ARGUMENTATION-BASED LEARNING: AN EXAMPLE OF MATHEMATICAL QUESTIONS THROUGH ONLINE INTERACTIONS AMONG PROSPECTIVE TEACHERS

ARGÜMANTASYON TABANLI ÖĞRENME: ÖĞRETMEN ADAYLARININ MATEMATİK SORULARI EŞLİĞİNDE ONLINE ETKİLEŞİMLERİ[1]

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Abstract: Argumentation is a process in which whether arguments are associated with data is constructed with warrants that they are based on. In this process, mathematical arguments are of vital importance especially in problem solving. Producing valid arguments or proofs and criticizing the arguments are inseparable parts of doing math. Therefore, if there is nothing done to develop reasoning skills, mathematics will become just following a sequence of operations and copying the examples without thinking about their meanings. It is necessary to direct argumentation-based learning environments well because students may find it hard to understand given arguments or have certain challenges due to misunderstandings and misunderstood arguments when sharing their ideas with other students or during the stage of invalidating the ideas. This is why good direction of the argumentation process depends on living and experiencing the process itself. This study is an application of teacher education developed in the light of the idea that prospective teachers need to experience their own processes of argumentation so that they could handle the argumentation-based learning approach in their future classrooms. Traces of pedagogical content knowledge exhibited by the prospective teachers in the online argumentation activities which were designed through possible student questions were examined, and the strengths and weaknesses of the learning environment were

Özet: Argümantasyon; iddiaların dayandırıldığı gerekçeler belirtilerek veriler ile ilişkili olup olmadığının yapılandırıldığı süreçtir. Bu süreçte matematiksel argümanlar özellikle problem çözümlerinde hayati bir öneme sahiptir. Geçerli argümanlar ya da ispatlar üretme ve argümanların kritik edilmesi, matematik yapmanın ayrılmaz parçasıdır. Bu nedenle, muhakeme becerileri öğrencilere kazandırılmazsa matematik, bir işlem dizisini takip etmek ve anlamını düşünmeden örnekleri taklit etmek olur. Argümantasyona dayalı öğrenme ortamlarının iyi yönlendirilmesi gerekmektedir. Bunun nedeni, öğrencilerin süreçte verilen argümanları anlamada güçlük çekebilmesi, öğrencilerin diğer öğrencilerle düşüncelerini paylaşmada ve birbirlerinin düşüncelerini çürütmede yanlış anlamalar yaşaması olabilir. Bu yüzden argümantasyon sürecinin iyi yönetilmesi, bu süreci yaşamaya ve deneyimlemeye dayalı olarak gerçekleşir. Bu çalışma, öğretmen adaylarının sınıflarında argümantasyon öğrenme yaklaşımını etkin biçimde uygulayabilmeleri için öncelikle kendilerinin argümantasyon sürecini yaşamaları gerektiği düşüncesinden yola çıkılarak geliştirilmiş bir öğretmen eğitimi uygulamasıdır. Çalışmada öğrenciden gelebilecek sorular üzerinden tasarlanmış online argümantasyon etkinliklerinde öğretmen adaylarının sergiledikleri pedagojik alan bilgisi izleri incelenmiş ve öğrenme ortamının güçlü ve zayıf yanları araştırılmıştır. Sonuç olarak, online

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investigated in the study. Consequently, online argumentation method was found to have positive impacts on the improvement of prospective teacher's pedagogical content knowledge and their own learning. It is anticipated that these presented preservice education components will shape future studies to be carried out in the field of teacher education.

Keywords: *Online argumentation, student questions, pedagogical content knowledge, mathematics education, teacher education*

argümantasyon yönteminin öğretmen adaylarının pedagojik alan bilgisi gelişiminde ve kendi öğrenmeleri üzerindeki olumlu etkileri ortaya çıkmıştır. Sunulan bu hizmet öncesi eğitim bileşenlerinin öğretmen eğitimi alanında gerçekleştirilecek gelecekteki olası çalışmalara yön vereceği düşünülmektedir.

Anahtar Sözcükler: *Online argümantasyon, öğrenci soruları, pedagojik alan bilgisi, matematik eğitimi, öğretmen eğitimi*

Introduction

According to the constructivist approach on which the philosophy of several curricula is based on, learning is an active process through which learners acquire information, interpret and make meaning of the information with their prior knowledge and experiences. The constructivist approach emphasizes the importance of research- and questioning-based learning. Such learning makes learners active and guides them toward question asking, problem solving and critical thinking. Argumentation (scientific discussions) has been one of the instructional techniques that support such a student-centered learning in recent years. According to the Argumentation-Based Scientific Learning approach, students form the information by asking questions, producing arguments and reinforcing these arguments in a learning environment (Keys, Hand, Prain & Collins, 1999). It is stated in the literature that environments of argumentation have a positive effect on learning and engagement because they offer the opportunity to learn by sharing the information with peers and teachers (Günel, Kıngır & Geban, 2012; Keys, Hand, Prain & Collins, 1999). Moreover, argumentation-based learning environments feature developmental characteristics of concepts and support social learning (Driver, Newton & Osborn, 2000).

Argumentation is a process in which whether arguments are associated with data is constructed with justifications that they are based on (Toulmin, 2003). According to Kuhn (1991), how individuals use statements supporting or rebutting the perspective of solving a problem refers to the act of doing argumentation on their own. In this sense, argumentation and learning are in fact interrelated and integral parts of the thinking process (Kuhn, 1991). In the educational literature, Toulmin's model is utilized for exploring and analyzing what kind of a discussion environment is created in the classroom (Wood, 1999) and how learning is progressing (Yackel, 2001). According to Toulmin (2003), a well-built, logical argument involves three main, interrelated elements: data, warrant, and conclusion/assertion. Three auxiliary components can be added to arguments. These components are not required but may empower the assertion. The Model Qualifier (M) component is added to the assertion in the form of adverbs (absolutely, probably, quite likely, *etc.*) and expresses the reliability degree of the conclusion. The Backing (B) component reinforces the warrant. Backing is needed where the strength and validity of the warrant cannot be clearly observed. With the addition of auxiliary components to the main components, Toulmin's model has a six-component structure. Conclusion/assertion (C) is the proposition which individual wants to argue the addressee into; it is the conclusion required to be confirmed. Data (D) is what confirms the Conclusion (C); in other words, the evidence of conclusion. Warrant (W) is the rule of concluding. This rule helps concluding from the data. It is the component which indicates that conclusion/assertion is valid and serves as a bridge between the data and conclusion. Six-component structure of Toulmin's model is presented in Figure 1.

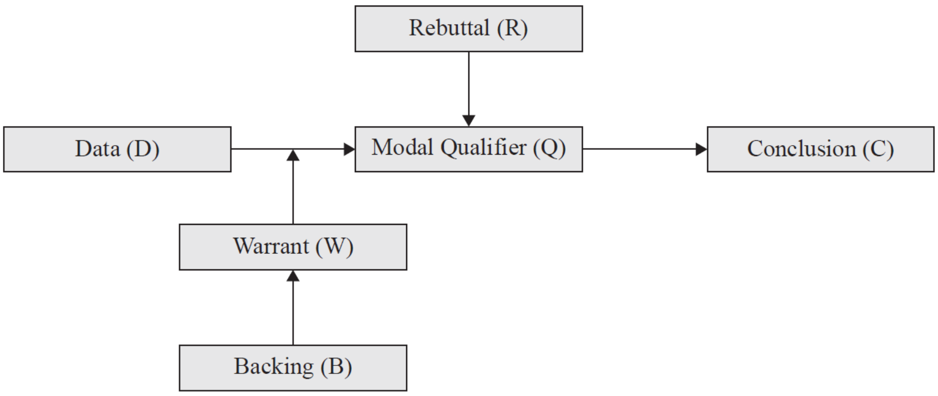


Figure 1. The Toulmin's model of argument (Toulmin, 2003, p. 97)

Inglis, Mejia-Ramos, Simpson (2007) stated that mathematical arguments are of vital importance especially in problem solving. The most studies using the argumentation method are focused on the science education, however, research has also been conducted in mathematics education (Brown & Reeves, 2009; Krummheuer, 2007). Brown and Redmod (2007) stated that use of cooperative argumentation increases students' desire to learn mathematics, and it is required to implement cooperative argumentation studies to enhance professional development in mathematical learning domains. Krummheuer (2007) emphasized that mathematical learning of students depends on their engagement in group discussions. Brown and Reeves (2009) stated that argumentation method helped students develop their mathematical skills and understandings, levels of using mathematical operations in problem solving, levels of expressing the problems mathematically and skills of developing new approaches in problem solving.

