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Investigation of Teacher Knowledge of Elementary Mathematics Teachers: Case of Probability

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Abstract

This study was conducted in order to examine the content knowledge of mathematics teachers and to determine their views on probability issues. In the study, the case study model was used from the qualitative research designs. The participants of the study were six elementary mathematics teachers who is Turkey's north-east province who served. Teachers participating in the study were selected by quota sampling method. The criteria of the study were determined as teachers' masters and professional seniority. The data of the study were collected by semi-structured interview form. In the form of semi-structured interviews, teachers were asked questions of cognitive and affective questions related to the adequacy of probability information. Content analysis method was used in the data analysis. The results of the current study shows that the teachers considered the probability content knowledge is sufficient, but did not find the pedagogical content knowledge sufficient. Another result of the study is that some teachers do not know the concept of discrete event and independent event. Therefore, teachers have difficulty in distinguishing between discrete event and independent event concepts.

Ortaokul Matematik Öğretmenlerinin Öğretmen Bilgilerinin İncelenmesi: Olasılık Örneği

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Öz

Bu çalışma ortaokul matematik öğretmenlerinin olasılık konusundaki alan bilgilerini incelemek ve olasılık konularına yönelik görüşlerini belirlemek amacıyla yapılmıştır. Çalışmada nitel araştırma desenlerinden durum çalışması modeli kullanılmıştır. Çalışmaya Türkiye'nin kuzey doğusunda yer alan bir ilde görev yapan altı ortaokul matematik öğretmeni katılmıştır. Çalışmaya katılan öğretmenler ölçüt örnekleme yöntemiyle seçilmiştir. Çalışmada ölçüt olarak öğretmenlerin yüksek lisans yapma durumları ve mesleki kıdemleri temel alınmıştır. Çalışmanın verileri yarı yapılandırılmış görüşme formu ile toplanmıştır. Yarı yapılandırılmış görüşme formunda öğretmenlere olasılık bilgilerinin yeterliğine yönelik bilişsel ve duyuşsal alan soruları sorulmuştur. Çalışmada toplanan verileri içerik analizi uygulanmıştır. Yapılan analizler sonucunda öğretmenlerin olasılık konusunda alan bilgilerini yeterli gördükleri ancak pedagojik alan bilgilerini yeterli bulmadıkları belirlenmiştir. Çalışmada ulaşılan bir diğer sonuç ise bazı öğretmenlerin ayrık olay ve bağımsız olayı kavramsal olarak bilmedikleridir. Bu nedenle öğretmenler ayrık olay ve bağımsız olay kavramlarını ayırt etmede güçlük yaşamaktadır.

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Introduction

Probability is one of the important subjects of mathematics that we encounter in every aspect of our lives. Students are likely to become informally aware from the first years of primary school. However, this acquaintance occurs only in formal ways when students arrive at the 8th grade in secondary school. The encounter of the students probably brings along many difficulties and misconceptions. In order for the students to overcome these difficulties and not have misconceptions, their teachers should have good pedagogical and content knowledge about probability. Secondary school mathematics teachers take Statistics and Probability-I and Statistics and Probability-II courses on probability in undergraduate education. The updated curriculum also includes teaching the courses of Probability and Statistics, as well as the teaching of these courses separately (Probability, Statistics). What is expected from these courses is that teachers should be trained as mentors who not only know the possibility but also the possibility teaching. In other words, it aims to train teachers who know how to look at the subject of probability with two-lens glasses (student glasses and teacher glasses). Although the Probability and Statistics Teaching Lesson was added to the new curriculum, teachers who were currently employed did not take this course at the undergraduate level. Therefore, how these teachers look at probability subjects should be examined how they interpret the relationship between probability courses at the undergraduate level and probability subjects in secondary school. Because, it is their current teachers and the information of the teachers that will directly affect the students' learning of probability. Accordingly, it is considered that it is important for the practitioners to reveal the probability content information of the teachers. In this respect, the study aimed to examine how mathematics teachers associate probability information with content information and pedagogical information.

Content Knowledge of Teachers

The concept of content knowledge can be defined as the knowledge of the concepts, principles and definitions of teachers in their subject areas (Yanpar-Yelken et al., 2013). When the content knowledge of mathematics teachers is mentioned, it is understood that teachers' knowledge about the subjects in the mathematics lesson. To be more specific, can be expressed as to know the rules about mathematics, to understand why these rules are available, to know the reasons for the operations by the teachers (Ball, Thames, & Phelps, 2008). For example, in the teaching of probability subject in the secondary school curriculum, the reason why the probability values can take values between 0 and 1 is a statement corresponding to the content information of the mathematics teachers.

