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On the determinants of portfolio choice: An experimental study via fractional programming

Mehmet Horasanlı[1]
Department of Quantitative Methods, Faculty of Business Administration
Istanbul University, Istanbul, Turkey

Abstract

This paper applies the algorithm proposed by Özdemir and Giresunlu [1] to the foreign stock markets as well as Istanbul Stock Exchange National-100 index and its constituent stocks in order to address the risk perceptions of investors from different markets.

An optimal value of the risk aversion constant, which corresponds to the minimum risky portfolio for each market is obtained by using fractional programming and the risk level of different markets are compared based on the risk aversion constant calculated. Stock markets are ordered according to the investors' risk-bearing attitude. The question of why the hot capital flows prefer Turkish market is answered in this context.

Keywords: *Dinkelbach method, Fractional programming, Markowitz model, Risk aversion constant*

Özet

Bu çalışmada Özdemir ve Giresunlu [1] tarafından önerilen algoritma, farklı piyasalardan yatırımcıların riske karşı tutumlarının karşılaştırılması amacıyla, yabancı hisse senetleri piyasaları ile birlikte İstanbul Menkul Kıymetler Borsası Ulusal-100 endeksine ait hisse senetlerine uygulanmıştır.

Kesirli programlama ile riskten kaçınma katsayısı için bir minimum değer elde edilmiş, bu değere karşılık gelen portföylerin minimum riskli portföyler olduğu gösterilerek farklı piyasalar için riskten kaçınma katsayısının minimum değeri hesaplanmıştır. Farklı piyasalardaki yatırımcıların riske karşı tutumları, riskten kaçınma katsayısı baz alınarak piyasalar riskliliklerine göre sıralanmıştır. Sıcak para akımlarının neden Türk piyasalarını tercih ettiği sorusu bu bağlamda cevaplandırılmıştır.

Anahtar Kelimeler: *Dinkelbach yöntemi, Kesirli programlama, Markowitz modeli, Riskten kaçınma katsayısı*

1. Introduction and alternative portfolio selection models

Markowitz mean-variance portfolio theory is one of the most widely used approaches in portfolio selection and is based on the idea that the investors seek higher investment returns and wish to minimize their risk. Markowitz [2] describes how rational investors can construct optimal portfolios under conditions of uncertainty. He associates the return and the risk of an investment with the expected return and variance of the portfolio respectively. Since high investment returns and low level of risk is contradictory, investors face the problem of balancing a trade off between risk and return.

[1] horasan@istanbul.edu.tr (M. Horasanlı)

Markowitz in his seminal work [2], which brings him a share of 1990 Nobel Prize in Economics, determined the way of finding the optimum allocation of wealth of an investor among different investment alternatives so called diversification by minimizing the risk at a certain level of expected return or alternatively maximizing the return at a certain level of risk. But the main idea Markowitz [2] put into action is to use the risk of an investment as standard deviation.

The diversification depends on the mean, variance and the interrelationship between the assets so called covariance parameters obtained from the historical data. In the world of Markowitz, return and risk of a portfolio consisting of security combinations are calculated as follows where μ_p is the return; σ_p^2 is the variance on the portfolio.

$$\mu_p = \sum_{i=1}^{n} \mu_i x_i \qquad (1)$$

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_i \sigma_j x_i x_j \rho_{ij} \qquad (2)$$

Also note that, μ_i stands for the expected return of asset i, σ_i denotes the standard deviation of the return of the asset i, and ρ_{ij} symbolizes the correlation between assets i and j. The objective is to minimize the risk of the portfolio for a given level of return, or alternatively maximize the expected level of return of the portfolio for a given level of risk. Both approaches lead to a quadratic programming model and are modelled below as QP1 and QP2.

QP1:

$$\text{Min} \quad \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_i \sigma_j \rho_{ij}$$

$$\mu_p \ge \mu_0$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \ge 0 \qquad ;\forall\, i = 1,2,\cdots,n$$

QP2:

$$\text{Max} \quad \mu_p = \sum_{i=1}^{n} \mu_i x_i$$

$$\sigma_p^2 \le \sigma_0$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \ge 0 \qquad ;\forall\, i = 1,2,\cdots,n$$

These models necessarily will not provide the efficient frontier. Varying the desired level of return (μ_0) in QP1 or alternatively, varying the acceptable level of risk (σ_0) in QP2, each quadratic program identifies the minimum variance portfolio for each μ_0 in QP1 and the maximum return portfolio for each σ_0 in QP2. These are the efficient portfolios composing the efficient set. By plotting the variance and the return respectively efficient frontier can be obtained in the mean-variance plane.

