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# Convergence of Multiset Sequences

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**Abstract —** In this paper, we introduce the concept of the multiset sequence and its convergence. A few special examples of multiset sequences, e.g. a prime identifier, are also given. A metric is defined in multisets for statistical convergences of multiset sequences. Wijsman and Hausdorff convergence of multiset sequences are discussed.

***Keywords* —** *Multiset, multiset sequence, prime identifier sequence, set theoretic limit, Wijsman convergence, Hausdorff convergence*

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## 1. Introduction

During the process of information retrieval, duplicates may occur at various stages of the process. In such situations, the need for multisets and multiset operations arises. For example, in a cyber investigation, hitting on a particular website and phone number in a tower on some time interval are some of such situations where multisets are more suitable than ordinary sets.

Multiset (in short mset) or Bag is a collection of objects in which repetition is allowed [1]. Multiset theory can be used in situations, where the classical set theory proves inadequate. Research in the multiset theory is still at the infant stage. The need for multisets was pointed out by Knuth in 1981 [1]. The papers [2–9] are on the multiset theory and its applications in Mathematics and Computer Science. The relations and operations with multisets [10], relations, and functions in multiset context [11], are some of the developments in this field.

The concept of convergence of sequences of real numbers has been extended by several authors to the convergence of sequences of sets [12–17]. Statistical convergence for sequences of sets and some basic theorems are established by Nuray and Rhoades [18]. These papers include topics, such as statistical convergence and ideal convergence of set sequences.

In this paper, we define mset sequences and investigate their various properties. There is also a comparison of mset sequences with set sequences. A few special examples of mset sequences, e.g. a prime identifier, are also given. Here we are attempting to extend the concept of convergence on classical set sequences to mset sequences. A metric is introduced on mset for statistical convergence, and making use of this metric, Wijsman and Hausdorff convergences are defined.

Sequences have been used in various fields, such as computer science, for a variety of purposes, and convergence of these sequences could be found as well. These wide range of applications of sequences and their convergences in different real-life situations is the motivation of our work.

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In Section 2, some of the preliminaries that are necessary for further sections of the paper are given. In Section 3, an extension of the sequence of sets into the multiset context is presented. The following section has some examples of the same process. The final section covers the convergence of these multiset sequences.

## 2. Preliminary

In this section, we recall some basic definitions and properties of multisets that are necessary for this paper.

**Definition 2.1.** [1] A collection of elements containing duplicates is called multiset. The word multiset is often shortened to mset. If the elements of a multiset are taken from a set $X$, then it is said to be drawn from $X$. A multiset $M$ drawn from $X$ can be considered a function $C_M : X \to W$, where $W$ is the set of non-negative integers. For each $x \in X$, $C_M(x)$ is the characteristic value or count value of $x$ in $M$ and indicates the number of occurrence of $x$ in $M$. Since characteristic value actually characterizes a multiset, most of our assertions are based on this characteristic value.

**Note 2.2.** Let $M$ be an mset drawn from $X$ with $x_1$ appearing $k_1$ times, $x_2$ appearing $k_2$ times, and $x_n$ appearing $k_n$times. Then, $M$ is written as $M = \{k_1|x_1, k_2|x_2, \cdots, k_n|x_n\}$. $C_M(x) = k$ is sometimes denoted as $x \in^k M$.

**Definition 2.3.** [8] Let $M_1$ and $M_2$ be two msets drawn from a set $X$. $M_1$ is a submultiset (shortly submset) of $M_2$ if $C_{M_1}(x) \leq C_{M_2}(x)$ for all $x$ in $X$ and is written as $M_1 \subseteq M_2$.

**Definition 2.4.** [8] Two msets $M_1$ and $M_2$ are *equal* if $M_1 \subseteq M_2$ and $M_2 \subseteq M_1$. In other words, $C_{M_1}(x) = C_{M_2}(x),\ \forall x \in X$.