Good level of teachers' content knowledge is important for mathematics teaching. Because when teachers' content information is not sufficient, teachers can have problems in transferring the knowledge and skills they have to students, identifying places where students will have difficulty in learning information and eliminating possible misconceptions of students (Gökkurt, Şahin, & Soylu, 2012). On the contrary, it is likely that teachers with a good knowledge of the content will be able to learn the subjects better and know which operations they are doing and why. Because students imitate their teachers and try to make a solution themselves as they do their solutions (Öztürk, & Kaplan, 2019; Rosenholtz, 1985). Content knowledge alone is not enough for teaching mathematics. Teachers should also have good knowledge of pedagogy along with their content knowledge (Ball, Thames, & Phelps, 2008). Pedagogy knowledge is information that the teacher knows how to teach a subject. This type of knowledge is the knowledge of the teacher about the teaching methods and the curriculum (at which class level which subject they will teach) (Yanpar-Yelken et al., 2013). Building bridges between pedagogy knowledge and content knowledge increases the quality of the teaching process and provides better educated students.

Combining pedagogy knowledge and content knowledge is defined as pedagogical content knowledge (Baştürk & Dönmez, 2011; Yanpar-Yelken et al., 2013). Pedagogical content knowledge includes teachers' preference of appropriate teaching methods by having content knowledge and teaching knowledge (Staley, 2004). Stevens et al. (2009) defined pedagogical content knowledge as knowledge of a specific subject area, knowledge of the strategy techniques necessary to teach it, knowledge of the possible misconceptions of students, and knowledge of the curriculum.

Probability is one of the mathematical concepts that students and teachers have difficulty (Çelik & Güneş, 2007; Vysotskiy, 2018). Because students and teachers have difficulty in thinking probabilistic situations and mathematically approaching everyday situations related to probability (Memnun, 2008). However, the fundamental difficulty with regard to probability is related to the nature of the probability concept. Because the probability is different from the concepts that can be perceived intuitively, such as length, space, volume. Due to the fact that reason probabilistic problems involve concepts of chance and change, being busy with probability is more difficult than dealing with numbers or shapes.

Students' thinking of possible situations is a more complicated structure than the transition from geometry to form or from arithmetic to algebra (Vysotskiy, 2018). There are many difficulties in the learning and teaching of probability in our country (Bulut, & Şahin, 2003; Gökkurt-Özdemir, 2017).

One of the reasons for these difficulties is the lack of content and pedagogical knowledge of teachers (Bulut, & Şahin, 2003). Researches revealed that teachers 'mathematical knowledge influences students' perspectives on mathematics. Therefore, it is important that teachers have good mathematics content knowledge. When teachers' knowledge of the content is sufficient and they support this knowledge with pedagogical knowledge, the quality of teaching is likely to increase (Stevens, 2009). When students' misconceptions about probability are taken into consideration, it is understood that simple and unified events, misconception, omission of the size of the sample cluster (Çelik, & Güneş, 2007) should be prioritized in terms of the independent event on the subject and the content knowledge on the discrete event. As a matter of fact, studies conducted in the literature emphasize the importance of discrete event and independent event concepts for the content knowledge of the teachers (Altun, 2015, p. 485; Gökkurt, Şahin, & Soylu, 2012; O'Connell, 1999).

Discrete Event and Independent Event

Let a random sample space be S and events from this sample space are A and B. If $A \cap B = \emptyset$, in other words, if $A, B \subset S$ events are not possible together, A and B events are called discrete events (Argün, Arıkan, Bulut, & Halıcıoğlu, 2014, p. 377; Demir, 2016, s). 34. Discrete events are also known as incompatible events (Demir, 2016, p. 34). For example, any instances that may occur when a non-loaded dice are discarded constitute the sample space (S = {1,2,3,4,5,6}). Let's take two events from this sample space. The first of these events is that "the numbers that come to the upper surface of the dice are smaller than 3", the second is defined as "the numbers that come to the upper surface of the dice are greater than 5" (A = {1, 2}, B = {6}). The intersections of these two events are empty and these two events can be defined as discrete events (Altun, 2015, p. 485).

