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A New Stochastic Search Method for Filled Function

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Abstract: Deterministic methods are used for optimum solution of many engineering and scientific problems. Filled function method, which is a deterministic method, is not trapped to local minimums with its ability to bypass energy barriers. In order to achieve this, the basin regions of the filled function should be located, and the filled function should be constructed in that region. However, classical search strategies used for finding the basin regions don't yield effective results. In this study, a new stochastic search approach is presented as a faster and more efficient alternative to classic filled function search strategy. An unconstrained global optimization method based on clustering and parabolic approximation (GOBC-PA) has been used as a stochastic method for accelerating the L type filled function as a deterministic method. The developed method has been tested against classical filled function using 11 benchmark functions. When the obtained results are examined, it is seen that the stochastic search approach has superiority over the mean error, standard deviation and elapsed time values according to the classical approach. These results show that the combination of deterministic and stochastic methods can be more successful in finding the global minimum against the classic deterministic method.

Keywords: Stochastic search; GOBC-PA; L type filled function; global optimization.

Filled Fonksiyonunda Yeni Bir Stokastik Arama Yöntemi

Öz: Deterministik yöntemler birçok mühendislik ve bilimsel problemin optimum çözümü için kullanılmaktadır. Deterministik bir yöntem olan filled fonksiyon metodu ise, enerji bariyerlerini aşabilme kabiliyeti ile yerel minimuma takılmamaktadır. Bunu başarmak için filled fonksiyonuna ait havza bölgeleri bulunmalı ve filled fonksiyon o bölgede inşa edilmelidir. Bununla birlikte, havza bölgelerini bulmak için kullanılan klasik arama stratejileri etkili sonuçlar vermemektedir. Bu çalışmada, yeni bir stokastik arama yaklaşımı, klasik filled fonksiyon arama stratejisine daha hızlı ve daha verimli bir alternatif olarak sunulmuştur. Deterministik bir yöntem olan L tipi filled fonksiyonunu hızlandırmak için stokastik bir yöntem olan kümeleme ve parabolik yaklaşım tabanlı kısıtsız global optimizasyon yöntemi (GOBC-PA) kullanılmıştır. Geliştirilen yöntem, 11 kıyaslama fonksiyonu kullanılarak klasik filled fonksiyonuna karşı test edilmiştir. Elde edilen sonuçlar incelendiğinde, stokastik arama yaklaşımının ortalama hata, standart sapma ve hesaplama süresi değerlerinde klasik yaklaşıma göre üstünlüğü görülmektedir. Bu sonuçlar, deterministik ve stokastik yöntemlerin kombinasyonunun, klasik deterministik yönteme karşı küresel minimumun bulunmasında daha başarılı olabileceğini göstermektedir.

Anahtar kelimeler: Stokastik arama; GOBC-PA; L tipi filled fonksiyonu; global optimizasyon.

1. Introduction

Global optimization is used in many areas such as engineering, sciences and architecture. It can be observed in theoretical or real-world problems of these fields. The main purpose of the global optimization is to find the globally best solution of the function.

An optimization process holds two conditions. First, the solution must be obtained at time N. Second, the probability of finding the best solution at a specified time M < N must be higher than a lower probability bound [1]. The optimization process works as follows. Firstly, x0* as a local minimizer is searched starting from x0 as an initial point using a minimization algorithm. Then, deeper local minimizers are searched until x0* becomes the global minimum.

In general, popular algorithms to solve the global optimization problems can be divided into two categories as deterministic and stochastic methods. Both of them have some advantages and disadvantages when they are compared. The typical examples of deterministic methods are the filled function method (FFM) [2], trajectory method [3], tunneling method [4] and covering method [5]. The best known algorithms of stochastic methods are the simulated annealing method [6], genetic algorithm (GA) [7], artificial bee colony algorithm (ABC) [8], particle swarm optimization (PSO) [9] and unconstrained global optimization method based on clustering and parabolic approximation (GOBC-PA) [10].

Global optimization problem that has multiple local minima may be hard to solve. Finding local minimums easier is among the features that make FFM superior to other methods [11]. FFM can make effective jumps from one basin to another over the energy barrier. But finding these basin regions are difficult with conventional methods, and if the step distance is not very small, the area can't be detected by passing. On the other hand, the stochastic methods are usually faster than the deterministic methods. So, the stochastic methods can help deterministic algorithms to find these basin regions.

Stochastic and deterministic methods are usually combined to make searching more efficient and not to trap to the local minimum. Stochastic methods use jumping over the energy barrier, which is the ability of deterministic methods for faster convergence and efficient search. Furthermore, deterministic methods can be avoided from local minimums by stochastic methods because each deterministic method ends at one local minimum [12].

There are studies in literature in which stochastic and deterministic methods are combined [13]. However, these studies reveal that either the stochastic and deterministic methods are fully combined, or combination is performed in a way that the other method steps in if one method fails in any iteration. On the other hand, searching for the basin regions in the methods such as FFM, which is used to jump through the energy barrier, is a gap in the literature.

