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Research Article

Mathematical Communication Process of Junior High School Students in Solving Problems based on APOS Theory

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Abstract

Role of mathematical communication in learning is needed. Mathematical communication could carry conceptual understanding, problem solving, and mathematics reasoning. This research aims to explain communication process of Junior High School students in solving problems based on APOS framework theory. In solving mathematics problem at school, it emphasizes more on outcome than process without considering students' reasoning process. To express mathematics ideas, the learning seems to emphasize on written than spoken. To solve that problem, there is a need of mathematical communication process connection to students' reasoning in solving problems. The reasoning skill of the students was reviewed based on APOS Theory. This research is a qualitative research. The instruments of collecting data were problem solving task and interview. The results showed that there were 10 students performing mathematical communication process by having pseudo drawing communication and 20 students by having pseudo mathematical expression communication criteria.

Keywords:

mathematical communication, APOS Theory, problem solving skills, mathematic education

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Introduction

Mathematical communication has important role in learning mathematics. Departemen Pendidikan Nasional (2003) states that one emphasized aspect in mathematical learning is communication. There are several researchers have similar arguments to importance of mathematical communication in learning mathematics (Jung & Reifel, 2011; Kaya & Aidyn, 2014; Kosko & Wikins, 2010; Lamibao, Charita, & Namoko, 2016; Rohid, Suryaman, & Rusmawati, 2019; Smieskova, 2017; Sumaji, Sa'dijah, Susiswo, & Sisworo, 2019; Viseu & Olivera, 2012). Mathematical communication facilitates to improve conceptual understanding so it could help in solving problem (Smieskova, 2017). In line with the arguments, (Kaya & Aidyn, 2014) and Jung & Reifel (2011) state that through mathematical communication, students' mathematical reasoning and conceptual understanding could be improved. Lamibao et al., (2016) stated that mathematical communication in solving problems could develop conceptual understanding. It is in line with Rohid et al., (2019) states that mathematical communication in learning mathematics contributes to comprehensive teacher - student analysis. It is in line with Kosko & Wikins, (2010) explaining that mathematical packaged in the form of group discussion could encourage students in improving conceptual understanding. To support the notion, Sumaji, Sa'dijah, Susiswo, & Sisworo, (2019) and Viseu & Olivera, (2012) states that communication could facilitate students understanding the process and taking decision in solving problem.

Studies related to mathematical communication have been frequently done. Those studies cover: *First*, open ended problem implementation in developing mathematics communication (Kongthip, Inprasitha, Pattanajak, & Inprasitha, 2012; Rodriguez & Bonner, 2018; Sumaji, Sa'dijah, Susiswo, & Sisworo, 2020; Thinwiangthong, S, Inprasitha, M, & Loipha, S, 2012). The findings were: (1) Rodriguez & Bonner (2018) state the use of open ended problem could improve teacher's skill in asking questions and creating dialogue between teacher and student more productive and better, (2) Thinwiangthong, Inprasitha, & Loipha (2012) revealed that mathematics class adopting lesson and transparent approach, could improve mathematical communication, (3) Kongthip et al., (2012) explained that schools in lesson study context and transparent approach allowed the students to have greater learning opportunity based on their potencies, thinking skills, actions, and expressions, and (4) Sumaji et al., (2020) found that (1) mathematical communication of students for both in written and spoken was on first level. It was in solving limited open ended problem on several concepts and (2) mathematical communication of students for both written and spoken was on level two. The students shared correct explanation while solving open ended problems. *Second*, dealing with mathematical communication representations were from Tiffany, Surya, Panjaitan & Syahputra (2017) and (Rohid et al., 2019). The findings were: (1) (Tiffany et al., 2017) stated that students could connect figures, diagrams,

and mathematical idea tables, (2) (Rohid et al., 2019) stated that one out of three students could express mathematics ideas by using terms, notation, and symbols. *Third,* dealing with effective mathematical communication was stated by Ryve, Nilsson, & Pettersson (2013). They showed that visual communication was effective in learning mathematics.

Based on those findings, there was theory discussing about the correlation between mathematical communication and students' thinking process reviewed based on APOS theory in solving problem. Arnon et al., (2014) and Syamsuri, Purwanto, Subanji, & Irawati, (2017) explains that APOS theory involves mental structures, such as action, process, object, and scheme which are connected to mental mechanism which consists internalization, coordination, reversal, encapsulation, de-encapsulation, and thematization. It is in line with Sutarto, Nusantara, Subanji, Hastuti, & Dafik (2018) explaining that APOS theory involving action, process, object, and scheme.In this research, the researchers gave students problems. Then, their thinking processes were reviewed by APOS theory and it was seen in term of their mathematical communications. Rosidin, Suyatna, & Abdurrahman, (2019) and Sastrawati, Rusdi, & Syamsurizal, (2011) argue that thinking is a mental operation process, such as classifying, inducting, deducting, and reasoning. Sukoriyanto, Nusantara, Subanji, & Chandra, (2016) stated that reasoning is an information permeating process by having scheme design within human's brain. Therefore, researcher wanted to know the mathematical communication process of the students in solving problems based on APOS theory.

Problem of Study

Mathematical communication plays important role in learning mathematics. Through mathematical communication, it could develop mathematics concept. Then, when students understand mathematics concept, it could ease them solving mathematics problems. Currently, learning mathematics emphasizes on outcome rather than process without considering students' reasoning process in solving problem. To express mathematics ideas, it is emphasizes on written than spoken activities so that students had difficulties to express orally their works.

