



SOME COMPOSITE MODELS AND THEIR APPLICATION TO TURKISH MOTOR INSURANCE DATA

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ABSTRACT

It is important for an insurance company to predict the future claims in order to evaluate premiums, to determine the reserve necessary to meet its obligation and probabilities of ruin, etc. as the claim data is highly positively skewed and has heavy tail, no standard parametric model seems to provide an acceptable fit to both small and large losses. Composite models that use one standard distribution up to a threshold and other standard distribution thereafter are developed and it is seen that these composite models provide better fit than the standard models when claim data involve small and high claims.

The aim of this study is to investigate the use of the composite models namely Exponential-Pareto, Weibull-Pareto and Lognormal-Pareto to model the Turkish Motor Insurance claim data. From the results obtained, it is concluded that the composite Weibull-Pareto model provides better fit to Turkish Motor Insurance claim data than the all other models considered.

Keywords: Composite models, Standard parametric models, Turkish motor insurance loss data

1. INTRODUCTION

The classical parametric models such as Exponential, Gamma, Pareto, Weibull and Lognormal do not provide a better fit when data are highly positively and heavy tailed. Therefore, the composite models made up by piecing together two weighted distributions at a specified threshold have been proposed to model this data. For an insurance company, it is extremely important to predict future claims in order to calculate the premium to be charged, to determine the reserves required to meet its obligations, to calculate the probability of ruin, etc. As the insurance loss data involve both small and high claims in other words they are highly positively skewed and heavy tailed, the composite models are widely used. Cooray and Ananda [1] proposed the composite Lognormal-Pareto model. Ciumara [2] considered Weibull density up to an unknown threshold and Pareto density thereafter. Preda and Ciumara [3] compared the composite Lognormal-Pareto and Weibull-Pareto models. Teodorescu and Vernic [4] presented the composite Exponential-Pareto model. Scollnik [5] improved the composite Lognormal-Pareto model by incorporating unrestricted mixing weight in each component. Teodorescu and Vernic [6] designed a composite model by mixing a truncated Exponential and Pareto distribution and composite Exponential-Type II Pareto model. Vernic et al. [7] studied the composite Lognormal-Lognormal model. Nadarajah and Bakar [8] developed the composite Lognormal-Burr model. Maghsoudi et al. [9] concluded that among composite Weibull-Gamma family, composite Weibull-Inverse Transformed Gamma model is better than other models. Composite models based on Stoppa distribution were developed by Calderin-Ojeda and Kwok [10]. Abu Bakar et al. [11] developed several new composite models based on the Weibull distribution. The fit of almost all of these composite models above to insurance loss data is investigated using Danish fire loss data. It is concluded that these models provide a better fit when the data is extremely skewed.

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The outline of this paper is as follows. Section 2 involves the construction and the basic properties including the probability function, distribution function and non-central moments of the composite models and in Section 3 the maximum likelihood and ad-hoc procedure based on percentiles used for the estimation of the unknown parameters are given. The composite models namely the Exponential-Pareto, Weibull-Pareto and Lognormal-Pareto and comparison of these composite models with the standard parametric models namely Exponential, Weibull, Pareto and Lognormal are given in Section 4. The results of the application carried out on a Turkish Motor Insurance claim data is presented in Section 5. Finally, conclusion is given in Section 6.

2. CONSTRUCTION OF COMPOSITE MODELS AND THEIR BASIC PROPERTIES

Assuming that the loss data (X) involves small and high claims and the small claims have a probability density function $f_1(x)$ usually a light-tailed distribution and high claims have a probability density function $f_2(x)$ usually a heavy-tailed, then the random variable X has the following probability density function:

$$f(x) = \begin{cases} c f_1(x), & x < \theta \\ c f_2(x), & x \geq \theta \end{cases} \quad (1)$$

where θ is threshold and c is the normalizing constant.