Deterministic and stochastic gradient-like algorithms' convergence for asynchronous distributed computation is calculated and a model is presented in one of the early studies [14]. Numerical simulation of the chemical reactions is studied using a method which adaptively selects stochastic or deterministic calculations. Reaction schemes are partitioned to stochastic and deterministic processes by this switching [15]. A hybrid stochastic-deterministic method based on the studies of Balsa-Canto et al. (1998) [16] and Carrasco and Banga (1998) [17] is presented. The efficient hybrid method uses combination of a sequential adaptive stochastic method [18]. The combination of the deterministic method with the stochastic models for speech enhancement is presented. A developed method is efficient due to the certain speech sounds having a more deterministic character according to the stochastic models [19]. A hybrid stochastic–deterministic algorithm for estimating the parameters of a synchronous generator is presented. The first process is performed by GA as a stochastic method then the final solution is determined by Gauss–Newton method which is a deterministic method [20]. In the same year, studies on the field of combination such as Cottereau et al. (2011) [21] that couples the deterministic model with a stochastic one and Alotto (2011) [22] which combines the techniques for higher performance of differential evolution (DE) method were also carried out. In the following years, hybridization studies on stochastic–deterministic algorithms continued with other studies. Hybrid algorithms are used to develop the input design method [23]

planar covering with ellipses [24] and optimal designing of chemical processes [25]. Especially in chemical process, hybrid approach integrates a feasible point strategy, chaotic dynamics, and the information theory. Collins and Carr (2014) presented a bayesian approach for simultaneous optimization over the space of detections and data associations. For this purpose, a hybrid method which uses reversible jump markov chain monte carlo sampling and deterministic polynomial-time algorithms is developed [26]. A hybrid optimization method based on deterministic algorithm combined with stochastic approaches is presented. A developed method uses direct multisearch method [27] features and has powered by PSO and DE. High accuracy rate has been taken for electromagnetic device optimization using hybrid method [28]. Later, Alb et al. (2016) uses hybrid deterministic-stochastic as pattern search-differential evolution [28] to improve the computational performance for magnetic-assisted medical procedure [29].

In recent years hybrid approaches have continued to develop. A combination of FFM and GA is presented for the video forensics as a nonconvex optimization problem [30]. The mixed integer linear programming is developed for solving the hybrid stochastic-deterministic unit commitment problem [31]. A hybrid approach for reasoning of a large semantic web data is proposed [32]. A hybrid method is proposed for improving the performance of the stochastic and deterministic optimal power flow problem [33]. A hybrid method combined DE with the Broyden–Fletcher–Goldfarb–Shanno quasi-Newton is presented [34]. A semi-autonomous particle swarm optimizer as another mixed method is developed for optimizing multimodal functions. This method uses gradient and diversity control [35].

In this study, a new stochastic search method is proposed as a faster and more effective alternative for classical searching on FFM. GOBC-PA has used as a stochastic-heuristic method for accelerating the L type FFM as a deterministic method. In this process, the role of GOBC-PA is to search the basin regions of FFM. The methods, used in this study, are preferred because of their speed, robustness and popularity. Searching basin regions is still an unsolved issue and the developed method is thought to cover this gap in the literature.

This paper is organized as follows: the first part of the paper is the introduction section and literature are discussed in it. The research methods are given in Section 2. In Section 3 the experimental studies and results are presented. Finally, we discuss the results in Conclusions Section.

2. Research Methods

2.1. Filled function method

The conception of the filled function was first proposed by Geatthe Dundee Biennial Conference on Numerical Analysis in 1983 and finally published in 1987 [36,37]. The FFM that is chosen as a deterministic method aims to construct an auxiliary function, which is called as filled function via the current local minimizer of the global optimization problem [38]. The better points are located on the filled function's basin location. To find the basin region is difficult with conventional methods, and if the step distance is not very small, the area can't be detected by passing [39]. Also, this approach costs lots of time. FFM has used various kinds of problems that need global optimization [40]. L type FFM used in this study because of their advantage to others [41]. L type FFM is given in Eq.(1).

$$L(x) = -\left\{ [f(x) - f(x_1)]^{1/m} + a\|x - x_1\|^2 \right\} \tag{1}$$

In Eq.(1), m is preferred as 3, and a is set to 0.05 as the weight factor.

For better understanding of the iterations of FFM, some examples can be given. Example of 1-dimensional optimization problem with the range of [-15 15] is given in Eq.(2).

$$f(x) = -1 * \left(\frac{2}{(x+8)^2+1} + \frac{5}{(x-1)^2+1} + \frac{8}{(x-8)^2+1} \right) \tag{2}$$

As seen in Eq.(2), optimization problem has a total of 3 local minimums, one of which is the global minimum. FFM iterations of this problem can be seen in Figure 1, Figure 2, and Figure 3.