Based on the problems, the researchers wants to review the process of students' mathematics communication in solving mathematics problem. Reasoning process of the students in solving problem based on APOS Theory.

Methods

Research Design

This research used qualitative research. Creswell (2012) explained about characteristics of qualitative research approach that research process will always dynamically develop. It means that each research process could be changed after

the researcher gets involved in research site to collect data. Technique to select the subject was purposive sampling.

Participants

This research was conducted at Public Junior High School 2 Rembang. There were 30 eighth graders of Junior High School 2 Rembang. The students were grouped into 2 criteria: (1) students undergoing mathematical communication process by having pseudo drawing communication and (2) students by having pseudo mathematical expression communication criteria. The steps in grouping the subjects are: (1) giving problem solving task, (2) promoting interview to confirm students' answers in written and to obtain uncollected information from problem solving written test, and (3) grouping students into two criteria: students undergoing mathematics communication process by having pseudo drawing communication and pseudo mathematical expression. Based on the explanation, it could be made plot of research subject as presented in Diagram 1.

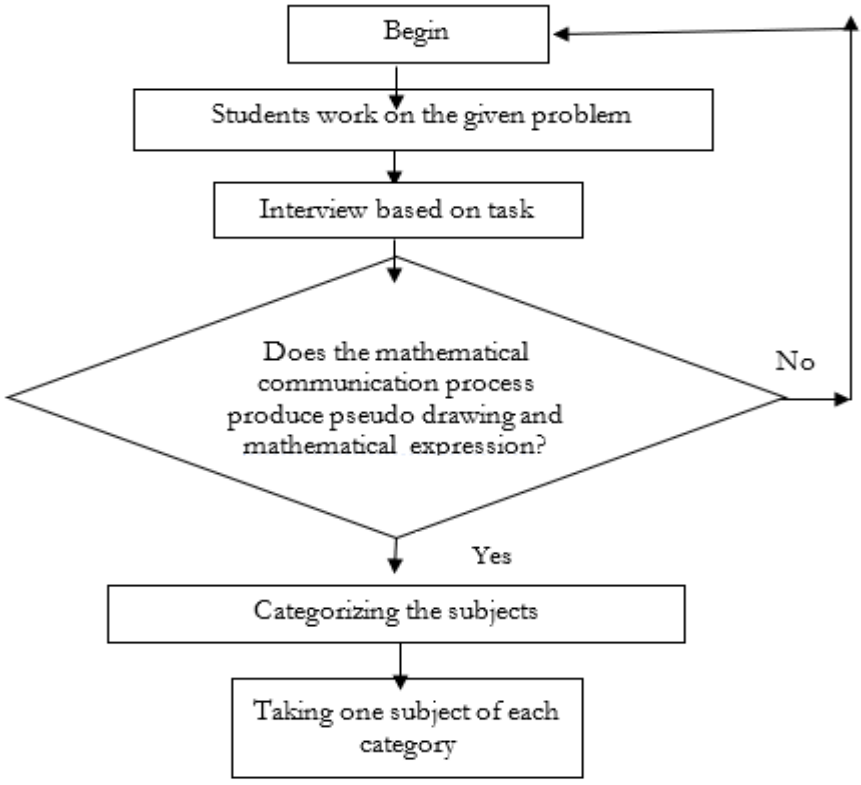


Diagram 1.

Research Subject Selection

Instruments

In this research, the students were asked to estimate logically the numbers of traveler candidates while jostling as presented in Figure as follows:

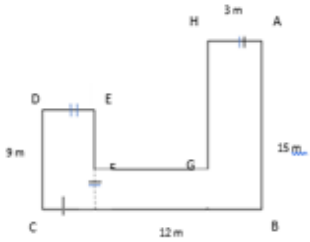


Figure 1.
Problem Figure and Picture

The traveler candidates were jostling at the ticketing queue as shown in Figure 1 department of transportation employed two officers, A and B, to estimate the numbers of traveler candidates whom are lining up at the ticketing place while providing the map of queue location as shown in Figure 2. It is done because the government, through department of transportation, will renovate the queue location in which is adjusted to the number of traveler candidate estimation. Thus, it will be able to cope all travelers without jostling each other. A and B have different ways to estimate it. A estimates the numbers of traveler candidates from second floor by taking estimated sample area in which is placed by each one individual. B estimates the numbers of the traveler candidates by estimating the numbers of individuals leaning back on the fence, as shown on Figure 1.

Questions

1. What is the most logic estimation of traveler candidate while jostling according to both officers' ways?
   *Clue: (a) make the most logic analogy of the occupied area by an individual while he is standing up and jostling to estimate the numbers of the traveler candidates as officer A does! (b) make the most logic analogy of occupied area by an individual while he is leaning back on fence and jostling to estimate the numbers of traveler candidates as officer B does!
2. Are the ways of A and B officers to estimate the numbers of traveler candidates logic? Share your answer!

Data Analysis

Data analysis technique of this research is qualitative. The researcher analyzed the written and spoken answers to obtain data about the students' mathematics communication process criteria based on APOS theory. The steps in analyzing the data are: describing data, reducing data by abstracting, arranging each data part and

grouping them to create code, creating mathematics communication process criteria based APOS theory, and concluding.

Results

Here is the explanation of the research's results concerning with communication process of the students in solving problems based on APOS theory. There were 30 students consisted of two criteria: 10 students conducting mathematical communication process with pseudo drawing communication criteria and 20 students conducting the communication with pseudo mathematical expression communication criteria. It could be seen below.

