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DISCRETE TIME SHOCK MODEL WITH VARYING SUCCESS PROBABILITY

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Abstract: Suppose a system fails if the time between two consecutive shocks falls below a fixed threshold $\delta \in \mathbb{N}$ and the lifetime of the system is measured as the time to the occurrence of this event. In this paper, we consider the interarrival times between $(i-1)$-th and i-th successive shocks follow a geometric distribution with mean $1/p_i$,where $p_i = \theta p^{i-1}$, $i = 1, 2, \ldots,$ $0 < \theta < 1$, $0 < p \leq 1$. Under the above considerations, the distribution of system lifetime is obtained. Probability generating function and than also moments of system are derived. The proportion estimates of distribution parameters are studied. A numerical example is also presented by using real data.

Key words: $q-$distributions; $\delta-$Shock model; Probability; Generating function
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1. Introduction

Shock models have aroused great interest in reliability theory [1]-[9]. Shock models are systems that experience shocks of random magnitudes at random times. There are three modes of shock models which are run shock model, extreme shock model and cumulative shock model. In an run shock model, the amplitudes of a specified number of consecutive shocks are considered a failure criterion. See, e.g., [3]. For extreme shock and cumulative shock model please see [1]-[3].

Let us consider, a system collapses when the time between two consecutive shocks falls below a fixed threshold δ. Furthermore, the system's lifetime is measured as the time to the occurrence of that event. Such systems called as δ-shock model. Since the δ-shock model take into account the time between two consecutive shocks instead of magnitudes, it can be considered as a forth mode in shock models. δ-shock models have been studied by [6]-[9].

Recently, Eryilmaz [9] studied the discrete time release of the δ-shock model. In this model, he assumed that the shocks occur according to a binomial process at all times and the interarrival times between successive shocks have a geometric distribution with mean $1/p$.

In this paper, we assume that the interarrival times between $(i-1)$-th and i-th successive shocks follow a geometric distribution with mean $1/q_i$,where $q_i = 1-\theta q^{i-1}$, $i = 1, 2, \ldots, 0 < \theta < 1, 0 < q \leq 1$. Studying such a geometric model in the context of this delta-shock model can be motivated as follows: Consider a unit that is subject to a sequence of shocks. Assume that the unit degrades

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after each shock. Since a shock threshold will be lower for degraded unit, the probability that the unit is subject to a shock will increase after each shock. If the unit is assumed to tolerate a shock after delta time period from the previous shock, then such unit's lifetime can be modeled by q-delta shock model.

In Section 2, a recursive formula is given to write a probability mass function (pmf) of system. The pmf of system is conducted by this recursive formula. The probability generating function (pgf), mean and variance of the system lifetime are derived in Section 3. In Section 4, proportion estimates of distribution parameters are provided and a simulation study is performed to investigate the properties of these estimates. A numerical example is also given to show the capability of new distribution for modelling any real data.

2. The system and its pmf

Suppose a system is subject to periodic external shocks that arrive according to following process. The period can be thought as minute, hour, day, month etc. What the important in this model is not the magnitude of the shocks but the length of the period between from one shock to next shock. Let us consider a sequence $I_1, \ldots, I_n$ of zero (there is a shock)–one (there is no shock) Bernoulli trials such that the trials of the subsequence after the $(i-1)$th shock until the ith shock are independent with failure probability

$$q_i = 1 - \theta q^{i-1}, i = 1, 2, \ldots,\ 0 < \theta < 1,\ 0 < q \leq 1,$$

which process studied by Charalambides, Yalcin and Eryilmaz (see [10], [11]). In other words, ith shock occurs with probability q_i and does not occur with probability $1 - q_i$. For $i = 1, 2, \ldots$, let X_i denotes the period length between $(i-1)$th and ith shocks, where X_0 be the period until the first shock. Then the random variables $X_0, X_1, \ldots$ are independent but not identical with the pmf

$$P\{X_i = x\} = q_i(1 - q_i)^{x-1}$$

for $x = 1, 2, \ldots$.

Under the above discussion, the lifetime of a system can be expressed under the discrete time $\delta-$shock model by

$$T_\delta = \sum_{i=0}^{M} X_i,$$

where the stopping random variable M is defined as

$$\{M = m\} \Longleftrightarrow \{X_1 > \delta, ..., X_{m-1} > \delta, X_m \leq \delta\},$$

for $\delta \geq 1$ and $m = 1, 2, \ldots$, (see Eryilmaz [9]).

