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Modelling the volatility in Istanbul Stock Exchange: shifting from Box-Jenkins to ARCH type models

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Abstract

Forecasting the volatility of financial markets is one of the important issues in empirical finance that absorbed the interest of many researchers in the last decade. As it is known, there has been many studies uncovering the properties of competing volatility models. In this study, both traditional (unconditional) and conditional volatility models, which have the implications for finance that investors can predict the risk, are analyzed. In this study, Box-Jenkins model (ARIMA) and ARCH-type models (ARCH-GARCH-EGARCH-TARCH and GARCH-M) are discussed for the time–dependence in variance that is regularly observed in financial time series and various classical volatility forecasting approaches are compared using ISE-100 Stock Index for the time period between the years 1987 and 2009. As a result, it is found that IMKB-100 returns series include; leptokurtosis, leverage effects, volatility clustering (or pooling), volatility smile and long memory and TGARCH (1,1) is the best fitting model for modeling the volatility of Ise-100 Index.

Keywords: *Volatility, ARIMA models, ARCH models, time series, ISE.*

İstanbul Menkul Kıymetler Borsası'nda volatilitenin modellenmesi: Box-Jenkins modellerden ARCH ailesi modellere geçiş

Özet

Finansal piyasaların oynaklığının tahmin edilmesi son zamanlarda uygulama alanında birçok araştırmacının dikkatini çeken konular arasında gelmektedir. Bilindiği üzere, volatilite modellerinin birbirlerine kıyasla üstün özelliklerini ortaya koymaya çalışan birçok araştırma bulunmaktadır. Bu makalede, yatırımcıların risklerini belirleyebilmelerinde kullanılan, birçok finansal uygulamaya konu olan, geleneksel (koşulsuz) ve koşullu varyans modelleri incelenmiştir. Ayrıca, finansal zaman serilerinde sıkça gözlemlenen zamana bağlı değişkenliği gözlemlemek için Box Jenkins ve ARCH ailesi modelleri (ARCH-GARCH-EGARCH-TARCH ve GARCH-M) ele alınmış ve 1987-2009 yılları arasında İMKB-100 Endeksi verilerinden hareketle çeşitli klasik oynaklık tahminleme modelleri göreceli olarak karşılaştırılmıştır. Araştırma sonuçları, İMKB-100 getiri serisinde kalın kuyruk probleminin bulunduğu, oynaklık kümelenmelerinin olduğu, negatif şokların etkisinin pozitif şoklara oranla daha etkili olduğu ve uzun sürdüğü, veri setinin uzun hafıza içerdiği

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ve ayrıca TGARCH (1,1)'in IMKB-100 Endeksi'nin oynaklağını tahminleyen en iyi model olduğunu ortaya koymuştur.

Anahtar Sözcükler: *Volatilite, ARIMA modelleri, ARCH modelleri, zaman serileri, İMKB.*

1. Introduction

Volatility is one of the essential concepts of modern finance in predicting the trade-off between risk and expected return, where risk is associated with some notion of price volatility. As such, measuring and forecasting volatility is arguably among the most important pursuits in empirical asset pricing finance and risk management (Andersen et al., 2005). However, researches in finance have devoted significant effort in the last two decades to coming up with better models to estimate volatility.

The frequency of financial time series is often high and many high-frequency financial time series have the property of 'long-memory' (the presence of statistically significant correlations between observations that are a large distance apart). Another distinguishing feature of many financial time series is the time-varying volatility or 'heteroscedasticity' of the data (Harris and Sollis, 2003: 213). As the high-frequency financial data exhibits volatility clustering, large (small) price changes tend to be followed by large (small) changes of either sign (Mandelbrot, 1963).

The statistical analysis of financial time series provides evidence of various stylized facts, among which volatility clustering has received considerable attention. Many models have been added throughout the years to the Autoregressive Conditional Heteroscedasticity (ARCH) family, following the seminal paper by Engle (1982), which capture the short-run dependency of the conditional variances.

This study focuses on modeling and forecasting the volatility with the ARCH-type volatility models in Turkish stock market, and compares the models performance. The purpose of this article is to quantify the afore-mentioned five models, using the daily closing prices of Istanbul Stock Exchange 100 Index (ISE-100) from the period July 3rd 1987 to July 3rd 2009, representing 5377 observations.

2. The Models

In this section, we introduce an Autoregressive Integrated Moving Average process (ARIMA) for the index returns. A basic assumption common to each of these models is that economic and financial data contain both permanent (represented by a random walk process) and transitory components (represented by a moving average process). Our model is established on Box Jenkins Technique (1976) and a brief explanation of AR-MA Models and Box-Jenkins Technique are given below.

Auto-Regressive (AR) Models

One common approach for modeling univariate time series is the *Auto-Regressive* (AR) model. In this application, if P_t represents the value of claims during day *t,* a general form of an Autoregressive model would be written as follows:

$$P_t = \delta + \Phi_1 P_{t-1} + \Phi_2 P_{t-2} + \ldots\ldots + \Phi_p P_{t-p} + u_t \qquad (1)$$

ε_t is a random (white noise) error term with zero mean and constant variance. Essentially, an Auto-Regressive model is a linear regression of the current value of the series against one or more prior values. The parameter *p* is referred to as the *order* of the AR model.