As an alternative approach, the return and risk in the objective function at the same time can be combined. Kallberg and Ziemba [3] explicitly trade risk against return in the objective function using the Arrow [4] - Pratt [5] absolute risk aversion constant (λ). The risk aversion constant is defined as follows,

$$-\frac{U''(w)}{U'(w)}=\lambda \tag{3}$$

where *w* is portfolio wealth and u' and u'' are the first and second order derivatives of a von-Neumann-Morgenstern utility function u. Utility function describes the relationship between risk and return for an investor. Each investor bares a different level of risk for an additional level of wealth. As the total return goes up, the investor is less and less willing to risk for an additional wealth. This requires the utility function given below to be non-decreasing, continuously differentiable and concave.

$$U(z)=1-e^{-\lambda z} \qquad ,\lambda>0 \tag{4}$$

Since every investor desires to maximize his total wealth, the expected value of the utility can be computed as follows [6] .

$$E\big(U(z)\big)=\int_{-\infty}^{\infty}\left(1-e^{-\lambda z}\right)\Phi(z)dz=\int_{-\infty}^{\infty}e^{\left(-\lambda z-\frac{1}{2}\left(\frac{z-\bar{z}}{\sigma}\right)^{2}\right)}dz=1-e^{\left(-\lambda\bar{z}+\frac{1}{2}\lambda^{2}\sigma^{2}\right)} \tag{5}$$

As the utility function described above is strictly increasing in z, it is equivalent to maximize the following quadratic model.

QP3:

$$\text{Max} \quad \sum_{i=1}^{n}\mu_i x_i-\frac{\lambda}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}x_i x_j\sigma_i\sigma_j\rho_{ij}$$

$$\sum_{i=1}^{n}x_i=1$$

$$x_i\geq 0 \qquad ;\forall\, i=1,2,\cdots,n$$

The efficient frontier is obtained by solving different instances of quadratic programs by varying the risk aversion constant (λ). Kallberg and Ziemba [3] suggest that, $\lambda \geq 6$ leads to risk-averse portfolios, $2 \leq \lambda \leq 4$ represents moderate absolute risk aversion and $0 \leq \lambda \leq 2$ leads to risky portfolios. There are dramatic changes in the optimal portfolio composition for even small changes in λ in risky portfolios. $\lambda = 4$ correspond approximately to pension fund management, typically, 60% stocks and 40%bonds. Kallberg and Ziemba [3] states that the most important problem is the selection of λ, risk aversion constant, in the mean-variance model. The importance of the risk aversion constant comes from the calculation of it. Since, λ is multiplied by the risk parameter, the weight of risk in the function $F(\lambda)$ becomes greater as λ becomes greater.

In Refs. [3,7,8] as well as many other researchers use randomly selected risk aversion constant to plot the efficient frontier. Naturally, it is important to solve the problem given by QP3 to obtain the different instances corresponding to different investor but Özdemir and Giresunlu [1] developed an algorithm for computing the optimum value for the risk aversion constant by using fractional programming or

Dinkelbach's method [9] alternatively. The algorithm they have proposed corresponds to the minimum risky portfolio as well.

2. Fractional programming for calculating the optimum risk aversion constant

Fiven two continuous functions f: $\Re^n \rightarrow \Re$ and g: $\Re^n \rightarrow \Re$ defined on a polyhedral set $S \subseteq \Re^n$ such that g(x) > 0 for all $x \in S$, the fractional programming problem is defined to find some point x^* which satisfies the following equality.

$$\lambda^* = \frac{f(x^*)}{g(x^*)} = \max_{x \in S} \frac{f(x)}{g(x)} \qquad (6)$$

An interesting result can be achieved if the maximum value of the program given by QP3 is assumed to be zero. Using $E(\mathbf{x}^*) = \sum_{i=1}^{n} \mu_i x_i^*$ and $V(\mathbf{x}^*) = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i^* x_j^* \sigma_i \sigma_j \rho_{ij}$ for simplicity and equalling the objective function in the QP3 program, following equation is obtained which is only valid at the optimum point (x^*).

