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TÜRKİYE CUMHURİYETİ'NİN YETMİŞBEŞ YILI

A GENERALIZATION OF THE PARETIAN LIBERAL PARADOX

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Özet. Bu makale, Paretian Liberal Paradoksu'nu geçişken olmayan tercihleri kapsayacak şekilde genelleştirmekte ve genelleştirilmiş paradoksun ortadan kalkacağı bazı koşulları belirlemektedir.

I. INTRODUCTION

A. Sen [1] has presented a startling impossibility theorem that seeks to establish a paradoxical conflict between the Pareto principle and liberalism. Sen's theorem is intended to show that liberal values that assign minimal rights to individuals in a society cannot possibly be combined with the condition of Pareto efficiency, given an unrestricted domain of individual preferences. If a society wants to maintain the Pareto principle and an unrestricted domain, then it cannot permit even minimal liberalism. That is, it cannot, for example, let more than one individual be free to read what she likes, sleep the way she prefers, dress as she cares to, etc., irrespective of the preferences of others in the community (Sen, [1]: 157).

There are a considerable number of works on Sen's paradox that have discussed its implications and explored possible ways to avoid it. Among these are Ng [2], Gibbard [3], Aldrich [4], Breyer [5], Pressler [6], Kelsey [7], Suzumara [8] and Kara [9],[10]. All of them (except Kara [9],[10]) deal with Sen's paradox under the assumption that all individual preferences are transitive in all social contexts. But whether individuals consistently possess transitive preferences has been the center of an ongoing controversy in the economic literature. Many researchers have called into question the axiomatic validity of the transitivity condition. Tversky [11]] has demonstrated systematic and predictable intransitivities under certain experimental conditions. May [12], McCrimmon and Larson [13], Fishburn [14] and Steedman and Krause [15] have indicated cases of consistent violations of transitivity in multidimensional choice contexts where the orderings of alternatives with respect to conflicting criteria often result in circularities in choices. Weinstein [16], Tversky [11] and Bar-Hillel and Margalit 17] have referred to the possibility of intransitivity as an 'instance' of bounded rationality. Epistemic and cognitive limits of human beings — such as limitations concerning information processing or imperfect sensitivity to the differences among some alternatives — could conceivably generate intransitivities in preference patterns. The more complex the choice situation, the more difficult it is to order alternatives transitively, and therefore the higher the information processing cost of obtaining an overall preference ordering.

How does the possibility of intransitivities in the preference patterns of individuals affect the presence or absence of a conflict between the Pareto principle and Liberalism in a social choice context? This paper seeks to provide an answer to this question by introducing intransitive individual preferences into the current formulation of Sen's paradox. The following section presents a general framework for the analysis of intransitive preference relations. The third section attempts to determine whether and under what conditions the conflict between the Pareto principle and Liberalism that plagues a society of individuals with transitive preferences continues (or ceases) to exist in social choice contexts with intransitive individual preferences. Our analysis in this third section will enable us to theorize about the comparative roles of rational (transitive) and irrational (intransitive) individual preferences in generating a conflict between Pareto efficiency and individual rights that induce irrational social choices. To undertake a comparative analysis of rational and irrational preferences in social choice processes, we will attempt to provide answers, in the context of Sen's paradox, to the following three questions:

(i) Are there social choice contexts where a group of irrational individuals (i.e., individuals with intransitive preferences) can make rational social choices?

(ii) Are there social choice contexts where a group of irrational individuals can make rational social choices better than a group of rational individuals?

(iii) Are there domain restrictions on individual preferences that eliminate irrational social choices in social choice contexts involving transitive and intransitive individual preferences?

The concluding section summarizes the main results of the paper.

II. THE GENERAL FRAMEWORK

Let E be the set of a finite number of individuals forming a society, and let Z be the set of mutually exclusive social alternatives. Assume that the cardinalities of E and Z, denoted by, respectively, $|E|$ and $|Z|$, are finite, and $|E|>1$, $|Z|>2$. Each individual i in the society has a preference relation R^i, which is a binary relation on Z such that $R^i \subseteq \{(x,y)$: *x, y are in Z and x, y are distinct}, and* $i=1,....,n$. For any x, y in Z, xR^iy will be interpreted as 'x is preferred to y' by individual i. Define strict preference (P^i) and indifference (I^i) relations on $\{x,y\}$ as follows:

xP^iy *if and only if* xR^iy *and not* yR^ix,

xI^iy *if and only if* xR^iy *and* yR^ix.

Given distinct x, y in Z, exactly one of the following four possibilities holds: xP^iy, yP^ix, xI^iy, and none of these. As such a preference relation can be specified by specifying P^i and I^i; specifically, xR^iy *if and only if* xP^iy *or* xI^iy. Thus, we will often employ a particular specification of R^i such that R^i *over an m-set in Z is a set, the elements of which are preferences over the pairs in that m-set*. For example, $R^i = \{xP^iy, yP^iz, xP^iz\}$ is a possible preference relation over a triple $\{x,y,z\} \subseteq V$ (i.e., an m-set, where m=3), which illustrates the particular specification proposed here.