Moving Average (MA) Models

Another approach for discerning patterns in univariate time series is the moving average (MA) model written generally as follows:

$$P_t = \mu + u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - .... - \theta_q u_{t-q} \qquad (2)$$

Where terms are defined as before (P_t is the value of claims in day *t*, μ is the mean value of the time series, ε_t are white noise terms in day *t*) and the θ_j (j = 1, 2, …q) are parameters of the model that are to be estimated by the statistical procedure. The value of *q* is called as the *order* of the MA model.

ARIMA (Box-Jenkins) Models

Box and Jenkins have developed a systematic methodology for identifying and estimating models that could simultaneously incorporate both AR and MA approaches. They begin by supposing that both Equations (1) and (2) can be applied to use their approach, however, it requires that the time series are "stationary".[4]

Model Estimation

Considering equations together, the identification step suggested the following generic form of ARIMA process:

$$\Delta P_t = \mu + \Phi_1 P_{t-1} + \Phi_2 P_{t-2} + ... + \Phi_p P_{t-p} + u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - .... - \theta_q u_{t-q} \qquad (3)$$

Where,

P_t, P_{t-1}, P_{t-2},….are the stationary price data;

u_t, u_{t-1}, u_{t-2}, …..are the present and prior forecast errors;

μ, Φ_t, Φ_{t-1}, Φ_{t-2}, …, θ_t, θ_{t-1}, θ_{t-2},…. are the parameters of the regression model

ARMA models are quite flexible because they include both AR and MA elements; however, building good ARMA models requires considerably more expertise than is necessary for more frequently-used statistical methods.

ARCH Model

The first model that provides a systematic framework for volatility modelling is the ARCH model of Engle (1982). The basic idea of ARCH model is that (a) the mean-corrected asset return a_t is serially uncorrelated, but dependent, and (b) the dependence of a_t can be described by a simple quadratic function of its lagged values (Tsay, 2002: 83). This modelling technique explicitly recognizes the temporal dependence suggested by the phenomenon of volatility clustering. According to the ARCH model the conditional error distribution is normal, but with conditional variance equal to a linear function of past squared observations[5]. Thus, there is a tendency for extreme values to be followed by other extreme values of unpredictable sign.

The basic ARCH (q) model can be expressed as:

$$\sigma_t^2 = h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + ... + \alpha_q u_{t-q}^2 \qquad (4)$$

---

[4] A stationary time series is one for which the mean, variance, and autocorrelation structure do not change over time. Conceptually, this may be visualized as a time series that is essentially "flat" over time (that is, without a trend) and for which the range of variations around the mean is also constant over time, and for which no periodic fluctuations (e.g., seasonality) are evident.

[5] Some formulations use past errors. See Bollerslev (1986).

where u_t is a sequence of independent and identically distributed random variables with zero mean and $\sigma_t^2 = 1$, α_0>0, α_1≥0 for the lags greater than zero.

$$\sigma_t^2 = h_t = \alpha_0 + \alpha_1 u_{t-1}^2 \quad (5)$$

This is an ARCH(1) model as it contains only one lag on the squared error term, however it is possible to extend this model by including more lags, if there are q lags it is termed as an ARCH(q) model.

GARCH Model

A GARCH(q,p) model is defined as a discrete time stochastic process u_t of the form:

$$u_t = w_t \sqrt{h_t} \quad (6)$$

$$u_t = v_t \sigma_t^2 = v_t \sqrt{h_t} \quad (7)$$

Where σ_t^2 is written as h_t and v_t has a zero mean and variance of one. We can then rewrite the conditional variance as:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \quad (8)$$

where $w_t \sim i.i.d.\, N(0,1)$, $h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + ... + \alpha_q u_{t-q}^2 + \beta_1 h_{t-1} + ...\beta_p h_{t-p}$, and q > 0, $p \geq 0$, α_0> 0, $\alpha_i \geq 0$ (i = 1,…,q) and $\beta_i \geq 0$ (i = 1,…,p). When p =0 we have an ARCH(q) model.

In most applications p=q=1 is found to suffice. In this case:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1} \quad (9)$$

where the stationarity condition is given by $\alpha_1 + \beta_1$ < 1.

ARCH-M Model

The standard ARCH effect in data implies 'volatility clusters' which can be captured to place appropriate structures to the volatility of the series, which may be otherwise highly unpredictable and difficult to interpret. The coefficients derived from estimated ARCH models are more efficient than those obtained from simple OLS method and offer a special ground for inference making.

As a variant to the general ARCH model described above, we employ here the ARCH-M model introduced by Engel et.al (1987) wherein each time the mean of the process is determined by additional information contained in standard deviation seen at the same time. The ARCH-M modeling is of special interest in studying financial time series as the conditional variance plays an important role in determining an explicit trade-off between expected returns and the variance or the covariance among returns. In the traditional capital asset pricing model (CAPM), for example, the expected excess return on the market portfolio is linear in its conditional variance, suggesting the usefulness of ARCH-M type models. In ARCH-M model it is assumed that the risk premium is an increasing function of the conditional variance of u_t (Enders, 2004). Mathematically, if h_t is the conditional variance of u_t the risk premium can be expressed as

$$\mu_t = \beta + \delta h_t \qquad \delta > 0$$

Where h_t is the ARCH (q) process:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 \quad (10)$$

Asymmetric GARCH Models

GJR-GARCH (T-GARCH)

Asymmetric GARCH models due to the leverage effect with asset prices, where a positive shock has less effect on the conditional variance compared to a negative shock. This can be incorporated into the GARCH model using a dummy variable. This was introduced by Glosten, Jangathann and Runkle (GJR), and showed that asymmetric adjustment was an important consideration with asset prices. The model is of the form:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta h_{t-1} + \lambda u_{t-1}^2 I_{t-1} \quad (11)$$

"I" is a dummy variable that takes the value of 1 when the shock is less than 0 (negative) and 0 otherwise. To determine if there is asymmetric adjustment depends on the significance of the last term, which can be determined using the t-statistic.