**Definition 2.5.** [1] Addition of two multisets $M_1$ and $M_2$ drawn from a set $X$ results in a new multiset $M = M_1 \oplus M_2$ such that $\forall x \in X,\ C_M(x) = C_{M_1}(x) + C_{M_2}(x)$.

**Definition 2.6.** [8] Subtraction of two multisets $M_1$ and $M_2$ drawn from a set $X$ results in a new multiset $M = M_1 \ominus M_2$ such that $\forall x \in X,\ C_M(x) = \max\{C_{M_1}(x) - C_{M_2}(x), 0\}$.

**Definition 2.7.** [11] For an mset $M = \{k_1|x_1,\ k_2|x_2, \cdots, k_n|x_n\}$, the set $S = \{x_1, x_2, \cdots x_n\}$ is known as the *root set* of $M$.

**Definition 2.8.** [1] The union of $M_1$ and $M_2$ is a multiset, denoted by $M = M_1 \cup M_2$, with the count value $C_M(x) = \max\{C_{M_1}(x), C_{M_2}(x)\}$, for every $x \in X$.

**Definition 2.9.** [1] The intersection of $M_1$ and $M_2$ is a multiset, denoted by $M = M_1 \cap M_2$, with the count value $C_M(x) = \min\{C_{M_1}(x), C_{M_2}(x)\}$, for every $x \in X$.

**Definition 2.10.** [11] For an mset $M$, the power mset $\tilde{P}(M)$ is the set of all the submsets of $M$.

**Definition 2.11.** [11]$[X]^m$ is the collection of all the msets derived from $X$ with multiplicity at most $m$ for every element of $x \in X$.

**Definition 2.12.** [11] The complement of a multiset $M \in [X]^m$ is the multiset $M^c$ with count value $C_{M^c}(x) = m - C_M(x)$.

**Definition 2.13.** Partition of a positive integer $n$ is a non-increasing sequence $(n_1, n_2, \cdots n_k)$ such that $n_1 + n_2 + \ldots n_k = n$, where the elements $n_i$ are positive integers $\forall i \in N$.

**Note 2.14.** Partition of a positive integer is a multiset and conversely, every multiset represents a partition of an integer. The mset $M = \{k_1|x_1, k_2|x_2, \cdots k_n|x_n\}$ is the partition of the integer $n = k_1x_1 + k_2x_2 + \cdots, k_nx_n$. In such case, we write $M = P(n)$.

## 3. Multiset Sequence

**Definition 3.1.** A sequence in which all the terms are sets is known as a set sequence. A set sequence is a function from $N \to P(X)$, where $N$ is the set of positive integers and $P(X)$ is the power set of a nonempty set $X$.

**Example 3.2.** If $A_n = \{1, 2, \cdots, n\}$, then $\{A_n\}$ is a set sequence.

**Definition 3.3.** A sequence in which all the terms are multiset is known as a multiset sequence or shortly mset sequence. An mset sequence can be considered a function from $N \to \tilde{P}(X)$, where $\tilde{P}(X)$ is the power mset of a nonempty set $X$. Many of the properties of set sequences are also satisfied by mset sequences with appropriate modification. Some of them are discussed here.

**Definition 3.4.** An mset sequence $\{M_n\}$ drawn from $X$ is said to be bounded from below if there exists an mset $A$ drawn from $X$ such that $A \subseteq M_n$ for each $n \in N$.

**Definition 3.5.** An mset sequence $\{M_n\}$ drawn from $X$ is said to be bounded from above if there exists an mset $B$ drawn from $X$ such that $M_n \subseteq B$ for each $n \in N$.

**Definition 3.6.** An mset sequence which is bounded from below and above is known as a bounded sequence.

**Definition 3.7.** Let $\{M_n\}$ be an mset sequence. This sequence is an expanding sequence or non-decreasing sequence, if $M_n \subseteq M_{n+1}$ for each $n$.

**Definition 3.8.** Let$\{M_n\}$ be an mset sequence. This sequence is a contracting sequence or non-increasing sequence, if $M_{n+1} \subseteq M_n$ for each $n$.

