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Generalized Fuzzy TOPSIS to Solve Multi-Criteria Decision-Making Problems

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Original Article

Abstract − Multiple Criteria Decision Making (MCDM) is a process in which we choose the best alternative from all feasible alternatives. In this paper, we study fuzzy sets with some basic concepts and fuzzy TOPSIS (technique for order preference by similarity to ideal solution) method. We proposed the TOPSIS method under a fuzzy environment and expressed the rating of each alternative and weight of each criterion in the form of a triangular fuzzy number. Finally, we used the proposed method for decision making in the garments industry for the selection of supplier.

Keywords *− Fuzzy set, triangular fuzzy number (TFN), fuzzy TOPSIS, MCDM*

1. Introduction

Nowadays TOPSIS is most familiar with MCDM in different fields. Hwang and Yoon [1] proposed the TOPSIS method to solve MCDM problems and choose the best alternative with the shortest distance from a positive ideal solution and farthest distance from the negative ideal solution. Many researchers used the TOPSIS method for decision making, medical diagnoses, and other different areas of life reported in the literature [2]–[10].

Later, Chen [11] introduced the concept of the vertex method to measure the distance among two TFN and extended the TOPSIS method under a fuzzy environment. For calculating fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) and ranking of all alternatives, he presented the closeness coefficient, according to the concept of TOPSIS. But in [12], the authors challenged the Chen fuzzy TOPSIS method and claimed that Chen's method is not appropriate, he claimed that the weighted normalized fuzzy ratings are not TFNs. To overcome these limitations, they proposed a new improved fuzzy TOPSIS method in which the membership functions for the weighted normalized fuzzy ratings were presented. They also proposed a simple

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method with a mean of relative areas for ranking of fuzzy numbers and established an improved fuzzy TOPSIS method by using presented ranking fuzzy numbers.

On the base of alpha level sets, a fuzzy TOPSIS method was proposed [12] by Wang and Elhag and discussed the relation among fuzzy TOPSIS and fuzzy weighted average. In [13], the authors gave the concept of a direct approach to the fuzzy extension of the TOPSIS method, they claimed that the proposed method more efficient than the previously proposed method and free of limitation. For group decision-making problems the extended TOPSIS method is based on fuzzy numbers presented in [14]. Parveen and Kamble [15] proposed the fuzzy TOPSIS method with hexagonal fuzzy numbers and compare with other MCDM problems, they also presented the difficulties faced by the women in society by using the newly proposed method. The authors proposed a decision-making method on an interval-valued fuzzy soft matrix [16] known as "interval-valued fuzzy soft max-min decision-making method". Zulqarnain et al. [17, 18] used the "interval-valued fuzzy soft max-min decision-making method" for decision making and medical diagnoses. In [19], the authors extended the TOPSIS method to Pythagorean fuzzy data for the solution of MCDM in which experts provided the feasible alternatives for assessment information. Mahmut used the fuzzy TOPSIS method for the selection of equipment in the mining industry and concluded that this method is very helpful for decision-makers to solve decision-making problems in the mining industry [20]. To improve the efficiency of the TOPSIS method in decision-making Ding constructed the integrated fuzzy TOPSIS method in [21].

Fuzzy TOPSIS use in different industries for hiring workers and also used for decision making, medical diagnosis for MCDM problems reported in the literature [22]–[26], the authors compare and decided that the fuzzy TOPSIS method is more efficient than classical TOPSIS. Ahmad and Mohamad [27] presented an evaluation among fuzzy TOPSIS and simplified fuzzy TOPSIS and detected that the fuzzy TOPSIS method is more suitable comparative to simplified fuzzy TOPSIS. In [28], the author used the fuzzy TOPSIS method for the evaluation of the power plant. The intuitionistic fuzzy TOPSIS method is used for the selection of the best choice for an auto company in [29]. Chu [30] used the fuzzy TOPSIS method for the selection of plant location. Fuzzy TOPSIS method is used for the selection of the best candidate for personnel selection according to the following criteria experience, education, technical skills, and relocation in [31] and Dual Hesitant Fuzzy Geometric Bonferroni Mean Operators and Diminishing Choquet hesitant 2-tuple linguistic aggregation operator are developed in [34, 35].

1.1 Motivation and Contribution

For the linguistic assessments, the technique of classical TOPSIS is used, but due to the uncertainty and imprecise nature of the linguistic assessments, we proposed fuzzy TOPSIS. In this paper, we discuss the fuzzy set with some operations and fuzzy TOPSIS. We presented the generalization of TOPSIS under a fuzzy environment and use the proposed method in the garments industry for supplier selection.

