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Constrained cost optimization under uncertainty for an incompletely-connected electric utility system

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Abstract

In this case study, we build a stochastic model to perform cost optimization and investment decision modeling for a system of interconnected power grids. The modeled utility system is composed of four independent power-generating regions. Each region has its own power demand and portfolio of types of power generating plants. Because of the variation in generating assets, the generation cost profile can vary dramatically by region. Power transmission lines connect the four regions subject to the constraints that some regions are not connected at all, and the amount of power that is allowed to flow over each boundary is limited. Furthermore, power plant availability and total generating capacity vary stochastically, as a function of many factors. First we mathematically define an optimization problem that allows us to meet the aggregate demand of all regions under these transmission constraints while minimizing the total cost. This is then implemented under the framework of uncertain generation capacity so we can make probabilistic statements about the costs and other relevant quantities. Ultimately, this optimization model can be used to guide and inform capacity and transmission expansion investment-related decisions. Our model is developed using Microsoft Excel and the @Risk and Evolver tools from Palisade's Decision Tools Suite.

Keywords: *Constrained Optimization, Utility Cost Modeling, Decision Support System, Investment Analysis*

Eksik-bağlı elektrik hizmeti sistemi için belirsizlik altında kısıtlı maliyet optimizasyonu

Özet

Bu vaka çalışmasında, birbirine bağlı enerji ağları sistemi için maliyet optimizasyonu ve yatırım kararı modellemesi gerçekleştirmek amacıyla stokastik bir model oluşturulmuştur. Modellenen hizmet sistemi dört adet bağımsız güç üreten bölgeden oluşmaktadır. Her bölge, kendi güç talebi ve güç üretme tesis türleri portföyüne sahiptir. Varlık üretmedeki değişkenlik sebebiyle, üretim maliyeti profili bölgelere göre önemli ölçüde değişiklik gösterebilmektedir. Enerji nakil hatları, bazı bölgelerin birbirine hiçbir bağlantısı olmadığına ve her bir sınır üzerinden geçecek güç miktarının sınırlı olduğuna dair kısıtlar dâhilinde bu dört bölgeyi birbirine bağlar. Ayrıca, güç üretme tesislerinin kullanılabilirliği ve toplam güç üretme kapasitesi tahminler doğrultusunda pek çok faktöre bağlı olarak değişkenlik göstermektedir. Öncelikle, bu nakil kısıtlamaları dâhilinde toplam maliyetleri de minimize eden ve tüm bölgelerin toplam talebini bize sunan bir optimizasyon problemi matematiksel olarak tanımlanmıştır. Sonrasında bu model belirsiz üretim kapasitesi çerçevesinde uygulanmış ve bu şekilde maliyetler ve diğer ilgili nicelikler hakkında olası

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açıklamalarda bulunulmuştur. Sonuç olarak, bu optimizasyon modeli, güç kapasitesinin ve naklinin genişletilmesine dair yatırımlar ile ilgili kararlarda bilgilendirici ve yönlendirici olmasıyla kullanılabilir bir modeldir. İlgili model, Microsoft Excel ve Palisade'ın Decision Tools Suite @Risk and Evolver araçları kullanılarak geliştirilmiştir.

Anahtar Sözcükler: *Kısıtlı Optimizasyon, Fayda Maliyet Modellemesi, Karar Destek Sistemi, Yatırım Analizi*

1. Introduction

Like any other business, electric utilities have to operate under cost constraints - even public utilities that don't need to make a profit. In fact, they can be constrained by more than costs in more ways than other businesses. If your local supermarket runs out of your favorite food, you're unhappy. If your power company runs out of power, it's an entirely different thing. When power prices rise dramatically (California, USA, 1990's), all hell breaks loose. Furthermore, power utilities are constrained by more than just costs:

- If a region's winter is especially warm & dry, there will be little runoff for dams the ensuing spring. If the utility relies on hydrological power for cheap baseload, this will raise costs.
- If a region's populace turns against gas transport pipelines, gas-burning CC / CT peaking units (basically a jet engine mounted on the ground) may have to be left off. The utility may have to purchase more off-network electricity at much higher prices - if even possible.
- If enough people are swayed by the arguments against nuclear power, clean-burning nuclear generators must be shut down. The utility will then have to burn more coal or gas - at higher costs.

In addition to usual operating costs, utilities often have further constraints on investments. If a utility's region expands outward dramatically, it can be much cheaper to extend high-voltage lines to the new outlying regions, instead of building new capacity. However, this requires extra wayleaves that landowners may fight. Furthermore, expansion by adding new power plants can face regulatory and environmental challenges, as well as the costs.

In this case study, we build a model to perform power generation cost optimization and inform investment decision modeling for a system of incompletely-connected power grids. We use Microsoft Excel with the @Risk and Evolver tools from Palisade's Decision Tools Suite. The modeled utility system in the UK is composed of four notional power-generating regions. Each region has its own power demand and portfolio of types of power generating plants. Because of the variation in generating assets, the generating cost profile varies dramatically by region. Power transmission lines connect the four regions subject to the constraints that

a) some regions are not connected at all, and

b) the amount of power that is allowed to flow over each boundary is limited.

Furthermore, power plant availability and total generating capacity vary stochastically, as a function of many factors.

As chronicled in Section 2, our model is built in three stages. We begin with a simple optimization problem that can be easily solved with linear programming. This model is next extended with a set of binary constraints that make it psuedo-linear. Finally, it is embedded within the framework of uncertain generation, and becomes highly non-linear. Section 3 details how the optimization model is implemented with Palisade's Decision Tools Suite, and also discusses its performance. We show results from our model in

Section 4, and use it to predict the impacts of different types of investments on generating costs, before concluding in Section 5.

2. The Constrained Optimization Model

We have a utility system of four independent power-generating regions, each with its own power capacity, demand, and generating cost. These four regions are designated as North Ireland, Scotland, England & Wales, and Republic of Ireland. As shown in Figure 1, some regions are connected so that power can flow across the boundary. The solid blue arrows indicate sub-sea cables under the Irish Sea. Note there is also a set of sub-sea cables coming from the European continent into England & Wales. Across each boundary, power can flow bi-directionally, constrained by the lines' capacities. We want to define an optimization problem that will allow us to meet the aggregate demand of all regions while minimizing the total generating cost, subject to these constraints.

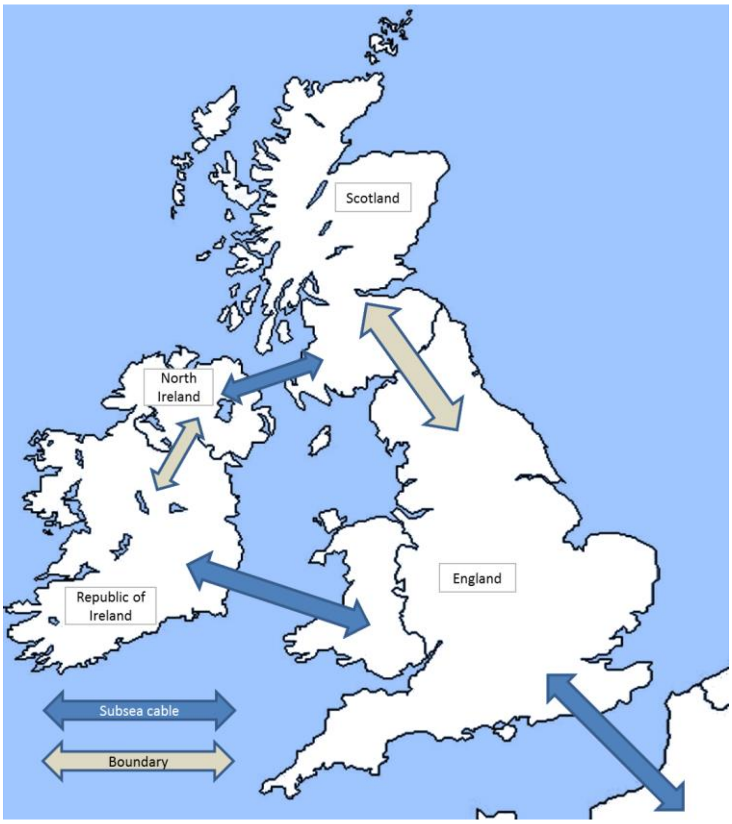

Figure 1 Power Flow Boundaries among Independent Generating Regions

In formulating our problem, first we define the region-specific variables:

- Unit Cost per region in £/MW: C_N, C_S, C_E, C_R;
- Generating capacity per region (MW): M_N, M_S, M_E, M_R;
- Peak power demand per region (MW): D_N, D_S, D_E, D_R.

We use $\boldsymbol{C}$ for the total system cost. The subscript characters indicate the region: N is for North Ireland, S is for Scotland, E is for England & Wales, and R is for Republic of Ireland. Next we define the maximum flow constraints (in megawatts) between all pairs of regions.

- North Ireland ↔ Scotland: M_{NSN},

- North Ireland ↔ England & Wales: M_{NEN},
- North Ireland ↔ Republic of Ireland: M_{NRN},
- Scotland ↔ England & Wales: M_{SES},
- Scotland ↔ Republic of Ireland: M_{SRS},
- England & Wales ↔ Republic of Ireland: M_{ERE}.

Note that even regions not physically connected are assigned a variable. In the optimization problem, these values (M_{NEN}, M_{SRS}) are set to 0. Finally, we define variables holding the total amount of power generated per region:

- $G_N = G_{NN} + G_{NS} + G_{NE} + G_{NR}$,
- $G_S = G_{SN} + G_{SS} + G_{SE} + G_{SR}$,
- $G_E = G_{EN} + G_{ES} + G_{EE} + G_{ER}$,
- $G_R = G_{RN} + G_{RS} + G_{RR} + G_{RR}$,

The first subscript indicates the generation region and the second indicates the demand region. For example, G_{SE} is the power generated by the Scotland region that flows into England & Wales; G_{ES} is the exact opposite. Remember, our power lines are allowed to carry power either way across the boundaries.

Thus, we want to solve the linear programming optimization problem shown in (1), subject to the inter-region flow constraints, by varying the G_{**} variables.

$$\begin{array}{ll}
\text{minimize} & \mathbf{C} = C_N G_N + C_S G_S + C_E G_E + C_R G_R \\
\text{subject to} & G_{**} \geq 0 \\
\text{flow limits} & \begin{cases} G_{NS}, G_{SN} \leq M_{NSN} \quad G_{NE}, G_{EN} \leq M_{NEN} \\ G_{NR}, G_{RN} \leq M_{NRN} \quad G_{SE}, G_{ES} \leq M_{SES} \\ G_{SR}, G_{RS} \leq M_{SRS} \quad G_{ER}, G_{RE} \leq M_{ERE} \end{cases} \\
\text{generation up to capacity} & \begin{cases} 0 \leq G_N \leq M_N \quad 0 \leq G_S \leq M_S \\ 0 \leq G_E \leq M_E \quad 0 \leq G_R \leq M_R \end{cases} \\
\text{demand is met} & \begin{cases} G_{NN} + G_{SN} + G_{EN} + G_{RN} \geq D_N \\ G_{NS} + G_{SS} + G_{ES} + G_{RS} \geq D_S \\ G_{NE} + G_{SE} + G_{EE} + G_{RE} \geq D_E \\ G_{NR} + G_{SR} + G_{ER} + G_{RR} \geq D_R \end{cases}
\end{array} \tag{1}$$

In the first constraint, G_{**} indicates all power generation quantities previously mentioned. This constraint forces power generation to be positive! The six constraints starting in the first block limit the amount of power that can flow over all possible boundaries. The next block of four constraints force the total power generated in each region to not exceed the capacity, and the last four constraints guarantee all regions' power demands are met. This model works, and can be solved extremely easily, but has one major shortcoming.

The major drawback of this optimization problem is that it will allow solutions with power flowing bidirectionally between two adjacent regions simultaneously. For example, maybe Scotland will generate some power to meet the demand in England & Wales, while England & Wales sends power to Scotland. The resulting minimum cost is the same as if the two flows simply netted out, but the solution requires some manual adjustment to be realistic. To solve this, let us define six binary constraints as shown here; the quantities G_{NEN}, G_{NRN}, G_{SES}, G_{SRS}, and G_{ERE} are similarly defined.

$$G_{NSN} = \begin{cases} 1 & \text{if } G_{NS} > 0 \text{ and } G_{SN} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Note that these binary constraints are all non-linear. By adding these six binary constraints to (1), we have an optimization problem that can no longer be called linear. Instead, we call the updated problem in (2) psuedo-linear.

$$\begin{array}{ll}
\text{minimize} & \mathbf{C} = C_N G_N + C_S G_S + C_E G_E + C_R G_R \\
\text{subject to} & G_{**} \geq 0 \\
\text{flow limits} & \begin{cases} G_{NS}, G_{SN} \leq M_{NSN} \quad G_{NE}, G_{EN} \leq M_{NEN} \\ G_{NR}, G_{RN} \leq M_{NRN} \quad G_{SE}, G_{ES} \leq M_{SES} \\ G_{SR}, G_{RS} \leq M_{SRS} \quad G_{ER}, G_{RE} \leq M_{ERE} \end{cases} \\
\text{generation up to capacity} & \begin{cases} 0 \leq G_N \leq M_N \quad 0 \leq G_S \leq M_S \\ 0 \leq G_E \leq M_E \quad 0 \leq G_R \leq M_R \end{cases} \\
\text{demand is met} & \begin{cases} G_{NN} + G_{SN} + G_{EN} + G_{RN} \geq D_N \\ G_{NS} + G_{SS} + G_{ES} + G_{RS} \geq D_S \\ G_{NE} + G_{SE} + G_{EE} + G_{RE} \geq D_E \\ G_{NR} + G_{SR} + G_{ER} + G_{RR} \geq D_R \end{cases} \\
 & G_{NSN}, G_{NEN}, G_{NRN}, G_{SES}, G_{SRS}, G_{ERE} = 0
\end{array} \tag{2}$$

Again, (2) should be relatively easy to optimize subject to the constraints given. However, we have not yet accounted for the uncertainty in generation capacity M_* and costs C_*.

For a power plant, the *nameplate capacity* is the maximum amount of power it can generate safely under optimal conditions. A utility system's *total nameplate capacity* is the sum of the nameplate capacities for all the plants. The maximum power a plant can generate at any given time will probably be less than its nameplate; the same can be said for the total utility system. A utility system's total generating capacity will generally vary from this theoretical maximum for reasons such as:

- planned outages (for maintenance) and / or unplanned outages (something breaks)
- if gas prices rise too high, gas-burning combustion turbine units may not be run
- if lower quality coal is used, a coal-burning plant may not be able to run at full power
- dam spillways maybe be partially closed during spawning season in spring to increase viability of fish roe (dam spillways and damn fish!)