To have a continuous and differentiable density, the following conditions at threshold must be imposed:

$$\begin{aligned} \text{Condition 1: } & f_1(\theta-) = f_2(\theta+) \\ \text{Condition 2: } & f_1'(\theta-) = f_2'(\theta+) \end{aligned} \quad (2)$$

Normalizing constant c in the Eq. (1) can be obtained from $\int_0^{\infty} f_x(x) dx = 1$ as

$$c = \frac{1}{\int_0^{\theta} f_1(x) dx + \int_{\theta}^{\infty} f_2(x) dx} \quad [1].$$

Denoting the cumulative distribution function of $f_i(x)$ by $F_i(x)$ for $i=1,2$, the cumulative distribution function of the probability density function in Eq. (1) is:

$$F_x(x) = \begin{cases} c F_1(x), & x < \theta \\ c [F_1(\theta) + F_2(x) - F_2(\theta)], & x \geq \theta. \end{cases}$$

The k^{th} non-central moment is given by

$$E(X^k) = c \left[\int_0^{\theta} x^k f_1(x) dx + \int_{\theta}^{\infty} x^k f_2(x) dx \right].$$

As this model has a fixed and priori known mixing weight, its default application and use for predictive modelling would seem ill-advised without very careful consideration in practical situations.

Scollnik [5] pointed out that the model given in Eq.(1) might also be written as convex combinations of two probability density functions:

$$f(x) = \begin{cases} rf_1^*(x), & x < \theta \\ (1-r)f_2^*(x), & x \geq \theta \end{cases} \quad (3)$$

where $0 \leq r \leq 1$ and $f_1^*(x)$ and $f_2^*(x)$ which are the adequate truncations of $f_1(x)$ and $f_2(x)$ are respectively as follows:

$$f_1^*(x) = \frac{f_1(x)}{\int_0^\theta f_1(x) dx} = \frac{f_1(x)}{F_1(\theta)}, \quad x < \theta$$

$$f_2^*(x) = \frac{f_2(x)}{\int_\theta^\infty f_2(x) dx} = \frac{f_2(x)}{1 - F_2(\theta)}, \quad x \geq \theta.$$

In this model, the mixing weight r can not be obtained from the integral $\int_0^\infty f(x) dx = \int_0^\theta rf_1^*(x) dx + \int_\theta^\infty (1-r)f_2^*(x) dx = 1$. It is a function of the parameters of $f_1(x)$ and $f_2(x)$ varying in the interval $[0,1]$ obtained by imposing the continuity and differentiability conditions at threshold in Eq. (2).

The cumulative distribution function of the probability density function in Eq. (3) can be given as:

$$F_X(x) = \begin{cases} r \frac{F_1(x)}{F_1(\theta)}, & x < \theta \\ r + (1-r) \frac{F_2(x) - F_2(\theta)}{1 - F_2(\theta)}, & x \geq \theta. \end{cases}$$

The k^{th} non-central moment is given by

$$E(X^k) = r \int_0^\theta x^k f_1^*(x) dx + (1-r) \int_\theta^\infty x^k f_2^*(x) dx.$$

3. PARAMETER ESTIMATION

As the maximum likelihood and ad-hoc procedure based on percentiles are two widely used methods for the estimation of the unknown parameters of the composite models, here these methods are mentioned briefly.

3.1. Maximum Likelihood Estimation

Let x_1, x_2, \dots, x_n be a random sample. If we assume this is an ordered sample ($x_1 < x_2 < \dots < x_n$) and the threshold is between m^{th} observation and $m+1^{\text{th}}$ observation ($x_m \leq \theta \leq x_{m+1}$), then the likelihood function for model in Eq.(1) is given by

$$L = c^n \prod_{i=1}^m f_1(x_i) \prod_{i=m+1}^n f_2(x_i).$$

So, the log-likelihood function is

$$\ln L = n \ln c + \sum_{i=1}^m \ln f_1(x_i) + \sum_{i=m+1}^n \ln f_2(x_i) \quad (4)$$

The likelihood function for model in Eq. (3) is given by

$$L = r^m (1-r)^{n-m} \prod_{i=1}^m f_1^*(x_i) \prod_{i=m+1}^n f_2^*(x_i)$$

So, the log-likelihood function is:

$$\ln L = m \ln r + (n-m) \ln(1-r) + \sum_{i=1}^m \ln f_1^*(x_i) + \sum_{i=m+1}^n \ln f_2^*(x_i) \quad (5)$$

As the score functions obtained taking partial derivatives of Eq.(4) or Eq.(5) with respect to unknown parameters can't be usually solved analytically, the numeric methods developed to find a solution to nonlinear system such as Newton-Raphson and Secant methods can be used.

3.2. An ad-hoc procedure based on percentiles

It is a procedure providing closed form expressions for the parameters to be estimated. In this procedure, threshold is estimated using percentile and the other parameters are estimated using maximum likelihood method. As in the maximum likelihood method, let us consider an ordered sample $x_1 < x_2 < \dots < x_n$ where $x_m \leq \theta \leq x_{m+1}$.