Producing valid arguments or proofs and criticizing the arguments are inseparable parts of doing math (Ross, 1998). If students cannot be equipped with reasoning skills, mathematics cannot go beyond following a sequence of operations and copying the examples without thinking about their meanings (Ross, 1998). Thus, it is necessary to direct argumentation-based learning environments well because students may find it hard to understand given arguments or have certain challenges due to misunderstandings and misunderstood arguments when sharing their ideas with other students or during the stage of invalidating the ideas. It is therefore possible to direct the argumentation process well by living and experiencing the process itself.

Great responsibility undoubtedly falls to mathematics teacher in bringing these skills to the students. Several studies have emphasized the efficient preservice and in-service teacher education for reinforcing the argumentation (Zeidler, 1997; Newton, Driver & Osborne, 1999; Driver, Newton & Osborne, 2000; Erduran & Jimenez-Aleixandre, 2007). This study's instruction was designed online in the consideration that educational technologies supported, argumentation-based instructional applications. The studies showed that argumentation-based instructional applications have been increasing in Finland (Kiili, 2013), Norway (Ludvingsen, 2012), Australia (Butchart, Forster, Gold, Bigelow, Korb, Oppy & Serrenti, 2009; Davies, 2009), United States of America (Hoffman, 2008) and United Kingdom (Okada, 2008) in recent years and such applications had positive impacts on the development of students' cognitive, affective and psychomotor skills. Teachers are needed to be informed of how to evaluate mathematical arguments, to determine the accuracy of mathematical propositions and to prove the accuracy of mathematical arguments which they think of as accurate. Pedagogical content knowledge (PCK) comes into play here. Teacher's pedagogical content knowledge is the body of knowledge and skills which a teacher will call on when a subject or concept is to become learnable for students (Baki, 2018). This is indeed how the teacher guides the student properly and effectively to acquire and embrace information and put it into practice.

Shulman (1987) defined PCK as follows: "It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction (p.8). PCK defined as the knowledge used for transferring content to students in more understandable forms (Geddis, Onslow, Beynon and Oesch, 1993; Grossman 1990; Marks 1990; Park and Oliver, 2008; Shulman 1986, 1987). In this regard, the development of PCK involves a shift in teachers' understanding "from being able to comprehend subject matter for themselves, to becoming able to elucidate subject matter in new ways, reorganize and partition it, clothe it in activities and emotions, in metaphors and exercises, and in examples and demonstrations, so that it can be grasped by students" (Shulman 1987,

p. 13). A large number of scholars have worked on PCK and tried to redefine the concept of PCK by modifying Shulman's components, but the focus of those various definitions is the idea that the transformation of subject matter knowledge for the purposes of teaching is the center of PCK. (e.g.; Banks, Leach and Moon, 2005; Cochran, deRutier and King, 1993; Fernandez-Balboa and Stiehl,1995; Grossman 1990; Hashweh 2005; Koballa, Graber, Coleman and Kemp, 1999; Loughran, Berry and Mulhall, 2006; Magnusson, Krajcik and Borko, 1999; Marks,1990; Park and Oliver 2008; Veal and MaKinster,1999). Grossman (1990) broadened the concept by defining four central components of PCK: (a) knowledge and beliefs about the purposes for teaching a subject, (b) knowledge of students' understanding, conceptions, and misconceptions of particular topics in a subject matter, (c) knowledge of curriculum and curriculum materials, and (d) knowledge of instructional strategies and representations for teaching particular topics. Park and Oliver (2008) elaborated Grossman's conceptualization and identified five components with adding "knowledge of assessment of student understanding." Although scholars handle PCK with different components, most scholars agree on Shulman's (1986) two key components of PCK: (a) knowledge of instructional strategies incorporating representations of subject matter and understanding of specific learning difficulties and (b) student conceptions with respect to that subject matter. In this study, considering the components accepted by all researchers and considering the compatibility with the learning environment, the focus in terms of PCK is 'Are students guided to right associations and reasoning with proper organizations according to student-teacher dialogs? Are students being able to be guided to desired association via proper methods, strategies, demonstrations, analogies and examples in accordance with the nature of the subject or concept?'

This study is an application of for teacher education developed in the light of the idea that prospective teachers need to experience their own processes of argumentation so that they could handle the argumentation-based learning approach in their future classrooms. Traces of pedagogical content knowledge (PCK) exhibited by the prospective teachers were observed in the online argumentations which were designed through possible student questions, and the strengths and weaknesses of the learning environment were investigated in the study.

Method

Participants

The participants of the study were 56 third-year prospective teachers attending an Elementary Mathematics Education program. In the study, due to the limitations in terms of time and labor conditions, appropriate sampling method was preferred in determining the sample. In accordance with participants' background within the scope of pedagogical content knowledge, the components which are: multiple representations of concept, student difficulties regarding concepts and misconceptions, concept assessment-evaluation, and teaching concepts in the curriculum were handled in the elective Mathematics Curriculum course. The preservice teachers performed basic computer-software functions in programs in the courses of Applications of Technology in Education and Instructional Technologies and Material Development courses. After addressing the methods and strategies for concept instruction in the Teaching Methods I course where the application was performed as the study topic, the preservice teachers performed microteachings regarding the goals specified in the curriculum.

Research Design and Procedure

This study research design is explanatory case study. This type of case study would be used if you were seeking to answer a question that sought to explain the presumed causal links in interventions (Yin, 2003). Explanatory case studies are used to provide information about a situation, to make unfamiliar situations familiar, to reveal the impact of interventions through qualitative data (observations, interviews, etc.), and to explain the connections to real-life situations (Baxter & Jack, 2008; Davey, 1991). In evaluation language, the explanations would link program implementation with program effects (Yin, 2003). The study, 8-week online argumentation contents were created in compliance with PCK. Course content of Teaching Methods II to be lectured following each weekly argumentation was shaped according to the discussions. Flow of the application process is presented in Figure 2.

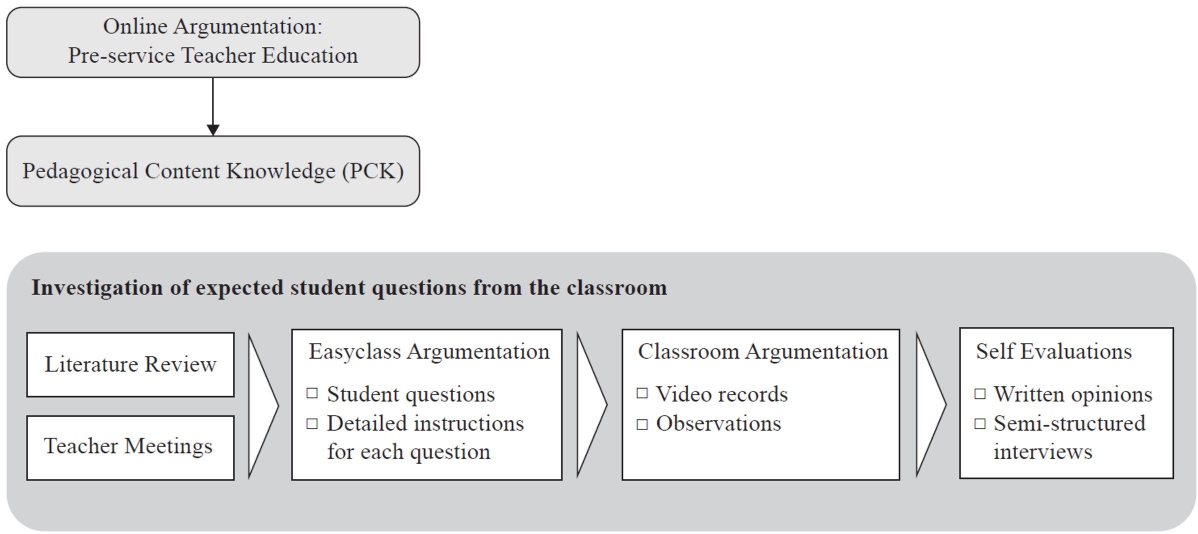


Figure 2. Online argumentation process in preservice teacher education

As seen in Figure 2, pedagogical content knowledge (PCK) was constructed for the discussions of possible student questions. The prospective teachers decided the questions about mathematical concepts that could be asked by students through literature review and teacher interviews according to their mathematical learning levels (6^{th} grade to 8^{th} grade). The literature review includes the related articles, books, dissertations etc. based on the learning areas.