The independent event is defined as if an event A does not affect the occurrence of a B event in any case or it is not affected by the event B then it is called an independent event (Demir, 2016, p. 64). Argün et al. (2014, p. 290) and Lipschutz and Lipson (2013, p. 92) emphasized that the two events should be in the identical (same) sample space in order to be an independent event. However, many studies in the literature do not use an expression of the necessity to have identical or identical sample spaces (Akdeniz, 2007, p. 77; Dekking, Kraaikamp, Lopuhaä, & Meester, 2005, p. 33; Demir, 2016, p. 64). In this study, it is searched that there should be identical sample spaces in the operational definition of the independent event. For example, when a non-loaded dice is thrown together with a fraudulent coin, all the situations that can occur are a sample space. T = Tura, Y = Let it be (S = {(1, T), (2, T), (3, T), (4, T), (5, T), (6, T), (1, Y), (2, Y), (3, Y), (4, Y), (5, Y), (6, Y)}). The number on the top of the dice is an odd number (S = {(1, T), (3, T), (5, T), (1, Y), (3, Y), (5, Y)}) and top events on the top of the coin (S = {(1, T), (2, T), (3, T), (4, T), (5, T), (6, T)}) they become independent events.

The above is described as an independent event with discrete event. As can be seen from the definitions, the discrete event and the independent event are actually very different concepts. However, these concepts are often confused with each other. In this context, the way to distinguish these two concepts is to look at the intersections of these events. If the interception of the two events are empty, the events are independent if it is different from the empty the events are independent (Lipschhutz, & Lipson, 2013, p. 92). Considering the above examples, there will be intersections of the events given in the discrete event example (($A = \{1,2\}, B = \{6\}, A \cap B = \emptyset$)). In other words, the intersections of the sets are empty. In the case of an independent event, the odds of the number one and top of the coin (S = {(1, T), (3, T), (5, T)} are possible and are $A \cap B \neq \emptyset$.

Literature Review

When the literature is examined, it is seen that the studies on the subject of probability in mathematics education have increased in recent years. Some of the studies are aimed at examining the probability information and misconceptions of middle and high school students (Barragués, Guisasola, & Morais, 2006; Engel, 1971; Munisamy, & Doraisamy, 1998), while others are aimed at examining the content knowledge of prospective teachers. In the studies conducted to examine the probability information of high school students, it was determined that the students had difficulty in probability subjects and their knowledge levels were not sufficient (Memnun, 2008). In the studies conducted to determine students 'misconceptions, students' representation, negative and positive effect, simple and unified events, representation short path, result approach, misconception of sample size were reached (Akkoç, & Yeşildere-İmre, 2015, p. 19; Çelik, & Güneş, 2007). Akkoç and Yeşildere-İmre (2015, p. 25) stated that not only students but also teachers had misconceptions. Teachers' misconceptions (Klymchuk, & Kachapova, 2012), pedagogical information (Schoen, LaVenia, Chicken, Razzouk, Kisa, 2019; Shin, 2011) and content knowledge were also examined in the studies conducted to examine the content knowledge of teacher candidates (Gökkurt-Özdemir, 2017). However, it has been determined that the studies on the content knowledge of the current teachers are quite limited. Gökkurt-Özdemir (2017) in the study to examine the probability knowledge of teacher candidates, examined the probability content knowledge of teachers through descriptive analysis. The study focuses on the knowledge of teacher candidates on disparities between discrete and non-discrete events and independent events and dependent events. As a result of the study, it was determined that most of the teacher candidates did not have enough knowledge about the discrete event and the independent event and confused the discrete event and independent event. The researchers analyzed the data by using descriptive analysis method. Descriptive analysis may limit the findings that may arise as a result of using an existing theoretical framework. In addition, the study was conducted with teacher candidates and there is a need for studies with teachers. Although it is important to examine teacher candidates and students, it is even more important to examine the content knowledge of current (on-duty) teachers. Therefore, it is important to examine teachers' content knowledge on probability. This study was carried out to determine the content knowledge of middle school mathematics teachers about probability. For this purpose, the following sub-problems were sought:

1. What is the opinion of middle school mathematics teachers about the adequacy of pedagogical content knowledge in probability?
2. What is the subject content knowledge of middle school mathematics teachers about the discrete event?
3. What is the subject content knowledge of middle school mathematics teachers about the independent event?

What kind of abstractions do middle school mathematics teachers make between separate and independent events?

Method

Research Design

In the study, the case study model of qualitative research designs was used. The case study is used in cases where one or more situations are examined in detail (Yıldırım, & Şimşek, 2013). In this study, a case study model is preferred because it is aimed to examine the teachers' knowledge on probability of middle school mathematics teachers in detail. Yıldırım and Şimşek (2013) stated that the case study is also important in terms of revealing the process. In this study, it is thought that it would be more appropriate to use this model as it will examine the awareness of teachers about discrete event and independent event concepts.

