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ANALYZING THE FRACTAL STRUCTURE OF STOCK RETURNS: EVIDENCE FROM ISTANBUL STOCK EXCHANGE

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Abstract: Efficient Market Hypothesis states that all new information is reflected in the market price fully and immediately. Security returns are essentially unpredictable since they follow a random walk. Therefore the impact of the new information is essentially unpredictable; it is as likely to be negative as positive.
Financial asset returns are often modeled with a series of small, normally distributed changes. Brownian motion asserts the independence of the changes but there are patterns or trends in capital market returns and they persist over time. Therefore security returns are not fully random.
This paper applies Hurst's R/S (Rescaled Range) analysis to XU030 and XU100 index within different time horizons. The analysis proceeded from two basic principles: dependence of each period in time series data and fractional Brownian motion of time series. The persistence behaviour of Istanbul Stock Exchange is investigated. The results show that each series taken into consideration exhibits a biased process characteristic of fractal Brownian motion.

Keywords: Brownian Motion, Fractal Brownian Motion, Hurst Exponent, Rescaled Range Analysis

HİSSE SENEDİ GETİRİLERİNDEKİ FRAKTAL YAPININ İNCELENMESİ: IMKB'DE BİR UYGULAMA

Özet: Etkin Pazar Hipotezi'ne göre güncel bilgiler hemen ve eksiksiz olarak pazara yansır. Esasen menkul kıymet getirileri rassal yürüyüş hipotezine uygun hareket ettiklerinden öngürülemezdir. Dolayısıyla yeni bilginin etkisi de öngörülemezdir; yani pozitif olabileceği gibi negatif yönde de olabilir.
Finansal varlık getirileri genellikle birbirinden bağımsız ve normal dağılan hata terimleri ile modellenmektedir. Brownian hareket, değişikliğin bağımsız olduğunu ancak yine de sermaye piyasaları getirilerinde bir düzen ya da eğilimin var olduğunu ve bu düzenin zaman içinde tekrarlandığını ileri sürer.
Bu çalışmada, Hurst [1,2] tarafından geliştirilen R/S analizi İstanbul Menkul Kıymetler Borsası XU030 ve XU100 endeksine farklı zaman dilimlerinde uygulanmaktadır. Analizde her bir dönemsel getirinin bağımsızlığı ve fraktal Brownian harekete uygunluğu gibi iki temel prensip incelenmektedir. Sonuç olarak İstanbul Menkul Kıymetler Borsası'nda işlem gören hisse senetleri getirilerinin Brownian harekete uygunluğu incelenmiştir. Sonuçlar, dikkate alınan her bir serinin fraktal Brownian hareket karakteri gösterdiğini ispatlamaktadır.

Anahtar Kelimeler: Brownian Hareket, Fraktal Brownian Hareket, Hurst Bileşeni, R/S Analizi

I. INTRODUCTION

It is well known that economic theories have traditionally been dominated by a linear modeling, based on concepts like Gaussian (normal) distributions and random walks, generating the so called Capital Market Theory built on the assumption of normally distributed returns and the Efficient Market Hypothesis (EMH), by which the markets follow a random walk.

But several developments in mathematics and the natural sciences over the last few decades yield important insights about the EMH and the behaviour of security returns. Furthermore, it is proven within several surveys that, the behaviour of the financial markets exhibit contrary results as behaving in a nonlinear fashion. While EMH and the random walk assumes the serial independence of the increments and normally distribution of the error terms, cycles and patterns reveal in the financial data indicating the trend in one direction.

R/S analysis first proposed by Hurst [1,2] and subsequently refined and applied to economic time series in a study by Mandelbrot [3]. From the analysis the most important parameter in the R/S analysis, namely Hurst exponent, was derived. Later on R/S analysis is modified [4] to handle the long-term dependence in stock prices.

In the present paper, Hurst's R/S (Rescaled Range) analysis is applied to XU030 and XU100 index within different time horizons. The informational efficiency of Istanbul Stock Exchange (ISE) is investigating by calculating the Hurst component using the XU030 and XU100 indexes. Using Rescaled Range-Hurst regression analysis long-term persistent dependence of Turkish market is investigated.