Table 1.

Communication Process Criteria of the Junior High School Students in Solving Problem

	Categorizing the Junior High School students' communication process in solving problem	Percentage
1	Mathematical communication process with pseudo drawing communication criteria	33,3%
2	Mathematical communication process with pseudo mathematical expression communication criteria	66,7%

The table 1 shows and explains two subjects. S1 was a student doing mathematical communication process with pseudo drawing communication criteria. S2 was a student doing mathematical communication process with pseudo mathematical expression communication criteria. Then, the next step of problem solving based on APOS theory is explained on Table 2. As follows:

Table 2.

Problem Solving Stage Based on APOS Theory

No	Problem Solving Stage	APOS Theory	
		Mental mechanism	Mental structure
1	Identifying problem components on question	Interiorization	Action
2	Processing correlation between length and width of queue location map so it forms area I, II, and III	Coordination	Process
3	Recalling formula of rectangular area $= p \times l$	Rehearsal	Process
4	Processing correlation of length and width to form area.	Coordination	Process
5	Processing connection of area estimation placed by an individual while jostling each other to obtain analogy of an occupied area by an individual as officer A estimates.	Coordination	Process
6	Determining the numbers of traveler candidates according to officer A by dividing the total area to estimate an occupied area of an individual.	Encapsulation	Object
7	Processing the traveler candidate conditions while jostling at each side of the place where each one of them leaning back on fence as officer B estimates.	Coordination	Process

8	Processing connection of each area on queue location map to obtain parameter.	Coordination	Process
9	Determining the numbers of traveler candidates according to officer B by dividing the parameter of queue location map to length of the place while occupied by an individual.	Encapsulation	Object
10	Explaining the ways used by both officers in estimating the numbers of traveler candidates.	Encapsulation	Object
11	Generalizing that the estimated area while occupied by an individual while jostling as obtained from estimation of the most logic length and width, and during jostling. Then, it was connected to current existing knowledge.	Thematization	Scheme

Mathematical Communication Process of S1 with Pseudo Drawing Communication

Mental action structure was indicated by S1 while sharing his spoken communication by reading aloud the question. S1 did interiorization by identifying the problem components in the form of spoken communication, such as: the travelers jostling up in a crowd, two officers – A and B estimating the numbers of travelers, officer A estimating the numbers of travelers by supposing the area was

located by one person, officer B supposing the length of one person side while standing near on the fence.

On mental process structure, S1 run coordination process in the form of written communication. Here is the S1 coordination in written form.

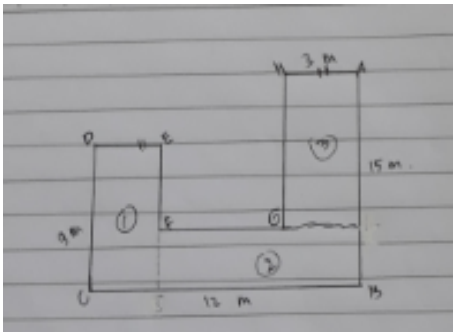


Figure 2.

Coordination Result of S1 in the Form of Written Communication by Drawing the Map of the Queue

Then, coordination process of S1 in the form of spoken communication. Here is the result of the coordination through interview.

S01: Before you find the area, did you draw the map?

S02: I did

S01: Tell me how did you draw and divide the map into three parts?

S02: Emmm…. It thought it was by connecting, sir.

S01: How did you do it?

S02: I don't know sir.

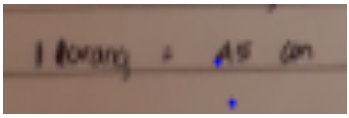
Based on the explanation, S1 run coordination process in each written from by drawing and dividing the queue location into three parts: side I, II, and III in the form of rectangle correctly. Based on confirmation result in the form of spoken communication through interview, S1 was found being unable to explain it. It proved that S1 did mathematical communication by pseudo drawing communication criteria.

On mental process structure, S1 did reversal process by recalling the formula of rectangle area to be re-coordinated by the already interiorization components. Here is the reversal process continued by coordination in the form of spoken communication through interview.

S01: How did you find the area of the location map of the queue? Please tell me

S02: Dividing the location of the map into 3 parts. They are I, II, and III. The first part is 9 m length and 3 m width. The area is $(9 \times 3 = 27)\text{m}^2$. The second part is $(12 - 3)\text{m}$ and 3 m width. The area is $(9 \times 3 = 27)\text{m}^2$. The third part is 12 m length and 3 m width. The area is $(12 \times 3 = 45)\text{m}^2$. The total of area is $90\ \text{m}^2$.

On mental process structure, S1 run coordination in the form of written and spoken communications. S1 did coordination in the form of written communication by estimating the area of a place stood by each person while he was jostling. Then, it was obtained the area of one person standing at the location was equal to 45 cm. It showed that S1 did coordination by revealing the written communication into mathematical expression. Here is the coordination in the form of S1's written communication.



1 person = 45 cm

Figure 3.

S1's Coordination Result in the Form of Written Communication by Determining One – Person Area Estimation

S1's spoken communication could be traced through interview as seen in this excerpt.

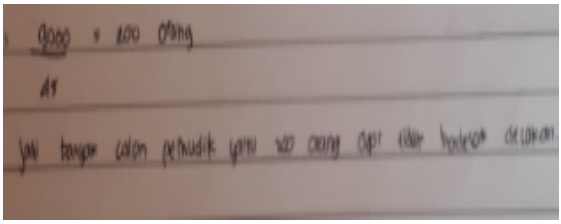
S01: What did you think to estimate the numbers of the travelers by estimating the area of one person placing the area from second floor?