The threshold parameter θ can be estimated using the smooth empirical estimate of p^{th} percentile as:

$$\tilde{\theta} = (1-h)x_m + hx_{m+1}$$

where $m = \lceil (n+1)p \rceil$, $h = (n+1)p - m$ and $p = F(\theta)$ [12].

4. COMPOSITE MODELS and THEIR BASIC PROPERTIES

This section involves the probability density function, cumulative distribution function and non-central moments of the composite Exponential-Pareto, Weibull-Pareto and Lognormal-Pareto models respectively. Here, the densities of these composite models are also illustrated for some parameter values for comparison purpose.

4.1.Exponential-Pareto Composite Model

Suppose in the Eq.(1) $f_1(x)$ has the form of a one-parameter Exponential density and $f_2(x)$ has the form of a two-parameter Pareto density given in the Eq.(6) and Eq.(7) respectively:

$$f_1(x) = \lambda \exp(-\lambda x), \quad x > 0, \lambda > 0 \quad (6)$$

$$f_2(x) = \frac{\alpha \theta^\alpha}{x^{\alpha+1}}, \quad x > \theta, \theta > 0, \alpha > 0. \quad (7)$$

By imposing the continuity and differentiability conditions in Eq.(2) at threshold θ to obtain a smooth composite density function, the following two equations are obtained:

$$\text{From the Condition 1} \quad \alpha = \lambda \theta e^{-\lambda \theta} \quad (8)$$

$$\text{From the Condition 2} \quad \lambda^2 e^{-\lambda \theta} = \frac{\alpha(\alpha+1)}{\theta^2}. \quad (9)$$

Using the Eq.(8) in the Eq. (9), the following equation is obtained:

$$\lambda\theta[e^{-\lambda\theta} - 1] + 1 = 0. \tag{10}$$

Using the numerical methods, the solution of the Eq.(10) is calculated as $\lambda\theta = 1.35$. So, $\lambda = \frac{1.35}{\theta}$. The parameter α is 0.35.

By imposing the condition $\int_0^\infty f(x) = 1$, the normalizing constant is calculated as

$$c = \frac{1}{2 - e^{-\lambda\theta}} = 0.574.$$

So, the composite Exponential-Pareto density function becomes

$$f(x) = \begin{cases} \frac{0.775}{\theta} e^{-\frac{1.35x}{\theta}} & , 0 < x \leq \theta \\ 0.2 \frac{\theta^{0.35}}{x^{1.35}} & , \theta \leq x < \infty. \end{cases}.$$

The cumulative distribution function of this composite model is

$$F(x) = \begin{cases} 0.574 \left(1 - e^{-\frac{1.35x}{\theta}} \right) & , 0 < x \leq \theta \\ 1 - 0.574 \left(\frac{\theta}{x} \right)^{0.35} & , \theta \leq x < \infty. \end{cases}$$

For the composite Exponential-Pareto model, the r^{th} non-central moment can be given by

$$E(X^r) = \frac{0.775\theta^r}{1.35^{r+1}} \Gamma(r+1, 1.35) - \frac{0.2\theta^r}{r-0.35}, \quad r < 0.35$$

where $\Gamma(s; z) = \int_0^z y^{s-1} e^{-y} dy$ is the incomplete Gamma function.

Using the Eq. 4), the likelihood function is obtained as

$$L(x_1, \dots, x_n; \theta) = k\theta^{0.35n-1.35m} e^{-1.35\theta^{-1} \sum_{i=1}^m x_i}$$

where $k = \frac{0.775^m 0.2^{n-m}}{\prod_{i=m+1}^n x_i^{1.35}}$ [4].

In Figure 1, the Exponential, Pareto and composite Exponential-Pareto densities are illustrated for $\theta = 10$. It is seen that composite Exponential-Pareto model has a heavier tail than the Exponential.

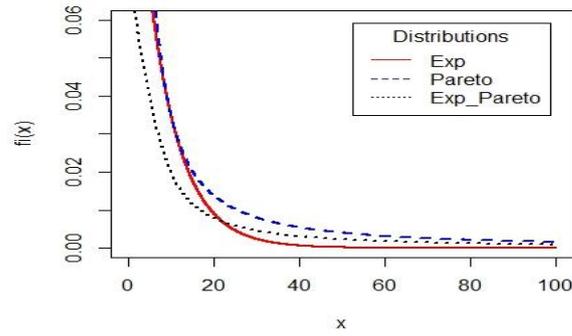


Figure 1. The Densities of Exponential, Pareto and Composite Exponential-Pareto Models for $\theta = 10$.