II. BROWNIAN MOTION AND FRACTAL BROWNIAN MOTION

Brownian Motion (BM) denoted by W for Wiener process is a stochastic process (in R^d) with continuous sample paths such that,

Condition1. $P(W_0=0)=1$;

Condition2. For any times $s > t$, W_s-W_t is normally distributed in R^d with zero mean and covariance matrix $(s-t)I$;

Condition3. for all times with $0<t_1< t_2<\ldots <t_n<\infty$, the random variables $W(t_0), W(t_1)- W(t_0), \ldots, W(t_n)-W(t_{n-1})$ are independently distributed.

The most important characteristic is the serial independence of the increments. This embodies some efficiency criterion: past changes do not help to predict future changes. Notice also that the variance is increasing (linearly) with time. This captures the obvious idea that it is harder to predict the whereabouts of the stock price or the particle farther out in the future.

The BM increment dW_t represents the unpredictable price change over the next instant due to news. Its marginal distribution is normal (Gaussian): $dW_t \sim N(0,dt)$. Furthermore, BM possesses a number of amazing properties that one may not suspect just from the definition. For instance, it is of unbounded variation, i.e. it travels an infinite distance over any time horizon, no matter how small. It is also nowhere differentiable and has some fractal-like properties.

Shaping BM with functions may be powerful to analyze the fractal behaviour. It exhibits the behaviour of a function drawn by Newtonian calculus. It asserts that when we focus on a function at a fixed point at narrower intervals, a straight line is obtained. This straight line can be regarded as the tangent line drawn at that point or calculating the first order Taylor series expansion. Since, a straight line can be obtained by zooming on differentiable functions; they can be built from straight-line segments. Therefore, they do not exhibit fractal property.

The next figure given in [5] queries the same behaviour of BM. Zooming in progressively, BM does not produce a straight line. Scaling the given time series each new graph exhibits the same behaviour as the BM. The self-similarity or the fractal property of BM can be seen from the Figure 2. Since a straight line cannot be obtained by zooming in, BM is continuous everywhere but not differentiable.

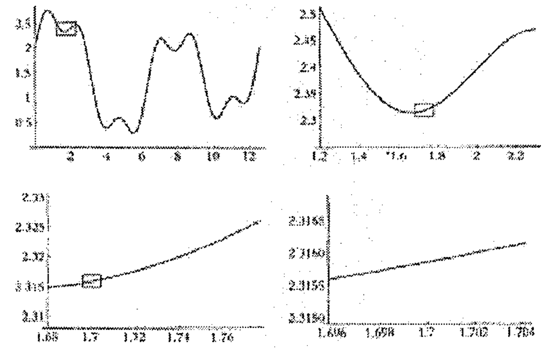


Figure.1. Progressive Magnification Around The Point 1.7

The lack of obtaining a straight line can be regarded as the lack of taking the derivative. BM is not a smooth function; therefore it is full of corners like the absolute value function has at the point zero. Since infinite number of tangent lines can be drawn at the point zero to the absolute value function, it is impossible to obtain a straight line working with BM.

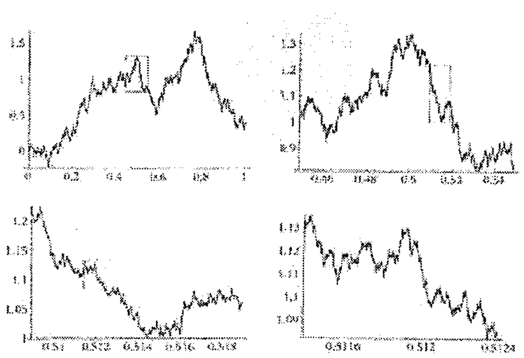


Figure.2. 'Zooming In' On Brownian Motion

Figure.2, given in [5] gives insights about the random behaviour of the stock price movements and the underlying fractal structure. Fractal structure can be regarded as the patterns or trends in the stock returns. Return behaviour in a period influence the subsequent ones, which contradicts the third condition of BM that embodies the independence of the increments.

Fractal Brownian motion (FBM) can be taken into consideration to eliminate this contradictory. FBM is a generalization of the BM by removing the last condition. So it is more likely to accept the behaviour of the stock returns as FBM.