A preference relation R^i *on Z is said to be* <u>*complete*</u> *if and only if* xP^iy *or* yP^ix *or* xI^iy *for all x, y in Z such that* $x \neq y$. R^i *on Z is* <u>*incomplete*</u> *if it is not complete.* R^i *on Z is* <u>*transitive*</u> *if and only if for all distinct x, y, z in Z,* (xP^iy *and* yP^iz *implies* xP^iz), *and* (xI^iy *and* yI^iz *implies* xI^iz). R^i *on Z is* <u>*intransitive*</u> *if it is not transitive.* R^i *on Z is* <u>*acyclical*</u> *over an m-set* $\{x_1,....,x_m\}$ *in Z if and only if the following condition holds: For all* $x_1,.....,x_m$ *in Z, if* [$x_1P^ix_2$, *and* $x_2P^ix_3$, *and...and* $x_{m-1}P^ix_m$], *then* $x_1R^ix_m$. R^i *is* <u>*cyclical*</u> *over an m-set* $\{x_1,...x_m\}$, *if, for* $x_1,....x_m$ *in Z,* $x_1P^ix_2$, *and* $x_2P^ix_3$ *and...and* $x_{m-1}P^ix_m$ *and* $x_mP^ix_1$.

Though we will relax the conventional requirement that individual preferences be transitive or acyclical, we will continue to assume that they are complete.

Let h be a collective choice rule, that is, a mapping from the set of preference relations $R^{ind} = \{(R^1,....,R^n)$: R^i *is an individual preference relation on Z,* $i = 1,....,n\}$ *into a set of preference relations* $R^{soc} = \{R$: *R is a social preference relation on Z} such that for any configuration of individual preference relations* $R^1,....,R^n$, *one and only one social preference relation R is determined, i.e., h:* $R^{ind} \rightarrow R^{soc}$ *such that* $R = h(R^1,....,R^n)$. *The social preference relation R is a binary relation whose strict preference and indifference parts are P and I.*

Over any triple $\{x,y,z\} \subseteq Z$, there are, with respect to any R^i, 27 possible relational outcomes that are complete. Of these, 13 are transitive and are referred to as orderings. The remaining 14 violate either the transitivity of strict preference or the transitivity of indifference. These are called non-orderings. Let T and S respectively denote the sets of orderings and non-orderings for some R^i. Hence, the set of all logically possible complete preference relations over a triple, written U, is:

$$U = T \cup S.$$

The elements of U are listed as follows:

(i) The elements of T (transitive preference relations):

$R1 = \{xP^iy, yP^iz, xP^iz\}$

$R2 = \{yP^iz, zP^ix, yP^ix\}$

$R3 = \{zP^ix, xP^iy, zP^iy\}$

$R4 = \{xP^iz, zP^iy, xP^iy\}$

$R5 = \{zP^iy, yP^ix, zP^ix\}$

$R6 = \{yP^ix, xP^iz, yP^iz\}$

$R7 = \{xP^iy, yI^iz, xP^iz\}$

$R8 = \{xI^iy, yP^iz, xP^iz\}$

$R9 = \{yP^iz, zI^ix, yP^ix\}$

$R10 = \{yI^iz, zP^ix, yP^ix\}$

$R11 = \{zP^ix, xI^iy, zP^iy\}$

$R12 = \{zI^ix, xP^iy, zP^iy\}$

$R13 = \{xI^iy, yI^iz, xI^iz\}$

(ii) The elements of S (intransitive preference relations)

$R14 = \{xP^iy, yP^iz, zP^ix\}$

$R15 = \{xP^iz, zP^iy, yP^ix\}$

$R16 = \{xP^iy, yP^iz, zI^ix\}$

$R17 = \{xP^iy, yI^iz, zP^ix\}$

$R18 = \{xI^iy, yP^iz, zP^ix\}$

$R19 = \{xP^iz, zP^iy, yI^ix\}$

$R20 = \{xP^iz, zI^iy, yP^ix\}$

$R21 = \{xI^iz, zP^iy, yP^ix\}$

$R22 = \{xP^iy, yI^iz, zI^ix\}$

$R23 = \{xI^iy, yP^iz, zI^ix\}$

$R24 = \{xI^iy, yI^iz, zP^ix\}$

$R25 = \{xP^iz, zI^iy, yI^ix\}$

$R26 = \{xI^iz, zP^iy, yI^ix\}$

$R27 = \{xI^iz, zI^iy, yP^ix\}$

A preference relation R^i over a triple $\{x,y,z\} \subseteq Z$ is said to be strongly strict if it has strict preference over every pair in $\{x,y,z\}$. It is said to be weakly strict if it has indifference over at least one pair in $\{x,y,z\}$. Of the elements of U above, 6 orderings (*R1 through R6*) and 2 non-orderings (*R14 and R15*) are strongly strict, and 7 orderings (*R7 through R13*) and 12 non-orderings (*R16 through R27*) are weakly strict.

The degree to which these preference relations represent, across individuals, compatible or incompatible orderings of alternatives will turn out to be crucial in our analysis of social choice paradoxes. Hence, we will define a few concepts to capture the 'relations of compatibility' among different preference relations: *A pair of preference relations R^i and R^j for any i and j are said to be incompatible (or in conflict) over a pair of alternatives $\{x,y\}$ iff xP^iy and yP^jx. For any i and j, R^i and R^j are said to be compatible over (x,y) if they are not incompatible over $\{x,y\}$. Define a conflict index $C^{i,j}(x,y)$ such that*

$$C^{i,j}(x,y) = 1 \text{ if } R^i \text{ and } R^j \text{ are incompatible over } \{x,y\}$$

$$= 0 \text{ otherwise}$$

With R^i and R^j defined over m alternatives of Z, there are $m(m-1)/2$ pairwise comparisons in an m-set. Let $C_m(R^i,R^j)$ be the sum of $C^{i,j}(x,y)$s over all alternatives $\{x,y\}$ for preference relations R^i and R^j over an m-set. $C_m(R^i,R^j)$ will be called the pairwise degree of conflict between R^i and R^j over an m-set. The value of $C_m(R^i,R^j)$ depends on how conflicted preference relations are and on the number of alternatives over which they are defined. For instance, over a triple, the maximum and minimum values of $C_3(R^i,R^j)$ are respectively 3 and 0.