1.2 Structure of Article

The following paper is organized as follows: in section 2, first, we discuss some basic definitions of fuzzy sets. In section 3, we study about fuzzy TOPSIS method and construct a graphical model for fuzzy TOPSIS. In section 4, we use the proposed method for the selection of suppliers in the garments industry. lastly, the conclusion is made in section 5.

2. Preliminaries

In this section, we recall some definitions of the fuzzy set with some operations.

Definition 2.1. [32] A fuzzy set A in M is characterized by a membership function $f_A(y_i)$ which associates with each object of M in the interval [0, 1], with the value of $f_A(y_i)$ where y_i representing the grade of membership of y in A.

Definition 2.2. [33] A fuzzy subset μ is convex, on the universal set $\mathbb{R}$ iff for all $c, d \in \mathbb{R}, \mu(\alpha c + \beta d) \geq \mu(c) \wedge \mu(d)$, where $\alpha + \beta = 1$.

Definition 2.3. [33] On the universal set V, a fuzzy subset μ is entitled as a normal fuzzy subset if here a subset c_i such that $\mu(c_i) = 1$.

Definition 2.4. [33] Stated upon the universal set S, a fuzzy number is a fuzzy subset that exists as together convex and normal.

Definition 2.5. [26] If C = (x_1, y_1, z_1) for all $x_1, y_1, z_1 \in \mathbb{R}$ is a fuzzy number with piecewise linear membership function defined as follows

$$\delta_C\ (t) = \begin{cases} \frac{t-x_1}{y_1-x_1} & if\ x_1 \leq t \leq y_1 \\ 1 & if\ t = y_1 \\ \frac{z_1-t}{z_1-y_1} & if\ y_1 \leq t \leq z_1 \\ 0 & Otherwise \end{cases}$$

Then C = (x_1, y_1, z_1) is called a triangular fuzzy number (TFN).

Definition 2.6 [11] If C = (x_1, y_1, z_1) and D = (x_2, y_2, z_2) are two TFN, then distance between them can be defined as

$$d\ (C, D) = \sqrt{\frac{1}{3}\ \left((x_1 - x_2)^2,\ (y_1 - y_2)^2, (z_1 - z_2)^2\right)}$$

3. Fuzzy TOPSIS Algorithm [11]

In this section, we present the fuzzy TOPSIS method with an algorithm and construct a model for the fuzzy TOPSIS method.

Let M = {M_1, M_2, M_3,…, M_m} be a set of *m* alternatives and N = {N_1, N_2, N_3,…, N_n} be a set of evaluation criteria and D = {D_1, D_2, D_3,…, D_l} be a set of *l* decision-makers.

Step 1: Fuzzy Rating Scale selection for Linguistic Variables

The criteria for linguistic variables and alternatives are given in table 1.

Step 2: Fuzzy linguistic ratings for alternatives and criteria of weights for decision-makers

"$\widetilde{x_{ij}^k}$ be a fuzzy rating for *k*th decision-maker for the *i*th alternatives and *j*th criterion, represented as follows

$$\widetilde{x_{ij}^k} = (a_{ij}^k, b_{ij}^k, c_{ij}^k)$$

The weight for *k*th decision-maker and *j*th criteria are given as follows

$$\widetilde{w_j^k} = (w_{j1}^k, w_{j2}^k, w_{j3}^k)$$

Step 3: Aggregated fuzzy ratings for the alternatives

$\widetilde{x_{ij}}$ be an aggregated fuzzy rating for the *i*th alternative w.r.t the *j*th criteria are given as follows

$$\widetilde{x_{ij}} = (a_{ij}, b_{ij}, c_{ij})$$

$$a_{ij} = \min_k \{a_{ij}^k\} \quad b_{ij} = \frac{1}{l}\sum_{k=1}^{l}\{b_{ij}^k\} \quad c_{ij} = \min_k \{c_{ij}^k\}$$

and

$\widetilde{w_j} = (\widetilde{w_{j1}}, \widetilde{w_{j2}}, \widetilde{w_{j3}})$ be an aggregated fuzzy weight for the j^{th} criteria represents in the following equation.

$$w_{j1} = \min_k \{w_{j1}^k\} \qquad w_{j2} = \frac{1}{l}\sum_{k=1}^{l}\{w_{j2}^k\} \qquad w_{j3} = \min_k \{w_{j3}^k\}$$

Step 4: Construction of Aggregated Fuzzy Decision Matrix (AFDM) and Aggregated Fuzzy Weight Matrix (AFWM)

