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META FUZZY INDEX FUNCTIONS

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Abstract. Meta-analysis was introduced to aggregate the findings of different primary studies in statistical aspects. However, in the proposed study, the term "meta" is used to aggregate different models for a specific topic with the help of fuzzy c-means clustering method. One of the motivations of the proposed method is based on the concept of indices. In the literature, there are numerous proposed indices under different conditions for a specific purpose. Our assumption is that each index has some information for a given dataset. Therefore, meta fuzzy index functions, which include each index in each function with a certain degree of membership value, are introduced in the proposed method. Currency crisis and process capability indices are chosen as applications in order to show that the proposed method can be useful tool in terms of indices.

1. Introduction

Meta-analysis was defined as a method of combining the results of multiple independent studies based on statistical methods on a given subject by Glass [16] in 1976. His aim was to aggregate 375 different psychotherapy outcome studies to reduce the confusion among different outcomes. Besides, DerSimonian and Laird [12] defined in their paper that meta-analysis is a collection of analytic results for integrating the findings to get more reliable results. Aforementioned advantages, the studies based on meta-analysis have become more popular in the last few decades. However, rather than aggregating different studies outcomes, the outcomes of different selected methods for a given dataset are aggregated in the study. Because there are numerous proposed indexes for a purpose, indexes are the focus of the paper.

Indexes represent the proportional changes of a simple or compound event in time or space. The expression of changes in percentage rather than absolute figures is preferred in terms of interpretation and understanding of events. In other

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words, a function of numerous indicators is defined as an index in statistics. Indices widely used for judging the pulse economy. Although they used to measure the effect of changes in prices in the beginning, today we use indices for industrial production, cost of living, agricultural production, currency crisis, process capability, etc. There are multiple proposed methods in literature for each title given above. Another motivation of Meta Fuzzy Index Functions (MFIFs) is the assumption that each method for an index has some information for a given dataset. Thus, our aim is to aggregate these methods in functions with the help of fuzzy c-means (FCM) clustering algorithm and call it MFIFs. Process Capability Indices (PCI) and Currency Crisis Indices (CCI) are selected as applications.

The PCI, which is defined as a statistical measure of process capability, tends to determine whether a production process is able to produce items within specification tolerance. There are several conditions, some of which are assumed to be normally distributed and to have a large sample size. Some of well-known PCIs are Cp, Cpk, and Cpm, which are introduced by [26, 19, 7]. For these PCIs, it is assumed that measured characteristic is normally distributed, and it is not, very often, possible to maintain the mean of the process on the center of the tolerance interval. Some of the well-known PCIs will be discussed in detail in Section 2.1.

A Currency crisis is defined as the crisis that consists of reduction in the reserves of an economy that uses fixed exchanged regime. The reduction in the reserves occurs when market actors change their national assets to international assets at the instant and, in parallel, as the consequence of the action of Central Bank's reserves. To forecast the crisis there are numerous currency crisis indices in literature, some of which are introduced by Kaminsky et al. [18] , Corsetti et al. [11] etc. Kaminsky's [18] index is composed of changes in nominal exchange rate and changes in foreign reserves while Corsetti's [11] index is composed of changes in nominal exchange rate and changes in international reserves. In short, there are different point of views for currency crisis' indicators for different researchers. Thus, the currency crisis indices are suitable for our purpose in the paper. The currency crisis indices will be given in detail in Section 2.2.

The assumption of the proposed method is that each index has some information about the process or the situation. Thus, we tried to aggregate indices in functions with the help of FCM algorithm. The first step of the MFIFs is the clustering the indices using FCM. Degree of membership values obtained from FCM for each cluster give coefficients of the indices in each function. Finally, the function that explains the process or the situation best is chosen as the meta-fuzzy index function. The MFIFs will be discussed in detail in Section 4.