Variability in the amount of power that plants can generate also affects the unit generating cost for the entire utility system. The cost to generate one megawatt of power from a power plant is largely, but not solely, a function of *fuel prices*. A generic generating cost hierarchy for some major plant types in the UK would be

Hydrological ≤ Nuclear ≤ Gas ≤ Coal ≤ Oil ≤ Diesel

In other places, such as the US, coal and gas may trade places. As generating capacity from power plants that are lower in the hierarchy is curtailed or completely unavailable, the total cost will be increased. Hence, since generating capacity (M_*) will vary from the nameplate capacity at any given period, so will unit cost (C_*). Power plant availability rates vary seasonally; what we used in this case study was estimated by subject matter experts as availability rates for the winter. For many plant types, generating capacity decreases in the summer. In this case study, we have several types of power plants, with capacity modeled as shown in Table 1.

Table 1 Stochastic Modeling of Plant Capacity; *N* is the Average Nameplate Capacity in MW, and *U* Indicates the Number of Plants[3]

Type	Cost (£/MW)	Capacity
Offshore Wind	-0.5	*Discrete(Uniform)×N*
Onshore Wind	-0.3	*Discrete(Uniform)×N*
Wave & Tidal	-0.7	*Binomial(n=U, p=0.20)×30*
Hydrological	-1.0	*Gaussian(μ=0.60, σ=0.04)×N*
Nuclear	0	*Binomial(n=U, p=0.70)×600*
France*	0	*Binomial(n=4, p=0.50)×500*
Imera/BritNed*	0	*Binomial(n=9, p=0.50)×500*
Base Gas	40	*Binomial(n=U, p=0.80)×500*
Base Coal	50	*Binomial(n=U, p=0.80)×500*
Biomass	55	*Binomial(n=U, p=0.85)×100*
Marginal Gas	70	*Binomial(n=U, p=0.80)×500*
Marginal Coal	70	*Binomial(n=U, p=0.80)×500*
Pumped Storage	120	*Binomial(n=U, p=0.90)×300*
Oil	200	*Binomial(n=U, p=0.80)×500*
Diesel (OCGT)	300	*Truncated Gaussian(μ=0.95, σ=0.03)×677*

The middle column shows the cost in pounds sterling per megawatt of power generated. In our model, we assume that the costs do not vary by region, and do not vary seasonally. Some plant types have slightly negative costs. These are for plants designed to run either intermittently (Wind) or predictably (Marine / Hydro), and contractual obligations and fixed costs are structured accordingly. Though the fuel costs are actually zero, there are commercial advantages for these plant to always run - hence the assumed negative cost. The two starred sources indicate submerged power cables coming from the European continent into the England & Wales region. In each region, some gas and coal units may be older and thermally less efficient. On a per MW basis, they would then be more expensive and should be used less frequently. These are indicated as *marginal*.

The right-most column indicates how we simulate generating capacity for each type of power plant. For all plant types, the left term in each multiplication is used to simulate how many plants are available, and the right term indicates the nameplate capacity. Across the entire UK National Grid utility system, we assume that the nameplate capacity *N* for any given type of power plant is constant and that only the number of plants *U* varies; the exception to this is for the wind and water turbines. The binomial distribution is often used to simulate availability for a set of *n* independent power plants, with *p* indicating the expected availability percentage. We use the Gaussian distribution for hydrological and diesel, as the availability can be continuously controlled. Since the mean

[3] Plant running costs and availability data is for illustrative purposes only, and does not represent information for any particular generating plant.

is so close to 100% for diesel, we use a truncated Gaussian to prevent capacity from going above 100%.

Generating capacity from wind turbines follows a complex set of curves computed from wind speed and turbine power curves. As a first-order approximation to the resulting empirical distributions, we use a two-step simulation procedure based on Table 2. In each simulation, we draw a random value uniformly from each of the intervals formed inclusively by the first two columns. These ten random values are then used with a generalized discrete distribution, such that each value is associated with the probabilities shown in columns 3 and 4. The resulting capacities are then scaled by *N* as appropriate for each region. The empirical distributions for wind power availability are shown in Figure 2.

Table 2 Table Used for Simulating Wind Power Availability

MW Interval		% Chance	
Min	**Max**	**Offshore**	**Onshore**
0	100	33%	40%
101	200	18%	17%
201	300	13%	12%
301	400	9%	9%
401	500	7%	7%
501	600	5%	5%
601	700	4%	4%
701	800	3%	3%
801	900	3%	2%
901	1000	5%	1%

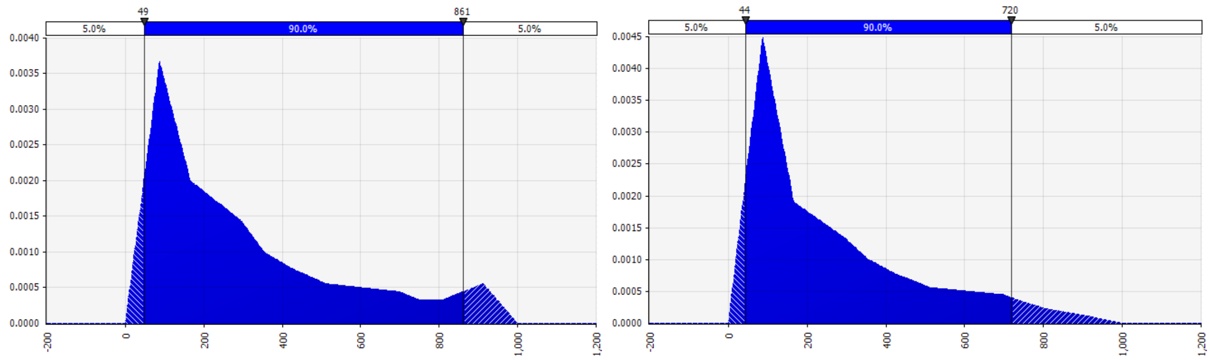

Figure 2 Empirical Distribution of Wind Generating Capacity, Will be Scaled by *N* as Appropriate *Left is Offshore, Right is Onshore*

Ordered by increasing cost and with capacities filled in, Table 1 is called a *Merit Order* when completed. The Merit Order for each of the four generating regions is specific. For example, North Ireland has only 3 gas plants and no coal-burning generators; England & Wales has 66 and 28, respectively. Each time we simulate the complete Merit Order for a region, we can sum the capacities and costs, then compute the average unit cost. For example, see Table 3, in which we compute M_E and C_E based on a partial Merit Order for England & Wales.