Fuzzy MCDM problem can be converted to an AFDM as follows

$$D = \begin{array}{c} M_1 \\ M_2 \\ \vdots \\ M_m \end{array} \begin{bmatrix} \widetilde{x_{11}} & \widetilde{x_{12}} & \cdots & \widetilde{x_{1n}} \\ \widetilde{x_{21}} & \widetilde{x_{22}} & \cdots & \widetilde{x_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{x_{m1}} & \widetilde{x_{m2}} & \cdots & \widetilde{x_{mn}} \end{bmatrix}$$

Moreover, the AFWM is defined as follows

$$\widetilde{W} = [\widetilde{w_1}, \widetilde{w_2}, \widetilde{w_3}, \ldots, \widetilde{w_n}]^T$$

Where $\widetilde{w_j}$ be an aggregated fuzzy weight for the j^{th} criterion.

Step 5: Normalization of the FDM (NFDM)

The NFDM is given as

$$\widetilde{R} = [\widetilde{r_{ij}}]_{m\times n} = \begin{bmatrix} \widetilde{r_{11}} & \widetilde{r_{12}} & \cdots & \widetilde{r_{1n}} \\ \widetilde{r_{21}} & \widetilde{r_{22}} & \cdots & \widetilde{r_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{r_{m1}} & \widetilde{r_{m2}} & \cdots & \widetilde{r_{mn}} \end{bmatrix} \text{ where } \widetilde{r_{ij}} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right) \text{ and } c_j^* = \max_k c_{ij} \text{ (benefit criteria)}$$

$$\widetilde{r_{ij}} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_j}\right),\ a_j^- = \min_i a_{ij}, \text{ where } a_j^- \text{ represents the cost criteria}$$

To normalize the decision matrix.

Step 6: Weighted Normalized Fuzzy Decision Matrix (WNFDM)

WNFDM gave as follows

$$\widetilde{V} = [\widetilde{v_{ij}}]_{m\times n} = \left[\widetilde{w_j(\cdot)\,\widetilde{r_{ij}}}\right] = \begin{bmatrix} \widetilde{w_1}(\cdot)\,\widetilde{r_{11}} & \widetilde{w_2}(\cdot)\,\widetilde{r_{12}} & \cdots & \widetilde{w_n}(\cdot)\,\widetilde{r_{1n}} \\ \widetilde{w_1}(\cdot)\,\widetilde{r_{21}} & \widetilde{\widetilde{w_2}(\cdot)\,\widetilde{r_{22}}} & \cdots & \widetilde{\widetilde{w_2}(\cdot)\,\widetilde{r_{2n}}} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{w_1}(\cdot)\,\widetilde{r_{m1}} & \widetilde{w_2}(\cdot)\,\widetilde{r_{m2}} & \cdots & \widetilde{\widetilde{w_n}(\cdot)\,\widetilde{r_{mn}}} \end{bmatrix}$$

Step 7: Determination of FPIS and FNIS

To find the FPIS and the FNIS we used the following equations

$$M^* = (\widetilde{v_1^*}, \widetilde{v_2^*}, \widetilde{v_3^*}, \ldots, \widetilde{v_n^*}) \qquad \text{where } \widetilde{v_j^*} = (c_j^*, c_j^*, c_j^*) \text{ and } c_j^* = \max_i \left\{v_j^{\widetilde{(3^{rd}\ component)}}\right\}$$

$$M^- = (\widetilde{v_1^-}, \widetilde{v_2^-}, \widetilde{v_3^-}, \ldots, \widetilde{v_n^-}) \qquad \text{where } \widetilde{v_j^-} = (c_j^-, c_j^-, c_j^-) \text{ and } c_j^- = \min_i \left\{v_j^{\widetilde{(1^{st}\ component)}}\right\}$$

Where $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$.

Step 8: Calculation of d_i^* and d_i^-

The distances from FPIS and FNIS of all weighted alternative $\widetilde{v_{ij}}$ where $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$.

d_i^* and d_i^- can be calculated as follows

$$d_i^* = \sum_{j=1}^{n} d(\widetilde{v_{ij}}, \widetilde{v_i^*}), \quad i = 1, 2, 3, \ldots, m$$

$$d_i^- = \sum_{j=1}^{n} d(\widetilde{v_{ij}}, \widetilde{v_i^-}), \quad i = 1, 2, 3, \ldots, m$$

Step 9: Determination of Closeness Coefficient CC_i

CC_i of alternatives can be calculated as follows

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \text{ for all } i = 1, 2, 3, \ldots, m\text{''}$$

Step 10: Ranking the alternatives

An alternative closeness coefficient's value is near to 1 represents that it is near to FPIS and away from FNIS.