2. Preliminaries

2.1. **Process Capability Indices (PCI).**

Process capability analysis is conducted to examine whether the products produced during the production process have the desired tolerances or not. In other words, it is the measure whether a process is capable of producing an item within specification limits. There are numerous PCIs in literature to measure the process potential and performance. At the beginning, it is assumed that the dataset of a process is normally distributed, and its tolerance limits are symmetric. Later, researchers proposed different PCIs under different conditions, such as processes with symmetric tolerance limits and normally distributed, asymmetric tolerance limits and normally distributed, and symmetric tolerance limits and asymmetric distribution. Some of the PCIs proposed are given below. The concerned univariate measurements will be denoted with the corresponding random variable by X. The expectation and standard deviation of X will be denoted by μ and σ respectively. When it is assumed that the measured characteristics of a process is approximately normally distributed and its tolerance limits are symmetric, following PCIs are commonly used by manufacturers. The C_p index introduced by Sullivan [26] is given in Equation 1,

$$PCI_1 = C_p = \frac{USL - ASL}{6\sigma} \tag{1}$$

and used when $\mu = M$ where $M = (USL + LSL)/6\sigma$. USL and LSL stand for upper specification and lower specification limits respectively. It is not always possible to maintain the process on the center of tolerance interval $[LSL, USL]$. In this case, C_{pk} and C_{pm} indices might be useful and given in Equation 2,3,

$$PCI_2 = C_{pk} = \frac{min(USL - \mu, \mu - LSL)}{3\sigma} \tag{2}$$

and

$$PCI_3 = C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - \tau)^2}} \tag{3}$$

When it is assumed that the measured characteristics of a process is approximately normally distributed and its tolerance limits are asymmetric, following PCIs are used by researchers. Some of the studies conducted for asymmetric tolerance and normal distributed of a process was proposed by Kane [19] , Boyles [5], Pearn and Chen [23], Chan et al. [7], and Chen et al. [8].

$$PCI_4 = C^{\star}_{pm} = \frac{d'}{3\sqrt{\sigma^2 + (\mu - T)}} \tag{4}$$

$$d' = \frac{(USL - T) + (T - LSL)}{2} \tag{5}$$

where T is the target value.

$$PCI_5 = C'_{pm} = \frac{d^{\star}}{3\sqrt{\sigma^2 + (A^{\star})^2}} \tag{6}$$

$$A^{\star} = max\left\{\frac{d^{\star}(\mu - T))}{D_u}, \frac{d^{\star}(T - \mu))}{D_l}\right\}, \tag{7}$$

$$D_u = USL - T, D_l = T - LSL \tag{8}$$

When the measured characteristics of a process has an asymmetric distribution and symmetric tolerance limits, some of the PCAs are introduced by Clements [9], Pearn and Kotz [25], Pearn and Chen [24], and Wright [28].

$$PCI_6 = \widehat{C_p} = \frac{USL - LSL}{U_p - U_l} \tag{9}$$

$$PCI_7 = \widehat{C}_{pk} = min\left\{\frac{USL - m}{m - L_p}, \frac{m - LSL}{U_p - m}\right\} \tag{10}$$

$$PCI_8 = \widehat{C}_{pmk} = min\left\{\frac{USL - m}{3\sqrt{[\frac{U_p - m}{3}]^2 + (m - T)^2}}, \frac{m - LSL}{3\sqrt{[\frac{m - L_p}{3}]^2 + (m - T)^2}}\right\} \tag{11}$$

$$U_p = \%99.865, L_p = \%0.135 \tag{12}$$

$$PCI_9 = \hat{C}'_{pk} = min\left\{\frac{USL - m}{[U_p - L_p]/2}, \frac{m - LSL}{[U_p - L_p]/2}\right\} \tag{13}$$

$$PCI_{10} = \hat{C}'_{pm} = \frac{USL - LSL}{6\sqrt{\left[\frac{U_p - L_p}{6}\right]^2 + (m - T)^2}} \tag{14}$$

$$PCI_{11} = \hat{C}'_{pmk} = min\left\{\frac{USL - m}{3\sqrt{\left[\frac{U_p - L_p}{6}\right]^2 + (m - T)^2}}, \frac{m - LSL}{3\sqrt{\left[\frac{U_p - L_p}{6}\right]^2 + (m - T)^2}}\right\} \tag{15}$$

$$PCI_{12} = C_s = \frac{(d - |\mu - T|)/\sigma}{3\sqrt{1 + [(\mu - T)/\sigma]^2 + |k_1|}} \tag{16}$$

Large value implies a better process for each index given above. The PCAs which are used in the application are given above.