**Definition 3.9.** An mset sequence which is either expanding or contracting is a monotone mset sequence.

**Note 3.10.** We can construct a monotone mset sequence from the given mset sequences.

**Definition 3.11.** Let $\{M_n\}$ be an mset sequence drawn from a set $X$. Consider the mset sequences $\{A_n\}$, $\{B_n\}$, $\{C_n\}$, and $\{D_n\}$, defined as

$$A_n = \cap_{i=1}^{n} M_n, \quad B_n = \cup_{i=n}^{\infty} M_n, \quad C_n = \cup_{i=1}^{n} M_n, \quad \text{and} \quad D_n = \cap_{i=n}^{\infty} M_n$$

Then, $\{A_n\}$ and $\{B_n\}$ are the contracting mset sequences, while $\{C_n\}$ and $\{D_n\}$ are the expanding mset sequences.

**Theorem 3.12. Distributive Laws:** Let $\{M_n\}$ be an mset sequence and $M$ be any mset, such that $M_n$, for all $n \in N$ and $M$ are elements of $[X]^m$ for a nonempty set $X$ and a positive integer $m$. Then,

(i) $M \cap (\cup_{n=1}^{\infty} M_n) = \cup_{n=1}^{\infty} (M \cap M_n)$

(ii) $M \cup (\cap_{n=1}^{\infty} M_n) = \cap_{n=1}^{\infty} (M \cup M_n)$

Proof.

(i) Let $M \cap (\cup_{n=1}^{\infty} M_n) = P$ and $\cup_{n=1}^{\infty} (M \cap M_n) = Q$. Then, $P$ and $Q$ are msets drawn from $X$. For an arbitrary $x \in X$, let $C_P(x) = k$. Then, $C_M(x) \geq k$, since $P$ is a subset of $M$.

<u>Case 1</u>: If $C_M(x) = k$, then $C_{M \cap M_j}(x) = k$ for those $j$ with $C_{M_j}(x) \geq k$ and $C_{M \cap M_j}(x) < k$ for those $j$ with $C_{M_j}(x) < k$. So, $C_Q(x) = k$.

<u>Case 2</u>: If $C_M(x) > k$, then $C_{M_n}(x) \leq k$ for each $n$ and there exists at least one $r$ with $C_{M_r}(x) = k$. Therefore, for each $n \in N$, $C_{M \cap M_n}(x) \leq k$ and in particular $C_M \cap M_r(x) = k$. Hence, $C_Q(x) = k$. Thus, in both cases, $C_P(x) = C_Q(x)$. Since $x$ is an arbitrary element, this is true for every element of $X$ and this proves $(i)$.

The proof of $(ii)$ is similar. □

**Theorem 3.13. De Morgan's Laws :** Let $X$ be a nonempty set and $m$ be a positive integer. For an mset sequence $\{M_n\}$, where each $M_n \in [X]^m$,

(i) $(\cup_{n=1}^{\infty} M_n)^c = \cap_{n=1}^{\infty}(M_n)^c$

(ii) $(\cap_{n=1}^{\infty} M_n)^c = \cup_{n=1}^{\infty}(M_n)^c$

PROOF. $(i)$ Let$(\cup_{n=1}^{\infty} M_n)^c = P$ and $\cap_{n=1}^{\infty}(M_n)^c = Q$. For $x \in X$, let $C_P(x) = k$. Then, $C_{\cup M_n}(x) = m - k$. $C_{M_n}(x) \leq m - k$ for each $n \in N$ and there exists at least one $M_r$ with $C_{M_r}(x) = m - k$, $C_{(M_n)^c}(x) \geq k$ for each $n \in N$, and in particular $C_{(M_r)^c}(x) = k$. So, $C_Q(x) = k$. This completes the proof of $(i)$.

$(ii)$ The proof is similar to that of $(i)$. □

## 4. Examples of Multiset Sequence

In this section, we introduce some multiset sequences that are of practical importance.