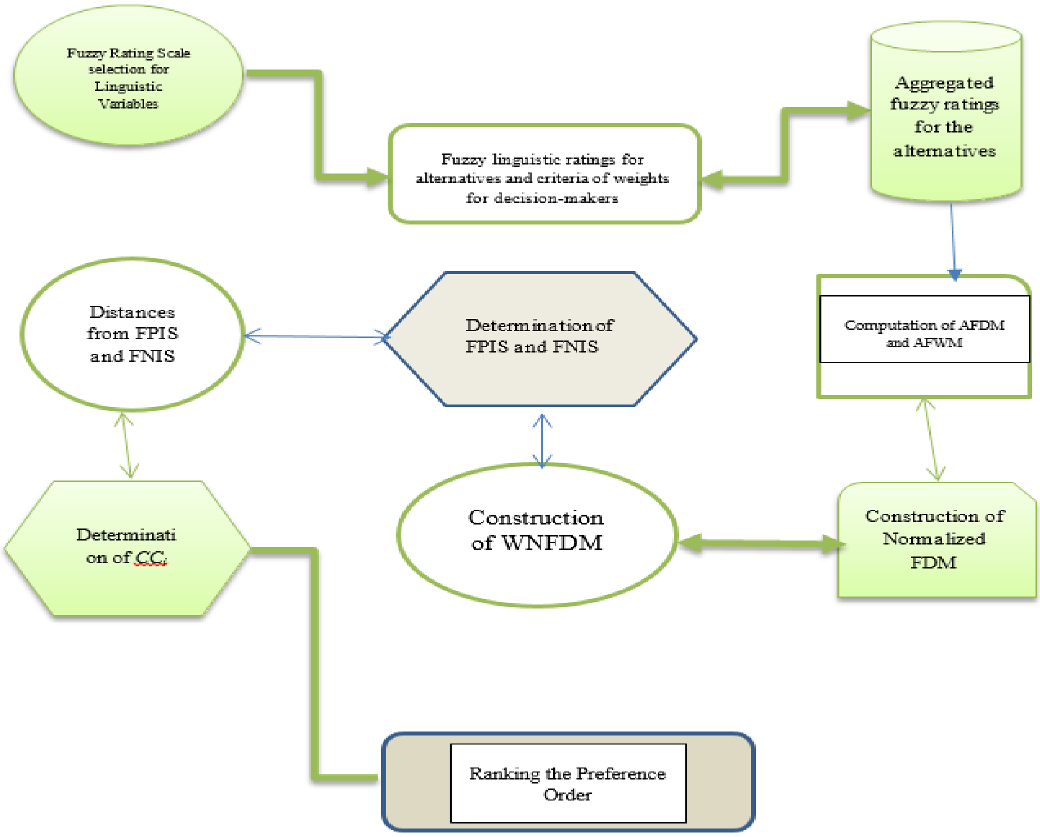

Figure 1. Algorithm of proposed Fuzzy TOPSIS

4. Application of fuzzy TOPSIS method

A garments industry wants to hire a supplier from out of two supplier's M = {M_1, M_2}. For the selection of the best supplier, the managing director of the industry hires a team of three decision-makers as follows D = {D_1, D_2, D_3}. The managing director of garments industry decided the evaluation criteria for the selection of the best supplier for the industry given as follows N = {N_1, N_2, N_3, N_4}

$$N = \begin{cases} Benifit\ Criteria & j_1 = \begin{cases} N_1 \\ N_2 \\ N_3 \end{cases} \\ Cost\ Criteria & j_2 = \{N_4 \end{cases}$$

Solution by Fuzzy TOPSIS

Step 1: Fuzzy Rating Scale selection for Linguistic Variables

The rating scale for linguistic variables given in the following

Table 1. Ratings for Linguistic Variables

Criteria Weights	Alternatives	TFN
L	VP	(1,1,3)
L	P	(1,3,5)
M	F	(3,5,7)
H	G	(5,7,9)
VH	VG	(7,9,9)

Where weights of criteria represent "very low (VL), low (L), medium (M), high (H), and very high (VH). Similarly, rating for alternatives VP, P, F, G, VG represents very poor, poor, fair, good, and very good" respectively.