III. Rescaled Range (R/S) Analysis And Hurst Exponent

From the second condition of Brownian motion, a generalized formula of the Brownian motion can be extracted. The conditions of Brownian motion imply the following equation where P denotes probability and 'a' is a real number such that $a \in \Re$.

$$P(X_{t+\Delta t} - X_t \le a) = 2\pi^{-1/2} \Delta t^{-H} \int_{-\infty}^{a} \exp(\frac{-x^2}{2(\Delta t)^{2H}}) dx \quad (1)$$

The distribution of the increments specified by fractal Brownian motion cannot have independent increments except the case of H=0.5. Therefore specifying H=0.5, one can easily obtain Brownian motion or random walk in other terms. As the definition suggests that the increments are normally distributed with mean zero, the following equation can be derived.

$$\begin{aligned} &E[(X_{t+\Delta t} - X_t)(X_t - X_0)] \\ &= 0.5 \left((t+\Delta t)^{2H} - t^{2H} - (\Delta t)^{2H}\right) \end{aligned} \quad (2)$$

The value of H can range between zero and one [6]. If H=0.5, the value of the equation (2) becomes zero implying independent increments or random walk in other terms. But if H is different 0.5 the value of the equation (2) is different from zero, thus the increments are dependent.

Specifically, if H is between zero and 0.5, anti-persistent behaviour exist which means there is a negative dependence between the increments. If a trend has been positive in the last period, it is more likely to be negative than positive in the next period [7]. But the H value greater than 0.5 indicates that some periods above or below the theoretical mean are extraordinarily long. If the value of H is greater than 0.5, the time series exhibit persistent behaviour. If the trend has been positive in the last observed period, it is likely to be positive in the next period. The level of persistence is measured by how far H is above 0.5.

To estimate the Hurst exponent, the range of the accumulated deviations from the average level has to be calculated first. This can be obtained by calculating the difference between the maximum and minimum cumulative deviations over N periods [8].

$$R_N = \underset{1 \le t \le N}{\mathrm{Max}}(X_{t,N}) - \underset{1 \le t \le N}{\mathrm{Min}}(X_{t,N}) \quad (3)$$

The range depends on the time period considered. It is expected that the value of R increase as N increases. In order to standardize the observations, R has to be divided by the standard deviation. Hurst found that the following empirical law could estimate R/S.

$$R/S = (N)^H \quad (4)$$

Using a logarithmic transformation, the equation becomes as follows. H can be estimated by performing an ordinary-least-squares regression between log(N) and log(R/S).

$$\mathrm{Log}(R/S) = H \log(N) \quad (5)$$

For very long series, or in other words by increasing N, H tends to converge to value 0.5. Therefore, the regression referred to above has to be performed on the data prior to convergence of H to 0.5. Also, the correlation between periods can be calculated as follows [9].

$$C_N = 2^{(2H-1)} - 1 \quad (6)$$

As a result of Brownian motion (H=0.5) C_N can be calculated as zero. If the time series exhibit persistent behaviour (H>0.5) C_N takes positive values and anti-persistent time series has negative correlation.

IV. EMPIRICAL RESULTS

This paper applies Hurst's R/S (Rescaled Range) analysis to XU030 and XU100 index returns within different time horizons. In order to investigate the persistence behaviour of Istanbul Stock Exchange, XU030 and XU100 indexes between 01.01.1997 and 31.12.2005 are taken into consideration and R/S analysis is applied to monthly data. Daily observations of the U.S. dollar and German mark against Turkish lira between July 2, 1981 and December 29, 1995 in [10] and found that the Turkish foreign exchange is anti-persistent or mean reverting. Findings of this paper are consistent with the ones obtained by [10]. Log returns for each time series is obtained first and R/S analysis is modelled on a spreadsheet. The VBA code implemented for the model is given in the Appendix.

Table.1 shows the regression results for XU030 and XU100 index returns. For XU030 series, H was estimated to be 0.592865 within a standard error of 0.003504. The high R square value (0.996346) and the low level of standard error (0.059673) illustrate the goodness of fit. Since the Hurst exponent is greater than 0.5, XU030 index returns exhibit persistent behaviour. If the trend has been positive in the last observed period, it is likely to be positive in the next period. Also, the correlation between periods can be calculated as 0.137392. The correlation coefficient implies that 13.7392 percent of the XU030 returns are influenced by the past returns.

For XU100 index returns, H was estimated to be 0.589913 within a standard error of the coefficient of 0.003464. Since the parameters are close to the ones obtained from XU030, similar explanations can be given.

The higher R square value (0.996393) and the low level of standard error (0.058415) illustrate the goodness of fit. Since the Hurst exponent is greater than 0.5, XU100 index returns exhibit persistent behaviour. Also, the correlation between periods can be calculated as 0.132747. The level of persistence can be examined by the correlation coefficient. The correlation coefficient implies that 13.2747 percent of the XU100 returns are influenced by the past returns. This value is smaller than the one obtained from XU030. Therefore XU100 is said to be closer to the unbiased random walk.