The concept of pairwise degree of conflict can be used to analyze the relations of compatibility within a set of preference relations. *A pair of preference relations R^i and R^j will be called harmonic over an m-set if $C_m(R^i,R^j)=0$ over that m-set. R^i and R^j will be said to be diverse over an m-set if $C_m(R^i,R^j) \neq 0$ over that m-set. A set of preference relations will be called harmonic if the pairwise degree of conflict between every pair of preference relations in the set is zero.* It is clear by inspection of the above list that the set S of intransitive preference relations over a triple can be decomposed into two disjoint harmonic subsets, say S_1 and S_2, such that

$$S_1 = \{R14, R16, R17, R18, R22, R23, R24\}$$

$$S_2 = \{R15, R19, R20, R21, R25, R26, R27\}$$

where $S_1 \cup S_2 = S$.

A close examination of S_1 and S_2 reveals certain relationships that exist between the preference relations in them. To formalize these relationships, let N_1 and N_2 be respectively the sets that contain those elements of preference relations in S_1 and S_2 that involve strict preference, that is,

$$N_1 = \{xP^iy, yP^iz, zP^ix\}$$

$$N_2 = \{yP^ix, xP^iz, zP^iy\}$$

Clearly, $N_1 \cap N_2 = \varnothing$.

Again by inspection of the above lists, any preference relation Ri in U that has a non-empty intersection with N_1 and an empty intersection with N_2 is in S_1,

i.e., *If* $Ri \cap N_1 \neq \varnothing$ *and* $Ri \cap N_2 = \varnothing$, *then* $Ri \in S_1$.

Similarly, any preference relation Ri in U that has a non-empty intersection with N_2 and an empty intersection with N_1 is in S_2,

i.e., *If* $Ri \cap N_2 \neq \varnothing$ *and* $Ri \cap N_1 = \varnothing$, *then* $Ri \in S_2$.

On the other hand, for every preference relation Ri in S_1, $Ri \cap N_2 = \varnothing$. Thus, any preference relation in U that has a non-empty intersection with N_2 is not in S_1,

i.e., *if* $Ri \cap N_2 \neq \varnothing$, *then* $Ri \notin S_1$.

Similarly, for every preference relation Ri in S_2, $Ri \cap N_1 = \varnothing$. Therefore, any preference relation in U that has a non-empty intersection with N_1 is not in S_2,

i.e., *if* $Ri \cap N_1 \neq \varnothing$, *then* $Ri \notin S_2$.

Using these relationships, Kara [9] proves a lemma that establishes a connection between the transitivity of a preference relation and N_1 and N_2:

Lemma 2.1: *A complete preference relation R^i over any arbitrary triple $\{x,y,z\} \subseteq Z$ is transitive if and only if:*

either $R^i \cap N_1 \neq \varnothing$ *and* $R^i \cap N_2 \neq \varnothing$,

or $R^i \cap N_1 = \varnothing$ *and* $R^i \cap N_2 = \varnothing$.

The importance of these relationships and lemma will become clear in some of the proofs presented below.

III. THE PARETIAN LIBERAL PARADOX

Sen's theorem establishing a conflict between the Pareto principle and Liberalism involves two central concepts, which we will define as follows (For reasons of convenience, we will rephrase some of the definitions and conditions of Sen's theorem without changing their substantive contents):

Definition: *An individual i is* <u>*decisive*</u> *for an ordered pair (x,y) if xP^iy implies xPy. The individual i is said to have a* <u>*right*</u> *over {x,y} if she has strict preference over {x,y} and is decisive for (x,y) and (y,x).*

The formal conditions of Sen's theorem are:

Condition U (Unrestricted Domain): *Every logically possible combination of individual orderings is included in the domain of the collective choice rule.*

Condition P (Pareto Efficiency): *Let {x,y} be any pair contained in Z. If for every i in E xP^iy, then xPy. If for every i in E xR^iy, and for some i in E xP^iy, then xPy. If for every i in E xI^iy, then xIy.*

Condition L (Liberalism): *There are at least two individuals in E, each of whom has a right over at least one pair of alternatives.*

Sen proves that given Condition U, Condition P and Condition L are incompatible, i.e., together they imply the possibility of cyclical social preferences. His theorem is based on the assumption that individuals possess transitive preference relations. We will relax this assumption and introduce the possibility of intransitive preferences. But, before undertaking a reformulation of Sen's theorem in the context of intransitive preferences, we will present five definitions, the first four of which characterize certain relations between rights of individuals, while the fifth formalizes the idea of a <u>conflict</u> between Pareto efficiency and individual rights (liberalism).