Step 2: Fuzzy linguistic ratings for alternatives and criteria of weights for decision-makers

Every decision-maker allocate some weight for each criterion given in the following table

Table 2. Criteria Weightage by the DMs $\widetilde{w_j^k} = (w_{j1}^k, w_{j2}^k, w_{j3}^k)$

	D_1	D_2	D_3
N_1	H (5, 7, 9) $\widetilde{w_1^1} = (\widetilde{w_{11}^1}, \widetilde{w_{12}^1}, \widetilde{w_{13}^1})$	M (3, 5, 7) $\widetilde{w_1^2} = (\widetilde{w_{11}^2}, \widetilde{w_{12}^2}, \widetilde{w_{13}^2})$	M (3, 5, 7) $\widetilde{w_1^3} = (\widetilde{w_{11}^3}, \widetilde{w_{12}^3}, \widetilde{w_{13}^3})$
N_2	VH (7, 9, 9) $\widetilde{w_2^1} = (\widetilde{w_{21}^1}, \widetilde{w_{22}^1}, \widetilde{w_{23}^1})$	H (5, 7, 9) $\widetilde{w_2^2} = (\widetilde{w_{21}^2}, \widetilde{w_{22}^2}, \widetilde{w_{23}^2})$	H (5, 7, 9) $\widetilde{w_2^1} = (\widetilde{w_{21}^3}, \widetilde{w_{22}^3}, \widetilde{w_{23}^3})$
N_3	VH (7, 9, 9) $\widetilde{w_3^1} = (\widetilde{w_{31}^1}, \widetilde{w_{32}^1}, \widetilde{w_{33}^1})$	H (5, 7, 9) $\widetilde{w_3^2} = (\widetilde{w_{31}^2}, \widetilde{w_{32}^2}, \widetilde{w_{33}^2})$	H (5, 7, 9) $\widetilde{w_3^3} = (\widetilde{w_{31}^3}, \widetilde{w_{32}^3}, \widetilde{w_{33}^3})$
N_4	M (3, 5, 7) $\widetilde{w_4^1} = (\widetilde{w_{41}^1}, \widetilde{w_{42}^1}, \widetilde{w_{43}^1})$	L (1, 3, 5) $\widetilde{w_4^2} = (\widetilde{w_{41}^2}, \widetilde{w_{42}^2}, \widetilde{w_{43}^2})$	L (1, 3, 5) $\widetilde{w_4^3} = (\widetilde{w_{41}^3}, \widetilde{w_{42}^3}, \widetilde{w_{43}^3})$

The aggregated fuzzy weights $\widetilde{w_j} = (w_{j1}, w_{j2}, w_{j3})$ for each criterion *"j = 1, 2, 3, 4"* are calculated as follows.

$$w_{11} = \min_k \{w_{11}^k\} = \min \{5, 3, 3\} = 3$$

$$w_{12} = \frac{1}{3} \sum_{k=1}^{3}\{w_{12}^k\} = \frac{1}{3} [7+5+5] = 5.667$$

$$w_{13} = \min_k \{w_{13}^k\} = \max \{9, 7, 7\} = 9$$

So,

$$\widetilde{w_1} = (w_{11}, w_{12}, w_{13}) = (3, 5.667, 9)$$

Similarly, we can get

$$\widetilde{w_2} = (w_{21}, w_{22}, w_{23}) = (5, 7.667, 9)$$

$$\widetilde{w_3} = (w_{31}, w_{32}, w_{33}) = (5, 7.667, 9)$$

$$\widetilde{w_4} = (w_{41}, w_{42}, w_{43}) = (1, 3.667, 7)$$

Therefore, the aggregated weight vector is

$$\widetilde{W} = [\widetilde{w_1}, \widetilde{w_2}, \widetilde{w_3}, \widetilde{w_4}]^T$$

$$\widetilde{W} = [(3, 5.667, 9), (5, 7.667, 9), (5, 7.667, 9), (1, 3.667, 7)]^T$$