In order to see the effect of the change in threshold θ , the densities of the composite Exponential-Pareto model for $\theta = 10, 20, 40$ are illustrated in Figure 2.

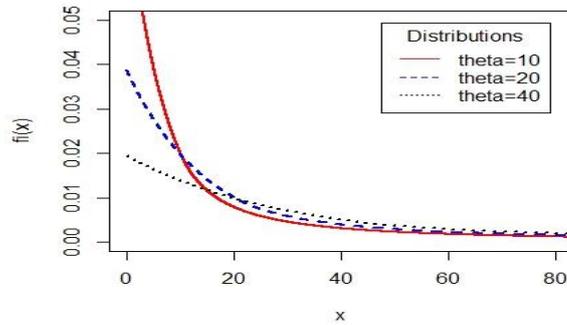


Figure 2. The Densities of the Composite Exponential-Pareto Model for $\theta = 10, 20, 40$.

As seen from Figure 2, as the θ increases, the tail of the composite Exponential-Pareto model becomes heavier.

4.2. Weibull-Pareto Composite Model

Suppose in Eq.(1) $f_1(x)$ has the form of a two-parameter Weibull density and $f_2(x)$ has the form of a two-parameter Pareto density in Eq.(11) and Eq.(7) respectively:

$$f_1(x) = \frac{\beta}{\gamma^\beta} x^{\beta-1} \exp\left[-\left(\frac{x}{\gamma}\right)^\beta\right], \quad x > 0, \gamma > 0, \beta > 1. \tag{11}$$

By imposing the continuity and differentiability conditions in Eq.(2) at threshold θ to obtain a smooth composite density function, the following two equations are obtained:

From the *Condition 1*

$$\alpha = \beta \left(\frac{\theta}{\gamma}\right)^\beta \exp\left[-\left(\frac{\theta}{\gamma}\right)^\beta\right] \tag{12}$$

From the Condition 2
$$\frac{\beta}{\gamma^\beta} \theta^{\beta-2} \exp\left[-\left(\frac{\theta}{\gamma}\right)^\beta\right] \left[\beta - 1 - \beta \left(\frac{\theta}{\gamma}\right)^\beta\right] = \frac{-\alpha(\alpha+1)}{\theta^2}. \quad (13)$$

Putting the Eq.(12) in the Eq.(13), one can obtain:

$$\alpha = \beta \left[\left(\frac{\theta}{\gamma}\right)^\beta - 1 \right]. \quad (14)$$

By equating the Eq.(12) to the Eq.(14),

$$t_0 - (t_0 + 1) \exp(-(t_0 + 1)) = 0 \quad (15)$$

where $t_0 = \left(\frac{\theta}{\gamma}\right)^\beta - 1$.

Using numerical methods, the solution of the Eq.(15) is obtained as $t_0 \approx 0.3499764854$. So, $\alpha = \beta t_0$ and $\gamma = \theta(t_0 + 1)^{\frac{1}{\beta}}$.

By imposing the condition $\int_0^\infty f(x) = 1$, the normalizing constant is calculated as $c = \frac{t_0 + 1}{t_0 + 2}$.

So, the composite Weibull-Pareto density becomes:

$$f(x) = \begin{cases} \frac{(t_0 + 1)^2}{(t_0 + 2)} \frac{\beta}{x} \left(\frac{x}{\theta}\right)^\beta \exp\left[-(t_0 + 1)\left(\frac{x}{\theta}\right)^\beta\right], & 0 < x \leq \theta \\ \frac{t_0(t_0 + 1)}{t_0 + 2} \frac{\beta}{x} \left(\frac{\theta}{x}\right)^{\beta t_0}, & \theta \leq x < \infty. \end{cases}$$

The cumulative distribution function of this composite model is

$$F(x) = \begin{cases} \frac{t_0 + 1}{t_0 + 2} \left[1 - \exp\left[-(t_0 + 1)\left(\frac{x}{\theta}\right)^\beta\right] \right], & 0 < x \leq \theta \\ 1 - \frac{t_0 + 1}{t_0 + 2} \left(\frac{\theta}{x}\right)^{\beta t_0}, & \theta \leq x < \infty. \end{cases}$$