E-GARCH Model

In this model, the conditional variance may be expressed as follows:

$$\ln(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left|\frac{u_{t-1}}{\sqrt{h_{t-1}}}\right| + \sum_{i=1}^{q} \gamma \frac{u_{t-1}}{\sqrt{h_{t-1}}} + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) \quad (12)$$

The form of the equation indicates that conditional variance is an exponential function of the variables under analysis, which automatically ensures its positive character. The exponential nature of EGARCH ensures that external unexpected shocks will have a stronger influence on the predicted volatility than TARCH. An asymmetric effect is indicated by the non-zero value of γ and the presence of a 'leverage effect' is shown by its negative value.

3. Literature Review

Time series realizations of returns often exhibit time-dependent volatility. These facts allow an alternative volatility specification based on non-linear models. Several authors have fitted time series models to obtain estimates of conditional or expected volatility from return data. This idea was first formalized in Engle's (1982) ARCH model, which is based on the specification of conditional densities at successive periods of time with a time-dependent volatility process.

The ARCH model is based on the assumption that forecasts of the variance at some future point in time can also be improved by using recent information. Since the publication of the original ARCH paper in 1982, these methods have been used by many researchers. Alternative formulations have been suggested and used and the range of applications has continually widened (see Bollerslev et al., (1992) and Bera and Higgins (1993) for a survey of these models). In the ARCH model (Engle, 1982) and its extension as generalized ARCH (Bollerslev, 1986), or exponential GARCH (Nelson, 1991) approximations, time series volatility is measured by means of the conditional variance of its unexpected component, that is, a distributed lag over squared innovations. Fitting GARCH models to stock price data provides an alternative way to estimate conditional volatility and has become standard in recent empirical applications.

Gökçe (2001) has studied ARCH-class models to estimate the appropriate model for forecasting volatility in ISE for the period of 02.01.1989-31.12.1997 with 2245 daily observation. It is found that the best fitting model is GARCH (1,1) for ISE 100-Index. Furthermore, he found a strong and positive relationship between daily trading volume and daily rate of return.

Aydın (2003) has examined the behavior of the Istanbul Stock-Exchange Index, ISE-30, which includes 30 leading Turkish companies. In the Ms. Thesis, it is observed that there are no normality, volatility clusters, negative skewness, large kurtosis, and autocorrelation in the financial time series data. Therefore, EWMA and Generalized ARCH models were applied to the index. It is found that the best fitting model is GARCH(1,1) and only one-day effect has observed both for EWMA and GARCH models.

In another study, Mazıbaş (2005) has examined the out-of-sample forecasting accuracy of fifteen symmetrical and asymmetrical GARCH models for daily, weekly and monthly volatility in composite, financial, services and industry indices of Istanbul Stock Exchange (ISE). Model estimations have demonstrated the existence of asymmetry and leverage effects in daily, weekly and monthly market data. In model forecasts, it has been found that weekly and monthly forecasts are more precise than daily forecasts. Moreover, it has also been found that due to high volatility in daily returns, ARCH-type models are incompetent in modeling daily volatility.

Akgün and Sayyan (2005) have examined the asymmetric response of stock returns in ISE-30[6] to news by using Asymmetric Conditional Heteroscedasticity models (EGARCH, GJR, APARCH, FIEGARCH, FIAPARCH) for the period 04.01.2000 to 25.04.2005. Their findings show that forecasting volatility in ISE-30 stock returns with Asymmetric Conditional Heteroscedasticity models especially APARCH and FIAPARCH models provides the most accurate volatility forecasts. Authors also claimed that using student-t or skewed student-t distribution instead of normal distribution is more appropriate in modeling and forecasting of financial data with negative skewness and large kurtosis.

Sarıoğlu (2006), in her PhD thesis, has tried to answer "how volatility of common stocks traded in Istanbul Stock Exchange (ISE) can be estimated" and analyzed ISE-100 Index for the time period January 1991 to December 2004. The models in the study have been examined for four cross sections by using two different statistical analyses. According to the regression analysis, conditional models were found more efficient and unbiased predictors than unconditional models in forecasting and modeling the volatility in ISE-100 Index. She also pointed out GARCH (1,1) and EGARCH (1,1) models are the best fitting models for ISE-100 Index.

Özden (2008), has researched the best fitting model for the Istanbul Stock Exchange (ISE) 100 Index's return volatility with ARCH, GARCH, EGARCH, TGARCH models. He has used daily closing values between the dates of 04.01.2000 and 29.09.2008 and found that the best model for the mean equation is ARMA(2,2). It is also found that TGARCH(1,1) is the more precise model to forecast stock returns volatility in ISE.