Alternative rating for decision-makers given as follows

Table 3. Rating of alternatives for DM

		D_1	**D_2**	**D_3**
N_1	**M_1**	F $\widetilde{x_{11}^{(1)}}=\left(a_{11}^{(1)}, b_{11}^{(1)}, c_{11}^{(1)}\right)$= (3, 5, 7)	F $\widetilde{x_{11}^{(2)}}=\left(a_{11}^{(2)}, b_{11}^{(2)}, c_{11}^{(2)}\right)$= (3, 5, 7)	F $\widetilde{x_{11}^{(3)}}=\left(a_{11}^{(3)}, b_{11}^{(3)}, c_{11}^{(3)}\right)$= (3, 5, 7)
	M_2	F $\widetilde{x_{21}^{(1)}}=\left(a_{21}^{(1)}, b_{21}^{(1)}, c_{21}^{(1)}\right)$= (5, 7, 9)	F $\widetilde{x_{21}^{(2)}}=\left(a_{21}^{(2)}, b_{21}^{(2)}, c_{21}^{(2)}\right)$= (3, 5, 7)	F $\widetilde{x_{21}^{(3)}}=\left(a_{21}^{(3)}, b_{21}^{(3)}, c_{21}^{(3)}\right)$= (3, 5, 7)
N_2	**M_1**	VG $\widetilde{x_{12}^{(1)}}=\left(a_{12}^{(1)}, b_{11}^{(1)}, c_{12}^{(1)}\right)$= (7, 9, 9)	VG $\widetilde{x_{12}^{(2)}}=\left(a_{12}^{(2)}, b_{11}^{(2)}, c_{12}^{(2)}\right)$= (7, 9, 9)	VG $\widetilde{x_{12}^{(3)}}=\left(a_{12}^{(3)}, b_{11}^{(3)}, c_{12}^{(3)}\right)$= (7, 9, 9)
	M_2	G $\widetilde{x_{22}^{(1)}}=\left(a_{22}^{(1)}, b_{22}^{(1)}, c_{22}^{(1)}\right)$= (5, 7, 9)	VG $\widetilde{x_{22}^{(2)}}=\left(a_{22}^{(2)}, b_{22}^{(2)}, c_{22}^{(2)}\right)$= (3, 5, 7)	G $\widetilde{x_{22}^{(3)}}=\left(a_{22}^{(3)}, b_{22}^{(3)}, c_{22}^{(3)}\right)$= (5, 7, 9)
N_3	**M_1**	P $\widetilde{x_{13}^{(1)}}=\left(a_{13}^{(1)}, b_{13}^{(1)}, c_{13}^{(1)}\right)$= (1, 3, 5)	F $\widetilde{x_{13}^{(2)}}=\left(a_{13}^{(2)}, b_{13}^{(2)}, c_{13}^{(2)}\right)$= (3, 5, 7)	P $\widetilde{x_{13}^{(3)}}=\left(a_{13}^{(3)}, b_{13}^{(3)}, c_{13}^{(3)}\right)$= (1, 3, 5)
	M_2	P $\widetilde{x_{23}^{(1)}}=\left(a_{23}^{(1)}, b_{23}^{(1)}, c_{23}^{(1)}\right)$= (1, 3, 5)	P $\widetilde{x_{23}^{(2)}}=\left(a_{23}^{(2)}, b_{23}^{(2)}, c_{23}^{(2)}\right)$= (1, 3, 5)	P $\widetilde{x_{23}^{(3)}}=\left(a_{23}^{(3)}, b_{23}^{(3)}, c_{23}^{(3)}\right)$= (1, 3, 5)
N_4	**M_1**	F $\widetilde{x_{14}^{(1)}}=\left(a_{14}^{(1)}, b_{14}^{(1)}, c_{14}^{(1)}\right)$= (3, 5, 7)	F $\widetilde{x_{14}^{(2)}}=\left(a_{14}^{(2)}, b_{14}^{(2)}, c_{14}^{(2)}\right)$= (3, 5, 7)	P $\widetilde{x_{14}^{(3)}}=\left(a_{14}^{(3)}, b_{14}^{(3)}, c_{14}^{(3)}\right)$= (1, 3, 5)
	M_2	P $\widetilde{x_{24}^{(1)}}=\left(a_{24}^{(1)}, b_{24}^{(1)}, c_{24}^{(1)}\right)$= (1, 3, 5)	P $\widetilde{x_{24}^{(2)}}=\left(a_{24}^{(2)}, b_{24}^{(2)}, c_{24}^{(2)}\right)$= (1, 3, 5)	F $\widetilde{x_{24}^{(3)}}=\left(a_{24}^{(3)}, b_{24}^{(3)}, c_{24}^{(3)}\right)$= (3, 5, 7)

Step 3: Aggregated fuzzy ratings for the alternatives

$\widetilde{x_{ij}}$ be an aggregated fuzzy rating for the i^{th} alternative w.r.t the j^{th} criteria can be calculated as follows