For the composite Weibull-Pareto model, the r^{th} non-central moment can be given by

$$E(X^r) = \frac{t_0 + 1}{t_0 + 2} \theta^r \left[(t_0 + 1)^{\frac{r}{\beta}} \Gamma\left(\frac{r}{\beta} + 1; t_0 + 1\right) + \frac{\beta t_0}{\beta t_0 - r} \right] \text{ for } r < \beta t_0.$$

Using the Eq.(4), the likelihood function is obtained as

$$L(x_1, \dots, x_n; \beta, \theta) = C^{WP} \beta^n \theta^{\beta((n-m)t_0 - m)} \frac{\prod_{i=1}^m x_i^\beta}{\prod_{i=m+1}^n x_i^{\beta t_0}} \exp\left(-\frac{t_0 + 1}{\theta^\beta} \sum_{i=1}^m x_i^\beta\right)$$

where $C^{wp} = \left(\frac{t_0 + 1}{t_0 + 2}\right)^n (t_0 + 1)^m t_0^{n-m} \prod_{i=1}^n x^{-1}$ [2].

In Figure 3, the Weibull, Pareto and composite Weibull-Pareto densities are illustrated for $\theta = 10$. It is seen that composite Weibull-Pareto model has a heavier tail than the Weibull.

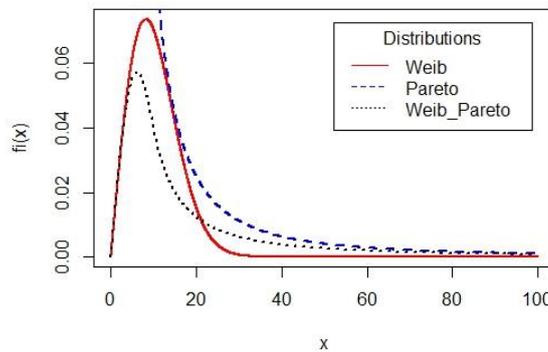


Figure 3. The Densities of Weibull, Pareto and Composite Weibull-Pareto Models for $\theta = 10$ and $\beta = 2$.

In order to see the effect of the change in threshold θ , the densities of the composite Weibull-Pareto model for $\beta = 2$ and $\theta = 10, 20, 40$ are illustrated in Figure 4.

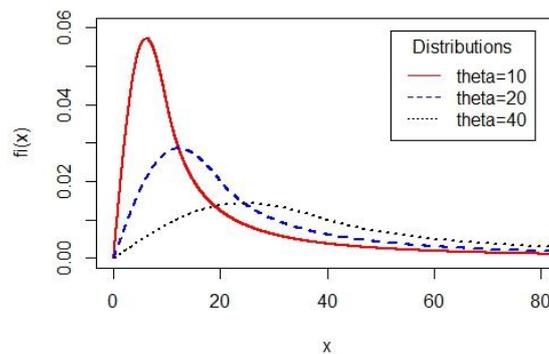


Figure 4. The Densities of the Composite Weibull-Pareto Model for $\beta = 2$ and $\theta = 10, 20, 40$.

As seen from Figure 4, as the θ increases, the tail of the composite Weibull-Pareto model becomes heavier.

4.3. Lognormal-Pareto Model

Suppose in Eq.(1) $f_1(x)$ has the form of a two-parameter Lognormal density and $f_2(x)$ has the form of a two-parameter Pareto density in Eq.(16) and Eq.(7) respectively

$$f_1(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right), \quad x > 0, \sigma > 0, \mu \in R. \quad (16)$$

By imposing the continuity and differentiability conditions in Eq.(2) at threshold θ to obtain a smooth composite density function, the following two equations are obtained:

From the Condition 1
$$\alpha = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{\ln \theta - \mu}{\sigma}\right)^2\right] \quad (17)$$

From the Condition 2
$$\alpha = \frac{1}{\sigma^2} (\ln \theta - \mu). \quad (18)$$

As seen form Eq. (18), $\mu = \ln \theta - \alpha\sigma^2$.

By equating the Eq.(17) to the Eq.(18), one can obtain:

$$\exp(-\alpha\sigma^2) = 2\pi\alpha^2\sigma^2. \quad (19)$$

Using numerical methods, the solution of the Eq. (19) is calculated as $k = \alpha\sigma \approx 0.372238898$.