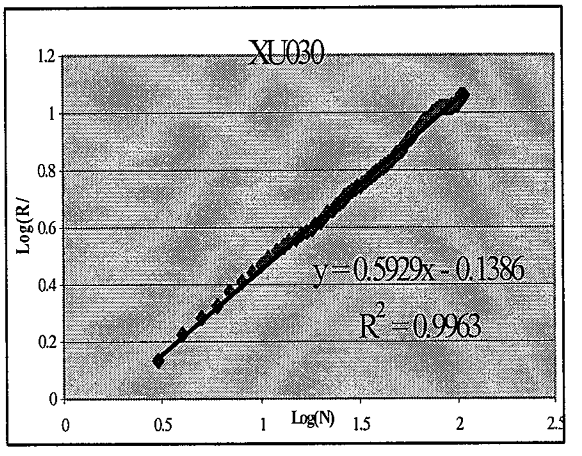
Table.1. Regression Results for XU030 and XU100

Parameters	XU030	XU100
R-Square	0.996346	0.996393
Standard Error	0.059673	0.058415
Constant	-0.13862	-0.13140
Standard Error of coef.	0.005889	0.005822
X coefficient(H)	0.592865	0.589913
Standard Error of coef.	0.003504	0.003464
C_N	0.137392	0.132747

Figure.3 illustrates the result of the regression for XU030 and XU100. The solid line shows an unbiased random walk within an H value of 0.5 whereas the actual returns deviate slightly from the regression line. The persistence in XU030 and XU100 index returns suggests difficulty in forecasting this market over the long term but not impossible.

Lower level of persistence for XU100 can be seen from the Hurst coefficient. Hurst coefficient for the series XU030 and XU100 do not seem so different. Another way to investigate the persistence behaviour of the two indices is to divide the series in two groups. The investment horizon can be divided in two as 01.01.1997-31.12.2000 and 01.01.2001-31.12.2005 in order to investigate the effect of the economic crisis on the Hurst component and the changes in the patterns of market returns.

Table.2 shows the regression results for XU030 and XU100 index returns separately for 01.01.1997-01.01.2001 and 01.01.2001-31.12.2005 horizons. Therefore, the time scales are consisting of 50 and 60 monthly returns. Main idea of dividing the series in two is to compare the behaviour of the market before and after the economic crisis. Changes in the pattern of the market can be examined by the change in the Hurst exponents of the naïve series.



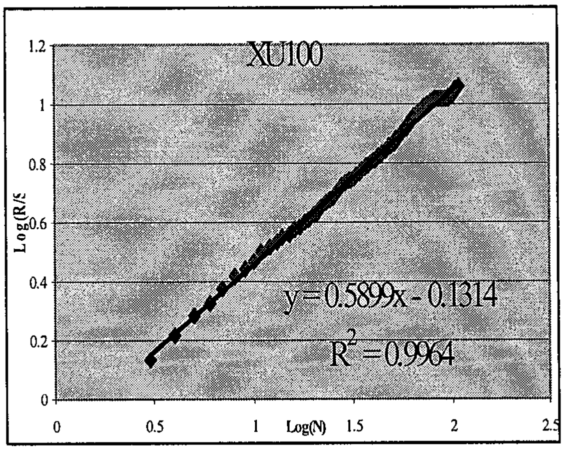


Figure.3. Rescaled Range Analysis For XU030 And XU100 Respectively

Table.2. Regression Results For XU030 And XU100 Within Different Time Horizons

Parameters	01.01.1997-31.12.2000		01.01.2001-31.12.2005	
	XU030	XU100	XU030	XU100
R-square	0.994814	0.995561	0.977240	0.969745
Standard Error	0.014430	0.012477	0.023231	0.022648
Constant	-0.156230	-0.158316	-0.021191	0.009761
Standard Error of coef.	0.013868	0.008956	0.013868	0.016257
X coefficient(H)	0.637407	0.645935	0.469126	0.443248
Standard Error of coef.	0.006861	0.006503	0.009654	0.011803
C_N	0.209838	0.224226	-0.041897	-0.075660

For the series XU030 within the first period, H was estimated to be 0.637407 within a standard error of 0.006861. The high R square value (0.994814) and the low level of standard error (0.014430) illustrate the goodness of fit. Since the Hurst exponent is greater than 0.5, XU030 index returns exhibit persistent behaviour. Compare to value calculated in the first step, H value increases. The strength of the persistence can be measured by how far the H coefficient is above 0.5. Therefore, the level of persistence is higher in the first period. Also, the correlation between periods is calculated as 0.209838. The correlation coefficient implies that 20.9838 percent of the XU030 returns are influenced by the past returns. Due to the increase in the correlation coefficient, strength of the persistence behaviour in the market can be asserted.