The first filled function has constructed at -7.995 point of the function in Figure 1. There are only two search directions because of 1-dimensional function. It is easy to search for basin regions using a gradient with an epsilon value very close to zero in 1-dimensional functions while it is more difficult in high degree functions.
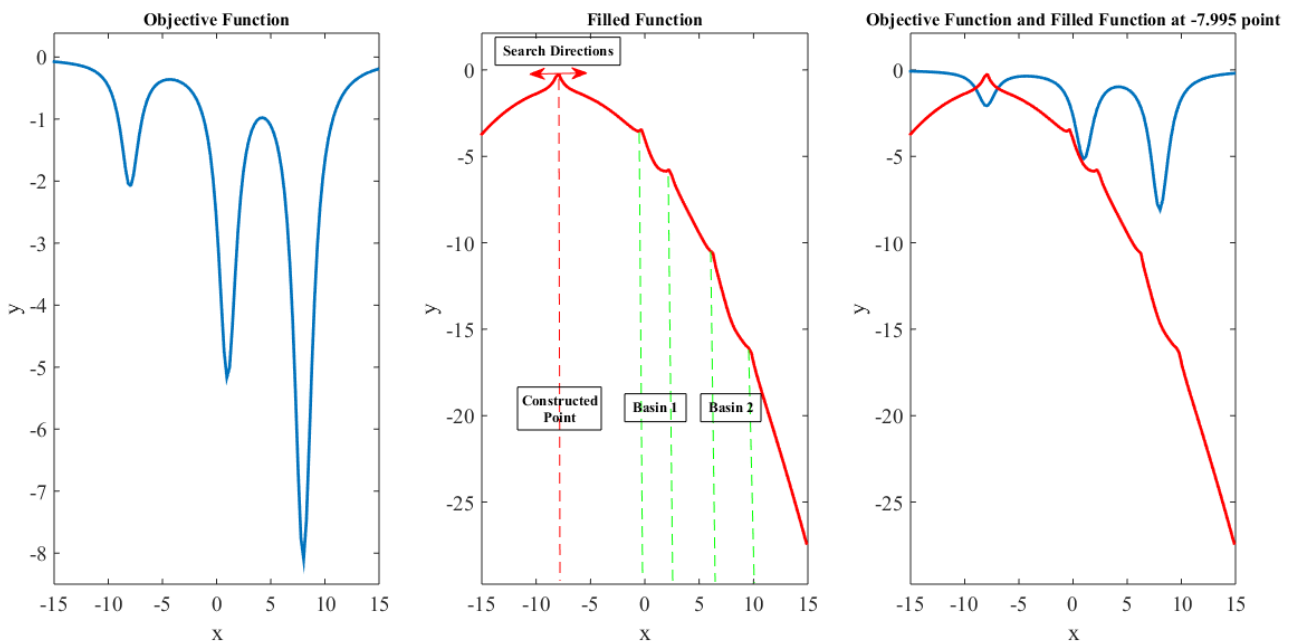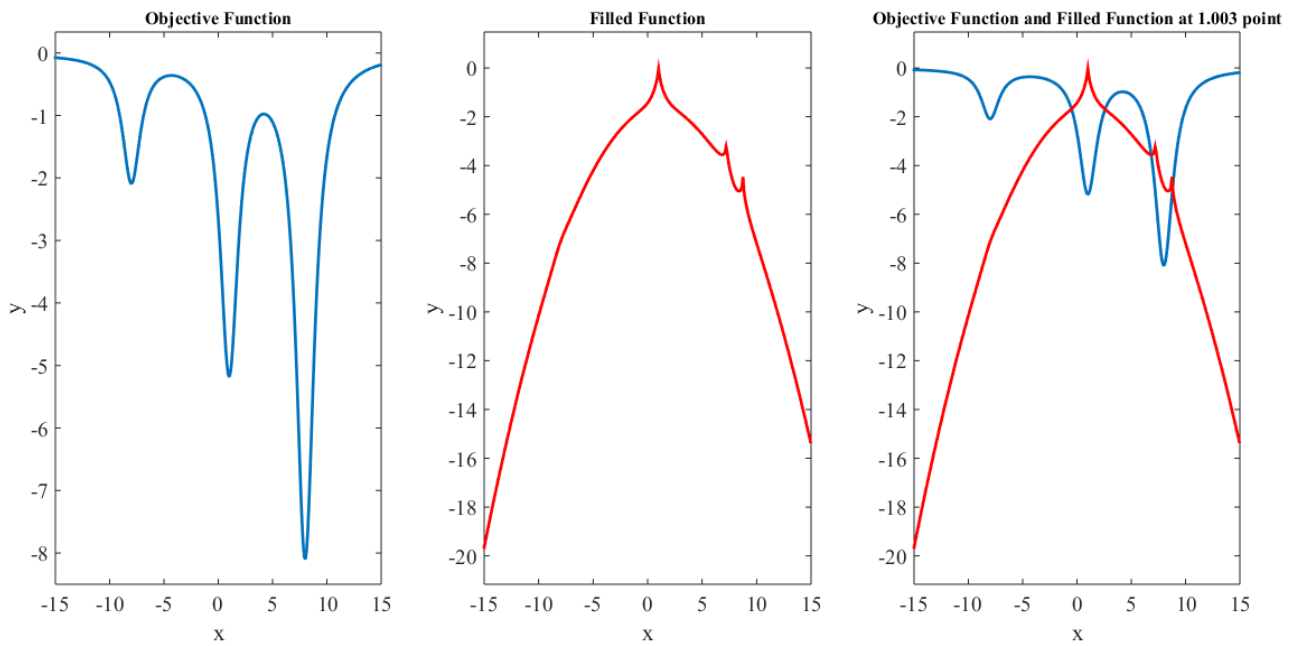

Figure 1. Example of FFM on 1-dimensional optimization problem at first iteration

The second filled function has constructed at 1.003 point of the function in Figure 2. There are not any signals at the range of [-15 0] in Figure 2 because FFM is below the region's local minimum which is located near at -7.995 point of the function. But it can be seen the basin region near the 8.0 point of the function in Figure 2 because there is a local minimum value below the value at FFM construction point.

There aren't any signals on FFM in Figure 3, so it can be accepted as a global minimum at this point of the function and process is terminated.