Table 3 Example England & Wales Partial Merit Order

Type	Unit Cost	Capacity	Total Cost
Nuclear	0	6,000	0
Base Gas	40	17,500	700,000
Base Coal	50	5,500	275,000
Biomass	55	1,100	60,500
Pumped Storage	120	2,100	252,000
		M_E =32,200	1,287,500
Average	C_E =39.89		

Hence we come back to our psuedo-linear optimization problem, which we modify to be non-linear. We allow each region's Merit Order to be dynamically generated, in that the optimization routine can vary binary variables to turn on / turn off individual plant types in each region independently. Thus, the generating capacity M_* and unit cost C_* in each region can be varied. To extend our example in Table 3, if the 2,100MW of expensive Pumped Storage is turned off, the generating capacity drops to 30,100MW and the unit cost to 34.40 £/MW. While this allows expensive power generators to be turned off to minimize the costs, this must be balanced against meeting all demand. Our final non-linear optimization problem is shown in (3), where M_*^d and C_*^d indicate that generating capacities and unit costs are *dynamic*.

$$\begin{array}{ll}
\text{minimize} & \mathbf{C} = C_N^d G_N + C_S^d G_S + C_E^d G_E + C_R^d G_R \\
\text{subject to} & G_{**} \geq 0 \\
\text{flow limits} & \begin{cases} G_{NS}, G_{SN} \leq M_{NSN} & G_{NE}, G_{EN} \leq M_{NEN} \\ G_{NR}, G_{RN} \leq M_{NRN} & G_{SE}, G_{ES} \leq M_{SES} \\ G_{SR}, G_{RS} \leq M_{SRS} & G_{ER}, G_{RE} \leq M_{ERE} \end{cases} \\
\text{generation up to capacity} & \begin{cases} 0 \leq G_N \leq M_N^d & 0 \leq G_S \leq M_S^d \\ 0 \leq G_E \leq M_E^d & 0 \leq G_R \leq M_R^d \end{cases} \\
\text{demand is met} & \begin{cases} G_{NN} + G_{SN} + G_{EN} + G_{RN} \geq D_N \\ G_{NS} + G_{SS} + G_{ES} + G_{RS} \geq D_S \\ G_{NE} + G_{SE} + G_{EE} + G_{RE} \geq D_E \\ G_{NR} + G_{SR} + G_{ER} + G_{RR} \geq D_R \end{cases} \\
 & G_{NSN}, G_{NEN}, G_{NRN}, G_{SES}, G_{SRS}, G_{ERE} = 0
\end{array} \tag{3}$$

While we allow generation capacity and costs to vary stochastically, note that demand is fixed. Demand in each region has been found to not have a significant effect on incremental generation costs. Hence, we set demand at a peak level (64,000MW), allocated to each region using historical proportions. Any changes in the overall system demand are assumed to affect each region such that the allocation remains the same.

3. Model Implementation

We implement all this in a Microsoft Excel spreadsheet model, using Palisade's Decision Tools Suite. Specifically, *@Risk* is used for the Monte Carlo simulation modeling and analysis, and *Evolver* is used for the constrained optimization. After each iteration of simulating the Merit Orders, the constrained optimization problem is solved. The modeling algorithm we designed follows these four steps:

i. At the beginning of the simulation, initialize the *Evolver* model and set user-specified parameters
ii. Simulate generation capacities for the Merit Order and copy values
iii. Execute the *Evolver* model to optimize the total system cost
iv. Record results

Steps ii through iv are iterated for 1,000 times. The *@Risk* model returns the empirical distributions of several optimized quantities:

- Regional Generating Capacity (M_*^d)
- Total System Generating Capacity ($\sum M_*^d$)
- Regional Unit Cost (C_*^d)
- Total System Cost ($\boldsymbol{C}$)
- Power Flow Across Boundaries (G_{**})

It also records information about the performance of the optimization model:

- Convergence Failure
- Total Number Trials & Number Valid Trials
- Time Required

Palisade's *Evolver* tool can use either the Genetic Algorithm [1, 2] or the OptQuest engine [3]. The OptQuest Engine incorporates metaheuristics to guide its search algorithm toward better solutions; OptQuest remembers which solutions worked well and recombines them into new solutions. It combines Tabu search, scatter search, integer programming, and neural networks into a single, composite search algorithm. Preliminary analysis showed that the genetic algorithm led to very poor results, so we let *Evolver* pick OptQuest. We let *Evolver* evaluate a maximum of 1,000 trials; with every trial, the current best is compared to the best from 200 trials previous. If the improvement in $\boldsymbol{C}$ is less than 0.001, it stops.

Running the entire modeling algorithm for 1,000 simulations took less than 3 hours running on virtualized Windows 7 with 8 GB RAM and 4 i7 CPU cores. We are certain this could be halved by further tuning of the *Evolver* parameters. In Figure 4, we see the empirical distribution of the time required for optimization. In 90% of the simulations, *Evolver* required between 6 and 9.5 seconds. However, note the very long right skew, with optimization taking up to nearly 13 seconds.

The time required for the optimization routine to converge is related to the proportion of trials in which all constraints could not be met so the optimization could not converge. In fact, in 36 of 1,000 simulations in our base scenario, *Evolver* was unable to find any trials in which all constraints were met. The empirical distribution of the proportion of valid (all constraints met) trials is shown in Figure 4. During an average run of the optimizer, only 38% of trials were valid, which means more than 2/3 of the time was wasted. The upper end of the 90% confidence interval was only 55%, and the maximum was still less than 80%. This attests to the complexity of meeting all the constraints in (3).

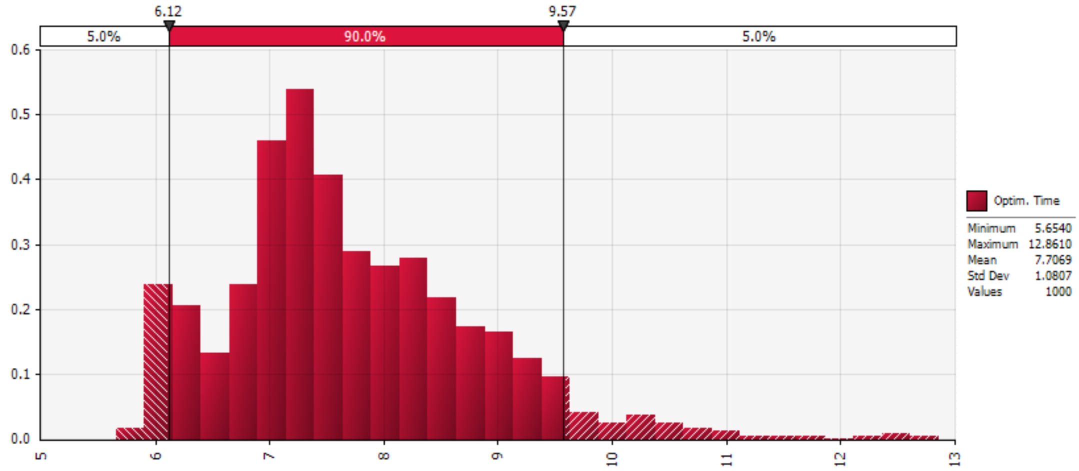

Figure 3 Empirical Distribution of Optimization Time

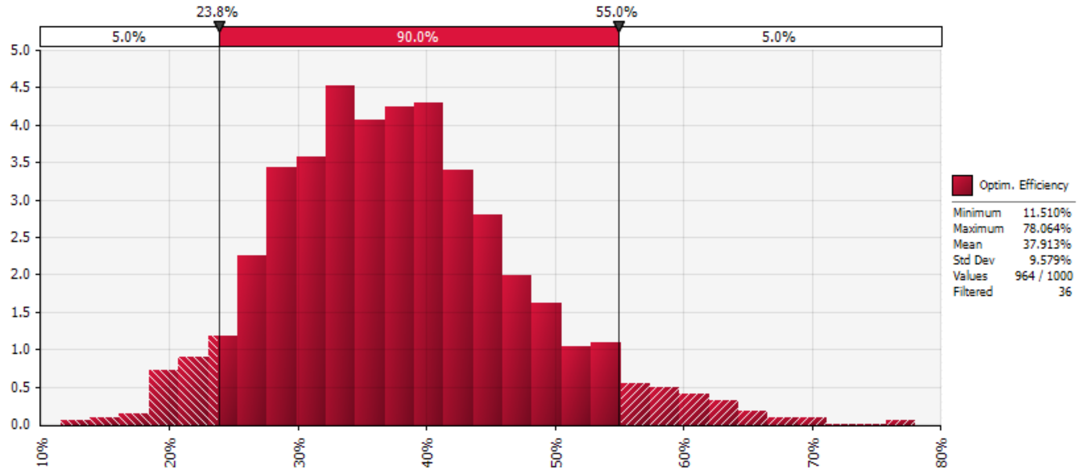

Figure 4 Empirical Distributions of Trials Efficiency

4. Numerical Results

4.1. UK National Grid Data

The data used in this case study are shown in Table 4 through Table 6. Please note again that plant running costs and availability data is for illustrative purposes only, and does not represent information for any particular generating plant. In Table 4, we show the demand in each region, based on a peak total system demand of 64,000MW. We also show the average generating capacity and cost per region, computed from the Merit Orders in Table 5. In said Merit Orders, the right-most column shows the expected value of simulated capacities. Finally, Table 6 shows the constraints controlling how much power is allowed to flow across each regional boundary. Note that the two pairs of disconnected regions are constrained to 0MW.