S02: By analogizing if 1 person's area is 45 cm.

S01: Were you sure? Please tell me

S02: I was, sir.

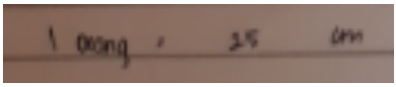
On mental object structure, S1 encapsulated by packaging the process into objects in the form of both written and spoken communications. Encapsulation in the form of written communication was done by calculating the estimation of the numbers of the travelers by dividing the area total of queue location map. Here is the encapsulation of the written communication form.



$(\frac{9000}{45} = 200)$ *people*
Thus, the numbers of the traveler candidates are 200 people to avoid jostling each other.

Figure 4.
The S1's Encapsulation Result in the Form of Written Communication in Determining the Estimation of the Numbers of the Travelers by Officer A

Then, the mental process structure was done by coordinating the written and spoken communications. The coordination was in the form of written form through connecting the interiorization components. They were the jostling travelers along the side of each person standing near the fence based on officer B. So, the analogy of one jostling traveler on the fence was 25 cm. Here is the coordination result in the form of written communication.



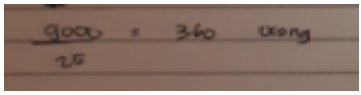
1 person = 25 cm

Figure 5.
S1's Coordination in the Form of Written Communication by Determining the Side Length of One Person

Here is the coordination in the form of spoken communication through interview.

S01: What did you think to estimate the numbers of the travelers by estimating the numbers of people standing near the fence assumed by officer B? Please tell!
S02: I estimated the length placed by one person was 25 cm.
S01: Could you share the reason why the length was 25cm? Please tell me!
S02: I did not know sir

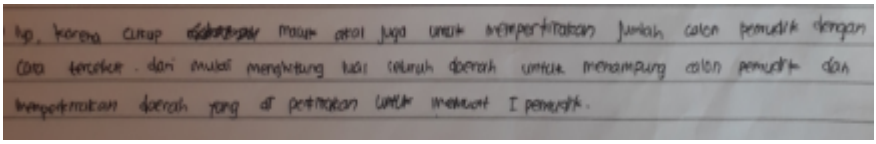
On mental object structure, S1 encapsulated both written and spoken communications. The revealed encapsulation in the communications was mathematical expression by estimating the numbers of travelers done by officer B – it was by dividing the total area with the length of one traveler placing the place. Here is the encapsulation result of S4's written communication.



$$(\frac{9000}{25} = 360\ )\text{people}$$

Figure 6.
The Revealed S1's Encapsulation Result in the Form of Written Communication in Determining the Estimateion of Travelers by Officer B

The mental process structure done by S1 coordinated the already interiorization. S1 coordinated them in the form of written and spoken communications by explaining the ways of A and B officers in estimating the numbers of travelers. S1 revealed that what B did was more logic than A in estimating the numbers of travelers while they were jostling. It was caused by officer B in estimating the numbers of travelers involved people whom were standing on fence. Here is the coordination process with S1 in written communication form.



Yes, it is since it makes sense to estimate the numbers of the travelers by such method, started by counting the area to cover all travelers and to estimate the area to cover a person.

Figure 7.
S1's Coordination Result in the Form of Written Communication by Written Text

Here is the S1's coordination process of spoken communication through interview discourse.
S01: Are the ways of officer A and B logic?
S02: B is more logic because B counted the people standing near fence.

S1 thematically revealed orally by generalizing the estimation of employed area by each person based on logic length and width while jostling. Then, S1 correlated the already existing knowledge: area of the rectangle. Then, it was obtained understanding if the area estimation of employed place by each people was smaller, then the estimation of the travelers would be higher. Here is the result of spoken communication trace through interview.

S01: Is smaller place employed by each person would result to increasing numbers of traveler estimation
S02: If each area of employed place by each person gets smaller, it means there are many people.

S01: The side length used by each person is getting longer, does it mean the numbers of traveler gets lesser?
S02: It does. The more people standing on the fence will result to lesser capacity

It shows that S1 thematically communicated orally by generalizing. Here is the mathematical communication analysis process by pseudo drawing communication criteria in solving problem based on APOS theory as shown in Diagram 2