1. $\{N_n\}$, where $N_n = \{1|1, 2|2, \cdots, n|n\}$ is an mset sequence in which the $n^{th}$ term contains $\frac{n(n+1)}{2}$ elements.

2. The prime factorises $n$ completely, and let $F_n$ be the mset of these factors, including 1. Then, $\{F_n\}$ is an mset sequence. For example,

$$F_1 = \{1\}$$
$$F_2 = \{1, 2\}$$
$$F_3 = \{1, 3\}$$
$$F_4 = \{1, 2, 2\}$$
$$F_{36} = \{1, 2, 2, 3, 3\}$$

3. For every positive integer $n$, define an mset $M_n = \{a_n|n, a_{n-1}|(n-1)\cdots, a_1|1\}$ , where $a_i = [\frac{n}{i}]$ , integer part of $\frac{n}{i}$.Then, $\{M_n\}$ is an multiset sequence with many properties, which are listed below. A remarkable one is that one can determine by using this sequence whether an integer is prime or not.

$$M_1 = \{1|1\}$$
$$M_2 = \{1|2, 2|1\}$$
$$M_3 = \{1|3, 1|2, 3|1\}$$
$$M_4 = \{1|4, 1|3, 2|2, 4|1\}$$
$$M_5 = \{1|5, 1|4, 1|3, 2|2, 5|1\}$$

**Properties of** $\{M_n\}$

- The root set of the $n^{th}$ term of $M_n$ is $\{1, 2, \cdots, n\}$.
- $M_1 \subset M_2 \subset M_3 \subset \cdots$. So, $\{M_n\}$ is an expanding sequence.
- $M_n \in [X]^n$, for each $n$.
- The number of elements in $M_n$ is $\sum_{k=1}^{n} n(D_k)$. Here, $D_k = \{m \in N : m \text{ divides } k\}$ and $n(D_k)$ denotes the number of elements in $D_k$.

  **Illustration:** For $M_6 = \{1|6, 1|5, 1|4, 2|3, 3|2, 6|1\}$, $D_1 = \{1\}$, $D_2 = \{1, 2\}$, $D_3 = \{1, 3\}$, $D_4 = \{1, 2, 4\}$, $D_5 = \{1, 5\}$, $D_6 = \{1, 2, 3, 6\}$. Then, $\sum_{k=1}^{6} n(D_k) = 14$.

- If $M_n \in P(k)$, then $M_{n+1} \in P(k + \sum j)$, where $P(n)$ is the partition set of $n$ and $j \in D_{n+1}$

  **Illustration :** For $M_5 = \{1|5, 1|4, 1|3, 2|2, 5|1\}$, $M_5 \in P(21)$. For $M_6 = \{1|6, 1|5, 1|4, 2|3, 3|2, 6|1\}$, $M_6 \in P(33) = P(21 + 12)$. Here, $\sum_{j \in D_6} j = 12$.

- $P(n) \subset \tilde{P}(M_n)$, where $P(n)$ is the partition set of $n$ and $\tilde{P}(M_n)$ is the power multiset of $M_n$.

  **Illustration :** For $M_3 = \{1|3, 1|2, 3|1\}$, $\tilde{P}(M_3) = \{\phi, \{1|1\}, \{1|2\}, \{1|3\}, \{1|2, 1|3\}, \{1|2, 1|1\}, \{1|3, 1|1\}, \{2|1\}, \{1|2, 1|3, 1|1\}, \{1|2, 2|1\}, \{1|3, 2|1\}, \{3|1\}, \{1|2, 1|3, 2|1\}, \{1|2, 3|1\}, \{1|3, 3|1\}, M_3\}$ and $P(3) = \{\{1|3\}, \{1|2, 1|1\}, \{3|1\}\}$. So, $P(3) \subset \tilde{P}(M_3)$.