Lemma 1. *For $0 < q \leq 1$, let us define*

$$B_q(r, s) = \sum_{\substack{y_1 + y_2 + \cdots + y_{r-1} = s \\ y_1 \geq 0, y_2 \geq \delta, \ldots, y_{r-2} \geq \delta, y_{r-1} \geq \delta}} \cdots \sum q^{y_2 + 2y_3 + 3y_4 + \cdots + (r-2)y_{r-1}},$$

where y_i s are integers. Then $B_q(r,s)$ obeys the following recurrence relation

$$B_q(r,s) = \begin{cases} \sum_{w=\delta}^{s-(r-3)\delta} q^{(r-2)w} B_q(r-1, s-w) & , r \geq 3, s \geq (r-2)\delta \\ 1 & , r = 2, s \geq 0 \\ 0 & , otherwise \end{cases}$$

PROOF. Considering the values that y_{r-1} can take, we have

$$\begin{aligned} B_q(r,s) &= \sum_{\substack{y_1+y_2+\cdots+y_{r-1}=s \\ y_1\geq 0, y_2\geq\delta,\ldots,y_{r-2}\geq\delta, y_{r-1}\geq\delta}} \cdots \sum q^{y_2+2y_3+3y_4+\cdots+(r-2)y_{r-1}} \\ &= \sum_{w=\delta}^{s-(r-3)\delta} q^{(r-2)w} \sum_{\substack{y_1+y_2+\cdots+y_{r-2}=s-w \\ y_1\geq 0, y_2\geq\delta,\ldots,y_{r-2}\geq\delta}} \cdots \sum q^{y_2+2y_3+3y_4+\cdots+(r-3)y_{r-2}} \\ &= \sum_{w=\delta}^{s-(r-3)\delta} q^{(r-2)w} B_q(r-1,s-w). \end{aligned}$$

for $r \geq 3$ and $s \geq (r-2)\delta$. The other parts of the recurrence are clear.

THEOREM 1. *The pmf of T_δ is*

$$P\{T_\delta = n\} = \sum_{i=2}^{\left[\frac{n+2\delta}{\delta+1}\right]} \left\{ \theta^{n-i} \left(\prod_{j=1}^{i} \left(1-\theta q^{j-1}\right) \right) \sum_{t=0}^{\min(\delta-1,n-i)} q^{(i-1)t} B_q(i, n-i-t) \right\} \tag{2.1}$$

for $n = 2, 3, \ldots$

PROOF. In order to find pmf of T_δ, Eryilmaz [9] used a random variable which is equivalent to T_δ. Let W_δ denotes the waiting time until two $0'$s are separated by at most "$\delta - 1$" failures in Bernoulli trials $I_1, I_2, \ldots$. Then

$$T_\delta \overset{st}{=} W_\delta$$

for $\delta \geq 1$. By using the above considerations, the pmf of T_δ can be obtained as follows: One of the typical patterns of length n including $i(\geq 2)$ $0'$s for the occurrence of the event $\{W_n = n\}$ is

$$\underbrace{1\ldots1}_{y_1\geq 0}0\underbrace{1\ldots1}_{y_2\geq\delta}0\underbrace{1\ldots1}_{y_3\geq\delta}0\ldots0\underbrace{1\ldots1}_{y_{i-1}\geq\delta}0\underbrace{1\ldots1}_{0\leq y_i<\delta}0.$$

Then pmf of T_δ is obtained by

$$\begin{aligned} &P\{T_\delta = n\} \\ &= \sum_{i=2}^{\left[\frac{n+2\delta}{\delta+1}\right]} \sum_{\substack{y_1+y_2+\cdots+y_i=n-i \\ y_1\geq 0, y_2\geq\delta,\ldots,y_{i-1}\geq\delta, y_i<\delta}} \cdots \sum \left(\theta q^0\right)^{y_1}\left(1-\theta q^0\right)(\theta q)^{y_2}(1-\theta q)\times\cdots\times\left(\theta q^{i-1}\right)^{y_i}\left(1-\theta q^{i-1}\right) \\ &= \sum_{i=2}^{\left[\frac{n+2\delta}{\delta+1}\right]} \left\{ \theta^{n-i}\left(\prod_{j=1}^{i}\left(1-\theta q^{j-1}\right)\right) \sum_{\substack{y_1+y_2+\cdots+y_i=n-i \\ y_1\geq 0, y_2\geq\delta,\ldots,y_{i-1}\geq\delta, y_i<\delta}} \cdots \sum q^{y_2+2y_3+3y_4+\cdots+(i-1)y_i} \right\} \\ &= \sum_{i=2}^{\left[\frac{n+2\delta}{\delta+1}\right]} \left\{ \theta^{n-i}\left(\prod_{j=1}^{i}\left(1-\theta q^{j-1}\right)\right) \sum_{t=0}^{\min(\delta-1,n-i)} q^{(i-1)t} \sum_{\substack{y_1+y_2+\cdots+y_{i-1}=n-i-t \\ y_1\geq 0, y_2\geq\delta,\ldots,y_{i-1}\geq\delta}} \cdots \sum q^{y_2+2y_3+3y_4+\cdots+(i-2)y_{i-1}} \right\} \end{aligned}$$

From Lemma 1, pmf of T_δ can be written by

$$P\{T_\delta = n\} = \sum_{i=2}^{\left[\frac{n+2\delta}{\delta+1}\right]} \left\{ \theta^{n-i} \left(\prod_{j=1}^{i} \left(1-\theta q^{j-1}\right) \right) \sum_{t=0}^{\min(\delta-1,n-i)} q^{(i-1)t} B_q(i, n-i-t) \right\}.$$

This completes the proof.