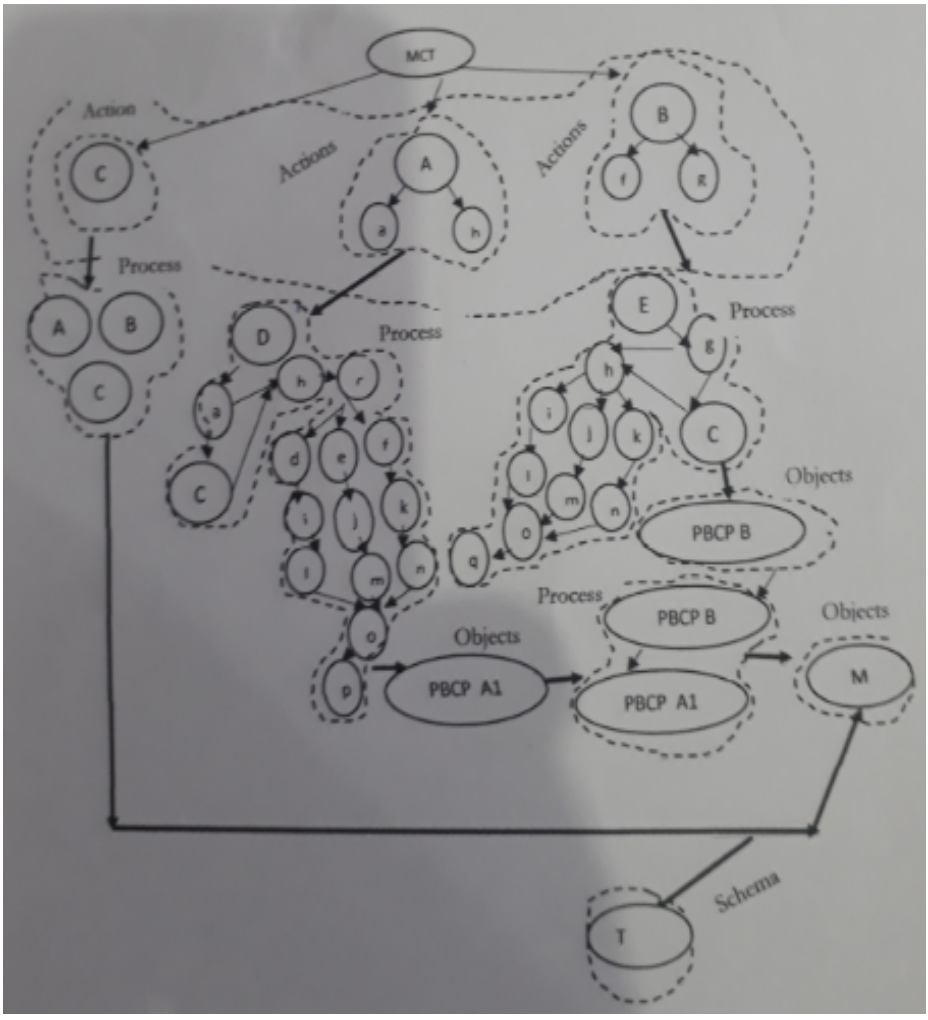
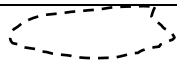


Diagram 2.
S1's Mathematical Communication Process Analysis with Pseudo Mathematical Expression Communication Criteria based on APOS Theory.

Table 3.

Code Remarks on Diagram 2

Code	Explanation	Code	Explanation
MCT	Mathematical Communication Test	j	size of ii part of the map
A	Ways used by officer A	k	Size of III part of the map
B	Ways used by officer B	l	First part map area
C	Traveler jostling	m	Second part map area
D	The numbers of travelers according to officer A	n	Third part map area
E	The numbers of travelers according to officer B	o	Area total
a	Officer A estimates the numbers of travelers from second floor by estimating the area placed by each person.	p	The total area divided by area estimation of one person
b	Area estimateion placed by one person	q	The total area divided by side length estimation of a person standing near the fence
c	Describing the queue location map into three parts in the form of rectangle.	M	The way of officer A and B estimating the numbers of travelers.
d	First queue location map	PBCP A	Estimation of the traveler numbers according to officer A
e	Second queue location map	PBCP B	Estimation of the
f	Third queue location map		travelers numbers according to officer B
g	Officer B estimates the numbers of the travelers based on the numbers of people standing near the fence.	T	Smaller area placed by a traveler will result many numbers of travelers.
h	Estimation of side length of each person standing near the fence.		Explanation of interiorizationed components.
i	First part map size		Mental object structure
	Conducting mental mechanism		Mental Structure

→	Moving to other mental structure
◯	Interiorizationed components.

Mathematical Communication Process of S1 with Pseudo Mathematical Expression Communication

The mental action structure was indicated by S2 revealed in the form of spoken communication through reading the question. S2 interiorizationed in the form of spoken communication by identifying the ways used by officer A and B in estimating the numbers of travelers and their jostling conditions.

On the mental process structure, S2 revealed both in written and spoken. The written communication of S2 described the queue location map and it was divided into three parts: A, B, and C, each of them shaped rectangular. It showed that the coordination was in the form of written communication, by drawing.

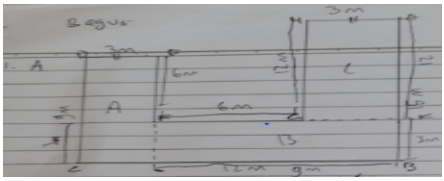
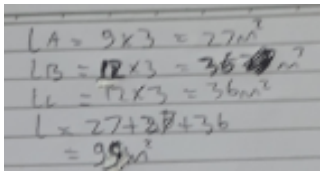


Figure 8.

Coordination Result of S1 in the Form of Written Communication by Drawing the Map of the Queue

On mental process structure, S2 did reversely by recalling his owned knowledge in the form of written communication. The reversal was area of rectangular. It was done because during coordination, there was lack of information found in interiorization dealing with area of rectangle. Then, the coordination was done by connecting the length and width of each side: A, B, and C. Here is the reversal process continued by coordination in the form of S2's written communication.



Area A = (9 x 3 = 27) m^2
Area B = (12 x 3 = 36) m^2
Area C = (12 x 3 = 36) m^2
Area Total = 99 m^2

Figure 9.

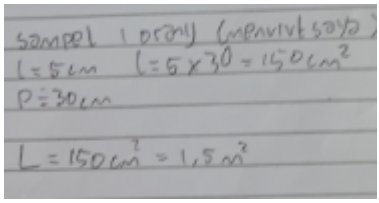
S2's Coordination Result in the Form of Written Communication by Determining the Area of Queue Location

S01: How is the size of Block A?
S02: Rectangle A is 9 m length and 3 m width. So the area is 27m square.
S01: How did you know it was 3 m?
S02: It was the estimation or it was equal with width of C. It has equivalent sign.