$$\sum_{i=1}^{n} \mu_i x_i^* - \frac{\lambda^*}{2} \sum_{i=1}^{n}\sum_{j=1}^{n} x_i^* x_j^* \sigma_i \sigma_j \rho_{ij} = 0$$

$$\sum_{i=1}^{n} \mu_i x_i^* = \frac{\lambda^*}{2} \sum_{i=1}^{n}\sum_{j=1}^{n} x_i^* x_j^* \sigma_i \sigma_j \rho_{ij}$$

$$2\frac{E\left(x^*\right)}{V\left(x^*\right)} = \lambda^*$$

This intuitive approach proposed in Ref. [1] is practical in sense because it provides solving the fractional programming model iteratively to obtain the optimal investment strategy. Or more crucially, it provides an optimum value for the risk aversion constant while solving the quadratic programming model given by QP3. A well-known approach for solving this problem is to convert the given model to a global optimization model described as follows.

$$F(\lambda) = \text{Max}\ \{\ f(\mathbf{x}) - \lambda g(\mathbf{x})\ \mid\ \mathbf{x} \in S\ ,\ \lambda \in R\} \qquad (7)$$

A parametric approach is proposed in Ref. [9] to solve the fractional program given by equation (6) which generates a sequence of values, λ_i's, that converge to the global optimum value of the function. The relationship between the optimization problem given by QP3 and the fractional program given by equation (7) can be stated with the following theorem:

Theorem: $\mathbf{x}^*$ solves the fractional programming problem given by equation (6) if and only if $\mathbf{x}^*$ solves the global optimization problem given by equation (7) with constant $\lambda^* = f(\mathbf{x}^*)/g(\mathbf{x}^*)$.

Dinkelbach's original iterative algorithm is based on the result of this theorem. The algorithm can be described as follows.

1. Select some $\mathbf{x}^o \in S$. Set $\lambda^o = f(\mathbf{x}^o) / g(\mathbf{x}^o)$ and k = 0
2. Solve the constrained global optimization problem given by equation (7) to get the optimal solution point x^{k+1}.
3. If $f(\mathbf{x}^{k+1}) - \lambda^k g(\mathbf{x}^{k+1}) = 0$, then set $\mathbf{x}^* = \mathbf{x}^{k+1}$ and $\lambda^* = \lambda^k$ and stop.
4. If $f(\mathbf{x}^{k+1}) - \lambda^k g(\mathbf{x}^{k+1}) > 0$, then set $\lambda^{k+1} = f(\mathbf{x}^{k+1}) / g(\mathbf{x}^{k+1})$, k = k +1 and go to step 2.

Also note that a test of the form $f(\mathbf{x}^{k+1}) - \lambda^k g(\mathbf{x}^{k+1}) < 0$ is not necessary since, for any fixed k,

$$f(\mathbf{x}^{k+1}) - \lambda^k g(\mathbf{x}^{k+1}) = \mathrm{Max}\{ f(\mathbf{x}) - \lambda^k g(\mathbf{x}) \mid \mathbf{x} \in S \} \geq f(\mathbf{x}^k) - \lambda^k g(\mathbf{x}^k) = 0 \qquad (8)$$

3. Data and Results

The algorithm described above is applied to weekly data between January, 2003 and December 2007, which makes 261 observations. In order to compare the risk-bearing attitudes of different investors, the algorithm is applied to data from various markets such as DowJones Industrial Index (DJI), EuroStoxx-50 (EuroStoxx) index, Hang-Seng index (Hang-Seng) and Istanbul Stock Exchange National-100 index (XU100). A limited number of stocks are chosen for simplicity thus, 20 stocks from each indexes are selected randomly. The symbols of the corresponding stocks as well as optimal portfolios at each iteration are given in Appendix A, Table 1 and Table 2. The algorithm started by assigning an equal weight (1/20=0.05) to each stock and at the end of iteration 4 the stability of weights is enhanced. Therefore, the weights given in the last column of Table 1 and Table 2 can be regarded as the optimal investment strategies. The portfolios in this column are the most well diversified portfolios according to Ref. [2].