Definition: *Two distinct pairs of alternatives are said to be* <u>*connected*</u> *if they share a common alternative, e.g., {x,y} and {y,z} are connected pairs while {x,y} and {z,w} are not.*

Definition: *Two rights assigned to two individuals are said to be* <u>*connected*</u> *if they are over connected pairs.*

Definition: *Individuals in a society are said to have* <u>*interconnected rights*</u> *if for every individual i in E who has a right over a pair, say {x,y}, in an arbitrary triple {x,y,z} of Z, there exists at least one individual j in E, j≠i, with a right over a connected pair in that triple, i.e., over {x,z} or {y,z}. No two individuals in E are allowed to have rights over the same pair in Z.*

Definition: *An individual i is said to have a* <u>*non-trivial right*</u> *over a pair {x,y}, if she has a right over that pair in the presence of some j in E whose preference is incompatible with that of i over {x,y}.*

Definition: *For a given configuration of individual preferences, a* <u>*conflict*</u> *is said to exist between Condition P and Condition L with respect to an m-set, m>2, in Z if the simultaneous (joint) application of both conditions results in a social preference relation R that violates acyclicity over that m-set while the individual application of each condition in the absence of the other does not.*

Given an unrestricted domain of individual preferences, Sen's theorem implies the existence of at least one m-set in *Z* with respect to which a conflict exists between Condition P and Condition L. We will first theorize about the conflict with respect to m-sets, where m=3, and then deal with the conflict with respect to m-sets, where m>3.

To clarify the content of this last definition, we will now give an example of a social choice context in which Condition L and Condition P together lead to a cyclical social choice:

Example 3.1: Consider a society of n individuals in which individuals 1 and 2 have the following preference relations over *{x,y,z}*.

$$R^1 = \{xP^1y,\ yP^1z,\ xP^1z\}$$

$$R^2 = \{yP^2z,\ zP^2x,\ yP^2x\}$$

Suppose that other individuals in the society have the same preference relations as individual 2. Let individuals 1 and 2 have rights, respectively, over *{x,y}* and *{z,x}*. Thus, Condition L is satisfied,

xP^1y implies xPy,

zP^2x implies zPx.

Since every individual in the society strictly prefers y to z, by Condition P, yPz. Thus, the social preference R over $\{x,y,z\}$ is:

$R = \{xPy, yPz, zPx\}$,

which is not acyclical. Hence, for this configuration of individual preferences, Condition L and Condition P are in conflict with respect to $\{x,y,z\}$.

Two features of this example, which also illustrates Sen's theorem, need to be noted. First, the preferences of all individuals in the society are transitive. Second, the preferences of individuals with rights over the connected pairs in $\{x,y,z\}$ have diverse preferences over that triple. That is to say, with some diversity in individual preferences, a society of individuals with transitive preferences faces a conflict between Condition L and Condition P that generates cyclical social choices.

Does the conflict that plagues a society of individuals with transitive preferences also exist in a society of individuals with intransitive preferences? It is straightforward to show that with an unrestricted domain of individual preferences, the conflict in question continues to exist regardless of whether individual preferences are intransitive or transitive. However, as shown in Theorem 3.1 below, which is proved in Kara (1999a), there are social choice contexts where the conflict disappears if individual preferences are intransitive, and surprisingly, the conditions that eliminate the conflict between Condition L and Condition P with respect to triples in Z in such contexts where individual preferences are intransitive continue to pose a conflict when individual preferences are transitive.

Theorem 3.1: *In a society of individuals with intransitive preferences, where individual rights are interconnected and individuals with rights over the connected pairs of any triple have diverse intransitive preferences over that triple, there exists no conflict between Condition L and Condition P with respect to any triple in Z.*

For a proof, see Kara (1999a). The following example illustrates the result presented by Theorem 3.1:

Example 3.2: Let individuals 1 and 2 in E have the following preference relations over $\{x,y,z\}$:

$R^1 = \{xP^1y, yP^1z, zI^1x\}$

$R^2 = \{yP^2x, xP^2z, zI^2y\}$.

Suppose that the rest of the individuals in E hold the same preference relations as individual 2 over $\{x,y,z\}$. Let individuals 1 and 2 have rights respectively over $\{x,y\}$ and $\{x,z\}$. Thus, Condition L is satisfied, and

xP^1y implies xPy,

xP^2z implies xPz.

Since yP^1z and every other individual in E is indifferent between y and z, by Condition P, yPz. Thus, the social preference R over $\{x,y,z\}$ is:

$R = \{xPy, yPz, xPz\}$,

which is transitive.

The conditions stated in Theorem 3.1 guarantee the absence of a conflict between Condition L and Condition P with respect to triples (i.e., m-sets, where m=3) in Z. With some additional assumptions, we can generalize this result to m-sets, where m>3. For instance, if we assume the existence of connected rights over every triple in Z, by Theorem 3.1, the social preference relation over those triples, provided that it is complete, will be transitive. Since a preference relation that is transitive over every triple in Z is also acyclical over every m-set in Z, under the assumptions specified above, there is no conflict between Condition L and Condition P with respect to any m-set in Z.

Though, in the presence of connected rights over every triple in Z, the conditions of Theorem 3.1 eliminate the conflict between Condition L and Condition P in social choice contexts where individual preferences are intransitive, they do not do so in contexts where individual preferences are transitive. For instance, in Example 3.1, rights assigned over the pairs in $\{x,y,z\}$ are connected, and individuals 1 and 2 with rights, respectively, over $\{x,y\}$ and $\{y,z\}$ have diverse preferences over $\{x,y,z\}$, but the resulting social preference relation over $\{x,y,z\}$ is not acyclical, i.e., there is a conflict between Condition L and Condition P with respect to $\{x,y,z\}$. Thus, Theorem 3.1, together with the additional assumptions specified above, establishes a surprising result by proving the existence of social choice contexts in which a society of individuals with intransitive preferences is not susceptible to the 'rights-efficiency' trade-off that plagues a society of individuals with transitive preferences. In other words, there exist cases where a group of people with intransitive preferences can better reconcile individual rights with Pareto efficiency compared to a group formed by individuals with transitive preferences. If transitivity is considered a reasonable condition for rationality, and individual rights and Pareto efficiency are desirable conditions for social choice, then the theorem exemplifies an interesting irony, for it proves the possibility that in certain contexts a society of irrational individuals can be more rational than a society of rational individuals.