By imposing the condition $\int_0^\infty f(x) = 1$, the normalizing constant is calculated as $c = \frac{1}{1 + \Phi(k)}$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

So, the Lognormal-Pareto composite density becomes:

$$f(x) = \begin{cases} \frac{\alpha\theta^\alpha}{(1 + \Phi(k))x^{\alpha+1}} \exp\left(-\frac{\alpha^2}{2k^2} \ln^2\left(\frac{x}{\theta}\right)\right), & 0 < x \leq \theta \\ \frac{\alpha\theta^\alpha}{(1 + \Phi(k))x^{\alpha+1}}, & \theta \leq x < \infty. \end{cases}$$

The cumulative distribution function of this composite model is:

$$F(x) = \begin{cases} \frac{1}{1 + \Phi(k)} \Phi\left(\frac{\alpha}{k} \ln\left(\frac{x}{\theta}\right) + k\right), & 0 < x \leq \theta \\ 1 - \frac{1}{1 + \Phi(k)} \left(\frac{\theta}{x}\right)^\alpha, & \theta \leq x < \infty. \end{cases}$$

For the composite Lognormal-Pareto model, the r^{th} non-central moment can be given by:

$$E(X^r) = \frac{1}{1 + \Phi(k)} \theta^k \left[\Phi\left(k - \frac{kr}{\alpha}\right) \exp\left(\frac{1}{2}\left(\frac{k}{\alpha}\right)^2 (r^2 - 2\alpha r)\right) + \frac{\alpha}{\alpha - r} \right] \text{ for } r < \alpha.$$

As mentioned in Section (3.1), the likelihood function of the Lognormal-Pareto composite model is:

$$L(x_1, \dots, x_n; \alpha, \theta) = C^{LP} \alpha^n \theta^{\alpha n} \prod_{i=1}^n x_i^\alpha \prod_{i=1}^n x_i^\alpha \exp\left(-\frac{\alpha^2}{2k^2} \sum_{i=1}^n \ln^2 \frac{x_i}{\theta}\right)$$

where $C^{LP} = \frac{1}{\left(\prod_{i=1}^n x_i\right) (1 + \Phi(k))^n}$ [1].

In Figure 5, Lognormal, Pareto and composite Lognormal-Pareto densities are illustrated for $\alpha = 1$ and $\theta = 10$. It is seen that composite Lognormal-Pareto model has a heavier tail than the Lognormal.

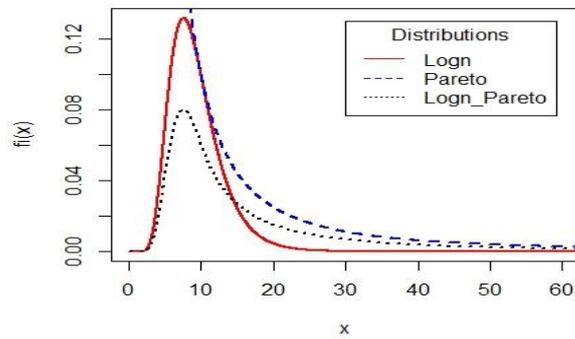


Figure 5. The Densities of Lognormal, Pareto and Composite Lognormal -Pareto Models for $\alpha = 1$ and $\theta = 10$.

In order to see the effect of the change in threshold θ , the densities of the composite Lognormal-Pareto model for $\alpha = 1$ and $\theta = 10, 20, 40$ are illustrated in Figure 6.

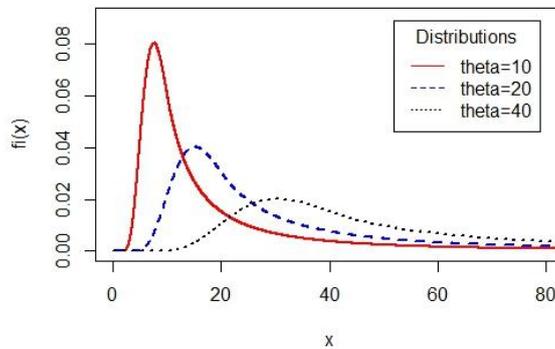


Figure 6. The Densities of the Composite Lognormal-Pareto Model for $\alpha = 1$ and $\theta = 10, 20, 40$.

As seen from Figure 6, as the θ increases, the tail of the composite Lognormal-Pareto model becomes heavier.

