation. All results simulated are discussed and compared with results available in the literature.

Keywords: Atmospheric Turbulence, Optical Communications, Laser Beam Propagation, Generalized Atmospheric Spectrum, Phase Screen Method

DOI:

1. INTRODUCTION

Atmospheric turbulence is random fluctuations of air currents in the form of eddy driven by large scale thermal gradients [8]. It is responsible for random variations in the laser beam intensity which is called scintillations. It leads to other effects such as beam spreading and beam wander [9-12]. Turbulent air motion represents a set of eddies of various scale sizes. They are extended from a large scale L_0 to a small scale size l_0 . Whereas the large scale is called the outer scale of turbulence, the small scale size l_0 which is called the inner scale of turbulence

[13]. L_0 ranges from tens to hundreds of meters and l_0 is on the order of millimeters. Turbulent eddies break down into smaller structures under the influence of inertial forces. It forms a continuous cascade of scale sizes between L_0 and l_0 . And it is known as the inertial range. Scale sizes with smaller than the inner scale belong to the dissipation range [13]. The size of the inertial range is related to the Reynolds number. It can be seen in Figure 1.

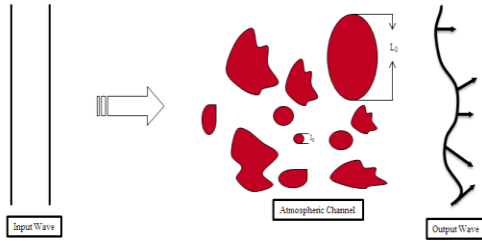


Figure 1. Kolmogorov cascade theory of turbulence, where L_0 depicts the outer scale and l_0 is the inner scale [13].

Atmospheric turbulence can be depicted by turbulence eddies and the structure parameter of the refractive index fluctuations C_n^2 . It is one of the most important parameters of the effects of atmospheric turbulence [14]. The turbulence theory is based on the study of A. N. Kolmogorov in the 1940s [15]. The mathematical description of turbulence is stated with the help of the statistics.

Under the hypothesis of homogenous medium, one can derive the index of refraction structure function:

$$D(n(r)) = C_n^2 r^{\frac{2}{3}}, l_0 < r < L_0 \quad (1.1)$$

where the term C_n^2 is the strength of the turbulence, $m^{-2/3}$, r is the scalar distance between two points in space. C_n^2 values near the ground in warm climates range between 10-14 and 10-12 $m^{-2/3}$ in general, the latter of which is considered strong turbulence [13]. Several C_n^2 profile models have been developed, but the most commonly used is the Hufnagel-Valley model described by

$$C(h)^2_n A \exp\left(-\frac{h}{100}\right) + 5.94 \times 10^{-53} \left(\frac{v}{27}\right)^2 h^{10} \exp\left(-\frac{h}{1000}\right) + 2.7 \times 10^{-16} \exp\left(-\frac{h}{1500}\right) \quad (1.2)$$

where h is in meters (m), v is the rms wind speed in meters per second (m/s), A is a nominal value of $C_n^2(0)$ at the ground in $m^{-2/3}$. This parameter increases as the temperature differences of the eddies increases. C_n^2

values near the surface vary from 10-17 $m^{-2/3}$ for weak turbulence to 10-13 $m^{-2/3}$ strong turbulence. It tends to be smallest before sunrise and after sunset whereas the biggest values tend to sunny days [16].

The power spectral density is explained by:

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-\frac{11}{3}} \quad (1.3)$$

This equation is the Kolmogorov power-law spectrum. This spectrum model is valid only over the inertial subrange $1/L_0 \ll \kappa \ll 1/l_0$ [17].

Tatarskii spectrum and the generalized modified atmospheric spectrum model used in this study are given below [13].

$$\begin{aligned} \varphi_n(\kappa) &= 0.033 C_n^2 \left[1 + 1.802 \left(\frac{\kappa}{\kappa_l}\right) - 0.254 \left(\frac{\kappa}{\kappa_l}\right)^{\frac{7}{6}} \right] \frac{\exp\left(-\frac{\kappa^2}{\kappa_l^2}\right)}{(\kappa^2 + \kappa_0^2)^{11/6}} \end{aligned} \quad (1.4)$$

This spectrum model is valid only over the inertial subrange $0 \ll \kappa \ll \infty$.

2. NUMERICAL MODEL APPROACH

The refractive index fluctuation of the atmosphere is defined a random process. Turbulence models give statistical averages such as the structure-function and power spectrum [18]. Random phase screens for simulating atmospheric turbulence are created by transforming computer-generated random numbers into two-dimensional arrays of phase values on a grid of sample points [18]. In this paper, we used the split step method mathematically for generating phase screens. Phase screens are equally placed along the propagation distance between transmitter and receiver. In literature, the common method is based on the Fourier-transformable (FT). This method is first introduced by McGlamery [19].