S01: How is the block B size?
S02: Emm… There is something wrong. The length is 9m and the width is 3m. The length 9 m is gained from 15m minus 3 m. Thus, the area is 36 m^2.
S01: How is the total area?
S02: $27 + 36 + 36 = 90\ \mathrm{m}^2$
S01: $27 + 36 + 36$ What is the unit of each of them?
S02: Meter square, sir.

Based on the explanation, S2 had mistaken in coordinating process in the form of written communication. It was when the length of block B was 12 m. Then, after being traced through spoken communication in the interview, S2 could provide explanation that block B was 9 m. It was obtained from $9\ \mathrm{m} = (12 - 3)\mathrm{m}$. It showed that S2 did communication process with pseudo mathematical expression communication.

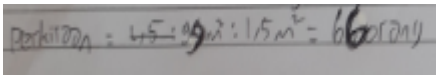
On mental process structure, S2 coordinated in the form of written communication by connecting the estimation of placed area of each person while jostling with area of rectangle formula: length multiplied by width by using ruler. So it was obtained the placed area of each person was equal to 5 cm x 30 cm. Here is the written communication coordination done by S2.



1 person, according to me:
Width = 5 cm
Length = 30 cm
Area = (5 x 30 = 150) cm^2
Area = 150 cm^2 = 1.5 m^2

Figure 10.
S2's Coordination Result in the Form of Written Communication by Determining One – Person Area Estimation

On mental object structure, S2 encapsulated by packaging the process into written and spoken communication forms. The encapsulation was in the form of written communication. It determined that the estimation of the traveler numbers according to officer A was done by dividing the area of second map by estimating the placed area of each person while jostling. It was $(99 \div 1.5 = 66)$ people. Thus, S2 estimated that the numbers of the travelers while jostling were 60 people. Encapsulation in the form of written communication was done by calculating the estimation of the numbers of the travelers by dividing the area total of queue location map. Here is the encapsulation of the written communication form.



Estimation: (99 ÷ 1.5 = 66) persons

Figure 11.
The S2's Encapsulation Result in the Form of Written Communication in Determining the Estimation of the Numbers of the Travelers by Officer A

Here is the interview result with S2.

S01: How is the estimation of the traveler numbers based on officer A? Please tell!

S01: $90 \div 1{,}5 = 60$, so 60 people

The estimation according to officer A was mathematical object resulted through encapsulating mental mechanism. S2 coordinated the objects by dividing the area of queue location by estimation of placed area by one person.

On mental process structure, S2 estimated the numbers of travelers by officer A through estimating the numbers of people standing near the fence. S2 coordinated in the form of spoken and written communications. In the written communication, he coordinated by connecting the interiorization components. They were the conditions of the travelers jostling on the each side of the fence. Thus, an analogy was obtained – the length of one person jostling near the fence was 30 cm. Here is the interview with S2.

S01: How did you estimate the numbers of the travelers based on officer B?

S02: Finding the perimeter then supposing the side length of one person is 30 cm.

On the mental process structure of officer B, S2 coordinated both in written and spoken communications. The spoken communication was done by connecting the length of each location side so a perimeter of queue location was obtained. Then, S2 conducted reversal process in the form of spoken communication by recalling the perimeter. It was done by adding all sides. The perimeter of the location based on S2 could be traced on this interview.

S01: Officer B estimated the numbers of people standing near the fence. What did you think?

S02: I thought about the perimeter.

S01: How was it?

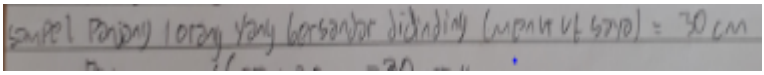
S02: 66 m = 6600 cm. It was obtained from the addition of all sides. $9 + 3 + 6 + 6 + 12 + 3 + 15 + 12 = 66\,\text{m}$

On mental process structure, S2 coordinated both in written and spoken communications in estimating the number of the travelers based on officer B. It was done by estimating the numbers of standing people near the fence. The coordination was in the form of written communication by connecting the conditions of jostling travelers along the sides of the fence. So, the analogy was obtained that one jostling person standing near the fence had 30 cm length. Here is the spoken communication coordination result by interviewing.

S01: How did you estimate the numbers of the travelers based on officer B?

S02: By finding the perimeter of the fence then supposing the length of one person was 30 cm.

Here is the written communication coordination done by S2

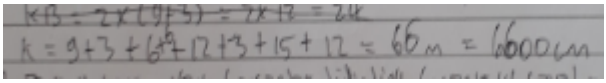


1 person standing near the wall is 30 cm length, according

Figure 12.

S2's Coordination Result in the Form of Written Communication Determining the Side Length Estimation of One Person

On mental structure process, S2 coordinated both in the form of written and spoken communications. The written communication coordination was done by connecting the side length of queue location map so it shaped a perimeter of the queue location. S2 did reversal in the form of written communication by recalling the perimeter. It was done by adding all sides. Here is the written communication coordination done by S2.



Perimeter = (9 + 3 + 6 + 2 + 12 + 3 + 15 + 12 = 66)m = 6600 cm

Figure 13.