Table 4 Peak Power Demand, for Total System Demand of 64,000MW and Average Generating Capacity and Cost per Region

Region	Demand (MW)	Avg. Capacity (MW)	Avg. Cost (£/MW)
North Ireland	2,193 (3.0%)	3,054	94.00
Scotland	7,360 (8.0%)	11,616	29.00
England & Wales	50,801 (83.0%)	55,562	44.00
Republic of Ireland	4,547 (6.0%)	7,335	67.00

Table 5 Full Merit Order for all Regions with Average Generation Capacities

Region	Type	Cost	#	Name-	Avg.	Capacity
North Ireland	Offshore wind	−0.5	NA	0.300	NA	91
North Ireland	Onshore wind	−0.3	NA	0.978	NA	260
North Ireland	Wave & Tidal	−0.7	2	30	20.0%	0
North Ireland	Hydro	−0.1	NA	4	60.0%	2
North Ireland	Base Gas	40.0	2	500	80%	1,000
North Ireland	Biomass	55.0	2	100	85%	200
North Ireland	Marginal Gas	70.0	1	500	80%	500
North Ireland	Oil	200.0	2	500	80%	1,000
Scotland	Offshore wind	−0.5	NA	3.185	NA	970
Scotland	Onshore wind	−0.3	NA	9.143	NA	3,048
Scotland	Wave & Tidal	−0.7	19	30	20%	120
Scotland	Hydro	−0.1	NA	1132	60%	679
Scotland	Nuclear	0.0	4	600	70%	1,800
Scotland	Base Gas	40.0	2	500	80%	1,000
Scotland	Base Coal	50.0	3	500	80%	1,000
Scotland	Biomass	55.0	1	100	85%	100
Scotland	Marginal Gas	70.0	2	500	80%	1,000
Scotland	Marginal Coal	70.0	3	500	80%	1,000
Scotland	Pumped Storage	120.0	3	300	90%	900
England & Wales	Offshore wind	−0.5	NA	13.375	NA	4,071
England & Wales	Onshore wind	−0.3	NA	0.659	NA	220
England & Wales	Wave & Tidal	−0.7	3	30	20%	30
England & Wales	Nuclear	0.0	17	600	70%	7,200
England & Wales	France	0.0	4	500	50%	1,000
England & Wales	Imera/Britned	0.0	9	500	50%	2,500
England & Wales	Base Gas	40.0	43	500	80%	17,000
England & Wales	Base Coal	50.0	15	500	80%	6,000
England & Wales	Biomass	55.0	13	100	85%	1,100
England & Wales	Marginal Gas	70.0	23	500	80%	9,000
England & Wales	Marginal Coal	70.0	13	500	80%	5,000
England & Wales	Pumped Storage	120.0	7	300	90%	1,800
England & Wales	Diesel (OCGT)	300.0	NA	677	95%	641
Rep. of Ireland	Offshore wind	−0.5	NA	0.325	NA	99
Rep. of Ireland	Onshore wind	−0.3	NA	3.493	NA	1,164
Rep. of Ireland	Wave & Tidal	−0.7	3	30	20%	30
Rep. of Ireland	Hydro	−0.1	NA	237	60%	142
Rep. of Ireland	Base Gas	40.0	4	500	80%	1,500
Rep. of Ireland	Base Coal	50.0	1	500	80%	500
Rep. of Ireland	Biomass	55.0	7	100	85%	600

Region	Type	Cost	#	Name-	Avg.	Capacity
Rep. of Ireland	Marginal Gas	70.0	4	500	80%	1,500
Rep. of Ireland	Marginal Coal	70.0	1	500	80%	500
Rep. of Ireland	Pumped Storage	120.0	1	300	90%	300
Rep. of Ireland	Oil	200.0	2	500	80%	1,000

Table 6 Power Flow Constraints Between all Regions

Regions	Constraint (MW)
North Ireland ↔ Scotland	500
North Ireland ↔ England & Wales	0
North Ireland ↔ Republic of Ireland	200
Scotland ↔ England & Wales	3,300
Scotland ↔ Republic of Ireland	0
England & Wales ↔ Republic of Ireland	500

Based on these tables, we first performed a base scenario of 1,000 simulations. Excluding the 36 simulations in which *Evolver* was unable to find a solution meeting all constraints, we can learn some basic characteristics of the utility system. These can be used as baselines for evaluating investment decisions. From Figure 5, we see that the mean total system generating capacity is 65,830MW, which is a mere 2.8% higher than peak demand; the 90% confidence interval ranges from 64,141MW to 69,180MW. With an extreme right-skew, even the upper bound here is less than 10% more than peak demand.