The conceptual questions were opened up for discussion via Easyclass digital learning environment (http://www.beyazpano.com) (Turkish version) and examined thoroughly by the researchers. Conceptual questions can be defined as questions that provide an opportunity to discuss the concept and its other related concepts, as well as to discuss the relationship of the concept with non-textual concepts. Next, explanations that can be presented for these questions and new questions that could be asked by students were shared by the prospective teachers in the Discussions section of Easyclass. The best possible styles of explanation were provided in accordance with these discussions in the Teaching Methods II course at the end of the Online Argumentation process. The classroom discussions were carried out within the framework of both main question and the best possible answers to new questions. The classroom discussions were videotaped, and observations were noted down. Opinions and ideas transferred into the digital environment were also recorded. Finally, written opinion form and semi-structured interviews were utilized for the participants to evaluate the process (the questions based on effects of argumentation process with student questions on mathematics teaching –learning and evaluation on the stages of implementation). The semi-structured interviews were performed with 3 prospective teachers randomly selected from each category of (poor-moderate-good) in academic average classification. PCK learning traces were tracked via Easyclass discussions, classroom discussion records and opinions.

Data Collection and Analysis

PCK learning traces were tracked via Easyclass discussions, classroom discussion records and opinions on participants' self-learning. Observation notes were taken in the course as the data collection instrument, and the written interview form and semi-structured interview were utilized at the end of the study.

Elements of argument were used both in the online argumentation via Easyclass and in classroom discussion was analyzed according to Toulmin's model. While one of the prospective teachers assumed the role of discussion leader, other prospective teachers presented questions that could be asked by students and possible explanations to be provided by teacher in both discussion environments. The leader prospective teacher focused on answering all questions in the discussions along with their instructional explanations. Moreover, the leader and other prospective teachers argued to decide the best possible instructional explanation. Instructional explanation that the most understandable without creating situations could cause misconceptions in students.

In the educational literature, Toulmin’s model is utilized for exploring and analyzing what kind of a discussion environment is created in the classroom (Wood, 1999) and how learning is progressing (Yackel, 2001). In this study, modal qualifier component of Toulmin’s model was not included in the analysis as it was not used by the prospective teachers in general. It was considered that a five-component structure of Toulmin’s model would be more appropriate to analyze argumentations of the prospective teachers. The version of Toulmin’s model used in the study is shown in Figure 3.

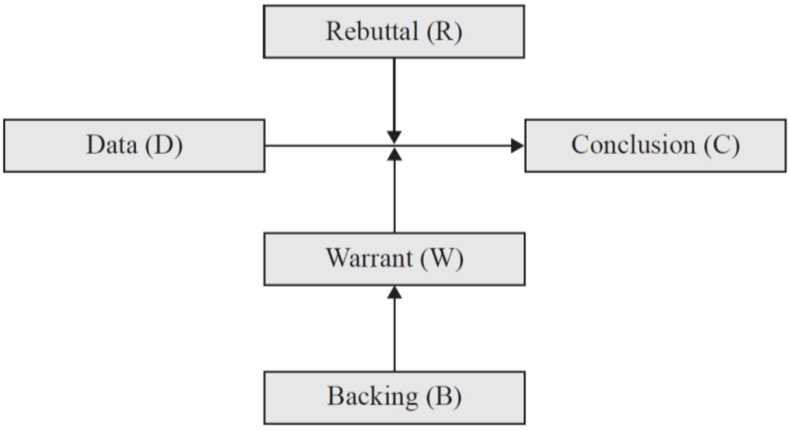


Figure 3. Adapted version of Toulmin's model of argument (Toulmin, 2003, p. 97) used in the study

An exemplary analysis of this study is presented below. The prospective teacher who was the discussion leader was encoded as “LTC” and other prospective teachers as “TC” and the researchers as “RE”.

> ***LTC:*** *The class could ask a question like “Why is the multiplication of two negative numbers a positive?” You produced nice ideas on Easyclass, thank you. Let us take it from here again.*
>
> ***TC1:*** *Teacher, I think it does not make any sense, multiplication of two negative numbers is a negative.*
>
> ***LTC:*** *Why do you think that?*
>
> ***TC1:*** *Because if multiplication is the shortcut of addition and both are negative, it is negative too.*

LTC: *But here, we are talking about adding (-2) pieces of (-3).*

TC2: *Why is it positive then, teacher?*

LTC: *For instance, we know 2 x (-5) = -10 because sum of 2 pieces of (-5) is -10. Let us make this pattern: (writes the operations on the whiteboard)*

2 x (-5) = -10	
1 x (-5) = -5	*: Makes the explanation "1 piece of -5 is -5, is not it, kids?"*
0 x (-5) = 0	*: "We know zero is the absorbing element, then the result is zero."*
(-1) x (-5) = ?	*: Asks the questions "Let us think together, does the second factor change?"*

TC3: *The multiplicand is always (-5), the multiplier is decreasing one by one.*

LTC: *How is the result part changing?*

TC3: *It is increasing five by five.*

LTC: *Accordingly, who can predict the result of (-1) x (-5) = ?*

TC4: *The result of the previous operation was zero; then, I add 5 to zero, and the result is 5.*

TC5*: Teacher, now the multiplication of two negative numbers is a positive.*

LTC: *You found it nice, you figured it out very nice, kids...*

TC6: *Teacher! It can be made discovered in a more different way, actually.*

RE: *How? What would be your explanation as a teacher?*

TC6: *I would explain it this way: (by coming to the whiteboard and explaining)*

2+ (-2)=0	*: We know the addition of a number to its negative is zero; now, let us multiply both sides of this equation by (-2).*
(-2) x [2 + (-2)]=(-2) x 0	*: With the distributive property of multiplication over addition...*
(-2) x 2 + (-2) x (-2)= 0	*: Here, we would say "we know (-2) x 2 = (-4), or if we started with 1+ (-1) = 0 and multiplied both sides*

	by (-1), *for (-1) x 1 = (-1), the result would be (-1) because the multiplication of a number by 1 is that number itself." About our question,*
(-4) + (-2) x (-2)= 0	*: Now, let us add 4 to both sides of the equation.*
(-2) x (-2)= 4	*They would see that the multiplication of two negative numbers is a positive.*

TC1: *I would prefer the first explanation if I were a student.*

TC4: *I would explain the both ways and think that some could comprehend one while others could comprehend the other.*

LTC: *Cansu's (TC6) explanation is effective, too. Like our friend said, helping students find it with both ways might be more effective so that they can make meaning of it.*

………….

The analysis of above argumentation: whether the argumentation took place or not was analyzed according to Toulmin's model.