2.2. **Currency Crisis Indices (CCI).** Although there is no consensus on defining currency crisis, there is more or less consensus on the indicators of a currency crisis. Usually, exchange rate, interest rate, and international reserves are considered as the indicators of the currency crisis. The definitions of CCIs are obtained using three or different pairwise combination of these indicators. The CCIs which will be used in the study are proposed by Eichengreen [15], Kaminsky [18], Corsetti et al. [10], Krkoska [21], Von Hagen and Ho [29], Bussierre [4], Yiu et al. [31], Alvarez-Plata and Schrooten [1], Bunda and Co-Zorri [6], and Johansen [17]. The definitions of CCIs given by the researchers introduced above are given in Equation 17-26 and cited from Ari and Cergibozan [2].

$$CCI1_t = \frac{1}{\sigma_{RER}}\Delta NER_{TR,t} - \frac{1}{\sigma_{RES}}\Delta RES_{TR,t} + \frac{1}{\sigma_{NIR}}\Delta(NIR_{TR,t} - NIR_{US,t}) \tag{17}$$

where NER is the nominal exchange rate, RES is international reserves, NIR is the nominal interest rate.

$$CCI2_t = ln\left(\frac{NER_t}{NER_{t-3}}\right) - \frac{\sigma^2_{NER}}{\sigma^2_{RES}} ln\left(\frac{RES_t}{RES_{t-3}}\right) \tag{18}$$

$$CCI3_t = \frac{1}{\sigma^2_{RER}}\Delta RER_t - \frac{1}{\sigma^2_{RES}}\Delta RES_t + \frac{1}{\sigma^2_{RIR}}(RIR_t - RIR_{t-1}) \tag{19}$$

where RER is the reel exchange rate, RIR is the reel exchange rate, σ^2_{RER}, σ^2_{RES}, and σ^2_{RIR} the standard deviations of the reel exchange rate, reserves and the reel interest rate respectively.

$$\begin{aligned} CCI4_t \quad = \quad & \frac{1}{\sigma_{NER}}\Delta NER_{TR_j} - \frac{1}{\sigma_{RES}}(\Delta RES_{TR_j} - \Delta RES_{US_j}) \\ & + \frac{1}{\sigma_{NIR}}(\Delta NIR_{TR_j} - \Delta NIR_{US_j}) \end{aligned} \tag{20}$$

where NIR_{TR} and NIR_{US} are the nominal interest rates, RES_{TR} and RES_{US} are the international reserves excluding gold, σ_{NER} is the standard deviation of nominal exchange rate, σ_{RES} is standard deviation of the change in the international reserves gap between TR and the US, and σ_{NIR} indicates the standard deviation of the difference between the nominal interest rate of TR and the US.

$$CCI5_t = \frac{1}{\sigma_{RER}}\Delta RER_t - \frac{1}{\sigma_{RES}}\Delta RES_t + \frac{1}{\sigma_{NIR}}(NIR_t - NIR_{t-1}) \tag{21}$$

$$CCI6_t = \triangle RER_t \tag{22}$$

$$CCI7_t = \Delta NER_t - \frac{\sigma_{NER}}{\sigma_{RES}}\Delta RES_t \tag{23}$$

$$CCI8_t = \Delta NER_t - \Delta RES_t + (NIR_t - NIR_{t-1}) \tag{24}$$

$$CCI9_t = \Delta NER_t - \frac{\sigma_{NER}}{\sigma_{RES}}\Delta RES_t + \frac{\sigma_{NER}}{\sigma_{NIR}}(NIR_t - NIR_{t-1}) \tag{25}$$

$$CCI10_t = 0.75\Delta NER_t - 0.25\Delta RES_t \tag{26}$$

2.3. **Fuzzy C-means (FCM).** Fuzzy set theory was introduced by Zadeh [32] in 1965. Fuzzy logic has been commonly studied topic by then. Fuzzy sets were used in many fields, engineering, health science, economics, statistics, etc. Fuzzy logic lets an object to become a member of different classes or clusters with a degree of membership value. In this sense, fuzzy c-means clustering algorithm is developed by Dunn [13] in 1973 and improved by Bezdek [3] in 1981. FCM is also very useful tool in many fields. The detailed steps of FCM algorithm are given in Step 3 in Section 3.