The level of difficulty of searching the basin regions can be shown by another example of 2-dimensional optimization problem similar to the last one with the range of [-15 15] is given in Eq.(3).

$$f(x,y) = -1 * \left(\frac{2}{(x+8)^2+(y-8)^2+1} + \frac{5}{(x-1)^2+(y+1)^2+1} + \frac{8}{(x-10)^2+(y+10)^2+1} \right) \tag{3}$$
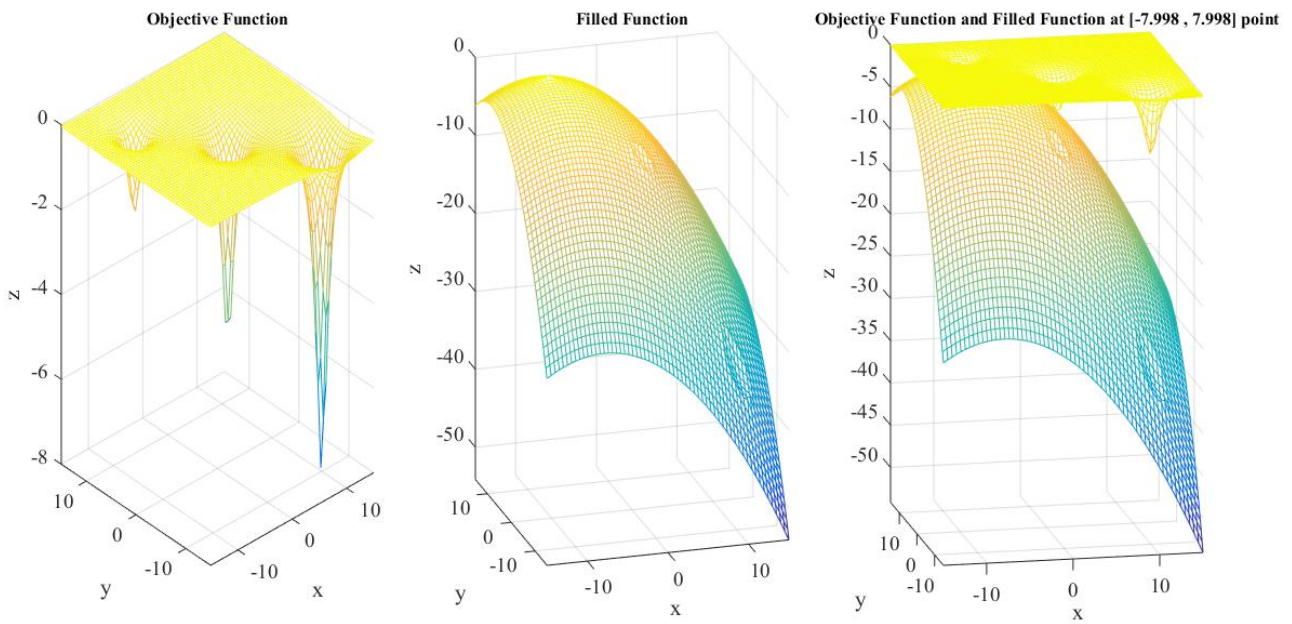
Figure 2. Example of FFM on 1-dimensional optimization problem at second iteration
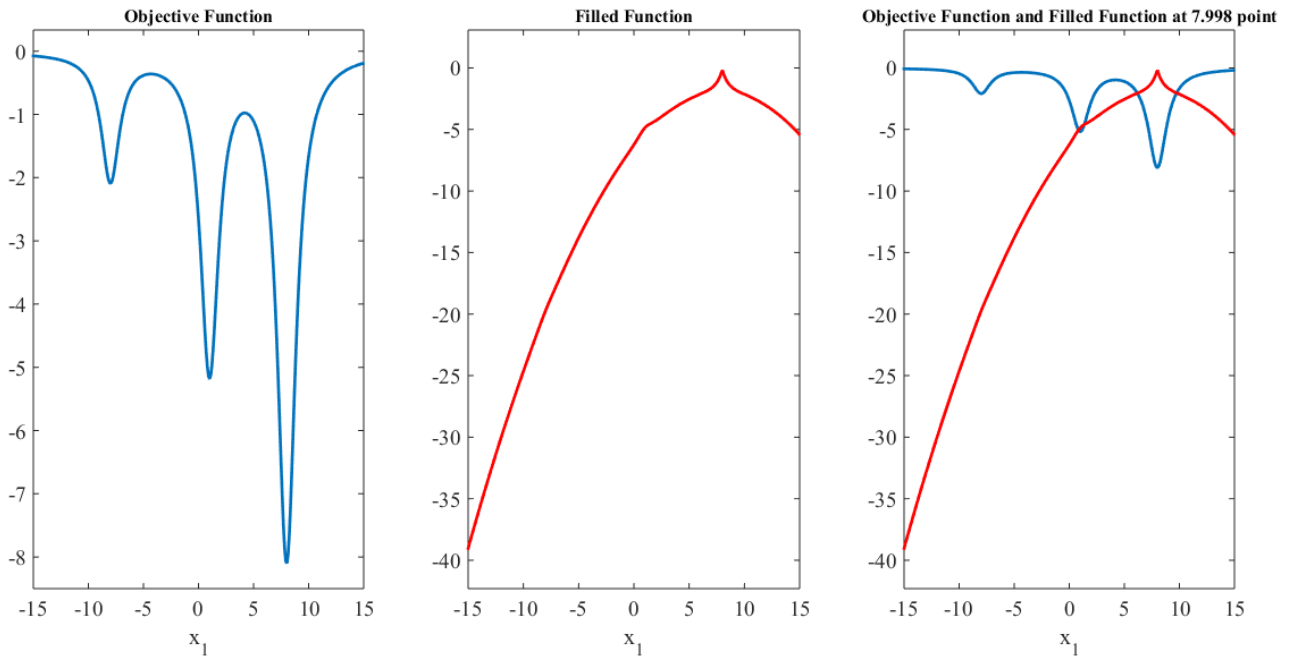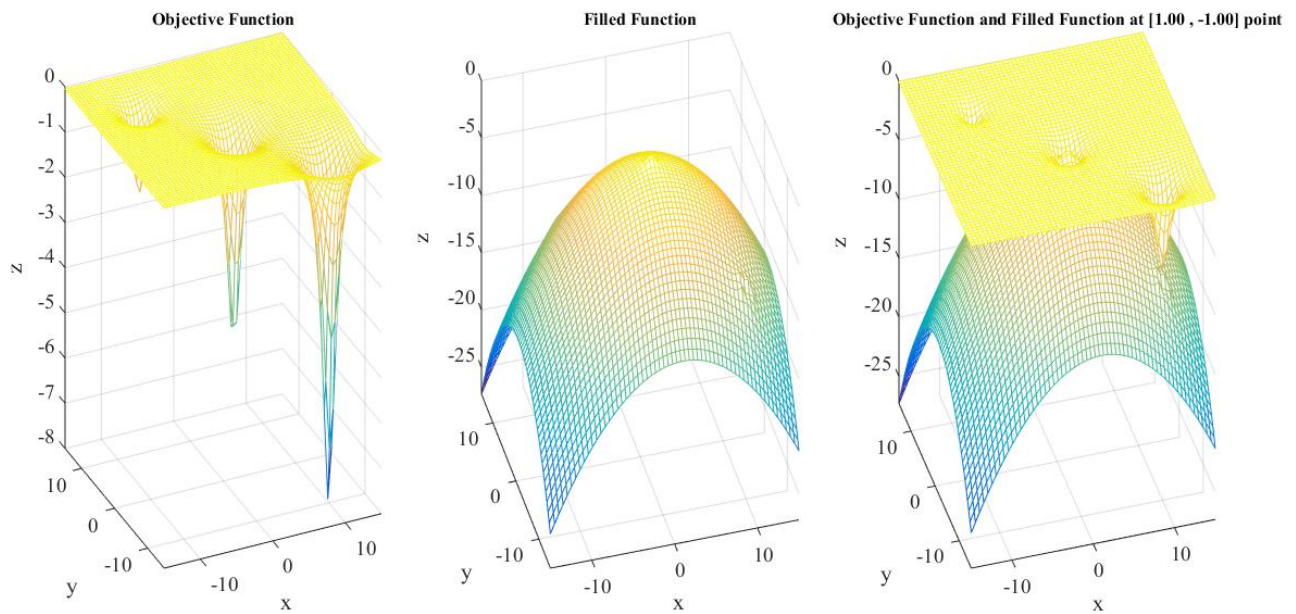