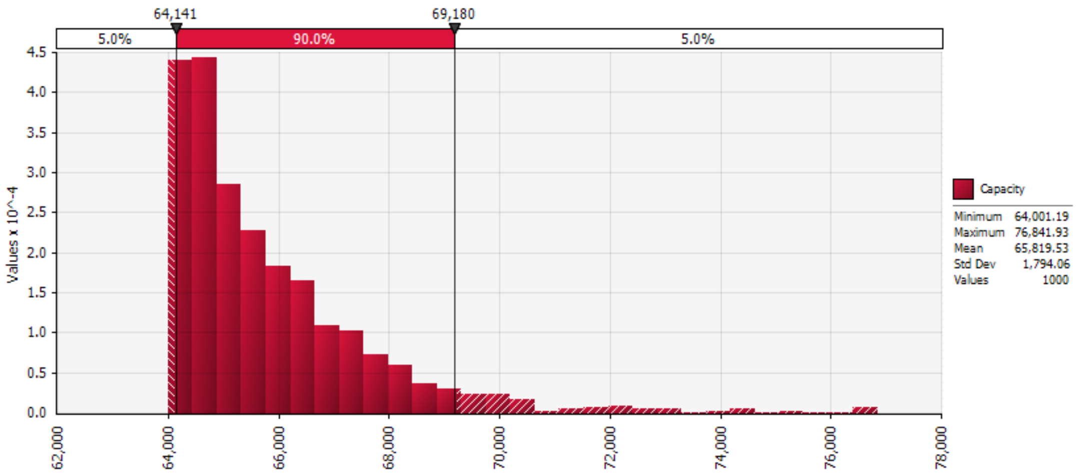

Figure 5 Empirical Distribution of Total System Generating Capacity

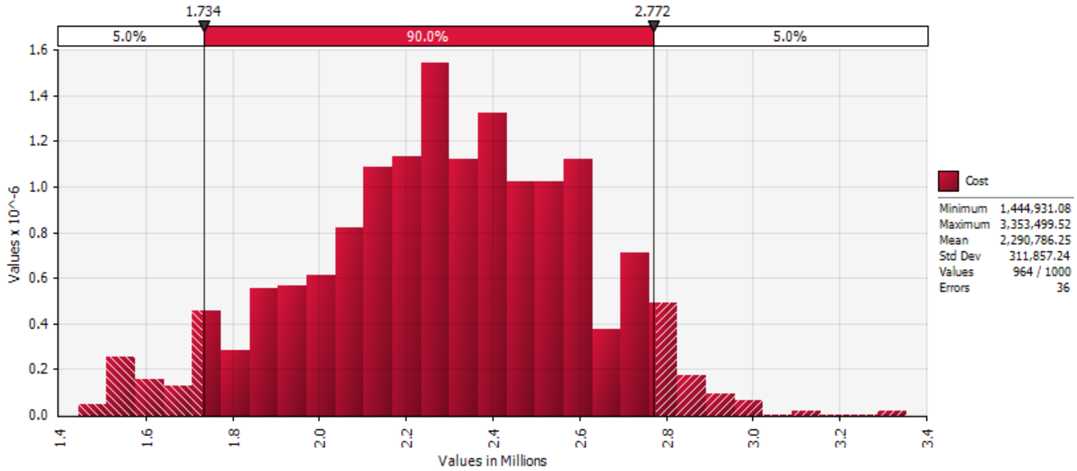

Figure 6 Empirical Distribution of Total System Cost

More directly relevant here, we have the distribution of the total system cost in Figure 6. Mean total system cost at peak demand is £2,290,786. The distribution is marginally skewed toward lower cost, and the 90% confidence interval is £(1,734,205, 2,771,690).

Investigating drivers of this cost, we see the average unit cost per megawatt in each region is:

- *North Ireland*: $C_N^d = £70.82$,
- Scotland: $C_S^d = £8.74$,
- England & Wales: $C_E^d = £70.82$,
- Republic of Ireland: $C_R^d = £37.93$.

Scotland is a clear outlier; C_S^d is so low because it can generate up to 12,300MW of wind power, has 19 tidal generators, and more than 1,100MW of nameplate hydro capacity - all with no fuel costs. The vast majority of UK National Grid's alternate clean energy plants are in this region.

North Ireland is also clearly an outlier, but with a unit cost nearly twice as high as the others. The distribution of C_N^d, in Figure 7, shows a clear bimodal structure, with the lower cluster averaging about like England & Wales or Republic of Ireland. Unlike Scotland, there are few low cost power plants; nearly all of North Ireland's power is generated by burning gas or oil. Furthermore, North Ireland's generating capacity is relatively low; it has an approximately 65% chance of being less than demand. Because of the high cost and this low capacity, North Ireland generates power for other regions infrequently and only in small amounts.

As shown in Figure 8, North Ireland generated on average 1,524MW of it's 2,183MW demand (70%) in the base scenario. Furthermore, the lower bound of the 90% confidence interval was 1,220MW; looking at the Merit Order in Table 5, we see that even in this "best case", North Ireland's base gas generators would have to be run.

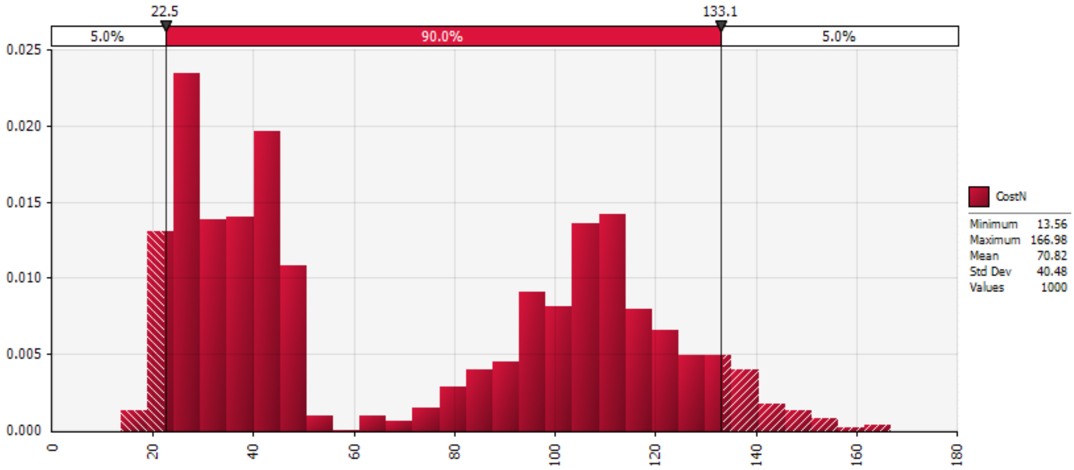

Figure 7 Empirical Distribution of North Ireland Unit Generating Cost

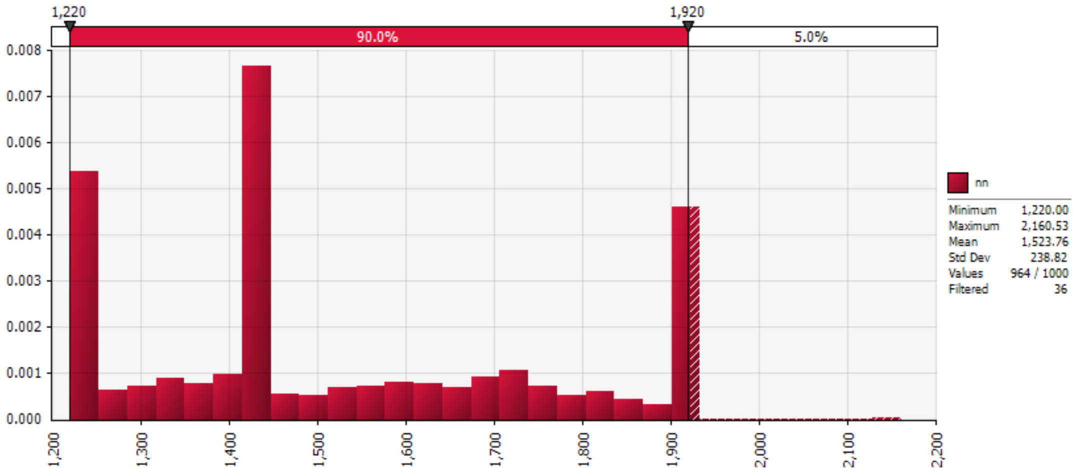

Figure 8 Empirical Distribution for North Ireland Meeting its own Demand - Base Scenario