The pmf of T_δ is given for different choices of θ, q and δ in Table 1 and Figs.1-3. From Fig. 1-3, it is concluded that pmf may be unimodal, bimodal or decreasing form. The distribution with pmf (2.1) will be called as $q\delta - Shock$ model.

TABLE 1. Pmf of T_δ for two different cases.

$\theta = 0.5,\ q = 0.6,\ \delta = 3$		$\theta = 0.7,\ q = 0.4,\ \delta = 5$	
n	$P(T_3 = n)$	n	$P(T_5 = n)$
2	0.3500	2	0.2160
3	0.2800	3	0.2117
4	0.1715	4	0.1651
5	0.0857	5	0.1203
6	0.0506	6	0.0855
7	0.0290	7	0.0599
8	0.0159	8	0.0422
9	0.0083	9	0.0297
10	0.0043	10	0.0208
≥ 11	0.0047	≥ 11	0.0488

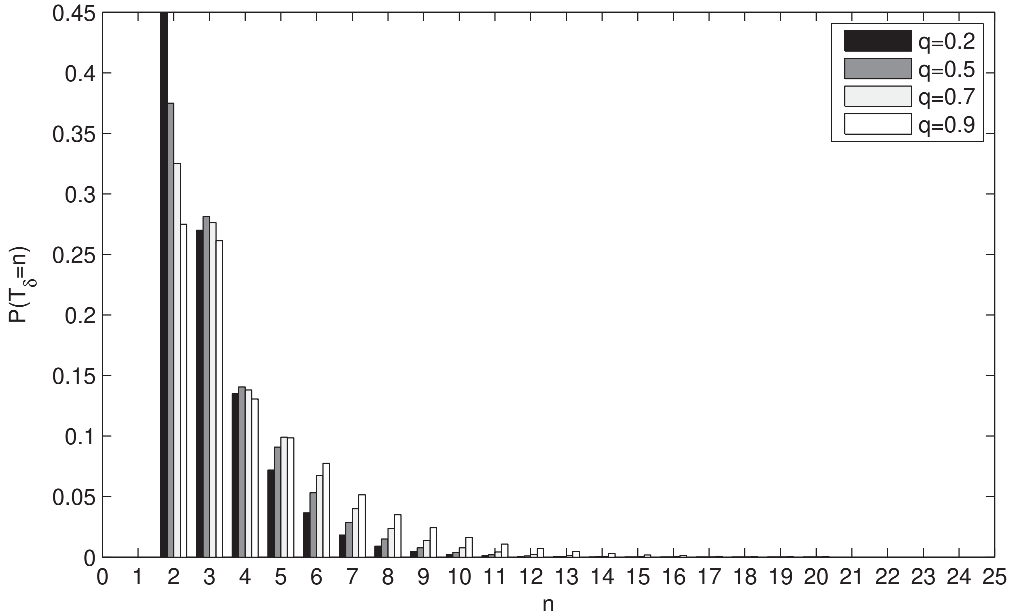

FIGURE 1. Pmf of T_δ for $\theta = 0.5, \delta = 4$ and some choices of q

REMARK 1. If $q = 1$, then

$$\sum_{t=0}^{\min(\delta-1,n-i)} B_q(i, n-i-t) = \binom{n-(i-2)\delta-1}{i-1} - \binom{n-(i-1)\delta-1}{i-1}$$