Theorem 3.1 considers only cases where all individual preferences are intransitive. However, the joint presence of transitive and intransitive preferences in social choice processes is a distinct empirical possibility. In this more general case, as illustrated in the example below, there may still exist a conflict between Condition L and Condition P.

Example 3.3: Let individuals 1 and 2 in E, respectively, hold transitive and intransitive preference relations over $\{x,y,z\}$ such that

$$R^1 = \{xP^1y,\ yP^1z,\ xP^1z\}$$

$$R^2 = \{xP^2y,\ yP^2z,\ zP^2x\}.$$

Suppose that xP^iy for all other i = 3,...,n. Let individuals 1 and 2 have rights respectively over $\{y,z\}$ and $\{x,z\}$. Thus, Condition L is satisfied, and

yP^1z implies yPz,

zP^2x implies zPx.

Since xP^iy for all i = 1,...,n, by Condition P, xPy. Hence the social preference R over $\{x,y,z\}$ is:

$R = \{xPy,\ yPz,\ zPx\}$,

which is not acyclical.

Though a conflict between Condition P and Condition L continues to exist in social choice contexts where transitive and intransitive preferences are jointly present, there are certain conditions under which the conflict disappears. The following theorem formulates domain restrictions on individual preferences that eliminate the conflict between Condition P and Condition L in the presence of both transitive and intransitive preferences.

Theorem 3.2: *Consider a social choice context in which individual rights are interconnected, and in which individuals with rights over the connected pairs of any triple have diverse preferences over that triple. If for any arbitrary triple $\{x,y,z\}$ in Z and any i,j = 1,...,n,*

$C_3(R^i,R^j) \neq 2$, when R^i and R^j over $\{x,y,z\}$ are both transitive and

2) $C_3(R^i,R^j) = 2$, when one of R^i and R^j over $\{x,y,z\}$ is transitive and one is intransitive, and individuals i and j have non-trivialrights over the pairs in $\{x,y,z\}$,

then there exists no conflict between Condition L and Condition P with respect to $\{x,y,z\}$.

Proof: The preferences of individuals who have rights over the pairs in any given triple can be all intransitive, all transitive, or some transitive and some intransitive. If all preferences are intransitive, as proved in Theorem 3.1, there is no conflict. We will prove, under the conditions stated in the theorem, the absence of a conflict in the other two cases.

(i) First suppose individuals with rights over the pairs in a triple have preferences, one of which is transitive and one is intransitive. In order for a conflict to exist over $\{x,y,z\}$, Condition L and Condition P have to lead to a cyclical social preference over $\{x,y,z\}$. We will show that if $C_3(R^i,R^j) = 2$ over $\{x,y,z\}$, where R^i is transitive and R^j is intransitive, Condition L and Condition P are bound to produce a transitive social preference, ruling out any conflict between them.

If $C_3(R^i,R^j) = 2$, in $\{x,y,z\}$, there is one pair, say $\{x,y\}$, over which R^i and R^j are compatible, and two pairs, say $\{y,z\}$ and $\{x,z\}$, over which they are incompatible. Since incompatibility is defined in terms of strict preference, individuals i and j have strict preferences over $\{y,z\}$ and $\{x,z\}$.

In order for Condition P to be applicable, there must exist at least one pair over which preferences of all individuals are compatible. Since $\{x,y\}$ is the only pair in $\{x,y,z\}$ over which R^i and R^j are compatible, let $\{x,y\}$ be that pair. Thus Condition P determines the social choice over $\{x,y\}$. To apply Condition L, let individuals 1 and 2, with preferences R^1 and R^2, have non-trivial rights over $\{x,z\}$ and $\{y,z\}$. Without loss of generality, let R^1 be transitive and R^2 be intransitive. By Condition L, the social choice over $\{y,z\}$ and $\{x,z\}$ will be determined as follows: Since R^2 is intransitive, it is either in S_1 or in S_2. If it is in S_1, then—since R^2 has strict preference over $\{y,z\}$ and $\{x,z\}$—it must be the case that yP^2z and zP^2x. Since individual 2 is decisive over one of these pairs, $R \cap N_1 \neq \emptyset$. Since R^2 is incompatible with R^1 over $\{y,z\}$ and $\{x,z\}$, it must be the case that zP^1y and xP^1z. Since individual 1 is decisive over one of these pairs, $R \cap N_2 \neq \emptyset$. Thus, the social preference R over $\{x,y,z\}$, which is complete over that triple, has a non-empty intersection with both N_1 and N_2, therefore by Lemma 2.1, R must be transitive. A similar argument applies if R^2 is in S^2.

(ii) Now suppose individuals with rights over the pairs of a triple have all transitive preferences. If $C_3(R^i,R^j) \neq 2$ over $\{x,y,z\}$, where R^i and R^j are transitive, then it has the value of either 1 or 3. Let us examine each case:

(a) $C_3(R^i,R^j) = 1$ over $\{x,y,z\}$: In order for Condition P and Condition L to lead to a conflict over a triple of alternatives, they need to induce a cyclical social preference over that triple. We will show that if $C_3(R^i,R^j) = 1$ over $\{x,y,z\}$, where R^i and R^j are both transitive, the simultaneous application of Condition P and Condition L is bound to produce a transitive social preference, ruling out any conflict between them.