The turbulence phase is an FT function and can be written in Fourier-integral representation as :

$$\phi(x, y) = \iint_{-\infty}^{+\infty} \psi(f_x, f_y) e^{i2\pi(f_x x + f_y y)} df_x df_y \quad (2.5)$$

where $\psi(f_x, f_y)$ is the spatial-frequency domain representation of the phase,

$$\phi(x, y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c_n c_m e^{i2\pi(f_{xn}x + f_{ym}y)} \quad (2.5)$$

where f_{xn} and f_{ym} are the discrete x and y directed spatial frequency and the $c_{n,m}$ are the Fourier series coefficients.

$$\langle |c_{n,m}^2| \rangle = \Phi_{\phi}(f_{xn}, f_{ym}) \Delta f_{xn} \Delta f_{ym} \quad (2.6)$$

The x and y grid sizes are L_x and L_y , respectively. Also, the frequency spacings are $\Delta f_{xn}=1/L_x$ and $\Delta f_{ym}=1/L_y$.

3. RESULTS and DISCUSSION

In our study, a Gaussian laser beam created with mathematical methods was assumed to propagate through a turbulent atmosphere [20-21]. A graphical interface was created to simulate the wave propagation in such medium. In this part, we presented examples of numerical results.

Table 1 gives parameters used in this condition.

Parameters	Symbol	Value
Beam Radius at Source	W_0	5 cm
Matrix Size	NxN	256
The Number of Samples	M	10
Wavelength	Λ	1070 nm
Outer Scale	L_0	10 m
Inner scale	l_0	0.001 m
Turbulence Structure Parameter	C_n^2	$1 \times 10^{-17} \text{ m}^{-2/3}$ $, 1 \times 10^{-13} \text{ m}^{-2/3}$
Effective Range	L	500 m, 1 km, 5 km
Laser Power	P	5 kW
Power law value	α	3.5

Table 1. Simulation parameters for the influence of different turbulence conditions.

Figure 2 shows the Gaussian laser beam profile at the source and the cross section of laser beam. On the other hand, Figure 3 demonstrates results after propagation throughout atmosphere for three propagation distances with different turbulent conditions. These conditions are weak refractive index parameter and strong refractive index parameter. It can be seen that strong refractive index parameter of atmosphere can make the intensity profile very noisy as can be seen from in Figure 4. It also demonstrates that the propagation distance affects the laser beam profile negatively. Therefore, it causes decrease of laser power for long distance propagation.

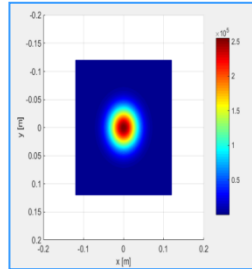


Figure 1. The cross section of laser beam.

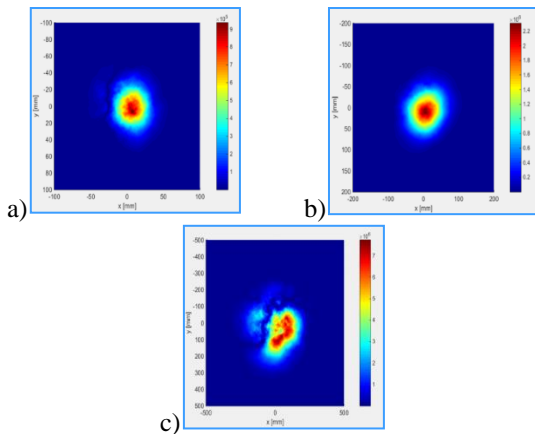


Figure 3. Gaussian beam profile through atmosphere with weak refractive index parameter (a) at 500 m, (b) at 1000 m, (c) at 5000 m.

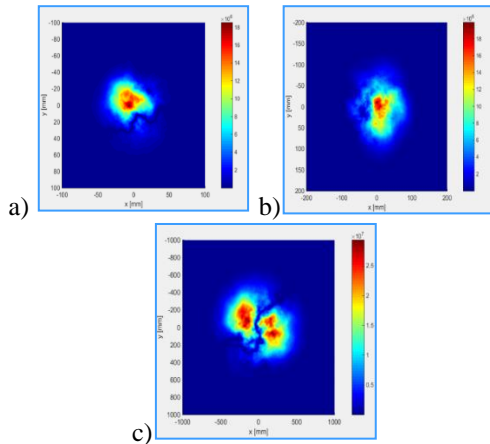


Figure 4. Gaussian beam profile through atmosphere with strong refractive index parameter (a) at 500 m, (b) at 1000 m, (c) at 5000 m.

stronger the refractive index parameter, the bigger the spot size at the target. Moreover, longer propagation distance influences the spot size at the target more significantly.