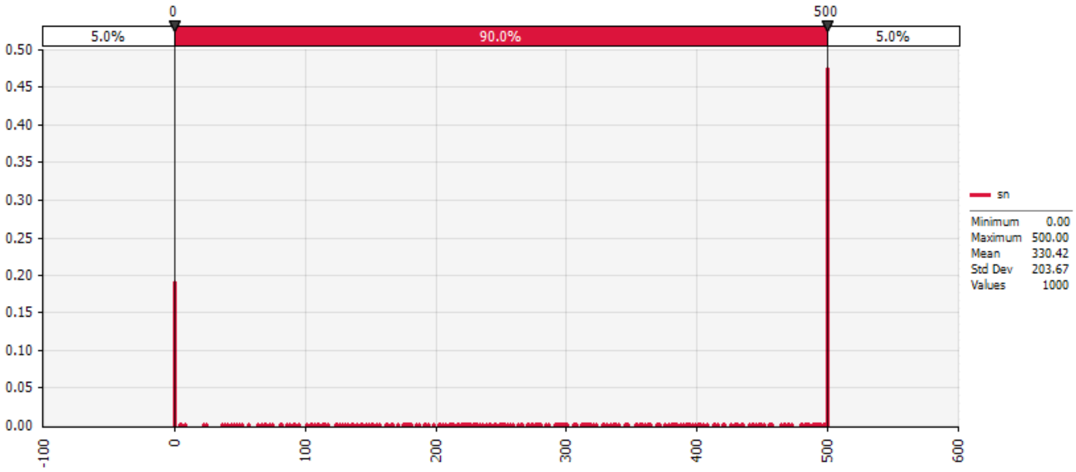

Figure 9 Empirical Distribution for Scotland Meeting North Ireland Demand - Base Scenario

Much of the remainder was generated by Scotland. However, as we can see in Figure 9, the amount of power generated by Scotland for North Ireland was limited by the 500MW constraint in nearly half of the base 1,000 simulations! Since Scotland (lowest generating cost) and North Ireland (highest cost) are connected, we would like to see as much power as possible flow from Scotland into North Ireland. This would allow us to turn off the expensive power plants in North Ireland. This leads us to our first investment analysis.

4.2. Investment - Increase Transmission Between Scotland and North Ireland

Here we test the scenario where investments are made to increase the transmission capacity between Scotland and North Ireland from 500MW to 2,000MW. We want to determine if we lower total system cost **C** by expanding this transmission capacity, and by how much. We ran another simulation with this constraint relaxed. Mirroring Figure 8 and Figure 9 from the base simulation,

Figure 10Figure 10 and **Figure 11** show the empirical distributions of both North Ireland and Scotland generating power to meet the demand in North Ireland.

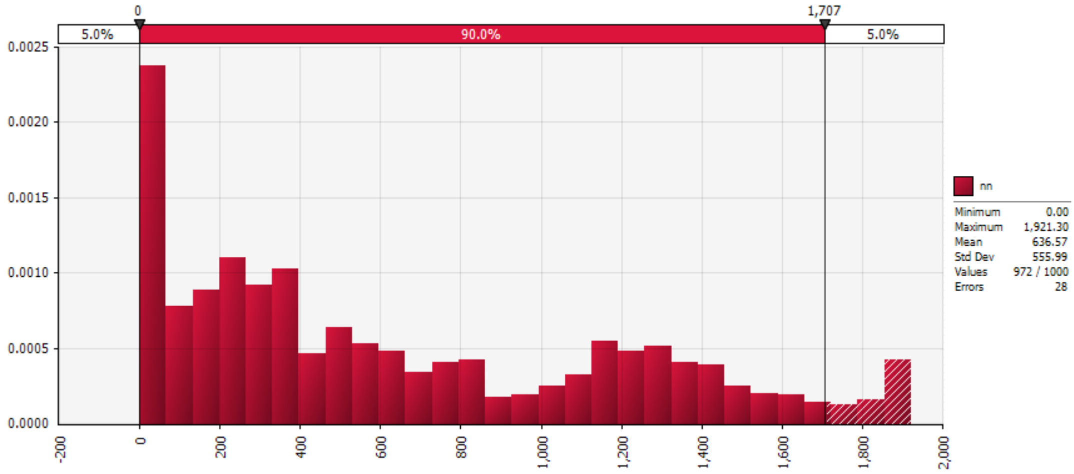

Figure 10 Empirical Distribution for North Ireland Meeting its own Demand - Increased Transmission Scenario

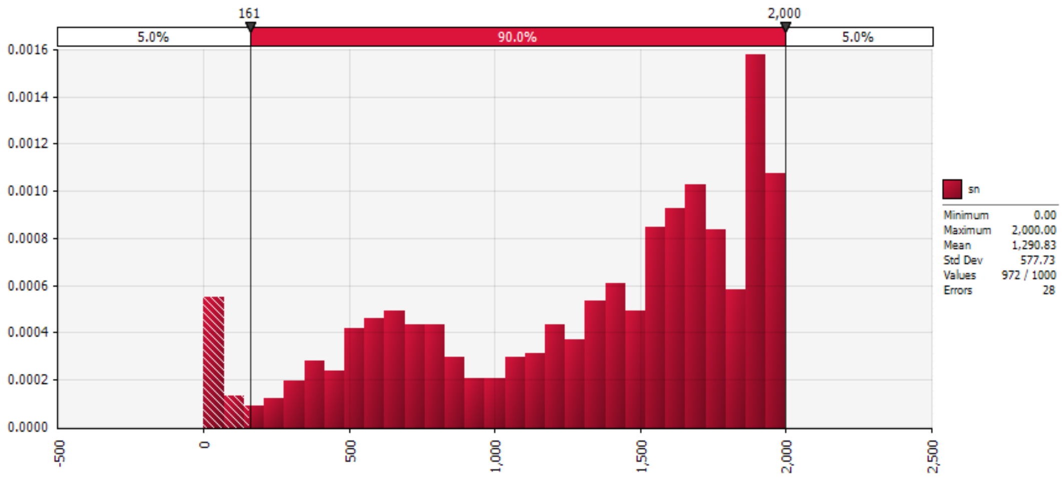

Figure 11 Empirical Distribution for Scotland Meeting North Ireland Demand - Increased Transmission Scenario

Recall that in the base scenario, North Ireland generated power to serve an average 70% of its own peak demand. With more robust transmission capacity to / from Scotland, this dropped dramatically to an average of 30% (637MW). The new 90% confidence interval lower bound is 0MW; the £40/MW generators can be left off. In

Figure 11, we see more directly the impact of the expanded transmission capacity. In nearly 90% of the simulations, more power than the original transmission constraint was allowed to flow from Scotland to North Ireland. Finally, Figure 12 shows the empirical distribution of **C**. With transmission constraints relaxed, the mean cost dropped by 2% to £2,245,495. The 5th percentile decreased by about 4%, and the 95th by 2%. As with the next two examples, the range of expected cost savings £(46,321, 71,011) could be used as inputs for an NPV analysis for expanding transmission capacity.

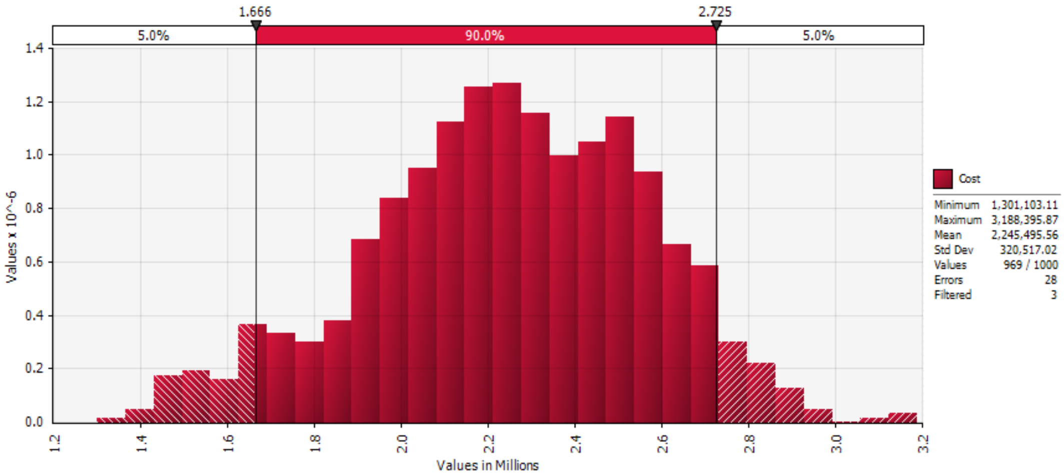

Figure 12 Empirical Distribution of Total System Cost - Increased Transmission Scenario

4.3. Investment - Increase Alternate Energy Capacity

UK National Grid has been proactive in contracting for and installing alternative power plants, including Wind, Tidal, and Nuclear. Offshore wind and Wave & Tidal specifically, still have much room for growth. Here we test the scenario where system-wide generation capacity from Offshore wind and Wave & Tidal plants is increased by approximately 50%. We assume this increase is spread evenly across all regions. The before and after nameplate capacities are shown in Table 7. In the base case, total nameplace capacity is 17,185MW for Offshore wind plants, and 810MW for Wave & Tidal. We evaluate the impact on total system cost of increasing these to 25,779 and 1,200, respectively.