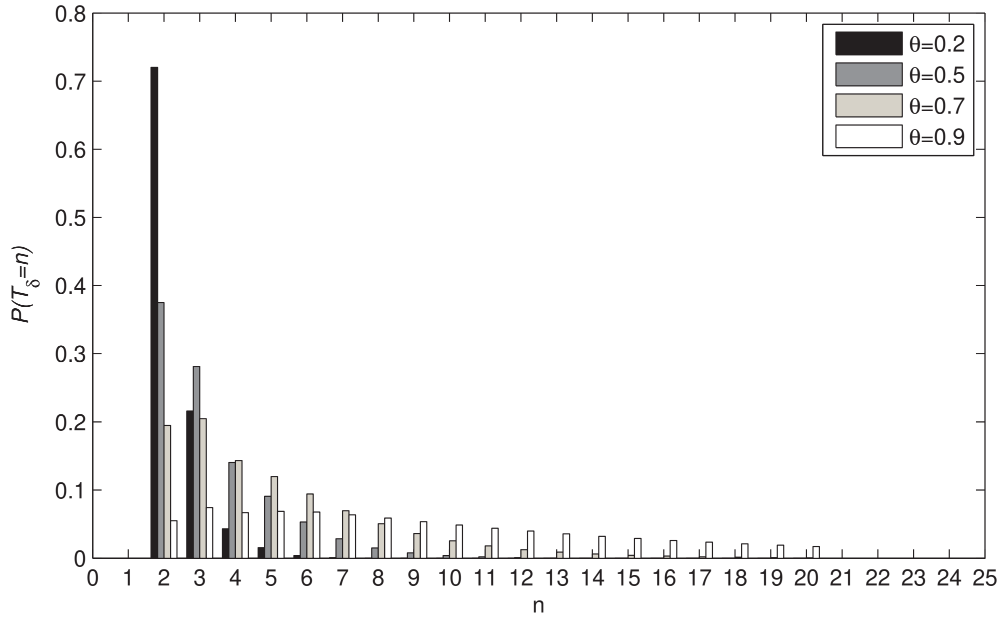

FIGURE 2. Pmf of T_δ for $q=0.3, \delta=4$ and some choices of θ

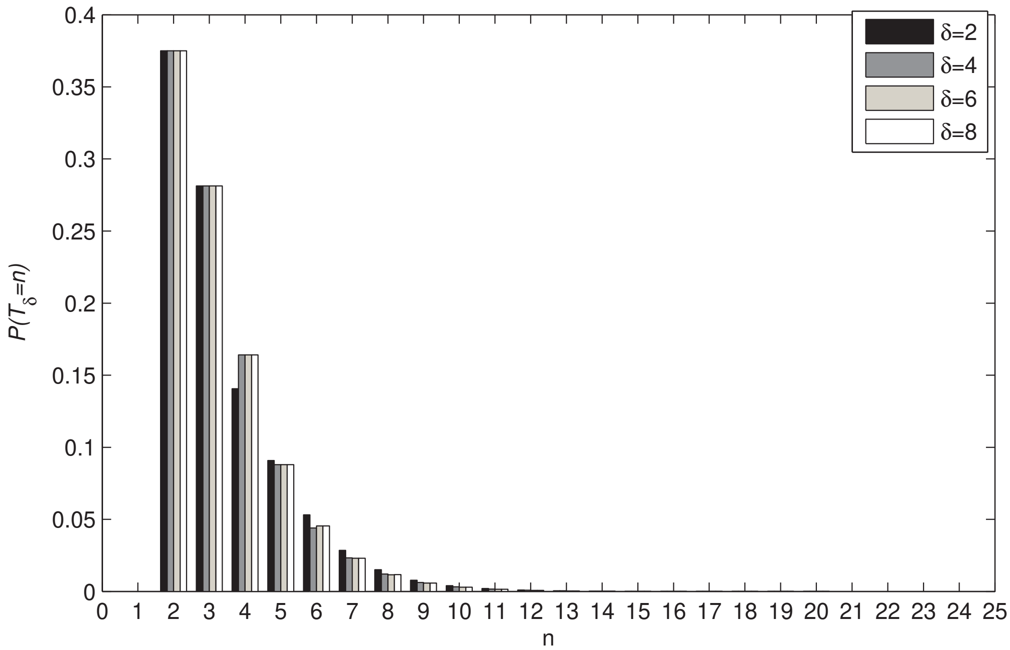

FIGURE 3. Pmf of T_δ for $q=0.5, \theta=0.5$ and some choices of δ

and

$$P\{T_\delta = n\} = \sum_{i=2}^{\left[\frac{n+2\delta}{\delta+1}\right]} \left[\binom{n-(i-2)\delta-1}{i-1} - \binom{n-(i-1)\delta-1}{i-1} \right] \theta^{n-i} (1-\theta)^i$$

which is result of Eryilmaz [9] for Binomial process with success probability $1-\theta$.

3. Probability generating function and moments

The pgf has interesting properties and can often greatly reduce the amount of hard work which is involved in analyzing a distribution. Since it is hard to calculate the first and second moments of T_δ from pmf given Eq. (2.1), these moments are obtained from pgf. Following theorem gives us pgf of T_δ.