Table 2. Simulation parameters for the influence of different turbulence conditions

Refractive index parameter, $m^{-2/3}$	Propagation distance, m	Spot size at the target, cm
1×10^{-17}	500	5,2
	1000	7,2
	5000	17,8
1×10^{-13}	500	5,6
	1000	11,2
	5000	34,9

3.2 The Influence of Different Beam Radius at Source

The beam radius at source is one of the significant parameters of laser systems. An optimum beam radius at the source provides the most important performance criterion on beam propagation. Therefore, in this paper, the effects of beam radius at source were investigated. In our graphical interface, the beam radius was changed and all calculations were done again. Table 3 gives the parameters used in this condition.

Figure 5 shows beam profiles at target through atmosphere with weak and strong turbulence parameter at different propagation distances. In these analyses, beam radius at source is calculated as 2.50 cm.

Table 2 shows the spot size versus propagation distance for two different turbulence conditions. The

Table 3. Simulation parameters for the influence of different turbulence conditions.

Parameters	Symbol	Value
Beam Radius at Source	W_0	2,5 cm
Matrix Size	$N \times N$	256
The Number of Samples	M	10
Wavelength	Λ	1070 nm
Outer Scale	L_0	10 m
Inner scale	l_0	0.001 m
Turbulence Structure Parameter	C_n^2	$1 \times 10^{-17} \text{ m}^{-2/3}$ $1 \times 10^{-13} \text{ m}^{-2/3}$
Effective Range	L	500 m, 1 km, 5 km
Laser Power	P	5 kW
Power law value	α	3.5

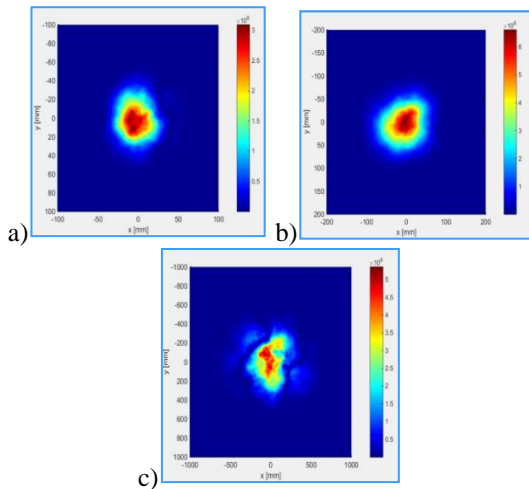


Figure 5. Gaussian beam profile through atmosphere with weak refractive index parameter (a) at 500 m, (b) at 1000 m, (c) at 5000 m.

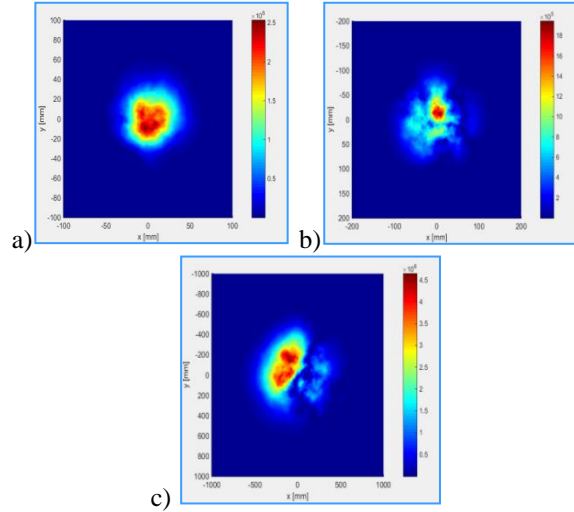


Figure 6. Gaussian beam profile through atmosphere with strong refractive index parameter (a) at 500 m, (b) at 1000 m, (c) at 5000 m.

Table 4 demonstrates the increase of beam size at the source. It is showed that the laser beam is more affected as the propagation distance increases because of increasing turbulence eddies. It can be seen that a small beam radius of the source affects the beam intensity profile at target more importantly because turbulence eddies in the air lead to more deterioration small beam radius.



Table 4. Simulation parameters for the influence of different turbulence conditions.