Table 7 Increase of Offshore Wind and Wave & Tidal Generating Capacity

	Offshore Wind (MW)		Wave & Tidal (MW)	
Region	**Bef.**	**Aft.**	**Bef.**	**Aft.**
North Ireland	300	450	2→60	3→90
Scotland	3,185	4,778	19→570	29→870
England & Wales	13,375	20,063	3→90	4→120
Republic of Ireland	325	488	3→90	4→120
Total	17,185	25,779	27→810	40→1,200

In Figure 13, we see that the addition of almost 9,000MW of Offshore Wind and Wave & Tidal power decreased the mean total system cost by 7.7%. Furthermore, the 5th percentile dropped by 20%, and the distribution is skewed more toward lower generating costs. We also note that the variability increased by 25% and the upper end of the 90% confidence interval didn't change much; this change has dramatically increased the uncertainty in the total system cost estimates.

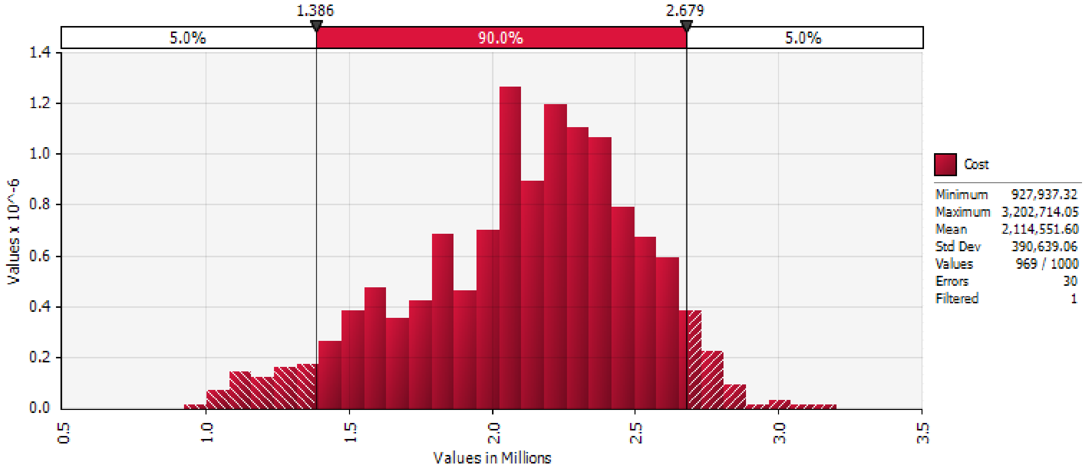

Figure 13 Empirical distribution of Total System Cost - Increased Alternate Energy Capacity Scenario

4.4. Investment - Decrease Peak Demand

Many utilities have been recently investing in programs to decrease demand - especially peak demand. This includes rebates for energy efficient smart appliances, education programs, interruptibility/curtailage agreements, and the Smart Grid. In our final scenario, we evaluate the impact on total system cost of a modest 6.25% decrease in peak demand. The new lower demand is 60,000MW, and we assume that if overall peak demand is reduced, all regions will be affected the same, so the relative proportions remain unchanged. The result of this decreased demand is shown in Figure 14.

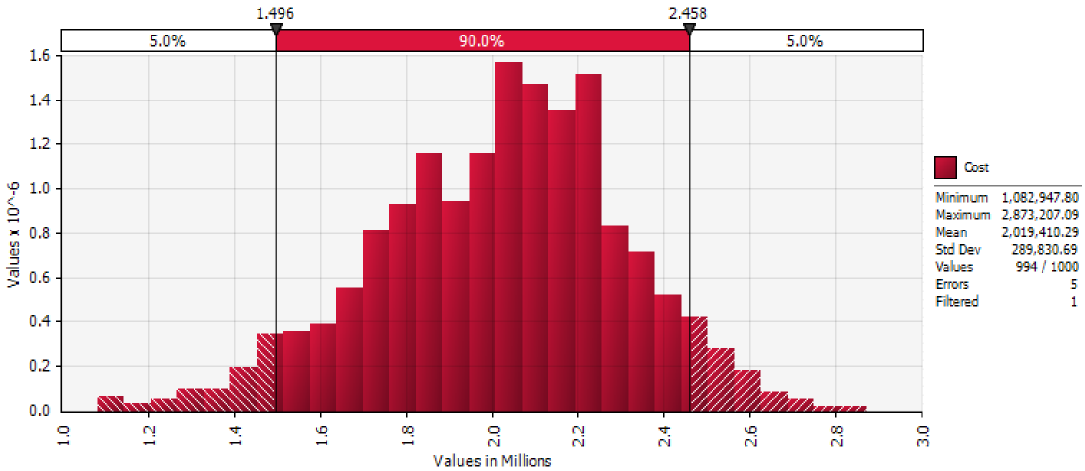

Figure 14 Empirical Distribution of Total System Cost - Decreased Peak Demand Scenario

Decreasing demand by 6.25% dropped the total system cost by 12% to £2,019,410. Unlike the previous scenario, both the 5th and 95th percentiles dropped (14% and 11% respectively). Furthermore, in this scenario, we see that variability also dropped by 7%, making our estimated costs less uncertain. As with the previous scenario, the main result

– that total system cost decreased is not necessarily surprising. What is possibly unexpected is the impact on uncertainty in total sysem cost.

5. Concluding Remarks

In this case study, we have built a stochastic model to perform cost optimization and investment decision modeling for a system of interconnected power grids. We began by mathematically defining an optimization problem that allows us to meet the aggregate demand of all regions under the transmission constraints while minimizing the total cost. This was then implemented under the framework of uncertain generation capacity, so the optimization could take into consideration the fact that capacity is a stochastic quantity.

The resulting Monte Carlo simulation model allows us to make probabilistic statements about regional generating capacity and cost, total system generating capacity and cost, and inter-regional power flows. Finally, we demonstrated how this optimization model can be used to guide and inform investment decisions. With this simulation model, we can evaluate and quantify the impact on total system generating cost from different types of investments. Not only does it give us information about how the system cost is expected to react, but we can quantify the impact on uncertainty and risk. Furthermore, for a given proposed investment, we could easily run the model for varying levels (i.e., programs to reduce demand by 4.25%, 6.25%, and 8.25%), and hence get a set of curves for the impact on total system generating cost. These results could be used as inputs to an investment model which would consider the capital, contractual, regulatory, and other costs associated with the invesment.

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