Over a triple of alternatives $\{x,y,z\}$, in order for Condition P to be applicable, there must exist at least one pair over which preferences of all individuals are compatible. Let $\{x,y\}$ be that pair. To apply Condition L, suppose that two individuals, with preferences R^1 and R^2, have rights over $\{x,z\}$ and $\{y,z\}$, which will determine, through Condition L, the social preference over those pairs.

Under the condition that $C_3(R^i,R^j) = 1$ over $\{x,y,z\}$, we will first establish two properties of social preference induced by Condition P and Condition L. First, the social preference relation R over $\{x,y,z\}$ is a strongly strict preference relation: Since, by inspection of the lists in Section II, there exist no two transitive preference relations R^i and R^j in U with $C_3(R^i,R^j) = 1$ over $\{x,y,z\}$ which have indifference over the same pair, if R^1 has indifference over $\{x,y\}$, then R^2 has to have strict preference over $\{x,y\}$. Hence, the application of Condition P over $\{x,y\}$ will produce strict preference over $\{x,y\}$. On the other hand, the social preference over $\{x,z\}$ and $\{y,z\}$ will be, by the definition of Condition L, one of strict preference. Thus, the social preference relation R induced by Condition P and Condition L is bound to be a strict preference over all pairs in $\{x,y,z\}$, i.e., R over $\{x,y,z\}$ is a strongly strict preference relation. Second, the social preference relation is compatible with either R^1 or R^2 over all pairs in $\{x,y,z\}$: Since $C_3(R^i,R^j) = 1$ over $\{x,y,z\}$, there exist two pairs, say $\{x,y\}$ and $\{y,z\}$, over which R^1 and R^2 are compatible and one pair, say $\{x,z\}$, over which they are incompatible. By assumption, Condition P determines the social choice over $\{x,y\}$. To apply Condition L, rights can be assigned, through Condition L, to individuals 1 and 2 in two ways: either individual 1 has a right over $\{y,z\}$ and individual 2 has a right over $\{x,z\}$, or individual 1 has a right over $\{x,z\}$ and individual 2 has a right over $\{y,z\}$. If individual 1 has a right over $\{x,z\}$, i.e., the pair over which her preference is incompatible with that of individual 2, then the social preference R will be compatible with R^1 over all pairs in $\{x,y,z\}$. If individual 2 has a right over $\{x,z\}$, then the social preference R will be compatible with R^2 over all pairs in $\{x,y,z\}$. In either case, the social preference R will be a preference relation that is compatible with either R^1 or R^2 over all pairs in $\{x,y,z\}$.

A close examination of the preference relations in U reveals that there exists no intransitive preference relation that simultaneously satisfies both of the properties of the social preference relation established above. There are only two strongly strict intransitive preference relations over $\{x,y,z\}$, namely, $R14$ and $R15$; and neither of these preference relations could be compatible with a transitive preference relation such as R^1 and R^2 that involves strict preference over at least two pairs in $\{x,y,z\}$. Thus, the social preference relation cannot be intransitive, it must be transitive.

(b) $C_3(R^i,R^j) = 3$ over $\{x,y,z\}$: Since, with this degree of conflict, there is no pair over which the preferences of all individuals are compatible, Condition P is not applicable. Hence, the question of a conflict between Condition P and Condition L does not arise.

Since the result is proved for any arbitrary triple of alternatives, the theorem holds true for any triple in the domain of alternatives that satisfies the restrictions imposed in the theorem.

QED.

The following examples provide illustrations for the result contained in the theorem:

Example 3.4: Let individuals 1 and 2 in E have the following transitive preferences that satisfy the condition that $C_3(R^i,R^j) = 1$ over $\{x,y,z\}$:

$$R^1 = \{xP^1y,\ yP^1z,\ xP^1z\}$$

$$R^2 = \{xI^2z,\ zP^2y,\ xP2y\}.$$

Suppose the rest of the individuals in E have the same preference relations as individual 2. Let individuals 1 and 2 have rights respectively over $\{x,z\}$ and $\{y,z\}$. Then, Condition L is satisfied, and

xP^1z implies xPz,

zP^2y implies zPy.

Since every individual in E strictly prefers x to y, by Condition P, xPy. Thus, the social preference relation R over $\{x,y,z\}$ is

$R = \{xPz, zPy, xPy\}$,

which is transitive.

Example 3.5: Let individuals 1 and 2 in E have the following preferences that satisfy the condition that $C_3(R^1,R^2) = 2$ over $\{x,y,z\}$:

$$R^1 = \{xP^1y, yP^1z, xP^1z\}$$

$$R^2 = \{yP^2x, xP^2z, zP^2y\}.$$

Then R^1 is transitive and R^2 is intransitive. Suppose that other individuals in E have the same preferences as individual 2. Let individuals 1 and 2 have rights respectively over $\{x,y\}$ and $\{y,z\}$. Then Condition L is satisfied, and

xP^1y implies xPy,

zP^2y implies zPy.

Since everyone in E strictly prefers x to z, by Condition P, xPz. Thus, the social preference relation R over $\{x,y,z\}$ is

$$R = \{xPz, zPy, xPy\},$$

which is transitive.

In Theorems 3.1 and 3.2 we made the assumption that the individual rights are interconnected. If we relax this assumption, then the result in Theorem 3.1 does not hold true.