Refractive index parameter, $m^{-2/3}$	Propagation distance, m	The increase of beam size for 5 cm beam radius at source, cm	
		The increase of beam size for 5 cm beam radius at source, cm	The increase of beam size for 5 cm beam radius at source, cm
1×10^{-17}	500	1,03 cm	1,44 cm
	1000	1,42 cm	1,78 cm
	5000	2,94 cm	5,93 cm
1×10^{-13}	500	1,1 cm	1,8 cm
	1000	2,2 cm	3,39 cm
	5000	5,77 cm	6,8 cm

4. CONCLUSION

In this paper, there is analyzed how the effects of atmospheric turbulence of Gaussian beam propagation through random media. The simulation is performed for weak and strong turbulence structure parameters for different propagation distances. It can be concluded that turbulence conditions can severely affect the beam profile. In this situation, the magnitude of beam size on the source is a critical parameter that can affect the peak value of the power intensity on the target. The effective beam radius on the target increases as the propagation distance increases both low and strong turbulence parameters. Also, the turbulence creates more degradation of beam size on target as beam size on source increases. The simulation and analytic results are in good agreement for all the investigated scenarios. In conclusion, this study demonstrates that the atmospheric turbulence has a great influence on the Gaussian beam transmission in the random media. We consider that the results could help to develop a basis for the laser-based applications as turbulence is one of the most challenge factors. Also this paper provides the quantitative evaluations of laser beam propagation in the turbulent

atmosphere. This situation offers the opportunity to compare with other models and experimental results. A possible future study is to justify the theoretical results with experimental data.

5. ACKNOWLEDGEMENT

F.Y. acknowledges the support of Roketsan Missiles Industries Inc.

6. REFERENCES

- [1] P. Burlamacchi, A. Consortini, L. Ronchi, G. di Francia, Laser beam propagation in the atmosphere, 259, Quant.Electron. (1967).
- [2] A. Consortini, L. Ronchi, Investigation of atmospheric turbulence by narrow laser beams, Appl. Opt. 9, 11, (1970) 2543-2547.
- [3] K.S. Shaik, Atmospheric propagation effects relevant to optical communications, TDA Prog. Rep., (1988) 42-94.
- [4] R.Z. Yahe, I. Last, Numerical simulation of laser beam propagation in three-dimensional random media: beam splitting and patch formation, Wave Random Media, (1992) 81-98.
- [5] Y. Cai, Propagation of various flat-topped beams in turbulent atmosphere, J. Opt. A-Pure Appl. Opt., 8, (2006) 6.
- [6] I. Toselli, L. C. Andrews, R. L. Phillips, and V. Ferrero, Angle of arrival fluctuations for free space laser beam propagation through non Kolmogorov turbulence, Proc. SPIE 6551,65510E (2007).
- [7] L.-Y. Cui, B.-D. Xue, X.-G. Cao, J.-K. Dong, and J.-N. Wang, Generalized atmospheric turbulence MTF for wave propagating through non-Kolmogorov turbulence, Opt. Express 18, (2010) 21269–21283.
- [8] S. K. Searles, G. A. Hart, J. A. Dowling, S. T. Hanley, Laser beam propagation in turbulent conditions, Applied Optics, 30, 4, (1991) 401-406.



- [9] A.A.B. Raj, Free space optical communication: system design, modeling and characterization and dealing with turbulence, Kings College of Engineering, 2016.
- [10] C. Gao, L. Su, W. Yu, Long-term spreading of Gaussian beam using modified atmospheric spectrum, Proceeding of 2015 IEEE, International Conference on Mechanics and Automation.
- [11] A. Prokes, Modeling of atmospheric turbulence effect on terrestrial FSO link, Radioengineering, 18, (2009) 42-47.
- [12] L. C. Andrews, R. L. Phillips, C. Y. Hopen, Laser Beam Scintillation with Applications, SPIE Press, 2001.
- [13] L. C. Andrews, R. L. Phillips, 'Laser Beam Propagation through Random Media', SPIE Press, 2005.
- [14] Z. I. Feizulin, Y. A. Kravtsov, 'Broadening of a laser beam in a turbulent medium', Radiophysics and Quantum Electronics, 10, (1967) 1.
- [15] U. Frisch, 'Turbulence: The legacy of A. N. Kolmogorov', Cambridge University Press, 1995.
- [16] H. Weichel, Laser beam propagation in the atmosphere, Bellingham, WA:SPIE Optical Engineering Press, 1990.
- [17] Army Research Laboratory, Characterization of optical turbulence data measured at the ARLA_LOT facility, ARL-MR-625, 2005.
- [18] J. D. Schmidt, Numerical Simulation of Optical Wave Propagation With Examples in MATLAB, 2010.
- [19] B. L. McGlamery, Restoration of turbulence-degraded images, J. Opt. Soc. Am. 57 (3), (1967) 293-U297.
- [20] J. D. Schmidt, Numerical simulation of optical wave propagation, with examples in MATLAB, SPIE Press 2010.
- [21] G. R. B. E. Römer, A. J. Huis in't Veld, Matlab Laser Toolbox, Physics Procedia, 5(0) 2010.