In her study Atakan (2009) has investigated the most appropriate method for modeling the volatility at the Istanbul Stock Exchange (ISE) by using the ARCH type models. The research spans the period of 1987-2008 of ISE-100 Index and daily closing data has used in the study. Author observed that the volatility of ISE-100 Index has the ARCH effect and the most appropriate model for forecasting the volatility of ISE-100 Index is GARCH(1,1). Moreover, it is found that during the crises and uncertain periods, the volatility increases. In the analysis it is observed that IMKB-100 shows volatility clustering in these periods.

[6] Index includes the 30 shares with the highest capitalization traded in Istanbul Stock Exchange

4. Data and Methodology

The analysis is based on daily returns of ISE-100 Index which includes the hundred shares with the highest capitalization traded in the Istanbul Stock Exchange (ISE), and the data set covers the time period from 3 July 1987 to 3 July 2009, representing 5377 observations. Data has been obtained from the ISE's web page. The data used in the study consisted of time series of daily stock-market index based on daily closing prices, in terms of local currency. The index did not include dividends and the returns in the market (Rt) were computed by the first difference of the natural logarithm of stock market index.

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) * 100 \qquad (14)$$

Where R_t shows continuously compounded percentage change of index for the time period t, P_t denotes price index at t (P_{t-1} is the same for preceding period) and ln is the natural logarithm. Minitab 14.0 and Eviews 6.0 statistical and econometric softwares are used in analyzing the data set.

5. Empirical Findings

One of the first things to look at when analyzing a set of financial values are descriptive statistics about financial data set, including the mean, standard deviation, skewness, and kurtosis. The skewness measures the symmetry of the distribution and the kurtosis measures the weight of the tails in the distribution. Histograms and box plots (which are graphical representation of the quantiles, usually the quartiles) are also useful to understand the shape and spread of the data.

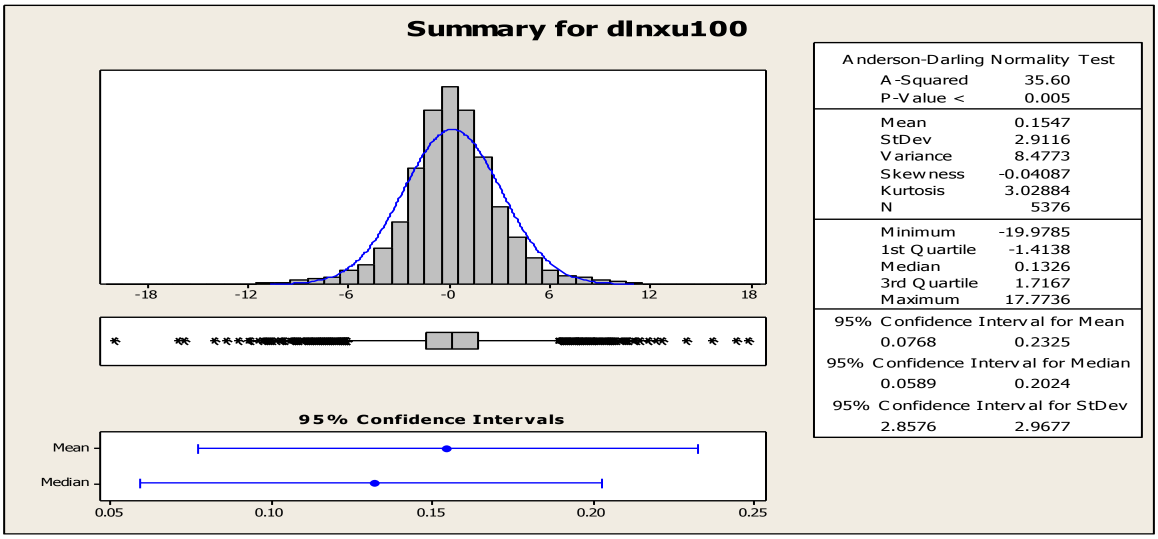
5.1. Testing For Normality

An important feature of the return series is the presence of departures from normality in the unconditional distribution. If the conditional distribution of the returns is i.i.d.[7] normal, then we would expect the unconditional distribution of the returns to be normal, too.

Table 1, given below, summarizes the basic properties of the data. The returns appear to be somewhat asymmetric, as it is seen by positive skewness estimates which mean there are more observations in the right-hand side (positive) tail than in left-hand tail. The mean return of the data set is positive and close to zero. Kurtosis value which is larger than 3 indicates a higher peak and fat tails. In any case, we can look at the skewness and the kurtosis of the unconditional distribution of the variables to try to get a first sense of how returns are distributed. Since we know that symmetric distributions have 0 skewness and that the normal distribution has kurtosis equal to 3, the empirical finding of our analysis seems to have unconditional third moment (skewness) not close to zero, so they have asymmetric unconditional distributions, and also they all present excess unconditional kurtosis. Due to the fact that a normally distributed random variable should have skewness and kurtosis near zero and three, respectively, we can say our data don't show a normal distribution.

[7] Independent and identically distributed

Table 1 The Basic Statistics of IMKB-100's Returns Data



Skewness and Kurtosis are based on the empirical data. The numerical methods for testing normality compare empirical data with a theoretical distribution. Widely used methods include the Kolmogorov-Smirnov (KS) test, Shapiro-Wilk test, Anderson-Darling test, and Cramer-von Mises test (SAS Institute 1995). The KS and Shapiro-Wilk (SW) tests are commonly used. It is widely known that the Kolmogorov type tests are more suitable to analyze a data sample which has some specific distribution. For n > 2000, it will be more appropriate to use KS to test the normality of the distribution[8].