Risk aversion constant at each iteration and parameters of the corresponding iterations are given in Appendix A, Table 3. It is evident from Table 3 that, the risk aversion constant monotonly increases and converges to its optimum value while $F(\lambda)$ converges to zero. According to Table 3, the optimm value for the risk aversion constant is calculated as 32.211778, 24.250188, 36.427928 and 17.472965 for DJI, EuroStoxx, Hang-Seng and XU100 respectively. As stated by Ref. [3] each market leads to risk-averse portfolios since the optimum value for the risk aversion constant is greater than 6 for each market.

Stock markets are drawn up relatively to their riskiness as XU100, EuroStoxx, DJI and Hang-Seng where XU030 is the most risky market and Hang-Seng is the most risk-averse. In other words, it can be stated as XU030 offers the greatest return for a unit of wealth. This conclusion may be the answer of why the hot capital flows prefer Turkish market.

Additionally, by using the risk and return parameters given in Table 3, efficient frontier can be obtained. For portfolios on the efficient frontier, seen in Figure 1 in the Appendix B, the balance between risk and return is constructed by the risk aversion constant. For this reason, using λ constants picked from E(x) / V(x) ratio in the model, various portfolio compositions are selected. Since each portfolio composition in Table 1 and Table 2 on Appendix A belongs to different investors, utility functions related to various risk aversion constant in these portfolios are shown in Figure 2 in Appendix B. It is evident from Figure 2 that, increasing the λ parameter, concavity of the utility function strenghtens, therefore investors are ranged from aggressive to conservative up to the bottom. In the figure, it can be seen that as the expected returns of the utility functions increase marginal utility becomes smaller, because, the slopes of the indifference curves are negative. This situation is clear from Figure 3 on Appendix B that $F(\lambda)$ function is decreasing and it approaches to zero asymptotically as λ gets higher values.

4. Conclusion

In this study, the algorithm proposed by Ref. [1] is applied to the foreign stock markets as well as Istanbul Stock Exchange National-100 index and its constituent stocks. The optimal value of the risk aversion constant, which corresponds to the minimum risky portfolio for each market is obtained by using fractional programming and the risk level of different markets are compared based on the risk aversion constant calculated.

Results show that Turkish stock market seems to offer the highest return for a unit of wealth and this is the answer why hot capital flows prefer Turkish market.

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Appendix A

Table 1: Optimal portfolios for DJI EuroStoxx stocks respectively

Optimal portfolios	Iterations				
	0	**1**	**2**	**3**	**4**
AA	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
AXP	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
BA	0.0500000	0.1222652	0.0686321	0.0632797	0.0632238
CAT	0.0500000	0.0494128	0.0077766	0.0036901	0.0036465
CVX	0.0500000	0.3153341	0.2245317	0.2152890	0.2151929
DIS	0.0500000	0.0000000	0.0179442	0.0196111	0.0196293
GE	0.0500000	0.0000000	0.0508755	0.0541875	0.0542221
GM	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
HPQ	0.0500000	0.0558880	0.0254563	0.0221385	0.0221027
INTC	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
JNJ	0.0500000	0.0000000	0.0668273	0.0737714	0.0738453
JPM	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
KO	0.0500000	0.0000000	0.0100176	0.0151523	0.0152061
MMM	0.0500000	0.0000000	0.0610124	0.0664285	0.0664871
MSFT	0.0500000	0.0000000	0.0070206	0.0098052	0.0098343
PFE	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
PG	0.0500000	0.4271816	0.4065085	0.4013301	0.4012749
T	0.0500000	0.0039395	0.0204788	0.0214592	0.0214700
WMT	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
XOM	0.0500000	0.0259787	0.0329184	0.0338574	0.0338652
AEGON	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
ALCTL-LUCENT	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
ALLIANZ	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
BASF	0.0500000	0.0915482	0.0919356	0.0900589	0.0900401
BAYER	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
CARREFOUR	0.0500000	0.0000000	0.0885489	0.0955195	0.0955893
DANONE	0.0500000	0.1005468	0.0953698	0.0941341	0.0941218
DEUTSCHE BNK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
DAIMLER	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
VINCI	0.0500000	0.1329706	0.1013847	0.0981530	0.0981207
E.ON AG	0.0500000	0.3245072	0.2670591	0.2614230	0.2613666
GEN. ASS	0.0500000	0.0000000	0.0976665	0.1062693	0.0063555
SAINT GOBAIN	0.0500000	0.0475046	0.0397523	0.0386307	0.0386195
ING GROEP	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
ARC. REG	0.0500000	0.1560317	0.1112303	0.1064049	0.1063566
ROY.PHILIPS	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
SAP	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
SIEMENS N	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
SCHN.ELECTRIC	0.0500000	0.0000000	0.0091063	0.0164236	0.0164968
VOLKSWAGEN	0.0500000	0.1468909	0.0979467	0.0929828	0.0929332