D*: Is the multiplication of two negative numbers positive or negative?*

C: *The multiplication of two negative numbers is definitely positive*

W_1:

2 x -5 = -10	
1 x (-5) = -5	*: 1 times -5 is -5.*
0 x (-5) = 0	*: Because zero is the absorbing element, the result is zero.*
(-1) x (-5) = ?	*: While the multiplicand does not change and the multiplier decreases by 1, the result increases by 5; then, the result is 5.*

W_2:

2 + (-2) =0	*: The addition of a number to its negative is zero. Let us multiply both sides of this equation by (-2).*
(-2) x [2+(-2)]=(-2) x 0	*: With the distributive property of multiplication over addition...*
(-2) x 2+(-2) x (-2) = 0	*: Here, we would say "we know (-2) x 2 = (-4), or if we started with 1+ (-1) = 0 and multiplied both sides by (-1),*

for (-1) x 1 = (-1), the result would be (-1) because the multiplication of a number by 1 is that number itself."

About our question:

(-4) + (-2) x (-2) = 0	*: Now, let us add 4 to both sides of the equation.*
(-2)x (-2) = 4	*: They would see that the multiplication of two negative numbers is a positive.*

***R:** Teacher, I think it does not make sense. The multiplication of two negative numbers is a negative. Because if multiplication is the shortcut of addition and both are negative, it is negative too.*

***V:** Algebraic rules and patterns.*

Whether the argumentation took place or not was analyzed according to Toulmin's model. Even though the rebuttal component could not be observed in some of the discussions, it was concluded that the argumentation took place. The researchers took Krummheuer's (1995) study as basis since Toulmin's model was used in mathematics education for the first time by Krummheuer (1995) who analyzed mathematical discussions in classroom. Krummheuer did not used the rebuttal and modal qualifier components in that analysis and did not feel the urge to add another component to the model. The studies following Krummheuer's (1995) study did not pay much attention to the rebuttal and modal qualifier components.

When the argumentation was analyzed according to Toulmin's model, PCK learning traces of the prospective teachers in this process were subjected to a qualitative analysis. The voice records were deciphered in the first place. When the data were being deciphered, the prospective teachers were consulted for ambiguous statements or comments deduced from the statements, ambiguous statements were clarified and approval was taken for comments deduced from the statements. Following the decipherment of the interview data, codes and categories were formed with raw data by the researcher. When forming the codes and categories, written and verbal statements of the prospective teachers were evaluated mutually. The data to be obtained was subjected to a content analysis which is a qualitative research technique. The data analysis may be shaped during the process in accordance with the depth and scope of the analyzed data by the nature of qualitative research (Miles & Huberman, 1994). The data were analyzed in four stages in general: (1) encoding, (2) thematic encoding, (3) organization of codes and themes, and (4) interpretation of the findings achieved (Yıldırım & Şimşek, 2008). In the analysis which would start with the data collection, these sub-procedures followed each other in a cyclical and interactive manner rather than in a linear order. As multiple researchers worked together in the data analysis, it is important to the reliability of the analysis that there is compliance among the codes determined by different researchers. Therefore, the codes to be determined by the researchers who encoded the same data set were numerically compared, and a reliability rate of 83% was achieved (Yıldırım & Şimşek, 2008). A general framework was created to define and describe the data in the third stage of the analysis, and the data were reviewed and made ready for description and interpretation with this framework. The last stage of the analysis included the relationships among these clearly described and defined data and the interpretation of these relationships within the conceptual framework. Interpretation of the data was available during the whole analysis; however, comments made toward the end of the analysis ensured that certain results were achieved beyond the data obtained previously.

Results

After whether elements of argument had been used in the online argumentation on Easyclass and in classroom discussion was analyzed according to Toulmin's model, PCK learning traces were focused on. The points taken into account in general for the learning traces were *"Are students guided to right associations and reasoning with proper organizations according to student-teacher dialogs? Are students being able to be guided to desired association via proper methods, strategies, demonstrations, analogies and examples in accordance with the nature of the subject or concept?"* How these points were considered is illustrated in Figure 4 along with an exemplary discussion in the study.

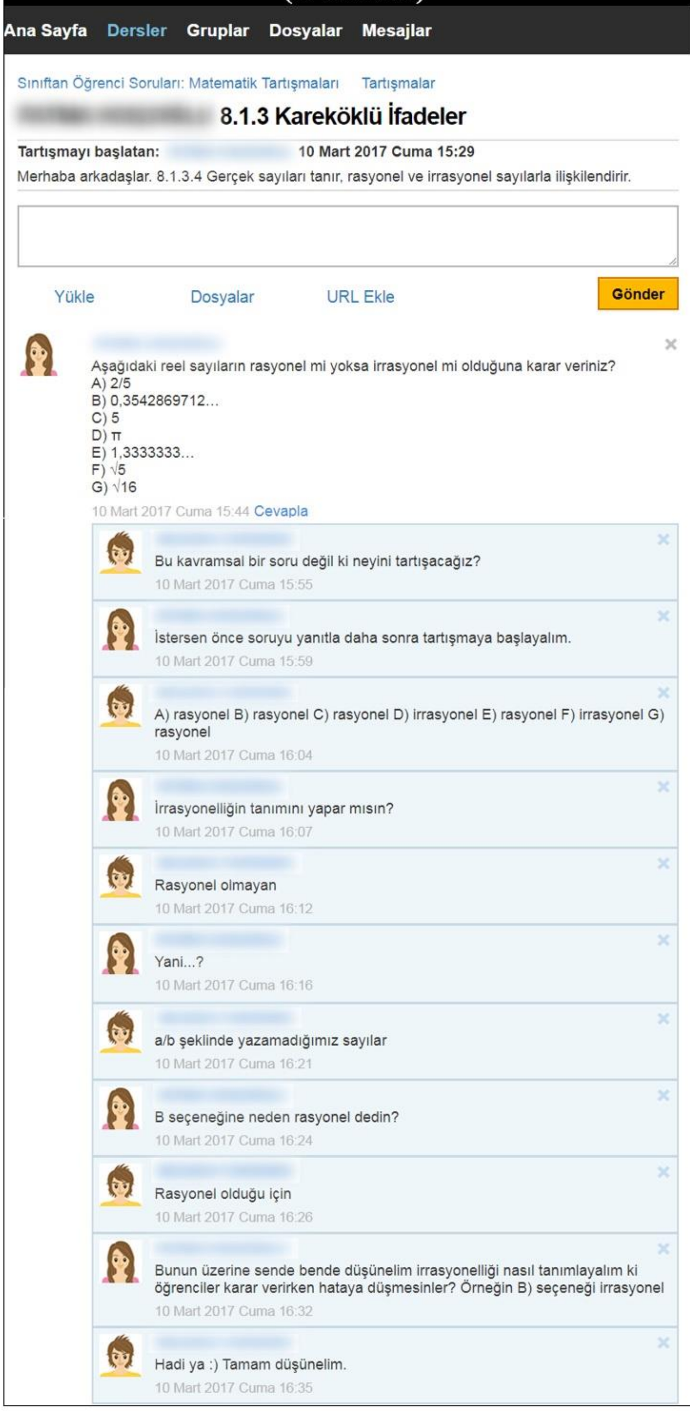
An example of an argumentation shaped with student questions (in Turkish)	Translated version of argumentation example (in English)
	***Leader Teacher Candidate (LTC) 8.1.3 Radical Expressions*** *Hi friends. 8.1.3.4 Understand real numbers and associate real numbers with rational and irrational numbers.* ***LTC:*** *Determine whether the following numbers are rational or irrational.* *A) 2/5* *B) 0.3542869712…* *C) 5* *D) π* *F) √5* *G) √16* ***TC07:*** *This is not a conceptual question. So, what are we going to discuss?* ***LTC:*** *It would be better to answer the question first. Then, we could discuss it.* ***TC07:*** *A) rational B) rational C) rational D) irrational E) rational F) irrational G) rational* ***LTC:*** *Could you please define irrational numbers?* ***TC07:*** *Numbers which are not rational.* ***LTC:*** *So…?* ***TC07:*** *The numbers that are not of the form a/b.* ***LTC:*** *Then, what is your point of view for saying the number B as rational?* ***TC07:*** *Because it is rational…* ***LTC:*** *Hmm. Let us think more about the definition of irrational numbers so that the students do not have difficulty in separating rational and irrational numbers. For example, B is irrational.* ***TC07:*** *Oh my god :). Let's think about it.*

Figure 4. Section of an exemplary online argumentation shaped by possible student questions

Classroom Discussions

A section of a classroom discussion is presented below.