THEOREM 2. *The pgf of T_δ is*

$$\psi(z)=\sum_{m=1}^{\infty}\frac{(1-\theta)\,z^2\left(\prod_{i=1}^{m-1}\frac{(1-\theta q^i)z\left(\theta q^i z\right)^\delta}{\left(\theta q^i\right)^\delta\left(1-\theta q^i z\right)}\right)(1-\theta q^m)\left(1-(\theta q^m z)^\delta\right)\left(\prod_{i=1}^{m-1}\left(\theta q^i\right)^\delta\right)}{(1-\theta z)(1-\theta q^m z)}.$$

PROOF. The pgf of T_δ can be written as

$$\psi(z)=E\left(z^{T_\delta}\right)=E\left(z^{X_0}\right)E\left(z^{\sum_{i=1}^{M}X_i}\right),$$

where

$$E\left(z^{X_0}\right)=\frac{(1-\theta)\,z}{1-\theta z}$$

and

$$\begin{aligned}
E\left(z^{\sum_{i=1}^{M}X_i}\right) &= \sum_{m=1}^{\infty}E\left(z^{\sum_{i=1}^{M}X_i}\mid M=m\right)P(M=m)\\
&= \sum_{m=1}^{\infty}\left(\prod_{i=1}^{m-1}E\left(z^{X_i}\mid X_i>\delta\right)\right)E\left(z^{X_m}\mid X_m\le\delta\right)P(M=m)\\
&= \sum_{m=1}^{\infty}\left(\prod_{i=1}^{m-1}\frac{(1-\theta q^i)\,z\,(\theta q^i z)^\delta}{(\theta q^i)^\delta\,(1-\theta q^i z)}\right)\frac{(1-\theta q^m)\,z\left(1-(\theta q^m z)^\delta\right)}{(1-\theta q^m z)\left(1-(\theta q^m)^\delta\right)}\\
&\quad\times\left(\prod_{i=1}^{m-1}\left(\theta q^i\right)^\delta\right)\left(1-(\theta q^m)^\delta\right).
\end{aligned}$$

The mean and variance of random variable T_δ given by using following identities

$$\begin{aligned}
E(T_\delta) &= \psi'(1),\\
Var(T_\delta) &= \psi''(1)+\psi'(1)-\left[\psi'(1)\right]^2.
\end{aligned}$$

In the following, mean of T_δ is given with explicit expression.

COROLLARY 1. *The mean of T_δ is*

$$E(T_\delta)=\sum_{m=1}^{\infty}\left\{\frac{\left(\prod_{i=1}^{m-1}\left(\theta q^i\right)^\delta\right)\left(A_1\sum_{j=1}^{m-1}\frac{\delta+1-\theta q^j\delta}{1-\theta q^i}+A_2\right)}{(1-\theta q^m)(1-\theta)}\right\},$$

where

$$A_1=(1-\theta)(1-\theta q^m)\left(1-(\theta q^m)^\delta\right),$$

and

$$A_2=2-\theta(1+q^m)-(\theta q^m)^\delta\left\{(1-\theta q^m)(1-\delta(1-\theta))+(1-\theta)\right\}.$$

The expected values of T_δ is given in Table 2 and Fig. 4 for different choices of θ, q and δ. Variance of T_δ is not given here since the formula is too large. However, we give the variance values for different points of θ, q and δ in Table 2 and Fig. 5 in order to observe how the parameters θ, q and δ affect the deviation in T_δ. From these tables and figs. it is conclude that $E(T_\delta)$ and $Var(T_\delta)$ are increasing in θ and q but decreasing in δ.

TABLE 2. Expected values and variances of T_δ.

(θ, q)		$E(T_\delta)$	$Var(T_\delta)$	(θ, q)		$E(T_\delta)$	$Var(T_\delta)$
$(0.2, 0.2)$	$\delta = 2$	2.2932	0.3640	$(0.2, 0.5)$	$\delta = 2$	2.3716	0.4899
	$\delta = 3$	2.2917	0.3564		$\delta = 3$	2.3621	0.4434
	$\delta = 4$	2.2916	0.3559		$\delta = 4$	2.3612	0.4369
$(0.5, 0.6)$	$\delta = 2$	3.5416	3.2353	$(0.7, 0.9)$	$\delta = 2$	7.2888	25.8703
	$\delta = 3$	3.4616	2.8589		$\delta = 3$	6.7188	19.8635
	$\delta = 4$	3.4384	2.7056		$\delta = 4$	6.4352	17.3293