5. APPLICATION OF COMPOSITE MODELS TO TURKISH MOTOR INSURANCE CLAIM DATA

In this section, in addition to the composite models given in Section 4 some standard distributions namely Exponential, Weibull, Pareto and Lognormal are applied to Turkish Motor Insurance claim data taken from an insurance company consisting of 976 losses paid in 2003 for the damages in the automobile insurance. The summary statistics of the data divided by one thousand and the histogram and boxplot are given in Table 1 and Figure 7 respectively.

Table 1. Summary Statistics of the Claim Data.

Summary Statistics	
Minimum	0.0875
First Quantile	0.7937
Median	1.4060
Mean	2.1274
Third Quantile	1.4060
Maximum	98.1953

As seen from Table 1, the first and third quantiles are far apart from the maximum value, meaning that the data is highly skewed. The histogram and boxplot in Figure 7 also verify this.

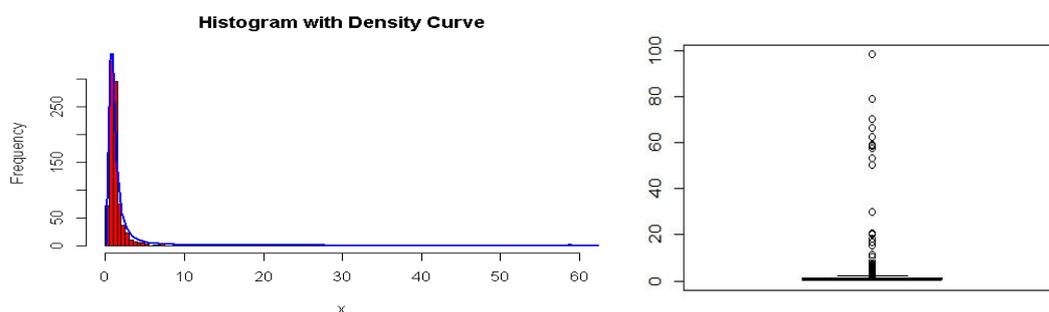


Figure 7. The Histogram with Density Curve and Boxplot of the Data.

Appropriate R optimization functions are used to estimate the parameters. The goodness of fit measures the Negative Log-Likelihood ($NLL = -\ln L$) and the Akaike Information Criterion ($AIC = 2k - 2\ln L$ where k is the number of the parameters of the fitted model) are used to measure the appropriateness of the fitted model along with the Kolmogorov–Smirnov (K-S) statistic. The smaller the NLL and AIC values and also K-S, the better the fit of the model considered. The results obtained are given in Table 2.

Table 2. The Estimated Values of the Fitted Models for the Turkish Motor Insurance Data.

Model	R optimization function	Estimated parameters	NLL	AIC	K-S
<i>Exponential</i>	-	$\lambda = 0.47$	1554.843	3111.686	0.293
<i>Weibull</i>	<i>nlm</i>	$\beta = 0.82 \quad \gamma = 1.75$	1489.753	2983.505	0.227
<i>Pareto</i>	<i>nlm</i>	$\alpha = 0.09 \quad \beta = 0.38$	1931.718	3867.436	0.419
<i>Lognormal</i>	-	$\mu = 0.12 \quad \sigma = 0.77$	1020.306	2044.612	0.626
<i>Exponential-Pareto</i>	<i>bbsolve</i>	$\theta = 58.62$	1457.397	2916.251	0.908
<i>Weibull-Pareto</i>	<i>uniroot</i>	$\theta = 0.96 \quad \beta = 4.15$	961.677	1927.355	0.096
<i>Lognormal-Pareto</i>	<i>uniroot</i>	$\theta = 0.91 \quad \alpha = 0.99$	1085.202	2174.403	0.169

The results in Table 2 suggest that the composite Weibull-Pareto model provides a better fit to this data than all other models considered. So, the composite models can be used to model Turkish Motor Insurance claim data. In Figure 7, the density curve belongs to the composite Weibull-Pareto model which has the best fit among all the models considered here.

6. CONCLUSION

In this paper, we investigated the fit of the composite Exponential-Pareto, Weibull-Pareto and Lognormal-Pareto models to the Turkish Motor Insurance claim data. After giving brief information about these composite models, all of these composite models along with the classical models namely Exponential, Weibull, Pareto and Lognormal are applied to the claim data taken from an insurance company. It is concluded that the composite Weibull-Pareto model provides a better fit than all other models considered. When the data is highly positively skewed and has heavy tailed, the necessity of the use of composite models is verified by the application.

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