> ***LTC:*** *Easyclass discussion was productive. Here, we can try to establish how we should teach the definition of irrationality.*
>
> …

TC22: *I think students do not get confused in the radical. They can make this inference: if it is subtracted from the radical fully, they say it is rational; if not, it is irrational.*

TC39: *Or they know that π is irrational.*

TC22: *Why? I think it is complicated, too. We say it can be written as 22/7 and then it is irrational. What is in child's mind contradicts the explanation "numbers that cannot be expressed in the form of a/b are irrational."*

....

TC7: *Teacher, I do not understand why 0.3542869712 is irrational.*

LTC: *Kids, let me make a brief explanation first: We should be able to express a rational number as the division of two integers on the condition that denominator is different from zero. So, is 5 rational?*

TC28: *Yes, because we can write it as 5/1.*

LTC: *What about 2/5?*

PT35: *It is very clear; it is written as the division of two integers. Of course it is rational.*

LTC: *So, are 1/3 and 22/7 rational or irrational?*

PT35: *Again, they are rational because they are written as the division of two integers, teacher.*

TC39: *22/7 is irrational because it is equal to π.*

TC16: *There is trouble with 1/3, too. Looking at the decimal expansion, it goes like 0.3333... I think it is irrational, too.*

LTC: *Why did going like 0.3333... make you think that?*

TC16: *At the end of the day, is not it irrational if it has an infinite expansion?*

LTC: *What do the class think about this issue?*

... (Students discuss between them.)

TC41: *The decimal part is confusing, teacher.*

LTC: *Let me ask you this then: compare 0.3333… with 3.14159265358979323… ?*

TC12: *Both are infinite decimals.*

TC25: *One is a repeating decimal; the other is a non-repeating decimal...*

LTC: *What is the difference between?*

TC38: *We can write the repeating one in the form of a/b, but we cannot the other.*

TC7: *How can we write the repeating one in the form of a/b?*

TC38: *We write the whole number as the numerator regardless of decimal point and take the non-repeating part out, and then we write 9 for repeating numbers after the decimal point and 0 for non-repeating numbers for the nominator...*

TC7: *Why?*

LTC: *(Explains over an example on the whiteboard) We can think like this:*

For example, let us find the value 0,33333…

Let 0,33333…= x.

Then, 10x = 3,33333...

10x = 3,33333…

x = 0,33333…

If we do subtraction on each side the result will be 9x=3, therefore x=1/3.

TC7: *I see. So, can we write 3.14159265358979323… in the form of a/b?*

LTC: *What do you think?*

TC7: *We cannot like you did on the whiteboard, we cannot write it teacher.*

LTC: *So, can we reach a conclusion?*

TC18: *Yes, teacher. Infinite repeating decimals are rational because they are written in the form of a/b. But infinite non-repeating decimals are irrational.*

LTC: *Well, what does the rest think?*

…(The class approves)

LTC: *So, we can express irrationality in two ways. First of all, a and b are integers; if b≠0, numbers that can be written in the form of a/b are called rational numbers, numbers that cannot be written in the form of a/b are called irrational numbers. Secondly, irrational numbers are infinite non-repeating decimals.*

…

It is understood from the discussion above that the leader prospective teacher used interrogative questions from basic to advanced so that students could make accurate associations and reasoning when learning the concept.

[Is 5 rational? What about 2/5? So, are 1/3 and 22/7 rational or irrational?]

The leader prospective teacher also utilized strategies compliant with the constructivist approach through the example selection and questions so that students could make the desired association. Accordingly, the leader prospective teacher also chose appropriate demonstrations for conceptual learning. For instance, the leader prospective teacher guided the students toward the desired association (irrational numbers are infinite non-repeating decimals) via interrogative questions such as *"Let me ask you this then: compare 0.3333… with 3.14159265358979323… ?"* The leader prospective teacher reinforced the conceptual learning with appropriate demonstrations as follows:

"For example, let us find the value 0,33333…

Let 0,33333… = x.

Then, 10x = 3,33333...

10x = 3,33333…

x = 0,33333…

If we do subtraction on each side the result will be 9x=3, therefore x=1/3."

In general, it was found that the prospective teachers reinforced the conceptual learning by paying attention to the possible dialogs between student and teacher when handling possible student questions in the first place and building it on a solid ground that students learn the given implicit concept or concepts by proper association in the Easyclass and classroom discussions. It was observed that the prospective teachers used elements to empower students' reasoning and association in the dialogs so that this conceptual learning could take place. It was found that the demonstrations, analogies, examples and materials used in instructional explanations via questions and all methods and techniques used in a wider scope reinforced the conceptual learning.

The elements addressed within the framework of PCK learning traces were "Are students guided to right associations and reasoning with proper organizations according to student-teacher dialogs? Are students be able to be guided to desired association via proper methods, strategies, demonstrations, analogies and examples in accordance with the nature of the subject or concept?" were also observed in prospective teachers' self-evaluations. Representative opinions of the prospective teachers are presented below:

TC52: *…[Answering a student question required us to have more knowledge than the one we need to possess to teach a subject. It gave us the chance to improve our limits mathematically. So much so that, answering the question impelled us to study critically and patiently as researchers. In this sense, the student questions paved the way for us to master the content of attainments and examine several examples of articles, videos, activities, instructions so that we could provide comprehensible instructions when teaching a subject, which is the main factor of the teaching profession. It had a great impact on us for acquiring experience and knowledge in our field.]*

TC13: *…[When I was thinking about how I should explain it or with which questions I can have them make the desired inference, I saw my learning improving, and most importantly, it helped me understand students' ways of thinking for my teaching profession. Because in this process, my instructions got very rich as I was thinking that students may think this, ask that question, how I can make an explanation] …*

Another finding in the discussions where PCK learning traces were tracked is that the prospective teachers explained the concepts which they had not previously known or had misunderstood to shed light on the change in their content knowledge present in their learning traces. This finding on the change in prospective teachers' content knowledge was also available in their self-evaluations. The following are representative opinions of the prospective teachers:

> ***TC45:*** *"This procedure brought lots of things to me other than methods and strategies I can use when I teach mathematics. I saw my deficiencies and the things I misunderstood in the subjects. For example, I did not know irrationality exactly. Indeed, there are points that I was wrong about. With my friends choosing them in their questions and addressing in the discussion, I learned them. I am going to be a mathematics teacher, but I did not know why the multiplication of a positive and a negative is negative. For example, the discussion whether zero multiplied by x equals to zero is an identity or an equation taught me the difference between them. I was enlightened about many concepts including this. You might think 'Our students are so ignorant', but I can easily admit it because I am happy now because I learned it. You were saying that learning improves anywhere, anytime; so, the right time was this course for me."*
>
> ***TC26:*** *... [Questions created in the study of possible student questions, being curious about the answers to the questions I was asked as a prospective teacher and my effort to find the answer at the end of meticulous research made important contributions to my learning. This way, I learned the concepts that I did not know. I learned that we should master the attainment within the question for answering the questions, recall the preliminary knowledge for explaining the questions, that the situations we know by rote learning but cannot give the answer while using it constantly as a rule are in fact inferences and they are based on a logic rather than being pure rote learning. Student questions helped me look at mathematics from a different perspective. We had an experience of getting into the foundation of a subject and questioning its smallest unit on the contrary to problems we solve without making mistakes. This stage gave the chance to scrutinize attainments of elementary mathematical attainments one by one in accordance with an objective, which had a significant and different effect on me for mastering the subjects which would instruct when we become teachers.]*

Opinions of the prospective teachers on "effects of student questions and argumentation on mathematics teaching" which were received through written opinion forms and semi-structured interviews were also examined in the study. Dominant themes exceeding 30% among the themes derived from the prospective teacher opinions on effects on mathematics teaching and representative prospective teacher opinions in these themes are given in Table 1.