$$\widetilde{x_{11}} = (a_{11}, b_{11}, c_{11}), \text{ where}$$

$$a_{11} = \min_{k} \{a_{11}^{k}\} = \min \{3, 3, 3\} = 3$$

$$b_{11} = \frac{1}{3} \sum_{k=1}^{l} \{b_{11}^{k}\} = \frac{1}{3} [5+5+5] = 5$$

$$c_{11} = \min_{k} \{c_{11}^{k}\} = \max \{7, 7, 7\} = 7$$

Therefore

$$\widetilde{x_{11}} = (3.000, 5.000, 7.000)$$

Similarly, we can find other values given in Table 4

Step 4: Construction of AFDM

Table 4. AFDM $\widetilde{D} = \widetilde{x_{ij}}$

	N_1	**N_2**	**N_3**	**N_4**
M_1	$\widetilde{x_{11}}$ = (3.000,5.000,7.000)	$\widetilde{x_{12}}$ = (7.000,9.000,9.000)	$\widetilde{x_{13}}$ = (1.000,3.667,7.000)	$\widetilde{x_{14}}$ = (1.000,4.333,7.000)
M_2	$\widetilde{x_{21}}$ = (3.000,6.333,9.000)	$\widetilde{x_{22}}$ = (5.000,7.667,9.000)	$\widetilde{x_{23}}$ = (1.000,3.000,5.000)	$\widetilde{x_{24}}$ = (1.000,3.667,7.000)

Step 5: NFDM

We can calculate the NFDM as follows

$$\widetilde{r_{11}} = \left(\frac{a_1^-}{c_{11}}, \frac{a_1^-}{b_{11}}, \frac{a_1^-}{a_{11}}\right), \text{ where } N_1 \text{ is the cost criteria}$$

$$\widetilde{r_{11}} = \left(\frac{3}{7}, \frac{3}{5}, \frac{3}{3}\right) \text{ where } a_1^- = \min_{i} a_{i1} = \min \{3.000, 3.000\} = 3$$

Similarly, we can get other values

Therefore, NFDM is given in the following table.

Table 5. NFDM $\tilde{R} = \widetilde{r_{ij}}$

	N_1	N_2	N_3	N_4
M_1	$\widetilde{r_{11}}$ = (0.429,0.600,1.000)	$\widetilde{r_{12}}$ = (0.778, 1.000,1.000)	$\widetilde{r_{13}}$ = (0.143,0.524,1.000)	$\widetilde{r_{14}}$ = (0.143,0.619,1.000)
M_2	$\widetilde{r_{21}}$ = (0.333, 0.474, 1.000)	$\widetilde{r_{22}}$ = (0.556,0.852,1.000)	$\widetilde{r_{23}}$ = (0.143,0.429,0.714)	$\widetilde{r_{24}}$ = (0.143,0.524,1.000)

Step 6: WNFDM

Now we get the WNFDM

Table 6. WNFDM $\tilde{V}$ = $[\widetilde{v_{ij}}]$

	N_1	N_2	N_3	N_4
M_1	$\widetilde{v_{11}}$ = (1,286, 3.400, 9.000)	$\widetilde{v_{12}}$ = (3.889, 7.667, 9.000)	$\widetilde{v_{13}}$ = (0.714, 4.016, 9.000)	$\widetilde{v_{14}}$ = (0.143, 2.270, 7.000)
M_2	$\widetilde{v_{21}}$ = (1.000, 2.684, 9.000)	$\widetilde{v_{22}}$ = (2.778, 6.531, 9.000)	$\widetilde{v_{23}}$ = (0.714, 3.286, 6.429)	$\widetilde{v_{24}}$ = (0.143, 1.921, 7.000)

Step 7: Determination of FPIS and FNIS

To calculate FPIS and FNIS given in the following table

Table 7. The calculated values of FPIS and FNIS

	FPIS			
M^*	$\widetilde{v_1^*}$ = (9, 9, 9)	$\widetilde{v_2^*}$ = (9, 9, 9)	$\widetilde{v_3^*}$ = (9, 9, 9)	$\widetilde{v_4^*}$ = (7, 7, 7)
	FNIS			
M^-	$\widetilde{v_1^-}$ = (1, 1, 1)	$\widetilde{v_2^-}$ = (2.778, 2.778, 2,778)	$\widetilde{v_3^-}$ = (0.714, 0.714, 0.714)	$\widetilde{v_3^-}$ = (0.143, 0.143, 0.143)

Step 8: Calculation of d_i^* and d_i^-

The distances from FPIS and FNIS of all weighted alternative $\widetilde{v_{ij}}$, where "*i = 1, 2, 3, …, m*" and "*j = 1, 2, 3, …, n*". d_i^* and d_i^- can be calculated as follows $d(v_{1j}, \widetilde{v_j^*})$ where *j = 1, 2, 3, 4*

for *j = 1*

$$d(v_{11}, \widetilde{v_1^*}) = d((1,286, 3.400, 9.000),\ (9, 9, 9))$$

$$d(v_{11}, \widetilde{v_1^*}) = \sqrt{\frac{1}{3}\ ((1,286-9)^2,\ (3.400-9)^2, (9.000-9)^2)} = 5.503$$