Participants

Participants of this study are the 6 middle school mathematics teachers who work in the cities located at the Northeast of the Turkey. All of the participants were selected from teachers who volunteered to participate in the study. Quota sampling method, which is one of the purposeful sampling methods, was used in the selection of the participants. The advantage of quota sampling is that it is more representative of the population than other purposeful sampling method (McMillan, & Schumacher, 2014, p. 153). The criteria of the study were based on the post-graduate status of teachers and their work seniority. In addition, at least two participants (1 female and 1 male) were included in the study. Two of the teachers participated in the study are studying their master degree. Yaşar (male) and Yasemin (female) code names were used for these teachers. Two of the teachers participated in the study have 10 years of experience. Kayra (male) and Kadriye (female) code names were used in these teachers. The other two teachers who participated in the study were chosen as new (0-2 years experience) teachers in the profession. Tarkan (male) and Tülay (female) code names were used for these teachers. The participants were informed about the study and it was stated that their names would be known only by the researcher, not to be shared anywhere, and the data obtained will be used only for scientific purpose.

Data Collection Tools

Semi-structured interview form was used as data collection tool. The reason for using the semi-structured interview form is to reveal the participants' content knowledge on the subject of probability. The researchers aimed to reveal the participants' content knowledge in detail by using probe questions in line with the answers from the participants. After the semi-structured interview form was prepared by the researchers, the expert opinion was obtained and finalized. In the preparation process of the semi-structured interview form, four questions were examined and six probes related to these questions were prepared. Then the prepared form was presented to two faculty members in the department of mathematics education and their opinions were taken. The experts stated that the questions were appropriate. However, he suggested that some corrections be made for the questions at the end and that one more question should be added to the questions. The corrected form was applied to a mathematics teacher working in middle school and to determine whether there are parts that are not understood in the questions. As a result of the application, it was determined that the questions were clear and understandable and there was no need for correction. The questions in the semi-structured interview form are as follows: *"Do you think that the education you received about the probability of your undergraduate education is sufficient?", "Can you describe the discrete event?", "Can you describe the independent event?", "Are the concepts of discrete and independent events identical or different?"* These questions include the following sample probe questions: *"If they are same, can you explain? If they are different, can you explain what the differences are? Can you show an example of the differences between these concepts?"*

Data Analysis

In the study, content analysis method was used from qualitative data analysis methods. For this purpose, interview data were analyzed and coded by the first researcher. Encodings were checked by the second researcher and the value of consistency between the researchers was examined. The inter-coding consistency was examined using the formula [(number of views / total views) X100] and the inter-encoder consistency value was calculated as 78%. This calculated value indicates that inter-encoder compatibility is sufficient (Miles, & Huberman, 2015).

Validity and Reliability

In this study, internal and external validity studies were conducted. For the external validity of the study, the working group is explained in detail and in the statement of the participant views, the line numbers in the transcript and the number of the transcripts were also given. For the internal validity of the study, participants were asked probe questions and give their supportive examples.

In this study, internal and external reliability processes were performed. In order to ensure the external reliability of the study, direct transfers were made from the views of the participants. For internal reliability, the research model, the working group, the data collection tools and the analysis of the collected data are prepared to be consistent. The compatibility between the coding of the researchers was investigated. In addition, opinions were taken from the same occupational group with common cohesion.

Findings

Findings Regarding the Critical Thinking Tendency

Five of the teachers who participated in the study stated that the probability course they had taken in the undergraduate education was sufficient. One of the participant teachers, Yaşar talked about "[00.14] *the basic concepts of probability at the University's probability course. We have seen examples of daily life in probability* (Line, 6-7)." It is understood that he thinks that the knowledge he has received in probability course at the university is sufficient. Similarly, teacher Tarkan said that "[00.10*] I think the probability course we have taken is sufficient. We even got a probability lesson at the advanced level. Our point of view widened in probability class* (Line, 5-6)". It was also determined that the content knowledge courses they took from the teachers' statements were very detailed (comprehensive). Teacher Kadriye stated that "[00.31] *The information that we learned at the university is very high level according to what we teach in secondary school* (Line, 9)" she had found the probability lesson in undergraduate education sufficient.