Table 1

Themes Derived from Prospective Teacher Opinions on Effects of the Procedure on Mathematics Teaching

Theme	Exemplary prospective teacher statements	f	%*
Learning by questioning	"Students can learn the information by questioning […]" "The same conclusion can be achieved with very different ways, we back what we have in some of them and rebut in others. I mean, students can be equipped with many skills by questioning in the conclusion process."	46	82
Cooperative learning-Social learning	"[…] students can discuss and learn with their classmates and discuss with their teacher too."	33	59
Meaningful learning	"Every student can learn the logic underlying each concept in this way."	39	70
Problem-solving skills	"They can look at the problems from different aspects as they learn by questioning. This will help them develop new solution methods […]	41	73
Reasoning skills	"Students' reasoning can improve by arguing the accuracy and validity of inferences."	24	43
Communication skills	"[…] Students need to use the mathematical language well when they express themselves, so they will improve using symbols and terms effectively and properly."	17	30
Association skills	"[…] As concepts and operations are associated, students can also associate concepts underlying the rules."	20	36

** The sum of percentages may exceed 100% as the prospective teachers provided more than one theme within the scope of the question.*

According to Table 1, the prospective teachers presented themes within the framework of constructivist approach and which emphasize the importance of research- and questioning-based learning. Given the evaluation of the procedure by the prospective teachers, 42 prospective teachers (75% of all participants) stated that interviews with the teachers were effective. They indicated the reason was that teachers could clearly explain what students find difficult to understand or which concepts they may have trouble with forming by giving examples because they had had the chance to examine and analyze conceptual thinking of students for years. On the other hand, 14 participants did not find these interviews efficient. Regarding the reasons, they reported that the teachers laid emphasis rather on operational errors or procedural concepts in possible student questions and did not mention about critical questions that would ensure conceptual learning. Majority stated that the literature review was effective because they observed more conceptual questions and contents in the theses, papers, books, etc. in the review.

Online stage of the procedure was found to be vital by the prospective teachers as it was where preliminary discussions took place, they had opportunities to investigate the concept, and they argued that such a discussion should be definitely performed before the classroom discussions. Exemplary questions addressed in the online argumentation and a screenshot of the discussion environment are shown in Figure 5.

Exemplary questions available in the Easyclass learning environment and addressed in the online argumentation could be listed as follows:

- *Is a square also a rectangle? Why?*
- *Are ratio and fraction the same things? Is ratio a division?*
- *When we tilt a square right prism, is it still a square right prism?*
- *Do you know why 0! equals to one?*
- *Is 0.x=0 an identity or an equation?*

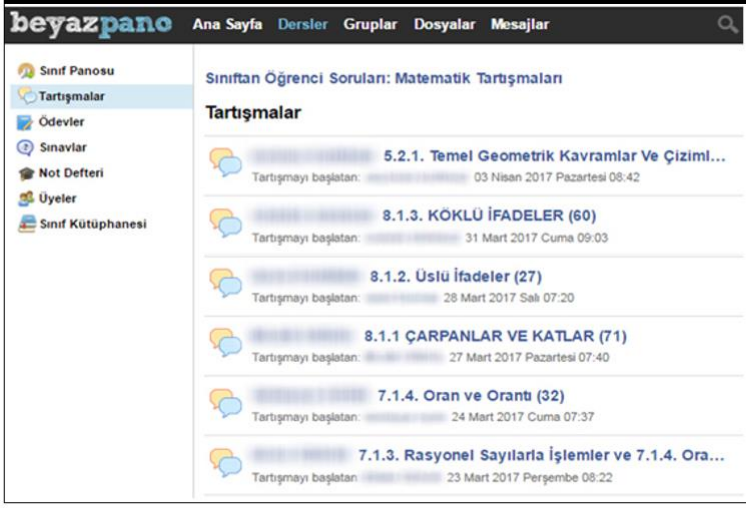
A view from argumentation website (in Turkish)	Translated version of concepts (in English)
	**Student questions from the class: Argumentations in mathematics** 5.2.1. Basic geometrical concepts and constructions 8.1.3. Radical expressions 8.1.2. Exponential numbers 8.1.1. Multiples and divisors of numbers 7.1.4. Ratio and proportion 7.1.3. Operations in rational numbers

Figure 5. A screenshot of the online argumentation shaped by possible student questions

Representative prospective teacher opinions chosen from the opinions which evaluated the online argumentation procedure are given below.

> ***TC17:*** *[First of all, I must say it did not have any negative effect. If I were to list its positive effects:*
> *I gained the experience of discussion around an objective in a virtual environment. I performed a qualified study to be able to answer the questions. I examined the attainment involving the question in the mathematics curriculum first. By this means, I was informed of the subject in a more detailed way. I learned the points in which I was deficient about the subject in the attainment.*
> *If I could not find answers to the questions first by scrutinizing in my mind, I consulted to several papers. Examining the articles was very instructive and entertaining. Scopes of the papers were very extensive, and studies conducted by experts and examining how they investigated the questions they were curious about contributed importantly to my own knowledge. But if there had been no such discussion before the classroom discussion, I could have participated in the classroom discussions with less knowledge.]*
> ***TC48:*** *[A positive effect of Easyclass on my teaching which was the responsibility of directing a discussion was an experience to be taken to the classroom setting. Directing a discussion and taking responsibility, encouraging participation in discussion contributed to me in many senses. When I examined the participations of my friends in a discussion about my teaching, I got the chance to see multiple different ideas and discovered different ways in the instruction of critical points in the subjects. Indeed,*

materials, papers, my friends' own opinions and activities shared in the discussions had a positive effect on me acquiring significant knowledge in mathematics.]

On the other hand, the platform on which the discussion was performed was found to be lacking from a few aspects. It was stated that Easyclass is an insufficient platform for this. It was reported that similar discussions were repeated as they lingered on and even which concept was being discussed was confused. It was stated that the prospective teachers needed different environments where student questions are given to them like a concept map and they could open titles on their own and discuss them from different perspectives. It was argued that the classroom discussions had a wrapping up function and were efficient in deciding the most effective instructional explanations. Representative opinions from which these findings were derived are given below.

TC53: *... [I think Easyclass discussions were definitely effective so that we could scrutinize the given concept, but means provided by Easyclass were insufficient. For examples, discussions were going well, but at some point, they were branching out. So, if there had been different regions like in a map, we had gone to the region of that concept which we wanted to discuss, more effective discussions would have taken place in every region…]*

TC8: *... [We were discussion in Easyclass, too many suggestions and strategies were being presented. It should have been ordered and constructed in the best productive way possible. That is why the classroom discussions went so well. We could have it exactly in our mind in which best possible way and which order we could do it.]*