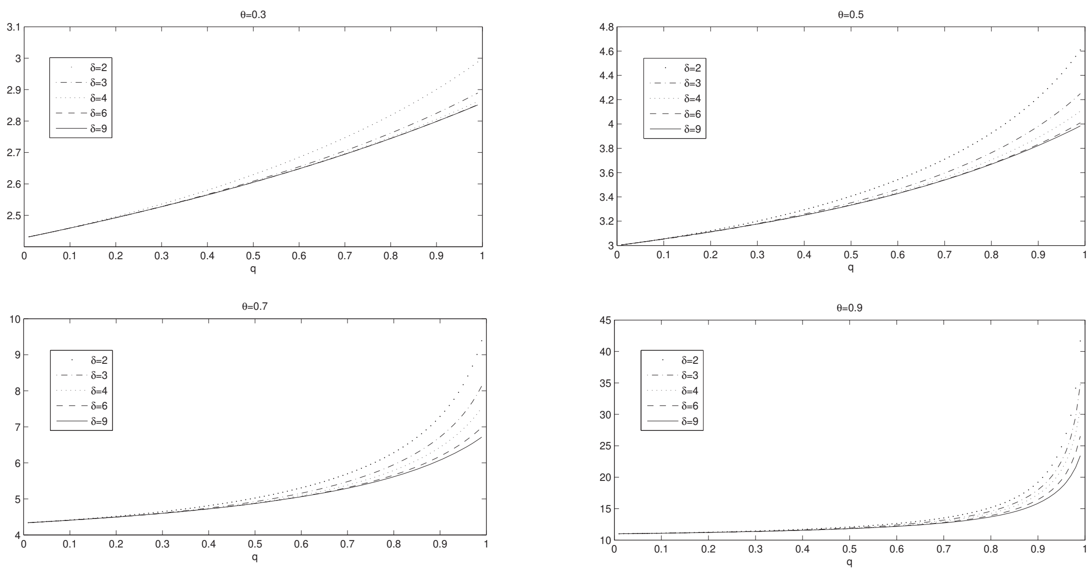

FIGURE 4. Expected value of T_δ for different points of θ, q and δ.

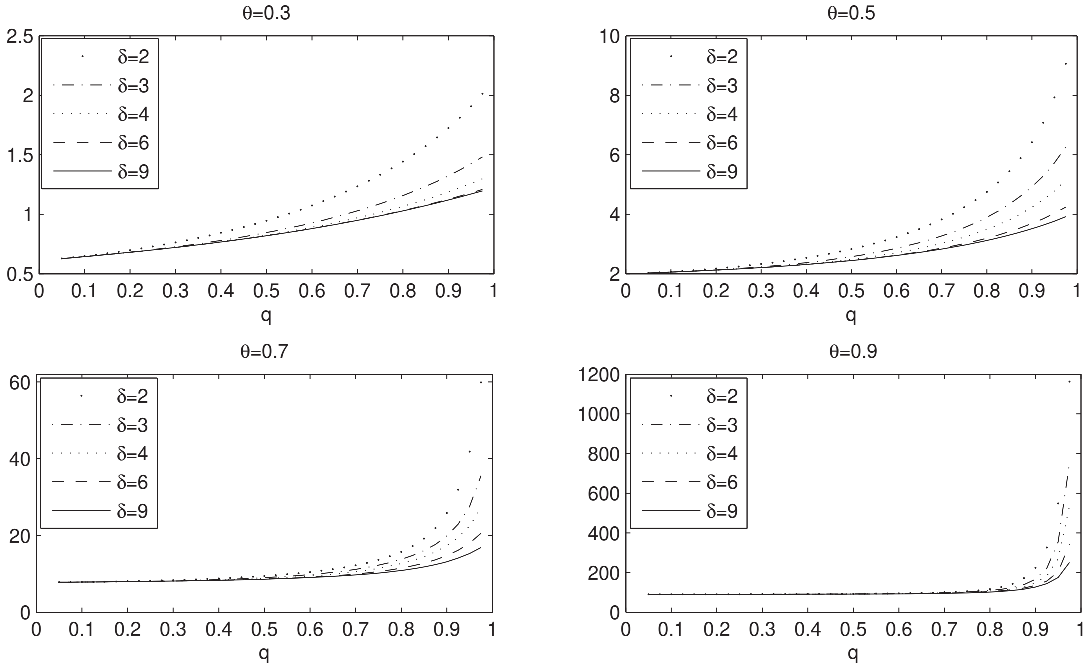

Figure 5. Variance of T_δ for different points of θ, q and δ.

4. Proportion estimates

Khan et al. [12] introduced a method of proportions to estimate the discrete Weibull distribution parameters. In this section, we used their method to estimate the parameters of introduced distribution of T_δ.

Let $T_{\delta,1}, T_{\delta,2}, \ldots, T_{\delta,n}$ be a sample from pmf (2.1). For $i = 1, 2, \ldots, n$, define the indicator functions $\nu_1(\cdot)$ and $\nu_2(\cdot)$ as

$$\nu_1(T_{\delta,i}) = \begin{cases} 1, & T_{\delta,i} = 2 \\ 0, & T_{\delta,i} > 2 \end{cases}$$

and

$$\nu_2(T_{\delta,i}) = \begin{cases} 1, & T_{\delta,i} = 3 \\ 0, & T_{\delta,i} \neq 3 \end{cases}.$$