The Coordination Result in the Form of Written Communication by Determining the Area of Queue Location Perimeter

Then, the mental object structure was done by encapsulating in the form of written and spoken communications. The encapsulation was found in spoken communication form. S2 revealed in written form that to estimate the numbers of travelers according to officer B by dividing the perimeter of queue location map perimeter by the placed length by one traveler. It was $6600\ \text{cm} \div 30\ \text{cm} = 220$ persons.

On mental process structure, S2 coordinated both in written and spoken communications. The written communication was done by explaining the ways of A and B. According to S2, the ways of A were more logic than B in estimating the numbers of the travelers while jostling because officer A compared the area of the queue with each area of one person. Meanwhile, the officer B only considered the perimeter. Here is the interview excerpt with S2.

S01: Are the ways used by officer A and B logic?

S02: A is more logic.

S01: Why?

S02: Because he compares the area of the queue and area of each person correctly.

S01: What about B

S02: He only considered the perimeter.

On mental scheme structure, S2 interiorizationed by revealing orally and generalizing the estimation of the logic placed area of each person while josting. Then, it was correlated to the already existing knowledge about the area of rectangle. Then, it was obtained an understanding if the placed area by each person was smaller, then the estimation of the traveler numbers would be greater. Here is the trace of spoken communication interview with S2.

S01: Is smaller placed area by a person will result to increasing number of the travelers?

S02: It is.

S01: Because lesser area of a place of each person will result to increasing capacity of the traveler numbers.

Here is the mathematical communication process with pseudo mathematical expression communication criteria in solving problem based APOS theory as shown by Diagram 3.

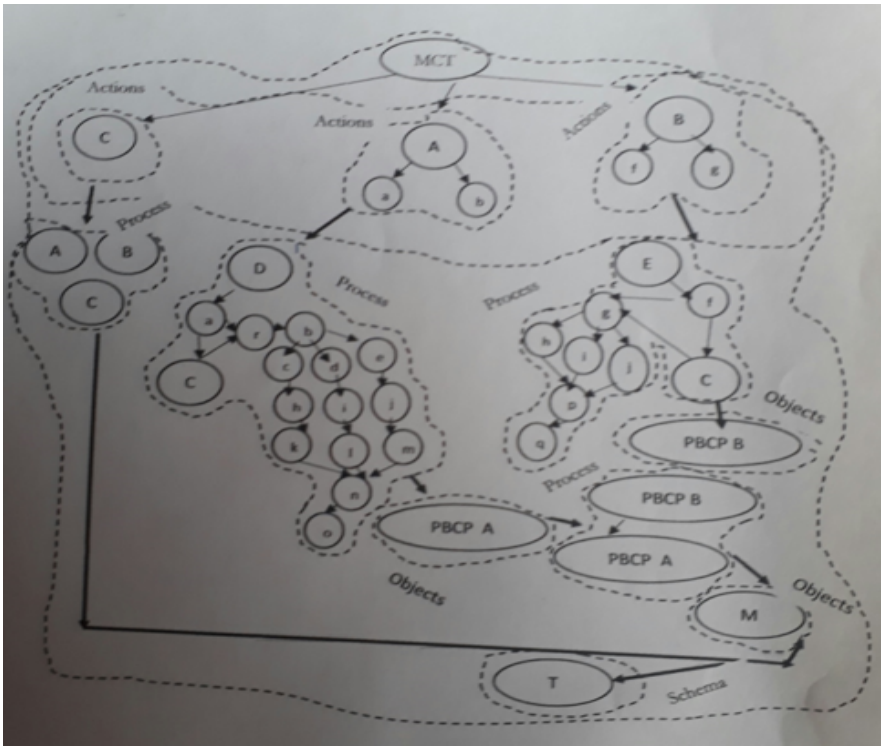
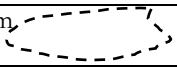


Diagram 3.

The Mathematical Communication Process Analysis of S2 with Pseudo Mathematical Expression Communication Criteria based on APOS Theory

Table 4.

Remarks of Diagram 3's Codes

Code	Explanation	Code	Explanation
MCT	Mathematical Communication Test	i	Size of ii part of the map
A	Ways used by officer A	j	Size of III part of the map
B	Ways used by officer B	k	First part map area
C	Traveler jostling	l	Second part map area
D	The numbers of travelers according to officer A	m	Third part map area
E	The numbers of travelers according to officer B	n	Area total
a	Officer A estimates the numbers of travelers from second floor by estimating the area placed by each person.	o	The total area divided by area estimation of one person
b	Area estimation placed by one person		Perimeter total
r	Describing the queue location map into three parts in the form of rectangle.	q	Total perimeter divided by side length estimation of a person standing near the fence
c	First queue location map	M	The way of officer A and B estimating the numbers of travelers.
d	Second queue location map	PBCP A	timation of the veler numbers cording to officer A
e	Third queue location map	PBCP B	timation of the velers numbers cording to officer B
f	Officer B estimates the numbers of the travelers based on the numbers of people standing near the fence.	T	Smaller area placed by a traveler will result many numbers of travelers.
g	Estimation of side length of each person standing near the fence.		Explanation of interiorization components.
h	First part map size		Mental object structure
	Conducting mental mechanism		Mental structure
	Moving to other mental structure		
	Interiorization components.		

Discussions

On mental action structure, the student shared response revealed in the form of spoken communication by reading the questions. It is in line with Arnon et al., (2014) they told that action is external in nature. It meant each transforming stage is done explicitly and guided by external instruction. Then, the subject interiorization in the form of spoken communication to identify the problem components in spoken communication form. It is in line with Mybert, Maharaj, & Brijlall (2013) they stated that interiorization is an action of understanding as object structure and using it in solving problem.