Contradictorily, in the second period the pattern changes dramatically. H value takes value of 0.469126, which is below 0.5. The market exhibits anti-persistent or mean-reverting behaviour in the second period, which means there is a negative dependence between the increments. Since the correlation coefficient takes negative value (-0.041897), if a trend has been positive in the last period, it is more likely to be negative than positive in the next period. Again, the high R squared value (0.977240) and the low level of standard error (0.023231) for the second period supports the claim.

Similar explanations can be done for the XU100 series. The economic crisis affects the pattern of the market dramatically, from persistent to mean reverting.

V. SUMMARY AND CONCLUSIONS

The primary contribution of this paper is that the returns in the Istanbul Stock Exchange market do not seem consistent with the implications of random walk hypothesis. Since higher the Hurst exponent, the stronger the persistence in the time series, XU100 index returns seem to be more random than the XU030. Also, the correlation coefficients support this claim.

The results of the R/S analysis show that the stock market returns in the Istanbul Stock Exchange are persistent before the economic crisis. Parallel to the economic reforms, the pattern in the market turns out to be mean reverting or anti-persistent. Long memory effects and the structure of Fractal Brownian Motion can be observed in the market. Also it seems likely that the stock market returns can be modeled using deterministic chaos.

Similar work can be done based on international stock indices, currencies, commodity prices and the Turkish foreign exchange.