It is easily seen that $Y = \frac{1}{n}\sum_{i=1}^{n} \nu_1(T_{\delta,i})$ and $Z = \frac{1}{n}\sum_{i=1}^{n} \nu_2(T_{\delta,i})$ denote the proportion of $2'$s and $3'$s in the sample, respectively. Then, the proportions Y and Z are unbiased and consistent estimates of the probabilities

$$f(2) = (1-\theta)(1-q\theta)$$

and

$$f(3) = \theta(1-\theta)(1-q\theta),$$

respectively. Hence the proportion estimates of θ and q are achieved for $\delta = 1$ by solving equations

$$\begin{aligned} (1-\theta)(1-q\theta) &= Y \\ \theta(1-\theta)(1-q\theta) &= Z \end{aligned}$$

simultaneously. Then proportion estimates of θ and q are obtained, respectively, by

$$\widehat{\theta} = \frac{Z}{Y} \text{ and } \widehat{q} = \frac{(Y^2 - Y + Z)\,Y}{Z\,(Z-Y)}.$$

Second version of the proportion estimates of θ and q are achieved for $\delta > 1$ by solving equations

$$\begin{aligned}(1-\theta)(1-q\theta) &= Y,\\ \theta(1-\theta)(1-q\theta)+(1-\theta)\theta q(1-q\theta) &= Z,\end{aligned}$$

simultaneously. A numerical method such as Newton-Raphson should be used for this aim.

Some simulation studies are performed to see the performances (mean squares errors (MSEs) and bias) of the proportion estimates given in this section. Simulation is done for different values of n, θ, q, δ and obtained average biases and MSEs of estimates based on 10000 repetitions are given in Table 3 and Table 4 respectively. It is observed that MSEs and Bias of estimates decrease when n increases for all cases discussed here.

Table 3. Average biases of proportion estimates

		$\delta = 1$		$\delta > 1$	
(θ, q)	n	$\hat{q}$	$\hat{\theta}$	$\hat{q}$	$\hat{\theta}$
$(0.5, 0.3)$	100	0.0489	−0.0144	−0.1274	0.0107
	200	0.1216	−0.0060	−0.1179	0.0169
	300	0.0678	−0.0043	−0.1159	0.0192
	400	0.0468	−0.0043	−0.1113	0.0201
	500	0.0362	−0.0010	−0.0939	0.0177
	1000	0.0170	−0.0006	−0.0657	0.0134
$(0.3, 0.5)$	100	−0.0373	−0.0047	−0.0198	−0.0220
	200	−0.0165	−0.0036	−0.0394	−0.0119
	300	−0.0124	−0.0016	−0.0567	−0.0060
	400	−0.0064	−0.0015	−0.0605	−0.0037
	500	−0.0029	−0.0011	−0.0616	−0.0016
	1000	−0.0009	−0.0008	−0.0815	0.0043
$(0.5, 0.5)$	100	0.0543	−0.0178	−0.0269	−0.0227
	200	0.0177	−0.0065	−0.0515	−0.0069
	300	0.0057	−0.0033	−0.0672	0.0011
	400	0.0036	−0.0028	−0.0658	0.0044
	500	0.0020	−0.0019	−0.0740	0.0073
	1000	0.0004	−0.0009	−0.0771	0.0118
$(0.1, 0.7)$	100	−0.1309	−0.0010	0.2090	−0.0330
	200	−0.0516	−0.0002	0.1765	−0.0263
	300	−0.0378	−0.0004	0.1573	−0.0227
	400	−0.0255	−0.0002	0.1510	−0.0208
	500	−0.0183	−0.0002	0.1383	−0.0195
	1000	−0.0112	−0.0001	0.0930	−0.0139

Table 4. Average MSEs of proportion estimates

		$\delta = 1$		$\delta > 1$	
(θ, q)	n	$\hat{q}$	$\hat{\theta}$	$\hat{q}$	$\hat{\theta}$
$(0.5, 0.3)$	100	0.5905	0.0184	0.2482	0.0070
	200	0.1216	0.0091	0.1913	0.0059
	300	0.0678	0.0063	0.1627	0.0053
	400	0.0468	0.0046	0.1446	0.0050
	500	0.0362	0.0036	0.1206	0.0044
	1000	0.0170	0.0018	0.0759	0.0030
$(0.3, 0.5)$	100	0.1459	0.0071	0.2502	0.0060
	200	0.0628	0.0035	0.2062	0.0044
	300	0.0377	0.0022	0.1830	0.0037
	400	0.0274	0.0017	0.1681	0.0034
	500	0.0228	0.0014	0.1555	0.0031
	1000	0.0107	0.0007	0.1280	0.0025
$(0.5, 0.5)$	100	0.8287	0.0250	0.2191	0.0077
	200	0.1591	0.0107	0.1733	0.0054
	300	0.1197	0.0068	0.1516	0.0049
	400	0.0551	0.0050	0.1367	0.0046
	500	0.0438	0.0041	0.1279	0.0044
	1000	0.0208	0.0020	0.0990	0.0037
$(0.1, 0.7)$	100	0.4452	0.0013	0.3931	0.0038
	200	0.1198	0.0007	0.3277	0.0027
	300	0.0751	0.0004	0.2842	0.0021
	400	0.0517	0.0003	0.2681	0.0019
	500	0.0389	0.0003	0.2496	0.0017
	1000	0.0187	0.0001	0.1979	0.0011