Table 2: Optimal portfolios for Hang-Seng and XU100 stocks respectively

Optimal portfolios	Iterations				
	0	1	2	3	4
0001.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
0003.HK	0.0500000	0.1847348	0.1824548	0.1817395	0.1817324
0006.HK	0.0500000	0.3203032	0.4115086	0.4222270	0.4223293
0011.HK	0.0500000	0.0000000	0.0874400	0.0999770	0.1000939
0013.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
0023.HK	0.0500000	0.0969962	0.0756556	0.0727677	0.0727400
0066.HK	0.0500000	0.0705648	0.0450016	0.0411224	0.0410832
0101.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
0267.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
0293.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
0330.HK	0.0500000	0.1051317	0.0644452	0.0592112	0.0591612
0386.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
0494.HK	0.0500000	0.0190755	0.0335118	0.0350855	0.0350995
0688.HK	0.0500000	0.0782057	0.0342052	0.0285880	0.0285347
0762.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
0857.HK	0.0500000	0.1081916	0.0650662	0.0592817	0.0592258
0941.HK	0.0500000	0.0167966	0.0007109	0.0000000	0.0000000
1038.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
1199.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
2388.HK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
AEFES	0.0500000	0.1054494	0.1085612	0.1085955	0.1085944
ALARK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
BEKO	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
BOYNR	0.0500000	0.0163482	0.0247480	0.0248641	0.0248526
CLEBI	0.0500000	0.0858874	0.0890520	0.0891456	0.0891348
DOHOL	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
ENKAI	0.0500000	0.284558	0.2689720	0.2682741	0.2683446
EREGL	0.0500000	0.1159483	0.0834864	0.0823890	0.0825011
GARAN	0.0500000	0.0218964	0.0000000	0.0000000	0.0000000
HURGZ	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
MIGRS	0.0500000	0.0073118	0.0356894	0.0361318	0.0360864
NTHOL	0.0500000	0.0707437	0.0604523	0.0601153	0.0601498
PTOFS	0.0500000	0.0000000	0.0388903	0.0399804	0.0398690
SAHOL	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
TCELL	0.0500000	0.0985086	0.1018319	0.1016581	0.1016768
TRKCM	0.0500000	0.0000000	0.0012884	0.0023570	0.0022476
TUPRS	0.0500000	0.1044502	0.1139314	0.1138972	0.1138998
ULKER	0.0500000	0.0888982	0.0730967	0.0725918	0.0726431
YKBNK	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000
ZOREN	0.0500000	0.0000000	0.0000000	0.0000000	0.0000000

Table 3: Parameters of the corresponding iterations of optimal portfolios

Parameters		Iterations 0	1	2	3	4
DJI	λ	16.664086	29.927351	32.185997	32.211778	32.211778
	F(λ)	0.0016917	0.0002239	0.249E-05	0.146E-07	0.147E-07
	E(x)	0.0026206	0.0038172	0.0031906	0.0031244	0.0031237
	V(x)	0.0003145	0.0002551	0.0019830	0.0001940	0.0001939
	U(x)	0.0427304	0.0957427	0.0975965	0.1079551	0.1079554
EuroStoxx	λ	14.504990	23.029853	24.237457	24.250187	24.250188
	F(λ)	0.0027203	0.0003083	0.319E-05	0.118E-08	0.955E-09
	E(x)	0.0046580	0.0073488	0.0061887	0.0060673	0.0060661
	V(x)	0.0006421	0.0006382	0.0005107	0.0005004	0.0005003
	U(x)	0.0653272	0.1367956	0.1368210	0.1392909	0.1556956
Hang-Seng	λ	19.007695	33.003916	36.367034	36.401601	36.427928
	F(λ)	0.0026080	0.0004491	0.791E-05	0.341E-05	0.171E-07
	E(x)	0.0064588	0.0061499	0.0048998	0.0047493	0.0047416
	V(x)	0.0067960	0.0037267	0.0002695	0.0002609	0.0002603
	U(x)	0.1155299	0.1586311	0.1587627	0.1632167	0.1836969
XU100	λ	11.907398	17.257496	17.450699	17.450701	17.472965
	F(λ)	0.0029676	0.0001009	0.0000116	0.308E-09	0.155E-10
	E(x)	0.0074445	0.0095726	0.0091174	0.0091121	0.0091134
	V(x)	0.0012504	0.0011094	0.0010436	0.0010420	0.0010444
	U(x)	0.0848290	0.1470135	0.1470334	0.1472662	0.1522751