Figure 3. Example of FFM on 1-dimensional optimization problem at third iteration

As seen in Eq.(3), 2-dimensional optimization problem has a total of 3 local minimums, one of which is the global minimum. FFM iterations of this problem can be seen in Figure 4, Figure 5, and Figure 7.

The first filled function has constructed at [-7.998, -7.998] point of the function in Figure 4. There are unlimited search directions occurred because of 2-dimensional function. To search basin regions using epsilon valued gradient can be more difficult on these functions. Searching direction degrees and step sizes must be carefully determined.
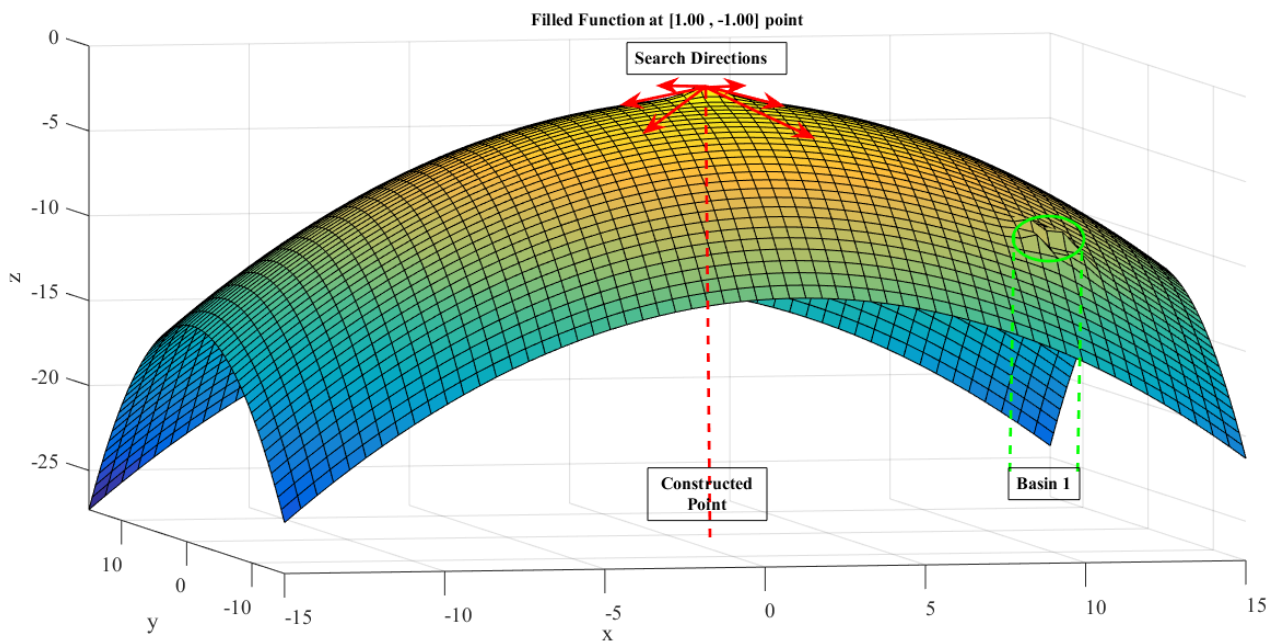
Figure 4. Example of FFM on 2-dimensional optimization problem at first iteration