Kolmogorov-Smirnov and similar tests use the below hypotheses:

Ho: data set is normally distributed vs.

Ha: data set is not normally distributed

KS Test which has been conducted by using MINITAB program, provides the approximate p-value < 0,010 and also leads us to assume normality of the data in our analysis. Since we usually have an alpha value of 0.05, we should reject the null hypothesis that the data follows a normal distribution. Finally, KS graph taken from outputs of the Minitab program, shows S trend. In this case, the sample values don't fall very close to the line, indicates that the data likely don't follow a normal distribution. As a result, we reject the null hypothesis in favor of the alternative hypothesis ("the data have a non-normal distribution" or "the factors are not independent").

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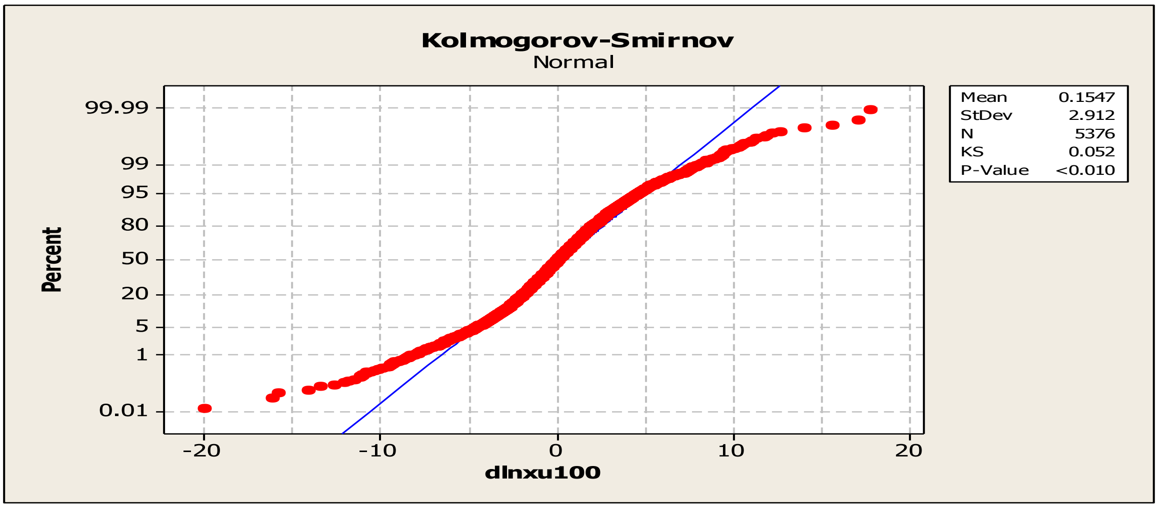
[8] SAS Institute recommends to use KS test instead of SW when observation number is greater than 2000.

Table 2 Kolmogorov-Smirnov Test Outputs



Augmented Dickey Fuller and Phillips-Perron Tests

We can also test the random walk hypothesis by using unit root tests and spectral analysis. Both the Augmented Dickey Fuller (1979) and Phillips-Perron (1988) tests can be used to examine the univariate time series properties of the data to see if the random walk hypothesis holds.

As it is known, a stochastic process $\{P_t\}$ is called a random walk if it follows:

$$P_t = \mu + P_{t-1} + u_t \quad (15)$$

In Equation (7) u_t is a white noise[9] with $E(u_t) = 0$ and Var(u_t) = σ^2. If $\mu \neq 0$, it is called a random walk with drift μ. It is easy to verify that a random walk without drift is a martingale:

$$E(P_t|P_{t-1}) = E(P_{t-1} + u_t|P_{t-1}) = P_{t-1} + E(u_t) = P_{t-1} \quad (16)$$

For EMH, only $E(u_t|\Omega_{t-1}) = 0$ is essential where Ω_{t-1} represents information set. There are also different versions of random walk hypothesis with respect to the distribution of u_t where u_t denotes the prediction error. If index return follows a random walk, then price changes are white noise. Therefore, testing whether returns are white noise is observationally equivalent to the test of random walk in index changes. Given r_t as the percentage change in P_t, the null hypothesis of market efficiency is thus formed as testing for the standard statistical properties of a homoscedastic white noise process as follows:

$$H_0 : E(r_t) = 0; \quad (17)$$

$$E(r_t r_t) = \sigma_r^2;$$

$$E(r_t r_s) = 0; \forall t \neq s.$$

[9] A time series r_t is called white noise if it is a sequence of independent and identically distributed ramdom variables with finite mean and variance. In particular, if r_t is normally distributed with mean zero and variance, the series is called a Gaussian white noise (Tsay, 2002:26-27)

In the analysis, a standard Augmented Dickey-Fuller (1979) unit root test (ADF) is employed on the Equation (18) and it is estimated with the lag length determined by Akaike information criterion. Moreover, the lag length chosen was sufficient to eliminate serial correlation in the error terms.

$$\Delta P_t = \alpha + \beta T + (\rho - 1)P_{t-1} + \sum_{t=1}^{n} \Phi_i \Delta P_{t-i} + u_t \qquad (18)$$

In Equation (18), P_t is the respective time series; T is a linear time trend parameter; Δ is the first difference operator; and u_t denotes the error process with zero mean and constant variance.