Discussion and Conclusion

Traces of pedagogical content knowledge exhibited by the prospective teachers in the online argumentation activities which were designed through possible student questions were examined in this study. Teacher's pedagogical content knowledge is the body of knowledge and skills which a teacher will call on when a subject or concept is to become learnable for students (Baki, 2018). This is indeed how the teacher guides the student properly and effectively to acquire and embrace information and put it into practice. This is about knowing how to make Socratic and dialogical discussions functioning. In this study, it was explored according to prospective teachers' pedagogical content knowledge traces that they reinforced the conceptual learning by paying attention to the possible student-teacher dialogs when addressing possible student questions in the first place and placing it in the center that students learn the given implicit concept or concepts by proper association in the Easyclass and classroom discussions. It was observed that the prospective teachers used elements to empower students' reasoning and association in the dialogs so that this conceptual learning could take place. It was found that the demonstrations, analogies, examples and materials used in instructional explanations via questions and all methods and techniques used in a wider scope reinforced the conceptual learning. As for prospective teachers' learning traces in the study, as stated by Baki (2018), "What kind of an organization should I make so that students can make proper associations and reasoning when learning a given concept? What kind of a dialog should we have? Which teaching method should I use by the nature of the subject or concept? Which demonstrations should I use? What kind of analogies and examples should I use and which questions should I ask so that students can make the desired association? Such questions are directly related to teacher's pedagogical content knowledge." Consequently, prospective teachers' PCK learning traces refer to positive developments which emphasize the importance of research- and questioning-based learning and within the framework of constructivist approach. But then, in the discussions in which the prospective teachers tried to find the most efficient answers to possible student questions by comparing the most appropriate techniques despite not having been included in the study, they explained the concepts which they had not previously known or had misunderstood to find out the changes in their content knowledge present in these learning traces. In this preservice teacher education procedure where the prospective teachers activated their knowledge on how to evaluate mathematical arguments, how to determine accuracy of mathematical propositions and how to prove accuracy of the mathematical propositions which they think of as accurate, improvement in their content knowledge is consistent with the literature given the same process they were through as students. Indeed, Brown and Reeves (2009) achieved the finding that argumentation method helped students enhance mathematical skills and understandings, levels of using mathematical operations in problem solving, levels of expressing the problems mathematically and skills of developing new approaches in problem solving.

Regarding the themes derived from the prospective teacher opinions on procedure's effects on mathematics teaching, their anticipations on the development of students' "learning by questioning, cooperative learning-social learning, meaningful learning, problem-solving skill, reasoning skill, communication skill, and association skill" coincide with the conclusions of theoretical and applied studies in the literature (Brown & Reeves, 2009; Driver, Newton & Osborn, 2000; Günel, Kıngır & Geban, 2012; Keys, Hand, Prain & Collins, 1999).

Producing valid arguments or proofs and criticizing the arguments are inseparable parts of doing math. On the other hand, it is necessary to direct argumentation-based learning environments well because students may find it hard to understand given arguments or have certain challenges due to misunderstandings and misunderstood arguments when sharing their ideas with other students or during the stage of invalidating the ideas. Directing the argumentation process well is made possible by living and experiencing the process itself. Great responsibility undoubtedly falls to mathematics teacher in bringing these skills to the students. This study is an application of teacher education developed in the light of the idea that prospective teachers need to experience their own processes of argumentation so that they could handle the argumentation-based learning approach in their future classrooms. This education was designed online in the consideration that educational technologies supported argumentation-based instructional applications have been increasing in recent years, and such applications had positive impacts on the development of students' cognitive, affective and psychomotor skills. Online stage of the procedure was found to be vital by the prospective teachers as it was where preliminary discussions took place, they had opportunities to investigate the concept, and they argued that such a discussion should be definitely performed before the classroom discussions. On the other hand, the platform on which the discussion was performed was found to be lacking. It was stated that Easyclass is an insufficient platform for this. It was also observed that similar discussions were repeated as they lingered on and even which concept was being discussed was confused. It was stated that the prospective teachers needed different environments where student questions are given to them like a concept map and they could open titles on their own and discuss them from different perspectives. Majority reported in the procedure evaluation that interviews with the teachers were effective. They indicated the reason was that teachers could clearly explain what students find difficult to understand or which concepts they may have trouble with forming by giving examples because they had had the chance to examine and analyze conceptual thinking of students for years. Minority of the participants, however, did not find these interviews efficient. Regarding the reasons, they reported that the teachers laid emphasis rather on operational errors or procedural concepts in possible student questions and did not mention about critical questions that would ensure conceptual learning. Here, qualifications of the teachers consulted for opinion are of importance; it is possible to say that many of the interviewed teachers attach importance to meaningful learning and some of them focus on operational learning. Therefore, it is understood that it is not required to choose interviewee teachers from among teachers who take conceptual learning in consideration. Majority stated that the literature review was effective because they observed more conceptual questions and contents in the theses, papers, books, etc. in the review. It was argued that the classroom discussions had a wrapping up function and were efficient in deciding the most effective instructional explanations.

Consequently, online argumentation method was found to have positive impacts on the improvement of prospective teachers' pedagogical content knowledge and their own learning. Moreover, online argumentation is not sufficient alone and needs to be reinforced with classroom discussions. While online argumentation environment is a learning environment that supports the emergence of introductory ideas and different ideas, active and face-to-face environments are needed to change and improve these ideas. Weaknesses of the Easyclass platform, as stated by the prospective teachers, is in question here as well. Thus, online discussions should be held on online learning platforms which are more efficient and involve different cognitive instruments. It is anticipated that these presented preservice components will shed light on future studies to be performed in the field of teacher education.

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Uzun Özet

Giriş

Günümüzde birçok öğretim programının felsefesinin dayandığı yapılandırmacı yaklaşıma göre öğrenme, öğrenenlerin bilgi edindikleri, bu bilgileri önceki bilgi ve deneyimleri ile yorumlayıp anlamlandırdıkları aktif bir süreçtir. Yapılandırmacı yaklaşım, araştırmaya ve sorgulamaya dayalı öğrenmenin önemini vurgulamaktadır. Bu öğrenme öğrenenleri aktif kılarak soru sormaya, problem çözmeye ve eleştirel düşünmeye yönlendirmektedir. Son yıllarda, böylesi bir öğrenci merkezli öğrenmeyi destekleyen öğretim tekniklerinden biri argümantasyondur (bilimsel tartışmalardır). Argümantasyon Tabanlı Bilim Öğrenme yaklaşımına göre öğrenciler bilgiyi bir öğrenme ortamında sorular sorarak, iddialar oluşturarak ve bu iddialarını destekleyerek oluşturmaktadırlar. İlgili alanyazında, argüman ortamlarının öğrenenlerin öğrenmeleri ve katılımları üzerinde, bilgiyi akranlarla ve öğretmenle paylaşarak öğrenme imkanı bulunduğundan, olumlu etki yarattığı belirtilmektedir. Aynı zamanda argümantasyona dayalı öğrenme ortamlarının kavramların gelişimsel özelliklerini ön plana çıkardığı ve sosyal öğrenmeyi desteklediği tespit edilmiştir.

Argümantasyon; iddiaların dayandırıldığı gerekçeler belirtilerek veriler ile ilişkili olup olmadığının yapılandırıldığı süreçtir. Bireylerin bir problemin çözümüne ilişkin bakış açısını destekleyen ya da çürüten ifadeler kullanması bireyin kendi kendine argümantasyon yaptığını göstermektedir. Bu durum göz önüne alındığında aslında argümantasyon ile öğrenme birbiri ile ilişkili ve düşünme sürecinin ayrılmaz bir parçasıdır. Eğitim literatüründe Toulmin modeli hem sınıf içinde nasıl bir tartışma ortamının oluşturulduğunu hem de öğrenmenin nasıl ilerlediğini ortaya çıkarmak ve analiz etmek için kullanılmaktadır.

Geçerli argümanlar ya da ispatlar üretme ve argümanların kritik edilmesi, matematik yapmanın ayrılmaz parçasıdır. Muhakeme becerileri öğrencilere kazandırılmazsa o zaman matematik yapma bir işlem dizisini takip etme ve anlamını düşünmeden örnekleri taklit etmeden öteye geçemez. Bu nedenle, argümantasyona dayalı öğrenme ortamlarının iyi yönlendirilmesi gerekmektedir çünkü öğrenciler süreç içerisinde verilen argümanları anlamada güçlük çekebilir ya da diğer öğrencilerle düşüncelerini paylaşmada ya da düşünceleri çürütme aşamasında yanlış anlamaları, yanlış anlaşılan argümanlardan dolayı bir takım zorluklar yaşayabilir. Bu yüzden argümantasyon sürecinin iyi yönetilmesi bu süreci yaşayarak ve deneyimleyerek mümkündür.