One of the teachers who participated in the study stated that a significant part of the probability subject was removed from the secondary school curriculum. Yaşar teacher mentioned that "[00.30] *In the middle school,* *probability is a little bit simpler and more basic* (Line, 10-11).” Kadriye Teacher who expressed similar views said that:

> “*In other words, the information we learned at university was indeed very high. In fact, the high school level was like the subjects we studied, but in secondary school, we've just shown how to write a sample space because they've reduced topics recently. We explain these situations, that is, the possibility that we learn in the university and the possibility that we aim to teach in schools is not very much related to each other*.” (Line 9-13)

These two statements show that teachers think that the curriculum has been simplified and that some subjects are removed from the program.

Some of the teachers who participated in the study think that the content knowledge they have learned at the undergraduate level and the probability information they aim to teach in secondary school are on the same basis. One of these participants, Tülay expressed his thoughts as follows:

> “[00.57*] I would say that there is a simple level, but when we think about it in a comprehensive way, we are inevitably going into some more details at the university. But in middle school, we explain it in a way that students can understand. But I can tell that, I think some of the subjects in the probabilities we took in the university are irrelevant. However, the basic logic is the same. It must already be built on the same logic. In this sense, I do not think that there is a big difference between the probability of university and the probability of middle school.”* (Line, 13-17)

It is understood from the teacher's statements that they think they have received a comprehensive probability education at the undergraduate level. On the other hand, because the probability subject in the middle school curriculum is given at a simple level, it is evident that the probability at the university and the probability subjects in the secondary school are only the same at the basic level.

It was determined that some of the teachers who participated in the study thought that their pedagogical knowledge was not sufficient despite the fact that the content knowledge was sufficient. Tarkan’s “[01.00] *The probability course at the university is an advanced course based on probability theories, not on probability teaching. So we didn't learn the possibility. We have learned what probability is used in daily life and where it is used* (Line, 14-16)” words have shown that the theoretical part of the probability course has been extensively described in the university. In addition, they do not take the course of teaching in the probability course. It was determined that one of the teachers who participated in the study thought that there was a gap between the teacher's knowledge of the content and their knowledge of teaching. The teacher Kayra with this thought supports this as;

> “*There are some differences between the probability course we took in the university and the probability lesson that we teach. As in the other courses, we have seen a more comprehensive course in the probability course than in the subjects we teach the students in middle school. Therefore, I think the probability courses we have taken in undergraduate education are sufficient*.” (Line, 20-23).

The distribution and frequency values of the teachers according to the codes obtained in the evaluation category for the pedagogical content knowledge are shown in Table 1.

Table 1. The distribution of the codes obtained according to the participants by the teachers for the evaluation of the pedagogical content knowledge

	Yaşar	Yasemin	Kayra	Kadriye	Tarkan	Tülay	f

Thinks that education he/she took is sufficient	X		X	X	X	X	5
Removed from the curriculum or simplified the curriculum	X			X			2
Probability subjects that are aimed to be taught in Middle School and probability subjects learned at university are the same.	X					X	2
His/her pedagogical knowledge is not enough				X	X		2
There is gap between content knowledge and teaching knowledge			X				1

When Table 1 is examined, it is determined that five of the teachers think that education is sufficient. This code is also the most repetitive code of this category. Following this, the most frequently repeated codes were removed from the curriculum or the curriculum was simplified, the pedagogical knowledge was not sufficient and the probability topics that were taught in middle school and the probability subjects learned at the university were the same basic codes (n = 2). The least repetitive code is the code between the field information and the teaching information.

Evaluations of Teachers about the Discrete Event Information on Probability

Four of the teachers who participated in the study explained the discrete event as events affecting each other. Tarkan teacher explained his idea as "[01.39] *... Discrete event requires two different events. The situation of two separate events, which do not affect each other, can be defined as a discrete event. That's how I can interpret this as far as I can remember* (Line, 22-24)". Similarly, Tülay teacher said that;

> "[01.28] *The discrete event is completely separate from each other. For example, in an event where the tossing a coin, the other event is to throwing a dice. These two events are discrete events. So the two are completely separate from each other. Separate cases, separate results*." (Line 19-21)

Some of the teachers who participated in the study described the discrete event as events with a single sample space or universe. Yaşar, from these teachers, said that "[00.45] *The discrete event is that you have a single space in your operations. For example, if you throw a dice, there is only one space, or if you toss a coin, there is only one space* (Line, 13-14)". Yasemin, who gave a similar opinion from the participants said that;