Figure 5. Example of FFM on 2-dimensional optimization problem at second iteration

The second filled function has constructed at [1, -1] point of the function in Figure 5. There is not any signal from the previous basin region because of FFM structure. Searching directions at this constructed point can be seen more easily in Figure 6.

Searching this basin region in Figure 6 is difficult with conventional methods, and if the step distance is not very small, the area can't be detected by passing. Also, this process costs lots of time.

Figure 6. Searching directions of FFM's basin regions on 2-dimensional optimization problem
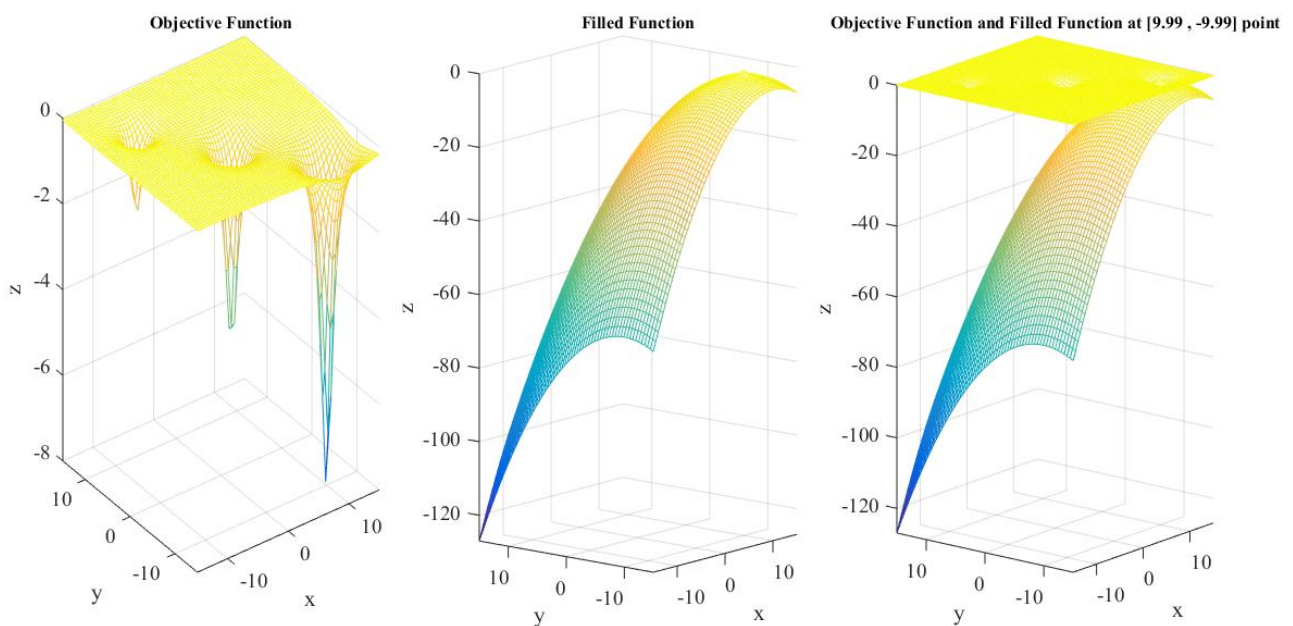
Figure 7. Example of FFM on 2-dimensional optimization problem at third iteration

There aren't any signals on FFM in Figure 7, so it can be accepted as the global minimum at this point and process is terminated.

2.2. GOBC-PA

GOBC-PA is a stochastic-heuristic method. The clustering technique and parabolic approximation are used in this method. In each step, the data are clustered and the cluster centers denote the local optimums. These centers are adapted with locally fitted parabolas [10].