3. Proposed Method

"Which index should we use for a given time series", "What are the characteristics of the dataset" lead us to come up with MFIFs. Thus, the aim of MFIFs is to propose a method that gives a way to aggregate the information of indices for a specific topic in functions by using FCM and get better outcomes. Thus, the inputs are MFIFs are the outputs of different indexes for a given dataset. Clustering the inputs by using FCM, the functions are obtained. In this case, a cluster represents a function in MFIFs. Thus, there are as many functions as the number of clusters.

Although there are many fuzzy clustering techniques like[20, 22], FCM[3] is used in the proposed method because of its easy-to-use structure and fame. The detailed steps of the proposed method are given below step by step.

- Step 1. Each index is calculated for the given dataset. When the scale of each index values is not the same, input matrix is standardized. Thus, the input matrix consists of the output of the indices.
- Step 2. Let c be the number of fuzzy sets (number of functions).
- Step 3. Using FCM algorithm, the degrees of membership values for each observation (indices) are calculated. In other words, the coefficients of indices in a function will be determined with the help of FCM.

  FCM Algorithm
  - Step 3.1. Initialize $\mu = [\mu_{ij}]$ matrix, determine the number of clusters and initial cluster centers.
  - Step 3.2. Calculate the membership value μ with the formula given in Equation 27.

$$\mu_{ik} = \left[\sum_{j=1}^{c} \left(\frac{d(z_k, v_i)}{d(z_k, v_j)} \right)^{\frac{2}{f_i - 1}} \right]^{-1}, i = 1, 2, .., c; k = 1, 2, .., n \tag{27}$$

    under the constraint; $\sum_{i=1}^{c} \mu_{ik} = 1, if \mu_{ik} < \alpha - cut$, then μ_{ik} value will be taken as zero. where Z is the input matrix, v is the cluster centers, $d(.)$ stands for Euclidean distance function, c is the number of clusters, and f_i is the fuzzy index value.
  - Step 3.3. Calculate the new cluster centers.

$$v_i = \frac{\sum_{k=1}^{n} \mu_{ik}^{f_i} z_k}{\sum_{k=1}^{n} \mu_{ik}^{f_i}} \tag{28}$$

  - Step 3.4. Repeat Step 2 and Step 3 until the difference of clusters between two iterations drops under some threshold or the number of iterations is reached.

- Step 4. Using the degrees of membership values, new vales of meta fuzzy index functions are calculated with the formula given in Equation 29.

$$MFIF_i = \frac{Z_i^T \mu_i}{\sum_{i=1}^{c} \mu_i}, i = 1, 2, ..c \tag{29}$$

where $MFIF$ stands for the index values of the proposed method and c stands for the number of clusters.

- Step 5. Step 4 is repeated for the number of cluster times.
- Step 6. The cluster that explains the purpose best is selected as the meta-fuzzy index function.

4. Applications

4.1. **Application of CCIs.** The first application of the proposed method is CCIs. CCIs are chosen because determining a currency crisis in a certain year is commonly studied topic by researchers and there are numerous proposed CCIs in literature. Each CCI has its own characteristic. However, our assumption is that each CCI has information about a dataset. Therefore, CCIs are suitable for MFIFs.

The data that were monthly observed were obtained from the Central Bank of Turkey. [27] The elements of the input matrix, which are observed from January of 1990 to June of 2014, are calculated using the given indices above. Summary of the input matrix is given in Table 1.

Table 1. Some observations of the input matrix

no	CCI1	CCI2	CCI3	CCI4	CCI5	CCI6	CCI7	CCI8	CCI9	CCI10	CCI11
1	-0.38	-0.08	-24.13	-0.65	-1.52	-0.12	-0.045	-0.09	-5.33	0.03	-0.06
2	-0.24	-0.04	-17.72	-0.54	-1.08	-0.01	-0.01	-0.03	-5.73	0.03	-0.02
3	0.16	0.04	6.62	1.73	0.56	0.03	0.08	0.09	-6.23	0.01	0.08
4	-0.08	0.06	-28.19	-0.17	-1.474	0.04	-0.01	-0.02	-21.13	0.02	-0.01
5	0.02	0.09	-1.19	0.6	-0.07	-0.02	0.02	0.03	-0.28	0.02	0.02
6	-0.28	-0.01	-14.01	-0.76	-1.07	-0.15	-0.06	-0.07	6.93	0.04	-0.06
.	.	.	.	.	.	.	.	.	.	.	.