5. A numerical example

In this section, it is showed that $q\delta-$ *Shock* distribution with pmf (2.1) can be used modelling real discrete data. Here, we consider the Phyo ([13]) records of the total number of decayed teeth (x_i) between the four deciduous molars on a sample of 100 children aged 10 and 11 years. The data are tabulated in Table 5, where o_i denote the observed frequency of observed value x_i. Since the random variable T_δ in $q\delta-$ *Shock* model can take the values greater or equal than 2, $X_i + 2$ will be used instead of X_i for the $q\delta-$ *Shock* model in χ^2 goodness of fit analysis. In Table 6, we

Table 5. Total number of carious teeth

x_i	0	1	2	3	4	Total
o_i	64	17	10	6	3	100

also reported the χ^2 goodness of fit test with proportion estimates of distribution parameters for Discrete Burr (DB) [14], Discrete Pareto (DP) [14], Discrete Weibull (DW) [15], Geometric (G), Poisson (P), $q\delta-$ *Shock* for $\delta = 1$ and $q\delta - $ *Shock* for $\delta = 2$ models. The proportion estimates of

(q,β) is $(0.4838, 0.2656)$ for $q\delta-$ *Shock* for $\delta = 1$. In addition, the proportion estimates of (q,β) is $(0.7617, 0.2265)$ for $q\delta-$ *Shock* for $\delta = 2$. Plug in estimation of expected values are calculated by 2.6546 and 2.5362 for $\delta = 1$ and $\delta = 2$ respectively. If models are descendingly ordered according to $p-$values, the first three models are determined as Discrete Weibull, $q\delta-$ *Shock* for $\delta = 1$ and Discrete Pareto. Fig. 6 shows also that the model $q\delta-$ *Shock* for $\delta = 1$ well fit the data. This results mean that $q\delta-$ *Shock* $(\delta = 1)$ model given with this paper is a competitive model for modelling some real discrete data.

Table 6. χ^2 goodness of fit test for some discrete distributions

		Expected frequencies						
x_i	o_i	DB	DW	DP	G	P	$q\delta$-Shock for $\delta=1$	$q\delta$-Shock for $\delta=2$
0	64	64	64	64	64	64	64.0031	64.0052
1	17	17	17	16.1959	23.0400	28.5624	16.9992	25.5397
2	10	6.9645	7.9926	6.8441	8.2960	6.3735	12.2279	5.7847
3	6	3.5924	4.2840	3.6325	2.9860	0.9481	4.2388	2.9650
4	3	2.1226	2.4664	2.1980	1.0750	0.1058	1.7182	1.1745
	χ^2	3.2294	1.3070	3.3308	8.4238	112.845	2.0938	11.8709
	p-value	0.1921	0.5202	0.3434	0.0380	0.0000	0.3510	0.0000

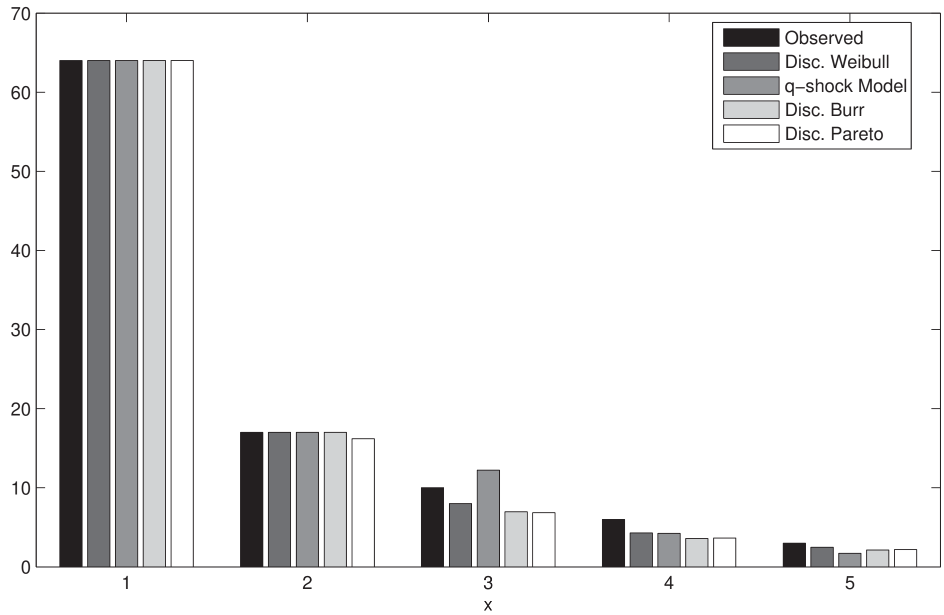

Figure 6. Empirical and fitted distributions based on the real data set

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