GOBC-PA is superior to other stochastic algorithms in terms of speed. Clustering of populations using their locations and objective function values can be seen in Figure 8.
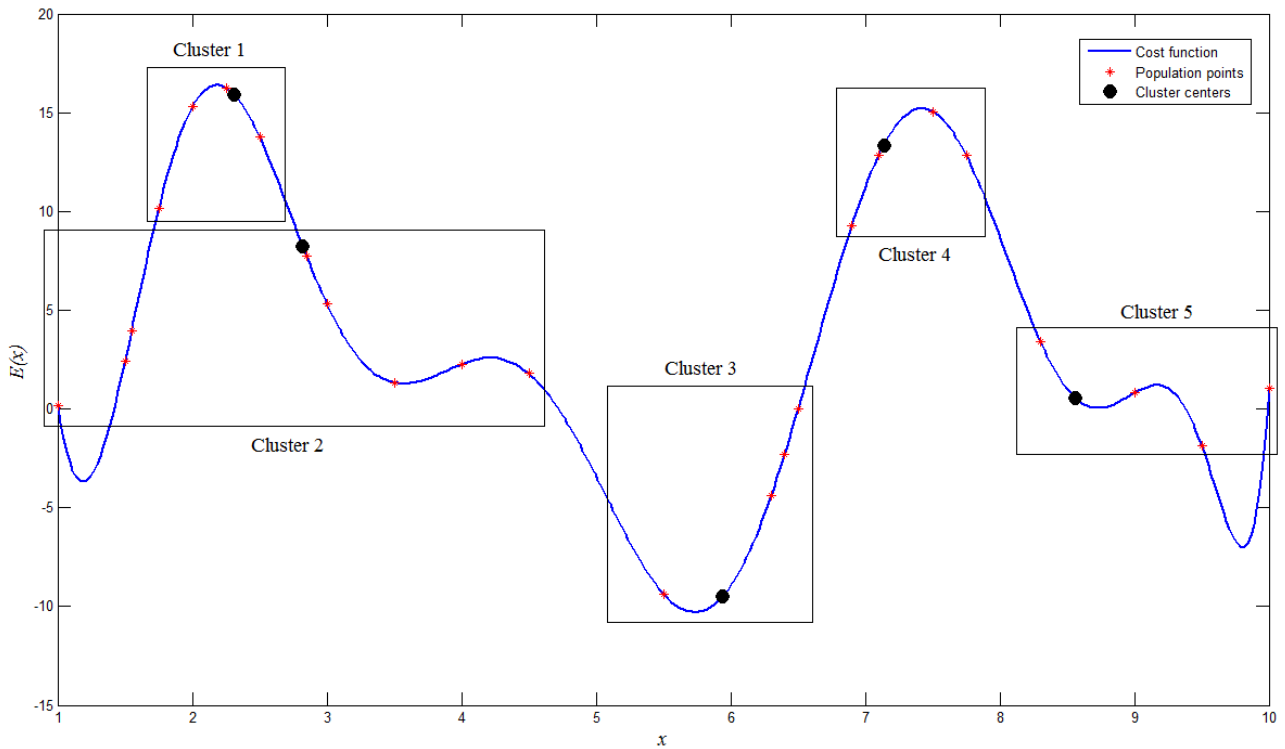
Figure 8. Clustering process using function values on GOBC-PA [10]

Curve fitting process to the clusters is performed using second order polynomials and can be seen in Figure 9. The vertex points of polynomials can stay on maxima or minima of the objective function [10].
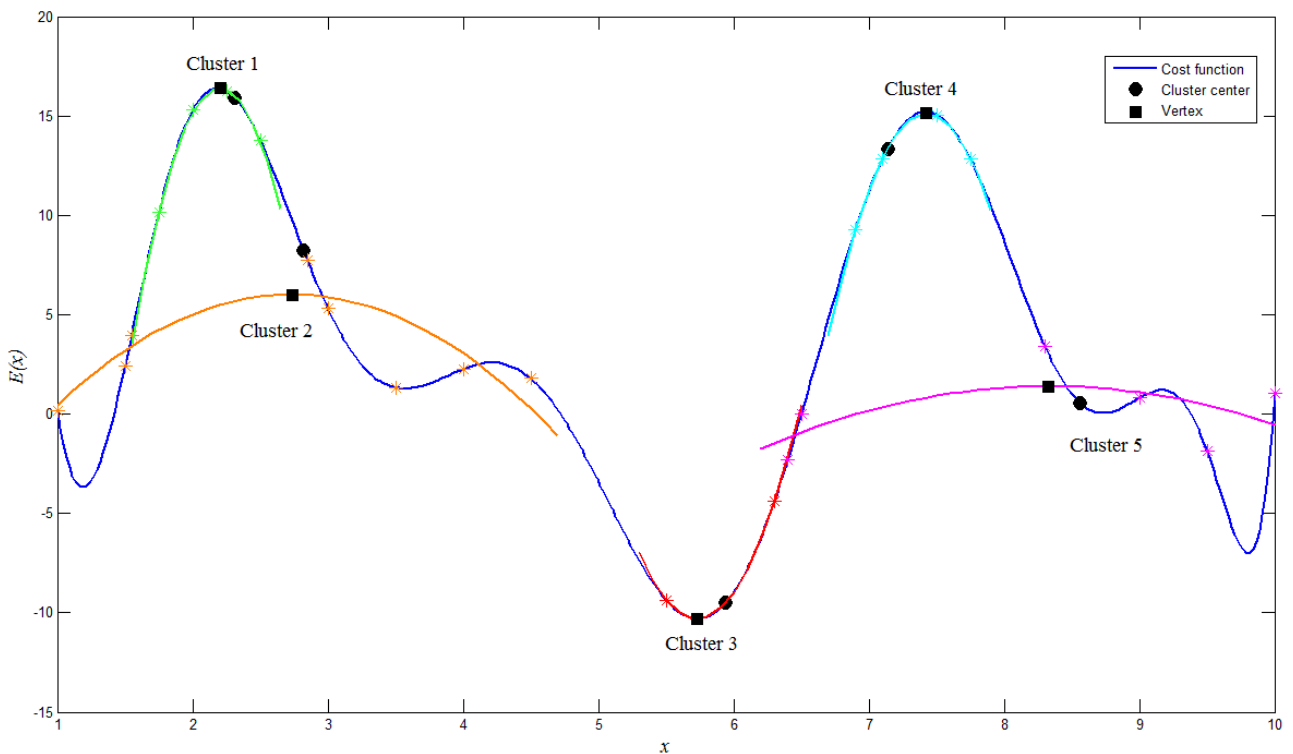
Figure 9. Curve fitting process to the clusters on GOBC-PA [10]

In this study, GOBC-PA method's objective function is determined as epsilon value of the gradient that gives the location of basin region. So, only purpose of the stochastic method is to find basin

region, not to find global optimum. The role of finding the global minimum has been left to the deterministic method. Parameters of GOBC-PA are set to 1000 and 60 as iteration number and population size, respectively.