Because the values of the indices are not in the same scale, the input matrix is standardized. The centers of the clusters are initialized randomly, and the number of clusters (functions) is taken as three. The $\alpha - cut$ value is taken as 0.1. The degree of membership matrix is given in Table 2.

Table 2 shows that which index belongs to which function with what degree of membership values. Using Equation 29, the following functions and graphs are obtained.

Using the first column of degree of memberships matrix and the CCIs, the first function is obtained as below.

TABLE 2. Degree of membership values of indices

Index	1	2	3
CCI1	0.12	0.02	0.86
CCI2	0.30	0.10	0.60
CCI3	0.01	0.00	0.99
CCI4	0.01	0.00	0.98
CCI5	0.00	0.00	1.00
CCI6	0.00	1.00	0.00
CCI7	0.08	0.01	0.90
CCI8	0.01	0.00	0.98
CCI9	0.96	0.01	0.04
CCI10	0.98	0.01	0.02
CCI11	0.00	0.00	1.00

$$MFIF_1 = (\mu_{11}CCI1 + \mu_{12}CCI2 + ... + \mu_{1n}CCIn)/\sum_{i=1}^{n}\mu_{1i} \tag{30}$$

$$MFIF_1 = (0.12*CC1+0.3*CCI2+...+0.98*CCI10)/(0.12+0.3+...+0.98) \tag{31}$$

Looking at $MFIF_1$, it is obvious that most contribution is made by $CCI9$ and $CCI10$, which means $CCI9$ and $CCI10$ reacts similar given the dataset. The graph of the first function is given in Figure 1. The time period of the currency crisis is determined if the points in a graph are 2 standard deviation away from the margin. In this case, the first function is capable of detecting three crisis that occurred in 1994, 2001, and 2008.

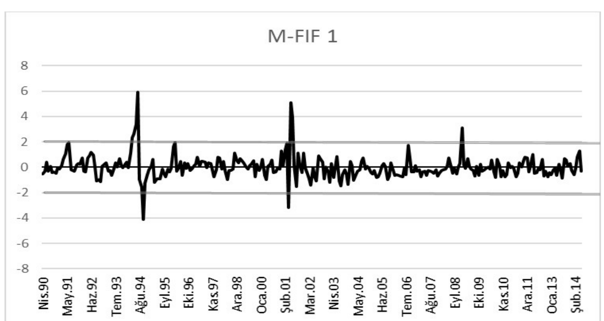

FIGURE 1. The first M-FIF for the crisis in Turkey

For the second function ($MFIF_2$), $CCI6$ dominates the other indices. The second $MFIF$ looks like as follows.

$$MFIF_2 = (\mu_{21}CCI1 + \mu_{22}CCI2 + ... + \mu_{2n}CCIn)/\sum_{i=1}^{n}\mu_{2i} \tag{32}$$

$$MFIF_2 = 1 * CCI6 \tag{33}$$

Figure 2 indicates 4 crises in Turkey, that occurred in 1994, 2001, 2006, and 2008.

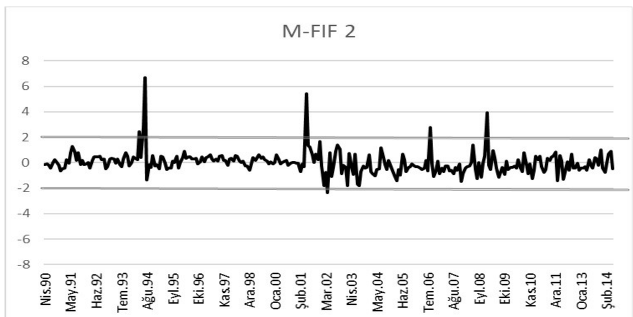

Figure 2. The second M-FIF for the crisis in Turkey

The third M-FIF contains the rest of the indices with higher degree of membership values.

$$MFIF_3 = (\mu_{31}CCI1 + \mu_{32}CCI2 + ... + \mu_{3n}CCIn)/\sum_{i=1}^{n}\mu_{3i} \tag{34}$$

$$MFIF_3 = (0.86 * CCI1 + 0.60 * CCI2 + ... + 1 * CCI11)/\sum_{i=1}^{n}\mu_{3i} \tag{35}$$

The third function indicates 2 crises that occurred in 1994 and 2001. The graph of M-FIF3 is given in Figure 3.