On mental process structure, the student coordinated both in written and spoken communication forms. The student coordinated in written communication by drawing the map of the collation and grouped them into three parts: I, II, and III with correct sizes. Based on the trace of spoken communication through interview, the student could not explain the ways of drawing the queue location map and its correct size. It showed that the student did pseudo drawing communication criteria. It is different with Premprayoon, Loipha, & Inprasitha (2014) explaining that written mathematical communication could be revealed in the form of figures, diagrams, and graphs. It was due to the student expressing his mathematical ideas both in written and spoken communications.

The student did reversal and coordinated by determining the area of I, II, and III. It showed that S1 doing so by pseudo mathematical expression of written communication form. It is in line with Lepak (2014) telling that mathematical ideas could be revealed in the form of written words, figures, and symbols (mathematical expressions). The student committed mistake in coordinating process in the form of written communication in determining the length of side II. Then, after being traced through spoken communication by interview, it was found that the student could explain correctly. It showed that the student had pseudo mathematical expression communication criteria. It is different with Tiffany et al., (2017) dan Nartani, Hidayat, & Sumiyati (2015) revealing that written communication of the student could be seen from mathematical expression. It was by revealing the situation into language or mathematical symbol. It was due to in this research, the student expressed his mathematical ideas both in written and spoken communications.

Then, the student coordinated both in spoken and written communications. The student coordinated in written communication by connecting estimation of placed area of one person jostling. It was obtained that the area made sense. It showed that the student coordinated in written form by mathematical expression. It is in line with Freeman, Higgins, & Horney (2016) that written mathematics communication could facilitate students with low level to improve their concepts.

On mental object structure, the student packaged the process into object in the form of spoken and written communication. He determined that to estimate the traveler numbers based on officer A, by dividing the area and estimation of placed area by one person. It showed that the student encapsulated both in spoken and written communications by mathematical expression. It is in line with Bicer et al., (2011) that written communication of mathematics could facilitate students mastering vocabulary, phrase, symbol, and mathematics meaning.

The student encapsulated into spoken communication by determining the total area of the map which was divided by estimation of one person – placed area. It is in line with Smieskova (2017) stating that spoken communications covered speaking, listening, asking, explaining, defining, discussing, drawing, and correcting/determining. On mental process structure, the student estimated the traveler numbers according to officer B. It was done by estimating the numbers of people standing near the fence. The student coordinated in written communication by connecting the interiorization components: the conditions of the people jostling on the side length. Thus, an analogy was obtained by estimating one person jostling near the fence. It showed that the student did mathematical expression communication criteria. It is in line with Rohid et al., (2019) arguing that notion, idea, and mathematics solution in written manner, such as numerical and algebra symbol uses, and visually by using diagram, figure, chart, table, and graphic.

On mental object structure, the student encapsulated both in spoken and written communication. It was realized into written communication by writing to estimate the traveler numbers based on officer B. It was done by dividing the area of the queue location by the length of one person placed location. It showed that the student encapsulated in written communication by mathematical expression. It is in line with Yuniara (2016) that written mathematical communication could be expressed through written communication into drawing which covers creating figure, table, diagram, and graphic. It consisted of creating figure, table, diagram, and graphs. Then, S1 looked back again to mental process. Then, he coordinated the interiorization components. The coordination was done in written communication in which based on the student, the ways used by officer B was more logic than A. It was due to officer B estimating the traveler numbers whom were standing near the fence. The coordination process in written communication was done by explaining in written words about the ways of officer A and B in estimating the traveler numbers. It was due to the ways of officer A was more logic than B since officer A compared the queue area to area of a person while officer B considered the perimeter. It showed that the student in coordinating in written communication was done it by written text. It is in line with Arnon, I et al., (2014) that process is fostered by one or two mental mechanism: interiorization or coordination. The student also coordinated which was revealed in spoken communication. It was done by providing reason/clarification. It is in line with Bal

(2015) telling that spoken communication could be explained as situation in which student understood a problem and could provide reason orally.

On mental scheme structure, the student thematically revealed orally by generalizing that the one person placed area estimation was obtained from logic width and length when people jostling. Then, he correlated to the already owned knowledge – the area of rectangle which was multiplication of legth and width. Then, an understanding that area placed by one person would be smaller resulted to increasing capacity was obtained. It is in line with It is in line with Osterholm et al., (2012) that spoken communication indicators consist of: speaking, listening, discussing, having dialog, and thinking while speaking.

Conclusion

Based on the results and discussion, it could be concluded that: (1) mathematics communication process with pseudo drawing communication criteria would occur when students solved problems and expressed it in written manner into figures with their sizes. However, after being investigated through spoken communication in the form of interview, there was not suitability between written and spoken communications. (2) Mathematics communication process with pseudo mathematical expression occurred when students solved the problems and revealed into written form by using algebraic symbol language or numeric. After being investigated through spoken communication in the form of interview, the students provided different explanations. The students' critical thinking process in solving problems was reviewed based on APOS theory. Through APOS theory, students could construct a concept then it was expressed into spoken and written communications. Moll, Trigueros, Badillo, & Rubio (2016) stated that implementation of APOS theory in learning mathematics could construct students' understandings.

In this research, the subjects were limited on junior high school students. Further research is suggested to have senior high school students or higher education students. It is suggested for further researches to investigate student mathematics miscommunication in solving mathematics problems.

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