Theorem 3.3: *The possibility of conflict between Condition P and Condition L is present in a society of individuals with intransitive preferences, provided that individual rights are not interconnected.*

Proof: To prove the theorem, it suffices to give an example. Let individuals 1 and 2 in E have the following preferences over $\{x,y,z,w\}$:

$$R^1 = \{xP^1y, yP^1z, zP^1x, xI^1w, yI^1w, zI^1w\}$$

$$R^2 = \{wP^2x, xP^2z, zP^2w, zI^2y, wI^2y, xI^2y\}$$

and suppose that other individuals in E have the same preferences as individual 2. Let individuals 1 and 2 have rights respectively over $\{x,y\}$ and $\{z,w\}$. Then Condition L is satisfied, and

xP^1y implies xPy,

zP^2w implies zPw.

Over $\{x,w\}$, xI^1w and every other individual in E strictly prefers w to x. Hence, by Condition P, wPx.

Over $\{y,z\}$, yP^1z and every other individual in E is indifferent between y and z. Hence, by Condition P, yPz.

Over $\{w,y\}$, every individual in E is indifferent between w and y. Hence, by Condition P, wIy. Thus, the social preference relation R over $\{x,y,z,w\}$ is

$$R = \{xPy, yPz, zPw, wPx, wIy\}$$

which is not acyclical over $\{x,y,z,w\}$. Thus, for this configuration of individual preferences, a conflict exists between Condition L and Condition P.

QED.

There are domain restrictions on intransitive individual preferences that eliminate the conflict between Condition L and Condition P in social choice contexts where individual rights are not interconnected. In the remainder of this paper, we will formulate those restrictions. First, we will introduce two concepts that will be used in the statement and proof of the relevant results.

Definition: *Let R^i be a weakly strict preference relation over an arbitrary triple $\{x,y,z\}$ of Z. A preference relation R^j over $\{x,y,z\}$ is said to be a diverse complement of R^i if R^j has strict preference over a pair (or pairs) over which R^i has indifference, and $C_3(R^i,R^j) \neq 0$.*

Definition: *A set of preference relations over a triple $\{x,y,z\}$ of Z is said to be iso-conflicted if it contains only preference relations that are incompatible over the same pair (or pairs) in $\{x,y,z\}$, and the preference relations that are compatible over every pair in $\{x,y,z\}$.*

To undertake a formulation of the domain restrictions in question, we will first consider those cases where all rights are over non-connected pairs of alternatives. In order for all rights to be over non-connected pairs, any triple of Z can have at most one pair with a right assigned over it, for otherwise at least some rights would be connected. The following theorem establishes, under the assumption of non-connected rights, the conditions for transitive and intransitive preferences under which there is no conflict between Condition P and Condition L with respect to an arbitrary triple of Z.

Theorem 3.4: *Let A^{IT} be a set of intransitive and transitive individual preferences $R^1,...,R^n$ over an arbitrary triple {x,y,z} of Z. Suppose that there is only one pair in {x,y,z} with a right assigned over it, and individual k, with preference relation R^k, has a non-trivial right over that pair. Suppose also that if R^k is intransitive, it is weakly strict and there exists at least one intransitive R^j in A^{IT} that is a diverse complement of R^k. If A^{IT} is an iso-conflicted set such that $C_3(R^k,R^j) \in \{0,1\}$, where R^k and R^j, j = 1,...,n, are in A^{IT}, then there is no conflict between Condition P and Condition L with respect to {x,y,z}.*

Proof: Individual k that has a right over a pair in *{x,y,z}* has either intransitive or transitive preferences over *{x,y,z}*. Let us examine each case in turn.

(i) R^k over *{x,y,z}* is intransitive: Without the loss of generality, let *{x,y}* be the pair over which individual k has a right. Since individual k's right over *{x,y}* is non-trivial, there must exist at least one individual, say j, with preference relation R^j, such that R^j is incompatible with R^k over *{x,y}*. Since A^{IT} is iso-conflicted such that $C_3(R^k,R^j) \in \{0,1\}$ for all j = 1,...,n, *{x,y}* is the only pair over which any two preference relations in A^{IT} could be incompatible. All preference relations in A^{IT} have to be compatible over the other two pairs *{y,z}* and *{x,z}* in *{x,y,z}*.

Since individual k has a right over *{x,y}*, his preference determines, through Condition L, the social preference over that pair. Since all preference relations in A^{IT} are compatible over *{y,z}* and *{x,z}*, Condition P determines the social preference over those pairs. Thus, social preference *R* over *{x,y,z}* is complete.

Since R^k is intransitive over *{x,y,z}*, it is either in S_1 or in S_2. Suppose that R^k is in S_1, then it must be the case that xP^ky, which implies, since individual k has a right over *{x,y}*, xPy. Thus, $R \cap N_1 \neq \varnothing$. By assumption A^{IT} includes at least one intransitive preference relation R^j that is a diverse complement of R^k. Since R^k is in S_1, and R^k and R^j are diverse, R^j must be in S_2. Being a diverse complement of R^k, R^j has strict preference over at least one pair over which R^k has indifference. Let *{y,z}* be that pair. Since R^j is in S_2, it must be the case that zP^jy. Since all preference relations in A^{IT} are compatible over *{y,z}*, none of them is incompatible with R^j over *{y,z}*, i.e., each of them either strictly prefers *z* to *y* or is indifferent between *z* and *y*. Thus, by Condition P, zPy. Thus, $R \cap N_2 \neq \varnothing$. Therefore the social preference relation *R*, which is complete over *{x,y,z}*, has a non-empty intersection with both N_1 and N_2. Hence, by Lemma 2.1, it is transitive. A similar argument applies when R^k is in S_2.