Implementing the proposed method to the currency crisis data set of Turkey, three different functions are obtained. The question arises as which function will be more useful for CCIs. In this case, the function which can detect more crisis is chosen as the M-FIF, which is the second function.

Overall, in the application of CCIs, we obtain three M-FIFs. Each M-FIF has different information about the crises in Turkey. The outcomes of MFIFs might be investigated by an economist in details. Our belief is that the M-FIFs is a useful tool in terms of CCIs.

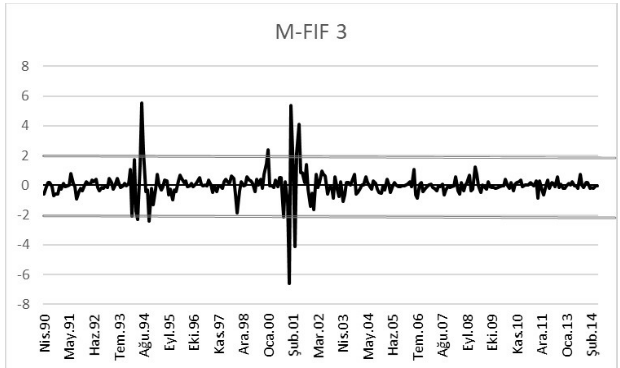

FIGURE 3. The third M-FIF for the crisis in Turkey.

4.2. **Application of PCIs.** PCIs are chosen as an application for the proposed method because there are numerous PCIs given under different conditions in literature. Some of them are explained in Chapter 3. Our belief is that each PCI has some information about a dataset no matter what distribution it has. Therefore, we tried to aggregate these indices in different functions. While some of the functions have more information about the process, some others have less information. Because there will be as many index functions as the number of clusters, we are looking for the best function in which indexes explain the process best. In this case, the best function that gives the highest value is selected the M-FIF. For the application, elastomer bearing sliding valve shaft dataset is chosen. The data set is obtained from [30].

The dataset is non-normally distributed, and its tolerance limits are asymmetric. At the beginning of the application, the bootstrap sampling method is used to determine how the indices react to the dataset. The bootstrap method, which was introduced by Efron [14], is a resampling technique with replacement used to estimate statistics/indexes on a population. Because there is just one value that is obtained from the indexes, we used bootstrap method to be able to learn the characteristics of an index. Ten bootstrap samples are obtained from the dataset and the index values of each sample are given in Table 3.

10 bootstrap samples of 11 indices are clustered using FCM. The degree of membership values of each index are given in Table 4.

Using Equation 26, we obtain the results in Table 5. Table 5 shows the M-FIF values for each bootstrap sample.

Table 5 gives the M-FIFs. Looking at Table 4, it is obvious that only $PCI12$ belongs to the first function with 1 degree of membership value. The function looks

Table 3. Values of indices and bootstrap samples

Index	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
PCI1	0.81	0.84	0.93	0.80	0.77	0.84	0.90	0.88	0.91	0.81
PCI2	0.69	0.73	0.81	0.73	0.71	0.77	0.78	0.77	0.83	0.71
PCI3	0.76	0.80	0.87	0.78	0.76	0.82	0.85	0.83	0.88	0.78
PCI4	0.64	0.66	0.73	0.62	0.59	0.65	0.71	0.70	0.70	0.64
PCI5	0.64	0.67	0.74	0.63	0.60	0.67	0.72	0.70	0.71	0.64
PCI6	1.10	1.04	1.15	1.01	1.05	1.13	1.06	1.04	1.10	1.02
PCI7	0.92	0.84	0.93	0.79	0.79	0.92	0.85	0.84	0.85	0.81
PCI8	0.91	0.84	0.93	0.77	0.76	0.92	0.85	0.84	0.84	0.80
PCI9	0.82	0.89	0.97	0.92	0.98	0.92	0.90	0.89	0.95	0.84
PCI10	1.10	1.04	1.14	0.99	1.02	1.13	1.05	1.04	1.10	1.02
PCI11	0.82	0.88	0.96	0.89	0.94	0.92	0.89	0.88	0.94	0.84
PCI12	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 4. Degree of membership values of indices