H_0: ρ-1=0, there is a unit root (*i.e.*, difference stationary)

H_a : ρ-1<0 ,

Secondly, we have used an alternative unit root test which has proposed by Phillips and Perron (1988). This test has the desirable feature that it allows for a weaker set of assumptions concerning the error process, specifically, the presence of dependence and heterogeneity in the error term. The presence of a unit root was tested using the Phillips-Perron (PP) procedure given as below:

$$P_t = \alpha + \beta(t - T/2) + \rho P_{t-1}\varepsilon_t \qquad (19)$$

In the equation, P_t denotes the respective time series; (t - T/2) is a time trend where T is the sample size; and ε is the error term. The hypothesis tested is;

H_0: ρ=1, the time series is nonstationary,

H_a : ρ<1 , the time series is stationary around a deterministic trend,

Tablo 3 The ADF P-P and KPSS Tests Outputs of Eviews 6.0

Test Critical Values	Constant	Constant & Trend	None
t-Statistic	-20.928*	-20.951*	-20.680*
1% level	-3.431	-3.960	-2.565
5% level	-2.862	-3.411	-1.941
10% level	-2.567	-3.127	-1.617
Test Critical Values	Constant	Constant & Trend	None
Adj. t-Stat	-65.535*	-65.531*	-65.638*
1% level	-3.431	-3.960	-2.565
5% level	-2.862	-3.411	-1.941
10% level	-2.567	-3.127	-1.617

Table 3 gives the result of Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests on the original series as well as the MacKinnon critical values for rejection of the hypothesis of the existence of a unit root at the 5% level of significance. Since the ADF and PP test statistics are larger in absolute values than the critical values, we reject the hypothesis of non-stationarity. As we know, if a time series has a unit root, then it follows a random walk; thus as per the results, we reject the hypothesis that says the IMKB-100 Index returns show a random walk.

5.2. Modeling the Market Return Applying ARMA Model

Before implementing the ARIMA models for estimating index returns yields, we have tested the return data for unit roots using the Augmented Dickey Fuller and the Phillips-

Perron (PP) unit root tests in Section 4.1.4. Sequential tests are conducted to analyze for the presence of the non-stationarity in both the levels and the first differences. As we know, if the equations are stationary then it can be modelled. Since the models are found to be stationary, our objective is to determine if the returns of index can be forecasted by its own past values.

Tablo 4 Parameters of ARMA Type Models

	AR(1)	**AR(2)**	**AR(3)**	**MA(1)**	**MA(2)**
α_0	0.154*	0.154*	0.154*	0.154*	0.154*
Φ_1	0.118*	0.118*	0.118*		
Φ_2		0.002	0.003		
Φ_3			-0.006		
θ_1				0.115*	0.118*
θ_2					0.017
θ_3					
F-stat	76.099*	38.042*	25.425*	74.585*	38.174*
AIC	4.961931	4.962483	4.962998	4.962045	4.962091
SIC	4.964383	4.966162	4.967903	4.964497	4.965768
	MA(3)	**ARMA(1,1)**	**ARMA(1,2)**	**ARMA(1,3)**	**ARMA(2,1)**
α_0	0.154*	0.154*	0.154*	0.154*	0.154*
Φ_1		0.134	-0.713*	-0.585***	-0.663*
Φ_2					0.105*
Φ_3					
θ_1	0.119*	-0.016	0.833*	0.704**	0.780*
θ_2	0.016		0.105*	0.085**	
θ_3	-0.014			-0.012	
F-stat	25.772*	38.053*	26.668*	20.099*	26.133*
AIC	4.962282	4.962299	4.961953	4.962251	4.962428
SIC	4.967185	4.965977	4.966857	4.968381	4.967332
	ARMA(2,2)	**ARMA(2,3)**	**ARMA(3,1)**	**ARMA(3,2)**	**ARMA(3,3)**
α_0	0.154*	0.167*	0.154*	**0.154***	0.155*
Φ_1	-0.634*	0.271	-0.571**	**-0.939***	0.045
Φ_2	-0.285*	0.716*	0.085**	**-0.745***	0.096
Φ_3			-0.017	**0.077***	0.499*
θ_1	0.750*	-0.152	0.689**	**1.060***	0.065
θ_2	0.363*	-0.727*		**0.868***	-0.089
θ_3		-0.103*			-0.526*
F-stat	20.400*	16.582*	19.939*	**18.608***	15.266*
AIC	4.962210	4.962338	4.962729	**4.960662**	4.961295
SIC	4.968341	4.969695	4.968861	**4.968020**	4.969879

ARMA models are combinations of Auto-Regressive and moving average models, so it may be helpful to examine each of these data given at Table 4[10]. Through an iteration process, we have found that ARMA (3,2) model is the best fitting model for forecasting the IMKB-100 returns. AIC criterion also shows us that ARMA(3,2) model fits the data

[10] Although we have tested up to 20 lags, we present only 3 lags.

quite well. We present in Table 4 the limited test results and estimated parameters of each ARMA model.