Appendix B

Figure 1: Efficient frontier

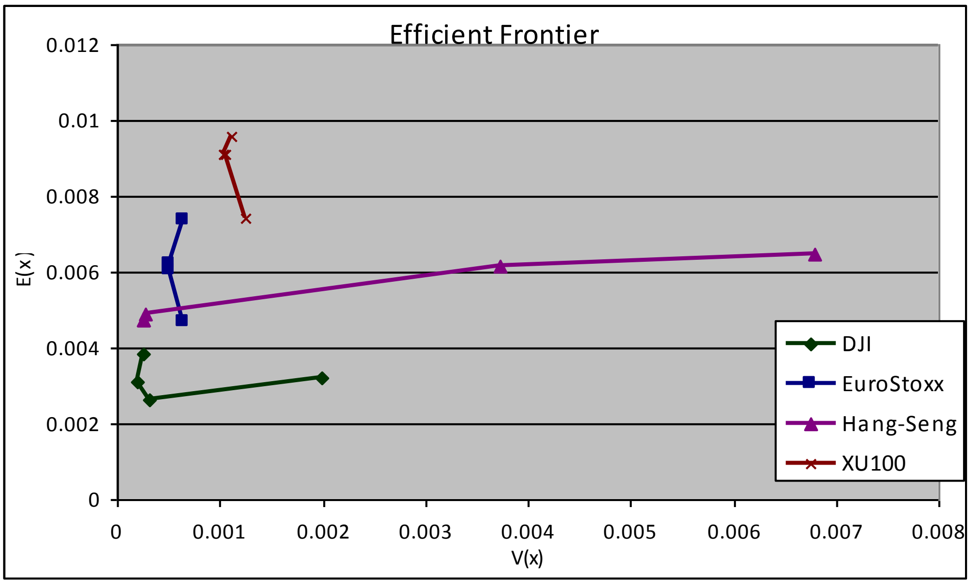

Figure 2: Utility functions for different investors

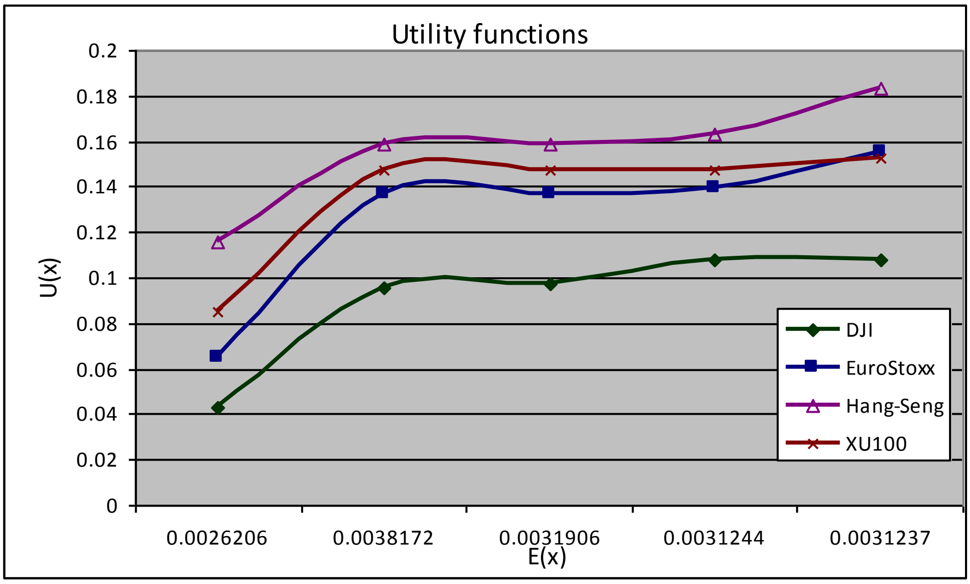

Figure 3: F(λ) function for various λ values