Öğrencilere bu becerilerin kazandırılmasında hiç şüphesiz matematik öğretmenlerine büyük sorumluluk düşmektedir. Birçok çalışmada argümantasyonu desteklemeye yönelik etkin hizmet öncesi ve hizmet içi öğretmen eğitiminin önemi vurgulanmıştır. Bu çalışma, öğretmen adaylarının sınıflarında argümantasyon öğrenme yaklaşımını etkin biçimde uygulayabilmeleri için öncelikle kendilerinin argümantasyon sürecini yaşamaları gerektiği düşüncesinden yola çıkılarak geliştirilmiş bir öğretmen eğitimi uygulamasıdır.

Bu çalışmada öğrenciden gelebilecek sorular üzerinden tasarlanmış online argümantasyonlarda öğretmen adaylarının sergiledikleri pedagojik alan bilgisi (PAB) izleri incelenmiş ve öğrenme ortamının güçlü ve zayıf yanları araştırılmıştır.

Yöntem

Çalışmanın katılımcılarını İlköğretim Matematik Öğretmenliği programının 3. sınıfında öğrenim gören 56 öğretmen adayı oluşturmaktadır. Açıklayıcı özel durum çalışması olarak tasarlanan bu araştırmada, 8 haftalık online argümantasyon içerikleri, PAB kapsamında hazırlanmıştır. Beyazpanoda gerçekleşen online argümantasyonda ve sınıf içi tartışmalarda argüman öğelerinin kullanılıp kullanılmadığı Toulmin modeline göre analiz edilmiştir. Argümantasyonun gerçekleşip gerçekleşmediği Toulmin modeline göre analiz edilirken, bu süreçteki öğretmen adaylarının PAB öğrenme izleri nitel analize tabi tutulmuştur. PAB öğrenme izleri, Beyazpano tartışmalarıyla, sınıf tartışma kayıtlarıyla ve kendi öğrenmeleri üzerine alınan görüşlerle takip edilmiştir. Aynı zamanda veri toplama araçları olarak derste gözlem notları tutulmuş ve çalışmanın sonunda yazılı görüş formu ve yarı-yapılandırılmış görüşme kullanılmıştır.

Bulgular

Beyazpano'da gerçekleşen online argümantasyon ve sınıf içi tartışmalarda argüman öğelerinin kullanılıp kullanılmadığı Toulmin modeline göre analiz edildikten sonra PAB öğrenme izlerine odaklanılmıştır. Öğrenme izlerinde genel olarak bakılan unsunlar *"Öğrenci- öğretmen diyalogları dikkate alınarak, uygun organizasyonlarla öğrenciler doğru ilişkilendirmelere ve akıl yürütmelere yönlendiriliyor mu? Konunun veya kavramın doğasına bağlı olarak uygun yöntem, strateji, gösterim, analoji ve örneklerle öğrenci istenilen ilişkilendirmeye yönlendirilebiliyor mu?"* dur.

Genel olarak, Beyazpano ve sınıf ortamındaki tartışmalarında öğretmen adaylarının öncelikli olarak öğrenciden gelebilecek öğrenci sorularını ele alırken öğrenci ile öğretmen arasında geçecek diyaloglara dikkat edip öğrencinin sorudaki saklı belli kavramı ya da kavramları doğru ilişkilendirerek öğrenmesini temele oturtturarak, kavramsal öğrenmeyi desteklediği bulgulanmıştır. Bu kavramsal öğrenmenin gerçekleşebilmesi için öğretmen adayları tarafından diyaloglarda öğrencilerin akıl yürütmelerini, ilişkilendirmelerini güçlendirecek öğeleri kullandıkları belirlenmiştir. Soru ekseninde düzenlenen öğretimsel açıklamalarda kullanılan gösterimlerin, analojilerin, örneklerin ve materyallerin daha geniş kapsamda kullanılan bütün yöntem ve tekniklerin kavramsal öğrenmeyi desteklediği tespit edilmiştir. Öğretmen adaylarının öz değerlendirmelerinde de PAB öğrenme izleri çerçevesinde ele alınan unsurlar, öğretmen adaylarının kendi cümlelerinde de yer almıştır.

PAB öğrenme izlerinin takip edildiği tartışmalarda diğer önemli bir bulgu ise öğretmen adaylarının kendi ifadeleriyle kavramsal olarak bilmedikleri ve yanlış bildikleri kavramları açıklayarak tespite ışık tuttukları öğrenme izlerinde mevcut olan, alan bilgilerindeki değişim bulgulanmıştır.

Öğretmen adaylarının, yazılı görüş formu ve yarı-yapılandırılmış görüşmeler aracılığıyla alınan 'öğrenci soruları ile argümantasyon sürecinin matematik öğretiminde etkileri' üzerine belirtikleri görüşleri incelenmiştir. Matematik öğretime etkileri üzerine alınan öğretmen adayı görüşlerinde elde edilen temalardan %30'u geçen baskın temalar ve bu temalarda temsili öğretmen adayı görüşleri belirtilmiştir. Öğretmen adaylarının araştırmaya ve sorgulamaya dayalı öğrenmenin önemini vurgulayan, yapılandırmacı yaklaşım çerçevesinde temalar sundukları görülmektedir. Uygulamanın öğretmen adayları tarafından gerçekleştirilen süreç değerlendirmesi ele alındığında ise 42 kişi (tüm öğretmen adaylarının %75'i) öğretmenlerle görüşmelerin etkili olduğunu belirtilmiştir. Bunun sebebi olarak öğretmenlerin yıllarca kavram bazında öğrenci düşünmelerini inceleme ve analiz etme fırsatları olduğu için kendilerine de öğrencilerin neleri anlamada zorlandıkları veya hangi kavramları oluştururken sıkıntı yaşayabileceklerini net bir şekilde ve örnekler vererek açıklayabildiklerini sunmuşlardır. Öte yandan 14 kişi öğretmenlerle görüşmeleri etkin bulmamıştır. Belirtilen nedenlere bakıldığında öğretmenlerin öğrencilerden gelebilecek sorularda daha çok işlemsel hatalar veya prosedürel kavramlar üzerinde durdukları fakat kavramsal öğrenmeyi sağlayacak kritik soruları belirtemedikleri görülmüştür. Alanyazın taramalarında tezlerde, makalelerde, kitaplarda… vb. daha kavramsal sorularla içeriklerle karşılaştıkları için çoğunluk alanyazın taramasının etkili olduğunu belirtmiştir.

Sürecin online gerçekleştirilen ayağı ön tartışmaların gerçekleştiği, kavramı araştırmaya fırsatların yakalandığı ve sınıf tartışmalarından önce mutlaka böyle bir tartışmanın yer alması gerektiği şeklinde öğretmen adayları tarafından elzem görülmüştür.

Sonuç ve Tartışma

Sonuç olarak, online argümantasyon yönteminin öğretmen adaylarının pedagojik alan bilgisi gelişiminde ve kendi öğrenmeleri üzerindeki olumlu etkileri ortaya çıkmıştır. Ancak online argümantasyon yöntemi tek başına yeterli olmayıp, online tartışmaların sınıf içi tartışmalarla desteklenmesi gerektiği bulgulanmıştır. Online argümantasyon ortamı başlangıç fikirlerinin ve farklı fikirlerin ortaya çıkmasında destekleyici bir öğrenme ortamı olmasına rağmen fikirlerin değiştirilip geliştirilmesi için aktif yüz yüze ortamlara da ihtiyaç duyulmaktadır. Oysa burada öğretmen adaylarının belirttiği üzere Beyazpano platformunun zayıf yanları da yer almaktadır. Bu nedenle online tartışmaların daha etkin, farklı bilişsel araçları içeren online öğrenme platformlarında gerçekleştirilmesine ihtiyaç vardır. Sunulan bu hizmet öncesi bileşenlerin, ileride öğretmen eğitimi alanında gerçekleştirilecek çalışmalara ışık tutacağı düşünülmektedir.