Index	1	2	3
PCI1	0.004	0.086	0.910
PCI2	0.003	0.024	0.973
PCI3	0.001	0.012	0.987
PCI4	0.034	0.105	0.861
PCI5	0.029	0.097	0.874
PCI6	0.001	0.984	0.015
PCI7	0.006	0.132	0.861
PCI8	0.005	0.103	0.891
PCI9	0.010	0.449	0.540
PCI10	0.001	0.988	0.011
PCI11	0.010	0.361	0.629
PCI12	1.000	0.000	0.000

like as below.

$$MFIF_1 = (\mu_{11}PCI1 + \mu_{12}PCI2 + ... + \mu_{1n}PCIn)/\sum_{i=1}^{n}\mu_{1i} \tag{36}$$

$$MFIF_1 = 1 * PCI12 \tag{37}$$

The second $M-FIF$ includes mainly two indices mainly but a few more with lower degree of membership values. $MFIF_2$ is given in Equation 38.

$$MFIF_2 = (\mu_{21}PCI1 + \mu_{22}PCI2 + ... + \mu_{2n}PCIn)/\sum_{i=1}^{n}\mu_{2i} \tag{38}$$

Table 5. Index values of M-FIFs

Samples	1	2	3
1	0.070	0.978	0.776
2	0.072	0.956	0.788
3	0.079	1.051	0.869
4	0.068	0.930	0.760
5	0.067	0.962	0.751
6	0.073	1.027	0.818
7	0.075	0.975	0.824
8	0.074	0.962	0.811
9	0.076	1.015	0.841
10	0.069	0.931	0.759

$$MFIF_2 = (0 * PCI1 + ... + 0.105 * PCI4 + ... + 0 * PCI12) / \sum_{i=1}^{12} \mu_{2i} \tag{39}$$

The third function includes the most of the indices more than 0.90 degree of membership values. The $MFIF_3$ looks like as below.

$$MFIF_3 = (\mu_{31} PCI1 + \mu_{32} PCI2 + ... + \mu_{3n} PCIn) / \sum_{i=1}^{n} \mu_{3i} \tag{40}$$

$$MFIF_2 = (0.91 * PCI1 + 0.973 * PCI2 + ... + 0 * PCI12) / \sum_{i=1}^{12} \mu_{3i} \tag{41}$$

Because we have three M-FIFs, the one that explains the process best is chosen as the M-FIF. Deciding which function will be MFIF is selected by looking at the mean of each cluster. The one which has the highest value is chosen as the M-FIF. In this case, the second function is chosen as the M-FIF for the process capability.

5. Conclusions

A new approach in terms of indices is proposed in the study. The need of the proposed method has arisen the question that which method we should prefer for a given dataset (i.e. normally distributed, linear, non-linear, semi-linear, etc.). Therefore, the methods that we might use for a given dataset are clustered with FCM algorithm. While the methods that perform better for a dataset are clustered in a function with higher degree of membership values, the methods that perform worse are clustered in a different function with higher degree of membership values. The advantages of the proposed method are discussed below.

- Because there are numerous methods in literature for a specific problem, there is also confusion which method will be chosen. With the help of the proposed method, the best method or methods are clustered in the same

function with certain degree of membership values. Thus, we are able to say that which method/methods are capable of dealing with a specific dataset.
- Because of the first advantage, we do not need to look for the assumptions of methods.
- MFIFs are able to aggregate the information of each index into functions.
- The M-FIFs approach is the first method in literature that aggregates the indexes into functions.

However, the major defect of the proposed method is the determination of the number and centers of the clusters. In the applications given above, the number of clusters is chosen as three and the centers are randomly initialized. Specifying the cluster centers with expert opinion might give better classification of the methods. Besides, increasing the number of clusters might give even better outcomes. This part of the proposed method is left for future work.

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