- $M_k \ominus M_{k-1} = D_k$

  **Illustration :** For $M_5 = \{1|5, 1|4, 1|3, 2|2, 5|1\}$ and $M_6 = \{1|6, 1|5, 1|4, 2|3, 3|2, 6|1\}$, $M_6 \ominus M_5 = \{1|6, 1|3, 1|2, 1|1\} = \{6, 3, 2, 1\} = D_6$.

- If $M_k \ominus M_{k-1} = \{1,\ k\}$, then $k$ is a prime number, otherwise, composite.

  **Illustration :** $M_{13} \ominus M_{12} = \{1,\ 13\}$. Thus, 13 is a prime number, but $M_{12} \ominus M_{11} = \{1,\ 2,\ 3,\ 4,\ 6,\ 12\}$, so 12 is not a prime number.

## 5. Convergence of Multiset Sequences

In this section, the convergences of multiset sequences are discussed. The concepts of Wijsman convergence, Hausdorff convergence, and statistical convergence are extended to mset sequences.

**Definition 5.1.** For an mset sequence $\{M_n\}$, $\cup_{k=1}^{\infty} \cap_{j \geq k} M_j$ is the limit infimum of $\{M_n\}$ and $\cap_{k=1}^{\infty} \cup_{j \geq k} M_j$ is the limit supremum of $\{M_n\}$.

**Definition 5.2.** If the limit supremum and limit infimum of an mset sequence are equal, then the sequence is said to be convergent and the common mset is known as the set theoretic limit or simply the limit of the sequence $\{M_n\}$.

**Proposition 5.3.**

(i) For a non-decreasing mset sequence, the set theoretic limit is $\cup_{n=1}^{\infty} M_n$, and that for a non-increasing mset sequence is $\cap_{n=1}^{\infty} M_n$.

(ii) For an mset sequence $\{M_n\}$, $\lim \inf M_n \subseteq \lim \sup M_n$.

**Definition 5.4.** Let $M$ be an mset derived from a metric space $(X, d)$. Then, $(M, d_M)$ is an mset metric space, if $d_M$ is a metric on $M$.

**Note 5.5.** The metric $d$ is also a metric on $M$. Since this metric is calculated without considering the multiplicity of elements, it is not treated as a good one.

Considering the multiplicity of each element of $M$, we can define a $d_M$ metric as follows:

**Definition 5.6.** Let $(X, d)$ be a metric space and $M$ be an mset drawn from $X$. Let $d_M : M \times M \to \mathbb{R}$ be a mapping defined by $d_M(x, y) = d(x, y) + |C_M(x) - C_M(y)|$ such that $\mathbb{R}$ is the set of real numbers.

**Proposition 5.7.** $d_M$ is a metric on $M$.

PROOF.

(i) For each $x, y \in M$, $d_M(x, y) \geq 0$. $d_M(x, y) = 0$, which means $d(x, y) = 0$ and $C_M(x) = C_m(y)$. That is, $x = y$ in $M$.

(ii) From the definition, $d_M(x, y) = d_M(y, x)$.

(iii) For $x, y, z$ in $M$,

$$\begin{aligned} d_M(x,y) &= d(x,y) + |C_M(x) - C_M(y)| \\ &\leq d(x,z) + d(z,y) + |C_M(x) - C_M(z)| + |C_M(z) - C_M(y)| \\ &= d_M(x,z) + d_M(z,y). \end{aligned}$$

□

**Definition 5.8.** Let $(M, d_M)$ be an mset metric space drawn from a metric space $(X, d)$ and $A$ be a submset of $M$. For any $x$ in $M$, $d_M(x, A) = inf\{d_M(x, a) : a \in A\}$.

**Definition 5.9.** Let $(M, d_M)$ be an mset metric space and $M_n, (n = 1, 2, 3, \cdots)$ are submsets of $M$. Then, the sequence $\{M_n\}$ is said to be Wijsman convergent to an mset $A \subseteq M$ if for each $x \in M$, $\lim_{n\to\infty} d_M(x, M_n) = d_M(x, A)$. In this case, it is written as Wlim $M_n = A$.