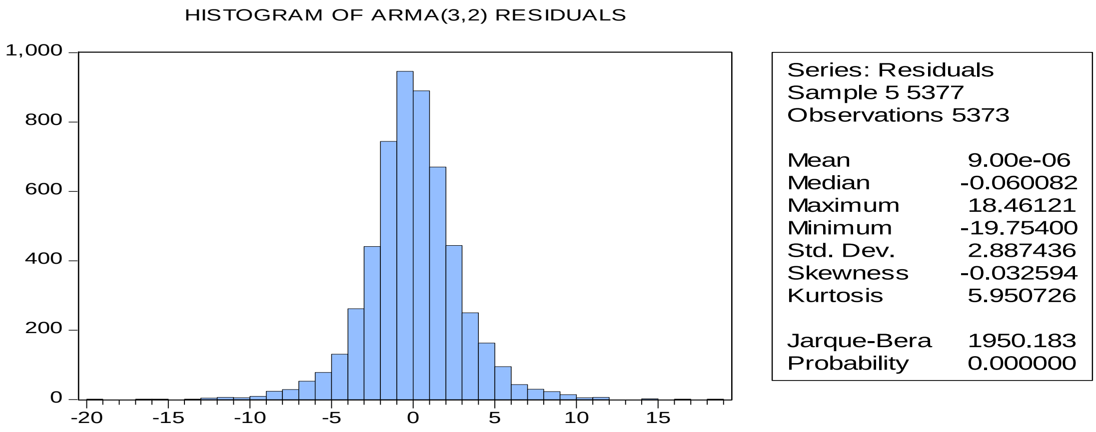
Table 5 Corelogram of residuals and squared residuals of ARMA (3,2) Model

Sample: 5 5377
Included observations: 5373
Q-statistic probabilities adjusted for 5 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.001	-0.001	0.0041	
		2	0.009	0.009	0.4254	
		3	0.009	0.009	0.9071	
		4	0.015	0.015	2.1813	
		5	-0.004	-0.004	2.2827	
		6	-0.008	-0.008	2.6187	0.106
		7	-0.006	-0.006	2.7893	0.248
		8	0.030	0.030	7.7620	0.051
		9	0.024	0.025	10.890	0.028
		10	0.031	0.031	16.129	0.006
		11	0.012	0.011	16.883	0.010
		12	0.011	0.009	17.580	0.014
		13	0.004	0.003	17.662	0.024
		14	0.003	0.002	17.696	0.039
		15	0.019	0.019	19.601	0.033
		16	-0.007	-0.007	19.876	0.047
		17	0.000	-0.001	19.876	0.069
		18	-0.001	-0.004	19.882	0.098
		19	0.014	0.012	20.970	0.102
		20	-0.008	-0.010	21.357	0.126

Sample: 5 5377
Included observations: 5373
Q-statistic probabilities adjusted for 5 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.280	0.280	420.60	
		2	0.252	0.188	761.95	
		3	0.175	0.074	927.59	
		4	0.134	0.038	1024.9	
		5	0.175	0.102	1189.0	
		6	0.125	0.030	1272.8	0.000
		7	0.087	-0.008	1313.8	0.000
		8	0.108	0.044	1377.0	0.000
		9	0.072	0.004	1405.0	0.000
		10	0.125	0.068	1489.4	0.000
		11	0.123	0.053	1571.3	0.000
		12	0.092	0.008	1616.6	0.000
		13	0.086	0.007	1656.5	0.000
		14	0.084	0.023	1694.8	0.000
		15	0.087	0.020	1735.1	0.000
		16	0.116	0.049	1807.1	0.000
		17	0.066	-0.011	1830.6	0.000
		18	0.069	0.004	1856.6	0.000
		19	0.068	0.013	1881.2	0.000
		20	0.079	0.027	1914.8	0.000



The adjusted R^2 value of this magnitude is statistically significant (implies good fit between actual data and model-estimated data) and absolute AIC value is higher according to the others. DW value, which is near 2 indicates that there is no significant autocorrelation in the residuals. But the LM test, computed for homoscedasticity of the residual variances, reveals statistically significant heteroscedasticity. It indicates that our model is a good fit with the data and the assumption of no residual autocorrelation is satisfied. On the other hand, homoscedasticity assumptions were not valid for our model. This means using heteroscedastic models like ARCH-GARCH will be more appropriate for modeling the IMKB-100 Index's returns and our model is not statistically reliable for the long run forecasts.

An ARMA model for observed time series is necessary to remove any serial correlations within the data. Above, the steps to obtain the ARMA model for IMKB-100 Index are shown clearly. From the observations made in previous step, for Istanbul Stock Index, the analysis conducted by using ARMA(3,2) model as mean equation:

Tablo 6 ARCH-LM Test Statistics

Heteroskedasticity Test: ARCH			
F-statistic	455.8219	Prob. F(1,5370)	0.0000
Obs*R-squared	420.3142	Prob. Chi-Square(1)	0.0000

After estimating the correct ARMA model, ARCH-LM test was applied to see whether there exists any conditional heteroskedasticity (ARCH effect) within the models. From *Table 7*, it can be seen that the F statistic for ISE-100 Index is significantly high (according to 5% sig.) so that we rejected the null hypothesis that there exists no ARCH effects within the models and went on selecting the appropriate ARCH type model. To decide on the best fitting GARCH model, we used the Akaike's Information Criterions (AIC). We selected the model which has the lowest value of AIC and F-statistic with the highest probability

We tested the data by using GARCH, TARCH, EGARCH and GARCH-M models. We tried to find out which model was the best for ISE-100 Index. We collected all the AIC values and LM-Test statistic values up to 5 lags for ISE-100. The results of estimation and statistical verification of ARCH (7), GARCH(1,1), TARCH(1,1), GARCH-M (1,1) and EGARCH(1,1) models are shown in Table 8. The results indicate that the GARCH components of the variance are statistically significant in all five models.