> "[02.19] *Let me tell you something better than the example. The discrete event is also the same as the universal set. Let me give you an example from the same event. When the dice are thrown in the discrete event, it is impossible to come both 2 and 4 at the same time. The sample space is the same in the discrete event but the two events are not happening at the same time*." (Line, 26-29)

Unlike Yasemin and Yaşar, participants from Kadriye described the discrete event as events in different sample spaces. The teacher's statements are: "[01.03] *The discrete event, I think, is the case of events in different spaces. So if we're talking about two events that aren't in the same space, I think these are discrete events* (Line, 15-17)". It was understood that the teacher thought that the discrete event was in different sample spaces. Some of the teachers who participated in the study described the discrete event as events without common outcomes. Kayra teacher described the discrete event as "[01.55] *Discrete event is two events without common outputs. When we throw a dice the result of odd number and even number is the discrete event with an odd number* (Line, 28-29)".

The last code reached from the teachers' statements about the discrete event is not possible at the same time. The teacher who identified this code, Yasemin;

> "[01.43] *... I remember the discrete event as an event that is not possible at the same time. So I threw the dice, for example, 2 and 3 at the same time on the top of the dice to come at the same time was impossible or discrete as far as I remember*." (Line, 22-24)

The distribution of the codes of the teachers related to the discrete event according to the participants is presented in Table 2.

Table 2. The distribution of the obtained codes according to the participants for the evaluation of the discrete event information on probability

	Yaşar	Yasemin	Kayra	Kadriye	Tarkan	Tülay	f
Events that do not affect each other			X	X	X	X	4
Events without common outputs			X			X	2
Having a single sample space or having the same universe	X	X					2
Having different sample spaces				X			1
Events that are not possible at the same time		X					1

When Table 2 is examined, it is determined that four of the teachers' opinions about the discrete event are "not affecting each other". This code is also the most repetitive code of this category. After this, the most common repetitive codes are events with no common outputs and a single sample space or having the same universe (n = 2). The least repetitive codes are the different sample spaces and the codes that are not possible at the same time (n = 1).

Evaluation of Teachers' about Independent Event Information

The result of one of the codes obtained in the standalone event category is the code that does not affect the other. Teachers from participants mentioned her thoughts about independent event as;

Tülay: "[01:58] *Independent events are two situations that do not affect each other. For example, to throw two different dice at the same time. One outcome doesn't affect the other, or if it comes 2 in one event it doesn't mean that it won't come again in the second event*." (Line, 23-25)

Kayra: "[02.05] *If the outcome of an event does not affect another event, these two events are independent of each other. When dice is thrown and a coin is tossed. The result of the two is independent of each other. Does not affect each other* (Line, 31-32)".

Kadriye: "[01.25] *We can say that independent events occur at the same time, the possibility of events that do not affect each other. For example, a dice-throwing experiment and a coin-tossing experiment are independent of each other. So the results here do not affect each other. I think so* (Line, 19-21)".

Tarkan:"[2.27] *In an independent case, for example, I pick the ball back from a bag, and I pick the ball again. The first situation does not affect the second situation. This is called an independent event*. (Line, 32-34)".

Yaşar Teacher, who approaches the independent event differently from the other participants "[01.00] *In an independent event you need to do more than one thing at the same time. So it's like throwing a dice at the same time as picking a ball or tossing a coin and throwing a dice* (Line, 16-17)". It is understood from the statement of the teacher that the independent event explains the occurrence of more than one event at the same time.

Some of the teachers who participated in the study described the independent event as events with different sample space. Participant teachers Yasemin said that "[02.19*] In the case of independent events, the spaces are different. For example, a dice is thrown and a coin is tossed. The dice and the coin have their own sample space. These two are independent events…*" (Line, 29-31)

Table 3 presents the distribution of the codes of the teachers as a result of the content knowledge on the independent event.

Table 3. The distribution of the obtained codes according to the participants for the evaluation of the independent event information on probability

	Yaşar	Yasemin	Kayra	Kadriye	Tarkan	Tülay	f

The result of one does not affect the other			X	X	X	X	4
Different sample space		X					1
Multiple events occurring at the same time	X						1

Table 3 pointed out that four of the teachers' opinions on the independent case is "do not affect the other". This code is also the most repetitive code of this category. The least repetitive codes are "different sample space" and the "occurrence of multiple events at the same time" (n = 1).