Similarly, for *j = 2*

$$d(v_{12}, \widetilde{v_2^*}) = d((3.889, 7.667, 9.000),\ (9, 9, 9))$$

$$d(v_{12}, \widetilde{v_2^*}) = \sqrt{\frac{1}{3}\ ((3.889-9)^2,\ (7.667-9)^2, (9.000-9)^2)} = 3.049$$

for *j = 3*

$$d(v_{13}, \widetilde{v_3^*}) = d((0.714, 4.016, 9.000),\ (9, 9, 9))$$

$$d(v_{13}, \widetilde{v_3^*}) = \sqrt{\frac{1}{3}\ ((0.714-9)^2,\ (4.016-9)^2, (9.000-9)^2)} = 5.582$$

for *j = 4*

$$d(v_{14}, \widetilde{v_4^*}) = d((0.143, 2.270, 7.000),\ (7, 7, 7))$$

$$d(v_{14}, \widetilde{v_4^*}) = \sqrt{\frac{1}{3}\ ((0.143-7)^2,\ (2.270-7)^2, (7.000-7)^2)} = 4.809$$

The remaining values $d(v_{2j}, \widetilde{v_j^*})$, $d(v_{1j}, \widetilde{v_1^-})$, $d(v_{2j}, \widetilde{v_j^-})$ for "*j = 1, 2, 3, 4*" are left for the sake of brevity and are given in the following Table

Table 8. Distances $d\ (M_i, M^*)$ and $d\ (M_i, M^-)$ from FPIS and FNIS for the alternatives M_i

	N₁	**N₂**	**N₃**	**N₄**
FPIS M₁	$d(v_{11}, \widetilde{v_1^*}) = 5.503$	$d(v_{12}, \widetilde{v_2^*}) = 3.049$	$d(v_{13}, \widetilde{v_3^*}) = 5.582$	$d(v_{14}, \widetilde{v_4^*}) = 4.809$
FPIS M₂	$d(v_{21}, \widetilde{v_1^*}) = 5.884$	$d(v_{22}, \widetilde{v_2^*}) = 3.864$	$d(v_{23}, \widetilde{v_3^*}) = 5.997$	$d(v_{24}, \widetilde{v_4^*}) = 4.926$
FNIS M₁	$d(v_{11}, \widetilde{v_1^-}) = 4.824$	$d(v_{12}, \widetilde{v_2^-}) = 4.613$	$d(v_{13}, \widetilde{v_3^-}) = 5.149$	$d(v_{14}, \widetilde{v_4^-}) = 4.145$
FNIS M₂	$d(v_{21}, \widetilde{v_1^-}) = 4.72$	$d(v_{22}, \widetilde{v_2^-}) = 4.195$	$d(v_{23}, \widetilde{v_3^-}) = 3.617$	$d(v_{24}, \widetilde{v_4^-}) = 4.089$

d_i^* be each weighted alternative from FPIS is computed as

$$d_i^* = \sum_{j=1}^{n} d(v_{ij}, \widetilde{v_j^*});\ i = 1, 2$$

Now d_1^* for the alternative M_1 form FPIS M^* is calculated as follows

$d_1^* = \sum_{j=1}^{n} d(v_{1j}, \widetilde{v_j^*}) = d(v_{11}, \widetilde{v_1^*}) + d(v_{12}, \widetilde{v_2^*}) + d(v_{13}, \widetilde{v_3^*}) + d(v_{14}, \widetilde{v_4^*})$ = 5.503 3.049 5.582 4.809 = 18.943

Similarly, we can find d_2^*, d_1^-, d_2^- and their respective values are given in the following Table

Table 9. The distance of each weighted alternative

$\mathbf{d_1^*}$	$\mathbf{d_2^*}$	$\mathbf{d_1^-}$	$\mathbf{d_2^-}$
18.943	20.671	18.731	16.621

Step 9: Determination of Closeness Coefficient CC_i

Finally, the closeness coefficient CC_i of alternatives "i =1, 2" calculated as follows

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}$$

$$CC_1 = \frac{18.731}{18.731+18.943} = 0.497$$

$$CC_2 = \frac{16.621}{16.621+20.671} = 0.445$$

Step 10: Ranking the alternatives

The ranking order for the alternatives is $M_1 > M_2$, i.e., M_1 is the best supplier according to the given criteria.

5. Conclusion

In this paper, we proposed a fuzzy TOPSIS method. By using crisp data it is more difficult to solve decision-making problems under an uncertain environment, to overcome such uncertainties fuzzy TOPSIS is more appropriate. Finally, to show the applicability and validity of the proposed technique with an illustrated example of the best supplier in the garments industry is presented. We consider this technique will be helpful in problem-solving and will expand the area of investigations for more accuracy in real-life issues.

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