For ISE-100 Index, we decided that the TGARCH(1,1) model is the best as it has both the lowest AIC statistic and the highest probability to accept the null hypothesis that there is no ARCH effect any more in the model. As shown in Table 8, the coefficient of γ value is positive but it is relatively small and the probability is significant at 5% critical level. The positive and significant γ value shows us that leverage effect exists, bad news increases volatility. When we sum up the arch (0.150) and GARCH values (0.802) we get 0.952**,** which is very close to 1. That means, the shock is persistent (is dying off slowly). The existence of a 'leverage effect' was confirmed in the case of TARCH model. But, although non-zero value of ɑ in E-GARCH equation indicates an asymmetric effect, the positive $u_{t-2}/\sqrt{h_{t-2}}$ value doesn't indicate presence of a 'leverage effect'.

It is found that for ISE-100 Index, the second best fitting model is GARCH-M (1,1) as shown in Table 7. The total of the α_1 (0.171) and β_1 (0.804) is 0.971. The coefficient of $\sqrt{h_t}$ term is 0.076, which is significant and positive. Therefore, the return is positively related to its past volatility. Additionally, there is a trade-off between the return and the risk. The higher risk the higher return since a rise in variance increases the mean of return.

Tablo 7 ARCH-Type Model Results

MEAN EQUATION					
	ARCH(7)	**GARCH(1,1)**	**GARCH-M**	**TGARCH**	**E-GARCH**
$\sqrt{h_t}$			0.076		
α_0	0.154	0.158	-0.012	0.139	0.158
Φ_1	0.825	0.902	-0.189	0.900	0.572
Φ_2	-0.052	-0.853	-0.954	-0.844	0.477
Φ_3	0.018	0.108	0.115	0.110	-0.058
θ_1	-0.705	-0.789	0.303	-0.785	-0.445
θ_2	-0.038	0.764	0.989	0.754	-0.543
VARIANCE EQUATION					
C	2.388*	0.272*	0.284*	0.291*	-0.00961
u_{t-1}^2	0.198*	0.167*	0.171*	0.150*	
u_{t-2}^2	0.172*				
u_{t-3}^2	0.094*				
u_{t-4}^2	0.053*				
u_{t-5}^2	0.133*				
u_{t-6}^2	0.076*				
u_{t-7}^2	0.0253**				
h_{t-1}		0.809*	0.804*	0.802*	
$u_{t-1}^2 * u_{t-1} < 0$				0.045*	
$\lvert u_{t-1}/\sqrt{h_{t-1}} \rvert$					0.357*
$\lvert u_{t-2}/\sqrt{h_{t-2}} \rvert$					-0.343*
$u_{t-1}/\sqrt{h_{t-1}}$					-0.031*
$u_{t-2}/\sqrt{h_{t-2}}$					0.033*
lnh_{t-1}					1.826*
lnh_{t-2}					-0.827*
AIC	4.755763	4.709717	4.709461	4.709034	4.733354
SIC	4.772932	4.72198	4.722951	4.722524	4.749297

ARCH LM Test					
F-statistic	0.772	1.485	1.403	1.398	2.351
Prob.	0.56	0.19	0.21	0.22	0.12
Obs*R-squared	3.863	7.424	7.015	6.989	2.351
Prob.	0.56	0.19	0.21	0.22	0.12
Root Mean Squared Error	2.891503	2.890127	2.890007	2.890193	2.891688
Mean Absolute Error	2.106011	2.105741	2.104896	2.10561	2.106573
Mean Abs. Percentage Error	126.8268	129.5082	125.2329	128.8367	126.8452

* denotes significance at 1 percent
** denotes significance at 5 percent

6. Conclusion

Modeling and forecasting stock price volatility has always been one of the important issues of financial theory and practice. This paper has represented an example of risk measurement that could be the input to a variety of stock returns. In this study, both Box Jenkins (ARIMA) and ARCH-type volatility models (ARCH-GARCH-EGARCH-TARCH and GARCH-M) are discussed under daily returns of Istanbul Stock Exchange 100 Index.

Asymmetric GARCH models and GARCH-M model are compared to the GARCH(1,1) model, which has found as best fitting model for ISE in prior studies, and we have rejected the common sense that none of the competing models are better than the GARCH(1,1). We have found that TARCH (1,1) is the best fitting model for forecasting the volatility of ISE-100 Index's. Our findings are not surprisingly, because the GARCH(1,1) model corresponds to a simple news impact curve, and a GARCH(1,1) process cannot generate a leverage effect.

The results of our study are coherent to the facts which have been reported in Bollerslev et al. (1994) and Pagan (1996). ISE-100 Index's returns include; leptokurtosis, leverage effects, volatility clustering (or pooling), volatility smile and long memory.

A large sum of the coefficients in the conditional variance equations implies that a large positive or a large negative return in ISE-100 Index will lead future forecasts of the variance to be high. Finally, for ISE-100, we can say that the leverage effect exists and bad news increases volatility.

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