**Definition 5.10.** Let $(M, d_M)$ be an mset metric space and $M_n \subseteq M$, for $n = 1, 2, \cdots$. Then the sequence $\{M_n\}$ is said to be a Wijsman Cauchy sequence, if for each $\varepsilon > 0$, there is a positive integer $N$ such that $|d_M(x, M_n) - d_M(x, M_m)| < \varepsilon$ for each $m, n > N$ and for each $x \in M$.

**Theorem 5.11.** Let $(M, d_M)$ be an mset metric space and $M_n, (n = 1, 2, 3, \cdots)$ are submsets of $M$. If the sequence $\{M_n\}$ is a Wijsman convergent sequence, then it is also a Wijsman Cauchy sequence.

Proof. Suppose $\{M_n\}$ is a Wijsman convergent sequence converging to $A$. Then, for each $x \in M$,

$$\lim_{n\to\infty} d_M(x, M_n) = d_M(x, A)$$

So, for given $\varepsilon > 0$, there is at least one positive integer $N$ such that

$$|d_M(x, M_k) - d_M(x, A)| < \frac{\varepsilon}{2}, \ \forall k \geq N$$

Choose $m, n > N$. Then,

$$|d_M(x, M_m) - d_M(x, A)| < \frac{\varepsilon}{2}$$

and

$$|d_M(x, M_n) - d_M(x, A)| < \frac{\varepsilon}{2}$$

Thus,

$$|d_M(x, M_m) - d_M(x, M_n)| \leq |d_M(x, M_m) - d_M(x, A)| + |d_M(x, A) - d_M(x, M_n)| < \ \varepsilon$$

Therefore, $\{M_n\}$ is a Wijsman Cauchy sequence. □

**Definition 5.12.** Let $(M, d_M)$ be an mset metric space and $M_n, (n = 1, 2, 3, \cdots)$ are submsets of $M$. Then, $\{M_n\}$ is said to be Hausdorff convergent to an mset $A \subseteq M$ if $\lim_{n\to\infty} sup_{x\in M}|d_M(x, M_n) - d_M(x, A)| = 0$

**Theorem 5.13.** Let $(M, d_M)$ be an mset metric space and $M_n, (n = 1, 2, 3, \cdots)$ are submsets of $M$. If $\{M_n\}$ is a Hausdorff convergent sequence, then it is also a Wijsman convergent sequence.

Proof. The proof is obtained directly from the definitions of the Wijsman and Hausdorff convergences. □

**Definition 5.14.** Let $(M, d_M)$ be an mset metric space and $M_n, (n = 1, 2, 3, \cdots)$ are submsets of $M$. Then, $\{M_n\}$ is said to be Wijsman statistically convergent to an mset $A \subset M$ if $\lim_{n\to\infty} \frac{1}{n}\{k \leq n : |d_M(x, M_k) - d_M(x, A)| \geq \varepsilon\} = 0, \forall x \in M$ and $\forall \varepsilon > 0$.

**Definition 5.15.** Let $(M, d_M)$ be an mset metric space and $M_n, (n = 1, 2, 3, \cdots)$ are submsets of $M$. Then, $\{M_n\}$ is said to be Hausdorff statistically convergent to an mset $A \subset M$ if $\lim_{n\to\infty} \frac{1}{n}\{k \leq n : sup_{x\in M}|d_M(x, M_k) - d_M(x, A)| \geq \varepsilon\} = 0, \forall x \in M$ and $\forall \varepsilon > 0$.

## 6. Conclusion

This paper attempts to probe into the development of the multiset theory in novel scenarios such as mset sequences. It delves into the sequences and their convergences to obtain various results and properties analogous to the set sequences. The work here only scratches the surface of the possibilities of convergence of msets. An expanded scope of the research in this paper can dive much deeper into the same, and further research can be conducted on their various applications in different fields.

## Conflicts of Interest

The authors declare no conflict of interest.

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