3. Experimental Studies

The developed search method has been tested against classical filled function method with 11 benchmark functions that are given in Table 1. All functions have 2-dimension and process is repeated 10 times. Classic FFM searches the basin every 3.6 degrees with [Range/2000] foot step. FFM is terminated after 10 basin jumps.

Table 1. Some 2-dimensional benchmark functions.

Function Name	Function	Range	Optimal
Shekel's Foxholes	$\left[\dfrac{1}{500} + \sum_{j=1}^{25} \dfrac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6}\right]^{-1}$	[-65536, 65536]	1
Sixhump Camelback	$\left(4 - 2.1x_1^2 + \dfrac{x_1^4}{3}\right)x_1^2 + x_1 x_2 + (4x_2^2 - 4)x_2^2$	[-5, 5]	-1.031628
Branin	$\left(x_2^2 - \dfrac{5.1}{4\pi^2}x_1^2 + \dfrac{5}{\pi}x_1\right)^2 + 10\left(1 - \dfrac{1}{8\pi}\right)\cos(x_1) + 10$	[-5, 10] [0, 15]	0.398
Goldstein-Price	$[1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_2^2 - 14x_2 + 6x_1 x_2 + 48x_2 - 36x_1 x_2 + 27x_2^2)]$	[-2, 2]	3
Beale	$[1.5 - x_1(1 - x_2)]^2 + [2.25 - x_1(1 - x_2^2)]^2 + [2.625 - $	[-4.5, 4.5]	0
Easom's	$-\cos(x_1)\cos(x_2)\exp\left(-(x_2 - \pi)^2 - (x_1 - \pi)^2\right)$	[-100, 100]	-1
Matyas	$0.26(x_1^2 + x_2^2) - 0.48x_1 x_2$	[-10, 10]	0
Schaffer	$0.5 + \dfrac{\sin^2\sqrt{x_1^2 + x_2^2} - 0.5}{1 + 0.001(x_1^2 + x_2^2)^2}$	[-10, 10]	0
Rastrigin	$x_1^2 + x_2^2 - \cos 18x_1 - \cos 18x_2$	[-1, 1]	-2
Bohachevsky	$x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1) - 0.4\cos(4\pi x_2) + 0.7$	[-100, 100]	0
Trefethen's Problem	$\exp(\sin 50x_1) + \sin(60e^{x_2}) + \sin(70\sin(x_1)) + \sin(\sin($	[-6.5, 6.5]	-3.3068686

The results of classic FFM and GOBC-PA FFM are given in Table 2. The main performance on functions is determined using the absolute error.

In Table 2, mean error indicates the average of 10-times run, elapsed time indicates the total elapsed time until the algorithm terminated. When the obtained results are examined in Table 2, it is seen that the new searching approach has superiority over the mean error, elapsed time and standard deviation values according to the classic search on FFM. Also, it can be seen GOBC-PA helps FFM to be faster than the classic search algorithm according to the elapsed times.

Table 2. Results of classic FFM and GOBC-PA FFM on benchmark functions.

Functions	Classic Filled Function			GOBC-PA Filled Function		
	Mean Error	Standard Deviation	Elapsed Time (Second)	Mean Error	Standard Deviation	Elapsed Time (Second)
Shekel's Foxholes	399.398	209.979	40.77	**0**	0	35.12
Sixhump Camelback	4.853e-08	1.067e-09	104.22	**4.810e-08**	5.222e-10	4.29
Branin	3.528	4.106e-10	7.36	3.528	1.294e-10	4.03
Goldstein-Price	4.347e-07	2.837e-07	26.58	**6.471e-08**	2.046e-07	2.61
Beale	0.183	0.237	27.71	**0.045**	0.145	3.03
Easom's	2.444e-09	7.979e-10	37.20	**2.031e-09**	1.065e-09	2.92
Matyas	6.337e-11	5.095e-11	10.60	**0**	0	2.92
Schaffer	**1.324e-09**	9.039e-10	44.94	1.382e-09	7.628e-10	3.46
Rastrigin	0.641	0.867	18.05	**2.883e-07**	3.937e-07	3.41
Bohachevsky	1.953e-08	1.453e-08	20.90	**0**	0	6.49
Trefethen's Problem	2.602	2.773	34.97	**0.839**	0.687	4.72

4. Conclusions

In this study, the results show that using stochastic method on searching the FFM can be more successful in finding the global minimum value according to classic deterministic method.

FFM as a deterministic method has various limitations such as searching basin regions. GOBC-PA FFM that is developed in this study can close this gap in the literature.

The superiority of GOBC-PA is to be really fast in training process. For this reason, GOBC-PA FFM will be a good option for the limited time problems. Today, Industry 4.0 is rising all over the world and companies are trying to get places in markets for themselves with technological products based on computer science. GOBC-PA FFM can be useful on any mathematical or industrial systems which are using deterministic or stochastic methods as an artificial intelligence. Therefore, the developed method has an important place in terms of speed and accuracy in today's Industry 4.0.

In future studies, it can combine the deterministic and stochastic methods completely, not only searching the basin regions. Another study can compare the performance of hybrid methods against the heuristic algorithms.

Acknowledgements

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