(ii) R^k over *{x,y,z}* is transitive: Assume, as in (i) above, that *{x,y}* is the pair over which individual k has a right. By the same argument as in (i), *{x,y}* is the only pair in *{x,y,z}* over which preference relations in A^{IT} could be incompatible. All preferences in A^{IT} have to be compatible over the other pairs *{y,z}* and *{x,z}*.

Since R^k is transitive, it is either a strongly strict preference relation over *{x,y,z}* or a weakly strict preference relation that has strict preference over two pairs and indifference over one pair in *{x,y,z}*. We will examine each case in turn.

(a) R^k is strongly strict: Since individual k has a right over *{x,y}*, *R* is identical to R^k over *{x,y}*. Since all preference relations in A^{IT} are compatible over *{y,z}* and *{x,z}* over which R^k has strict preference, by Condition P, the social preference relation *R* has the same strict preference as R^k over *{y,z}* and *{x,z}*. Thus, over all three pairs in *{x,y,z}*, *R* is identical to R^k, i.e., $R = R^k$. Since R^k over *{x,y,z}* is transitive, so is *R*.

(b) R^k is weakly strict: Without loss of generality, let R^k have strict preference over *{x,y}* and *{x,z}*, and indifference over *{y,z}*. As in (a) above, since individual k has a right over *{x,y}*, *R* is identical to R^k over *{x,y}*, and, by Condition P, *R* is identical to R^k over *{x,z}*. Over *{y,z}*, there are two possibilities. First, if all preference relations in A^{IT} have indifference over *{y,z}*, then, by Condition P, *R* will have indifference over *{y,z}*. Thus, over *{x,y,z}*, $R = R^k$. Since R^k over *{x,y,z}* is transitive, *R* is too. Second, if at least one preference relation R^j in A^{IT} involves strict preference over *{y,z}*, by Condition P, *R* has the same strict preference as R^j over *{y,z}*. Since R^k involves indifference over *{y,z}*, it will be compatible with *R* over that pair. Thus, *R* is a strongly strict preference relation over *{x,y,z}* that is compatible with R^k over all three pairs in *{x,y,z}*. Since, by inspection of the lists in Section II, there exists no strongly strict intransitive preference relation over *{x,y,z}* that is fully compatible with a weakly strict transitive preference relation, such as R^k, with strict preference over two pairs and indifference over one pair in *{x,y,z}*, *R* cannot be intransitive. It must therefore be transitive.

QED.

If a triple contains no pair over which someone has a right, then the question of conflict between Condition P and Condition L does not arise with respect to that triple. If a triple contains one pair over which a right is assigned, then, under the conditions imposed in Theorem 3.4, the social preference *R* over that triple is transitive. Thus, if all rights are non-connected, the conditions of Theorem 3.4

suffice to eliminate the conflict between Condition P and Condition L with respect to any triple in the entire domain of alternatives.

Theorem 3.1 and Theorem 3.2 consider cases in which triples involve only connected rights, while Theorem 3.4 considers cases where there are no connected rights. In between there are cases in which some triples involve connected rights, and some do not. In those cases involving a mixture of triples with connected and non-connected rights, a proper combination of the conditions imposed in Theorems 3.1, 3.2, and 3.4 guarantees the absence of a conflict between Condition P and Condition L with respect to any triple in *Z*. For instance, in social choice contexts that involve both transitive and intransitive individual preferences, and both connected and non-connected rights, the conditions of Theorem 3.2 secure the transitivity of social preference over triples with connected rights, and the conditions of Theorem 3.4 do likewise for triples involving non-connected rights. Thus, in such social choice contexts, the conditions of Theorems 3.2 and 3.4, if jointly imposed, suffice to eliminate the conflict between Condition P and Condition L with respect to any triple in *Z*.

Theorems 3.2 and 3.4 deal with the conflict between Condition P and Condition L with respect to triples (i.e., m-sets, where m=3) in *Z*. The results of these theorems can be generalized to m-sets in *Z*, where m>3, in the same way as for Theorem 3.1. With proper combinations of the conditions of these theorems, it is possible to secure the acyclicity of the social preference relation over such m-sets. For instance, in social choice contexts with both transitive and intransitive individual preferences, if there are either connected or non-connected rights assigned over some pairs of every triple in *Z*, the conditions of Theorems 3.2 and 3.4 ensure the transitivity of social preference, respectively, over triples involving connected and non-connected rights provided that the social preference relation over those triples is complete. Thus, if every triple in *Z* involves either connected or non-connected rights, under the assumptions specified above, the social preference *R* over every triple in *Z* is transitive. Since it is obvious that any preference relation that is transitive over every triple in *Z* is also acyclical over every m-set, m>3, in *Z*, *R* is acyclical over every m-set, m>3, in *Z*. Therefore, there is no conflict between Condition P and Condition L with respect to any m-set, m>3, in *Z*.

IV. CONCLUSION

The four theorems developed in Section III answer the three questions posed in Section I of the paper. Theorem 3.1 provides answers to the first and second questions by proving the existence of social choice contexts in which irrational individuals make rational social choices by reconciling Pareto efficiency with individual rights, and they do so better than rational individuals. The class of social choice contexts for which the result of Theorem 3.1 holds are characterized by a set of assumptions, one of which is the interconnectedness of rights. If we relax this assumption, as in Theorem 3.3, and thus assume the existence of some non-connected rights, the social choice contexts with intransitive individual preferences also become susceptible to the conflict between Pareto efficiency and individual rights. However, there are domain restrictions on individual preferences that eliminate the conflict in question in a variety of contexts involving non-connected and/or connected rights. Theorems 3.2 and 3.4 formulate these restrictions, and thus provide an affirmative answer to the third question posed in Section I.

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