Teachers' Evaluation about Probability Concept

It was determined that some of the teachers who participated in the study had negative thoughts about probability. Kadriye Teacher who was determined that she has negative automatic thought:

> "[03.15] *I'm having a hard time with probability subject. I just want to say that. Especially in the 10th class this topic is being studied. To be honest I'm having difficulties when it comes to high school level questions. I don't know about university education. I don't think it's about university education. In general, we may not have been taught very well in this regard. So I think that this is the level of problem until I come to university, so maybe we cannot think of multifaceted. I don't think it's right to just relate it to the university*." (Line, 40-45)

It was determined that one of the teachers who participated in the study had difficulty in distinguishing between discrete and independent events. Yasemin Teacher who mentioned this opinion said that:

> "[04.11] *Probability is a subject that I have been struggling with throughout my life. Because of that I can't get too close to it. I'm not very interested with it. To me, the probability is an extreme abstract concept. The subject of probability should be reduced a little, and in the 8s, 6s, 5s, it has to be put into the curriculum, but how is it like in the 8s? The students in 5, 6 and 7th grade don't have it. All of a sudden, at 8th grade, what's independent event? What's descrete event? They're dealing with such abstract possibilitiy subject. I think it should be given to children in the 5th grade, but in the 5th grade there should be try and fail, material and play. It should be considered more concrete. This subject is a very important; I think it can be studied at the first level in middle school. For example 2-3 hours of attainment time, then 4th, 5th grade. And also, sometimes they are removing from the curriculum sometimes put it back. I mean, I don't think the possibility should be removed from the program. Statistics should not be removed. But it is not very appropriate to be given only in the 8th grade. This is my opinion. Children do not understand it when they come to 8th grade. I do not think that teachers, including me, have much control over the probability subject*." (Line, 47-60)

Table 4 shows the distribution of the codes of the teachers as a result of their evaluations about the probability concept according to the participants.

Table 4. The distribution of the obtained codes according to the participants for the evaluation of the probability concept in terms of probability

	Yaşar	**Yasemin**	**Kayra**	**Kadriye**	**Tarkan**	**Tülay**	**f**
Have negative automatic thoughts about probability		X		X		X	3

Have difficulty to distinguish between discrete and independent events			X	1
Probability is extreme abstract concept	X			1
States that he does not think critically multifaceted.		X		1

When Table 4 is examined, it is determined that three of the teachers have negative automatic thoughts about probability. This code is also the most repetitive code of this category. The least repetitive codes are "have difficulty to distinguish discrete and independent phenomena", "the probability is excessive abstract concept", and "states that he does not think critically multifaceted" (n = 1).

Discussion and Conclusion

In this study, which was conducted in order to examine the content knowledge of middle school mathematics teachers about probability, it was determined that the teachers' probability content knowledge was good but their pedagogical knowledge was not sufficient. In addition, it was found that most of the participant teachers had difficulty in distinguishing discrete event and independent event and could not conceptually understand discrete event and independent event. Furthermore, it was found that teachers who had participated in the study had negative thoughts about probability. When the opinions of the teachers about the evaluation of probability information are examined, it is understood that most of the teachers found the education they received sufficient. On the other hand, they pointed out that there was a constant change in the curriculum and, along with these changes, the probability issues were greatly reduced in the curriculum. Although it was criticized by the teachers in the elimination of advanced topics in the curriculum, studies conducted during the years when the probability issues were heavily involved in the curriculum indicated that it was necessary to simplify the curriculum and to include firstly conceptual understanding activities (Munisamy, & Doraisamy, 1998). Another issue that teachers emphasize is the probability subjects that are aimed to be taught in middle school and the probability subjects learned in the university are on the same basis. The mathematics content courses that the teachers have learned at the university are expressed as content knowledge and their knowledge about the middle school mathematics courses that the teachers will teach is explained to us as pedagogical content knowledge. Baki's (2018, p.3) "the teacher should have at least one upper level of the curriculum that he / she has to teach" expression points out the knowledge of mathematics that the teachers have learned in the university and the mathematics knowledge they aim to teach in secondary school are the same. Some of the teachers stated that their pedagogical knowledge is not sufficient. Lim and Guerra (2013) examined the pedagogical content knowledge of pre-service teachers in terms of numerical calculation, probability and statistics, geometry and measurement, and algebraic expressions. In a study conducted by Danişman and Tanışlı (2017), he determined that teacher candidates think that they do not find themselves pedagogically sufficient in probability teaching and they think that they need improvement.