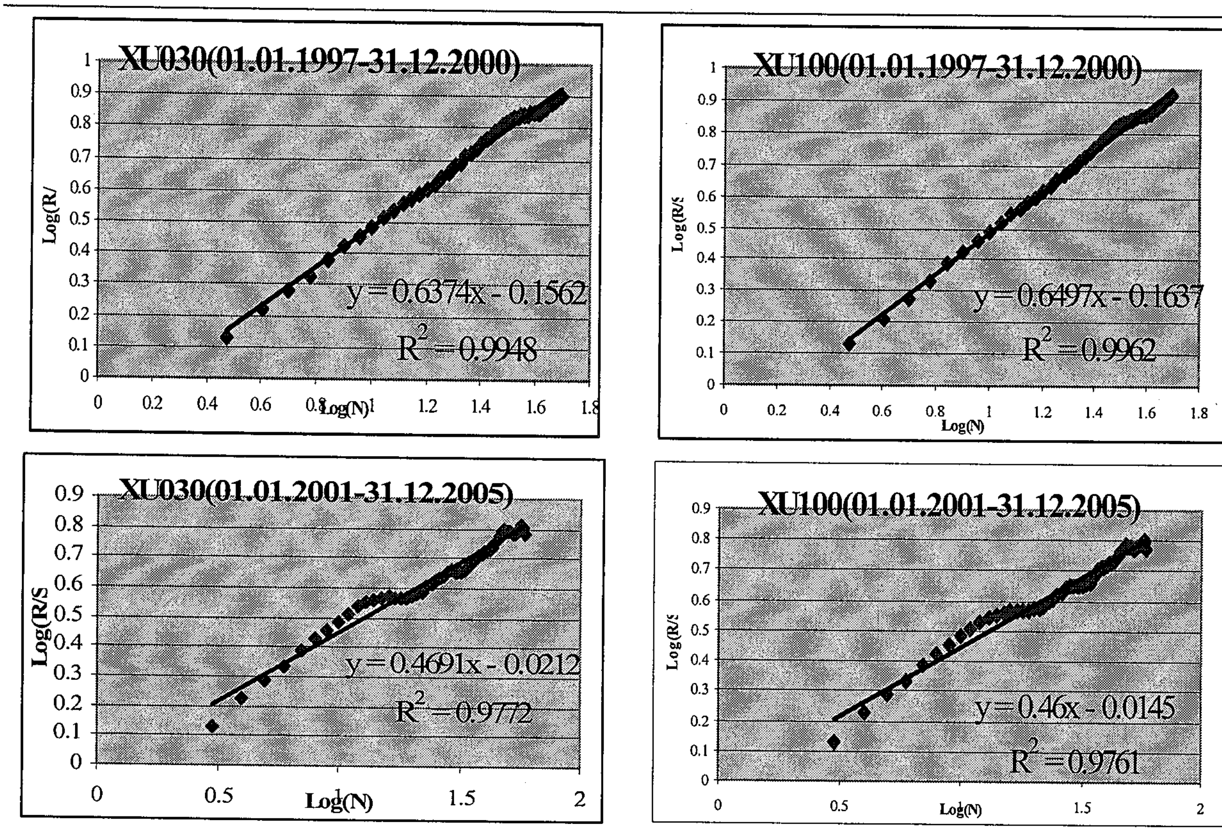


Figure.4. Rescaled Range Analysis For XU030 And XU100 Within Different Time Horizons

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Appendix: VBA Code for Rescaled Range Analysis

```
Option Base 1
Sub Hurst()
   Dim Data()
   Dim Array1()
   Dim Array2()
   Dim Mean
   Dim Result()
   Dim NoOfDataPoints As Integer
   Dim NoOfPlottedPoints As Integer
   Dim PlottedPointNo As Integer
   Dim NoOfPeriods As Integer
   Dim PeriodNo As Integer
   Dim N As Integer
   Dim i As Integer
   Dim m As Integer
   Dim logten
   Dim R
   Dim S
   Dim RS
   Dim SumSquared
   logten = Log(10)
   Worksheets("Data").Range("C3").Value = Null
   Worksheets("Data").Range("D:D").Value = Null
   Worksheets("Data").Range("E:E").Value = Null
   NoOfDataPoints = Worksheets("Data"). _
                                  Range("C1").Value
ReDim Data(NoOfDataPoints)
i = 1
counter = 1
Do While counter <= NoOfDataPoints
     Set curCell = Worksheets("Data").Cells(i, 1)
If    Application.WorksheetFunction.IsNumber( _
        curCell.Value)  Then
       Data(counter) = curCell.Value
       counter = counter + 1
End If
i = i + 1
Loop
        NoOfPlottedPoints = NoOfDataPoints - 2
        ReDim Result(NoOfPlottedPoints, 2)
For N = 3 To NoOfDataPoints
     totalR = 0
     totalS = 0
     NoOfPeriods = NoOfDataPoints - N + 1
        For PeriodNo = 1 To NoOfPeriods
        ReDim Array1(N)
                ReDim Array2(N)
        For i = 1 To N
           Array1(i) = Data((PeriodNo - 1) + i)
                 Array2(i) = 0
              Next i
              Summ = 0
                SumSquared = 0
                For i = 1 To N
                        Summ = Summ + Array1(i)
SumSquared = SumSquared + ((Array1(i)) * (Array1(i)))
Next i
Mean = Summ / N
S = Sqr((SumSquared - (Summ * Summ) / N) / N)
For i = 1 To N
Array1(i) = Array1(i) – Mean
Next i
For i = 1 To N
For j = 1 To i
Array2(i) = Array2(i) + Array1(j)
Next j
Next i
Maxi = Array2(1)
Mini = Array2(1)
For i = 1 To N
If Array2(i) > Maxi Then Maxi = Array2(i)
If Array2(i) < Mini Then Mini = Array2(i)
Next i
R = Maxi - Mini
totalR = totalR + R
totalS = totalS + S
Next PeriodNo
R = totalR / NoOfPeriods
S = totalS / NoOfPeriods
  RS = R / S
  PlottedPointNo = N - 2
  Result(PlottedPointNo, 1) = (Log(N)) / logten
  Result(PlottedPointNo, 2) = (Log(RS)) / logten
  Next N
   Sumx = 0
   Sumy = 0
   Sumxy = 0
   Sumxx = 0
   For i = 1 To NoOfPlottedPoints
   Worksheets("Data").Cells(i + 6, 4).Value = Result(i, 1)
   Worksheets("Data").Cells(i + 6, 5).Value = Result(i, 2)
      Sumx = Sumx + Result(i, 1)
      Sumy = Sumy + Result(i, 2)
      Sumxy = Sumxy + (Result(i, 1)) * (Result(i, 2))
      Sumxx = Sumxx + (Result(i, 1)) * (Result(i, 1))
Next i
H = (Sumxy - ((Sumx * Sumy) / NoOfPlottedPoints)) / _
(Sumxx - ((Sumx * Sumx) / _ NoOfPlottedPoints))
Worksheets("Data").Range("C3").Value = H
End Sub
```