In this context, it can be said that teachers' thoughts about not being sufficient on pedagogical knowledge is supports literature. The teachers who participated in the study stated that there is a gap between their content knowledge and pedagogical knowledge. In other words, teachers stated that there is a disconnection between field information and teaching information. De Vaul (2017) stated that mathematics teachers take content knowledge courses in undergraduate programs however; there may be deficiencies in teaching information due to insufficient pedagogical content knowledge. Studies in the literature have emphasized that the bridging of pedagogical knowledge and the content knowledge of teachers is important for teaching (Baştürk, & Dönmez, 2011; Yanpar-Yelken et al., 2013). It can be said that the finding obtained in this context supports the literature.

When the knowledge of the mathematics teachers about the discrete event was evaluated, it was determined that their teachers had some misinformation about the discrete event. Most of the teachers who have false information about the discrete event are defined as events that do not affect the discrete event. This definition is not sufficient and it is more accurate to identify with the independent event. In other words, teachers explain the discrete event as events that do not affect each other. Demir (2016, p. 64) stated the independent events as events that do not affect each other. One participant stated that discrete events are occurring in different sample spaces. This statement of the teacher shows that he has a false knowledge. Because discrete events must be made in the same sample space according to the definition (Argün et al., 2016, p. 377; Demir, 2016, p. 34). Some of the teachers who used the correct expression about the discrete event described the discrete event as events without common output. Altun (2015, p. 485), in his example, pointed out that there would be no common output of discrete events.

It was determined that the participants who used similar expressions defined the discrete event as events that could not be realized at the same time. Some participants who use the correct expressions of the discrete event have stated that for a discrete event, events must be in a single instance space or in the same universe. Argün et al. (2014, p. 377) and Demir (2016, p. 34) stated that discrete events are on the same sample space.

Most of the teachers participating in the study used the correct expression about the independent event. These teachers described the independent events as the events that did not affect the other. This expression coincides with the definitions in the literature (Demir, 2016, p. 64; Lipschhutz and Lipson, 2013, p. 92). One of the teachers who participated in the study was not focused on the key concept in the definition but gave the correct answer. Although the participant who describes the occurrence of more than one event at the same time as an independent event does not make any statement about the outcomes of the events, the statement he uses is correct. Altun (2015, p. 485) benefited from this statement when explaining the independent events. A teacher who responded incorrectly from the teachers who participated in the study stated that independent events took place in different sample spaces. Argün et al. (2014, p. 290) and Lipschutz and Lipson (2013, p. 92) emphasizes that independent events should be on identical spaces. In this context, it can be said that the expression used by the teacher is inaccurate.

Considering the students' evaluations on the concept of probability, it was determined that half of the teachers participating in the study had negative thoughts about probability. Although there is no clear indication that teachers have negative thoughts about probability in the literature, it is stated that teachers have experienced reservations, fears or concerns when approaching the subject of probability (Danişman, & Tanışlı, 2017). It was determined that the teachers who participated in the study had difficulty in distinguishing discrete and independent events. Gökkurt-Özdemir (2017), in his study, found that mathematics teacher candidates have difficulty in distinguishing between discrete event and independent incident. In this study, it was found that teachers could not fully understand the concept of discrete event and the concept of independent phenomenon in the findings obtained for the concepts of discrete event and independent event. Difficulty of distinguishing between discrete event and independent concepts can be related to this reason. In this context, it can be said that the findings of the study support the literature and the study. Another assessment of teachers about the concept of probability is related to the excessive abstractness of the concept of probability. A teacher indicated in his probability class that he did not think critically. Vysotsky (2018) emphasized that the subject of probability is different and more complex than algebra and geometry. The researcher stated that the possibility included the problems of chance and change to make it abstract. In this context, it can be said that the findings obtained support the literature. The findings of the study indicated that the teachers did not emphasize the basic difference of the discrete event and the independent event (the fact that the two events were empty and no intersections).

This study, which aims to examine the probability content knowledge of middle school mathematics teachers, has certain limitations. The first limitation was the number of participation teachers in the study which is six. This limitation relates to the model of the study. A case study model was used in the study, and the participants of the study were limited and detailed interviews were conducted because the case study included in-depth data collection with a small number of participants. The results of the study showed that teachers' content knowledge and pedagogical knowledge is not sufficient. For this reason, future researchers may design experimental research to increase teachers' content knowledge and pedagogical knowledge. Another finding reached in the study is that teachers have negative automatic thoughts about probability subject. Future researchers can carry out mixed-method research that will reveal teachers' negative auto-thoughts about probability. In addition, this study was carried out only on probability, and future researchers may also examine the field information of teachers or prospective teachers in the field of statistical learning.

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