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New Exponential Stability Criteria for Certain Neutral Integro-Differential Equations

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Abstract

In this work, the exponential stability of zero solution of some neutral integro-differential equations of the first order (NIDE) with discrete and distributed time-varying delays is discussed. A new result that has sufficient conditions on the exponential stability of zero solution is proved by aid a new Lyapunov-Krasovskii functional, the Leibniz-Newton formula and a matrix inequality. The result of this paper extends and improves some former results on the topic in the literature.

Keywords: Neutral differential equation, first order, exponential stability, time-varying delay, the direct method of Lyapunov

Bazı Neutral İntegro- Diferansiyel Denklemler için Yeni Üstel Kararlılık Kriterleri

Özet

Bu çalışmada, birinci basamaktan ayrık ve değişken gecikmeli neutral integro- diferansiyel denklemlerin (NIDE) sıfır çözümünün üstel kararlılığı incelenmektedir. Uygun bir Lyapunov-Krasovski fonksiyeli, Leibniz-Newton formülü ve matris eşitsizliği yardımıyla ele alınan denklemin sıfır çözümünün üstel kararlılığı için yeter şartlar içeren yeni bir sonuç ispatlanmaktadır. Bu çalışma literatürdeki konuyla ilgili önceki sonuçları genişletmekte ve geliştirmektedir.

Anahtar Kelimeler: Neutral diferansiyel denklem, birinci mertebe, üstel kararlılık, değişken gecikme, Lyapunov ‘un doğrudan metodu

1. Introduction

It is known that neutral differential equations are retarded systems that often appear in many various scientific areas such as physics, biology, chemistry, biophysics, mechanics, aerodynamics, economy, atomic energy, control theory, information theory, population dynamics, electrodynamics of complex media and so on. Therefore, qualitative behaviors of solutions NIDEs, stability, boundedness, convergence, instability, integrability, globally existence of solutions, etc., have been extensively investigated in the literature by this time. For a comprehensive treatment of these qualitative properties of solutions of NIDEs and some applications, we refer the readers to the papers or books of Hale et al (1993), Boyd et al (1994), Agarwal and Grace (2000), El-Morshedy and Gopalsamy (2000), Fridman (2001), Fridman (2002), Gu et al (2003), Park (2004), Kwon and Park (2006), Sun and Wang (2006), Kwon and Park (2008), Park and Kwon (2008), Deng et al (2009), Liao et al (2009), Li

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(2009), Nam and Phat (2009), Rojsiraphisal and Niamsup (2010), Chen et al (2011), Chen and Huabin (2012), Tunç and Altun (2012), Tunç (2013), Pinjai and Mukdasai (2013), Li and Fu (2013), Keadnarmol et al (2014), Chatbupapan et al (2016), Gözen and Tunç (2017a and b), Tunç and Mohammed (2017), Gözen and Tunç (2018), Tunç and Tunç (2018), Hristova and Tunç (2019), Slyn'ko and Tunç (2019) and the references can be found in these sources.

In a particular case, we should mention the following related paper on the qualitative behaviors of the solutions of NIDEs .

Chatbupapan et al (2006) considered a scalar NIDE with mixed interval time varying delays:

$$\frac{d}{dt}[x(t)+px(t-\tau(t))]=-ax(t)+b\tanh x(t-\sigma(t))$$
$$+c\int_{t-\rho(t)}^{t}x(s)ds,\ t\geq 0. \qquad (1.1)$$

Chatbupapan et al. (2006) discussed the delay-dependent exponential stability of solutions of NIDE (1.1). Based on a class of Lyapunov-Krasovskii functionals, a model transformation, the decomposition technique of constant coefficients, the Leibniz-Newton formula and linear matrix inequality (LMI), sufficient conditions are established to guarantee the exponentially stability of the zero of NIDE (1.1) by Chatbupapan et al. (2006) In addition, in this paper a few numerical examples are given to illustrate the effectiveness and applicability of the established conditions.

Motivated by the results of Chatbupapan et al. (2006) and the above mentioned works, we consider the following scalar NIDE with variable delays of the form

$$\frac{d}{dt}[x(t)+px(t-\tau(t))]$$
$$=-h(x)+b(t)\tanh x(t-\sigma(t))$$
$$+\int_{t-\rho(t)}^{t}C(t,s)x(s)ds, \qquad (1.2)$$

where $t\in R^{+}$, $R^{+}=[0,\infty)$, $b(t)$ is positive continuous function, $C(t,s)$ is a continuous function for $0\leq s\leq t<\infty$ such that $|C(t,s)|<1$, p is a real constant with $|p|<1$, $\tau(t)$, $\sigma(t)$ and $\rho(t)$ are continuously differentiable functions such that

$$0\leq\tau_1\leq\tau(t)\leq\tau_2,\quad \dot{\tau}(t)\leq\tau_d<\infty,$$

$$0\leq\sigma_1\leq\sigma(t)\leq\sigma_2,\quad \dot{\sigma}(t)\leq\sigma_d<\infty,$$

$$0\leq\rho_1\leq\rho(t)\leq\rho_2,$$

where $\tau_1,\tau_2,\sigma_1,\sigma_2,\rho_1,\rho_2,\tau_d$ and σ_d are given positive real constants.

Let

$$h_1(x)=\begin{cases}\dfrac{h(x)}{x}, & x\neq 0,\\ h'(0), & x=0,\end{cases}$$

such that

$$h'(x)\geq 1,\ x\neq 0.$$

For each solution $x(t)$ of (1.2), we assume the initial condition

$$x_0(t)=\phi(t),\ t\in[-w,0],$$

where $\phi\in C([-w,0],R)$ and $w=\max\{\tau_2,\sigma_2,\rho_2\}$.

In this paper, we prove an exponential stability theorem, which included sufficient conditions, such that the zero solution of NIDE (1.2) is exponential stability. To prove that theorem we benefit from the Lyapunov second method. For this define a new Lyapunov functional such that it is positive definite and its derivative along the solutions of NIDE (1.2) is negative definite. By this work, our aim is extend and improve some related results in the literature.

Definition 1.1 (Kwon and Park (2006). The zero solution of equation (1.2) is exponentially stable, if there exist positive real constants α,β such that, for each $\phi(t)\in C([-w,0],R)$, the solution $x(t,\phi)$ of equation NIDE (1.2) satisfies

$$\|x(t,\phi)\|\leq\beta\|\phi\|e^{-at},\ t\geq 0.$$

Lemma 1.1 (Gu et al. (2003), (Jensen's inequality)). For any symmetric positive definite matrix Q, positive real number h and vector function $\dot{x}(t):[-h,0]\to R^n$ the following integral is well defined

$$-h\int_{-h}^{0}\dot{x}^T(s+t)Q\dot{x}(s+t)ds$$

$$\leq -\left(\int_{-h}^{0}\dot{x}(s+t)ds\right)^T Q\left(\int_{-h}^{0}\dot{x}(s+t)ds\right).$$

Lemma 1.2 (Gu et al. (2003)). For any constant symmetric positive definite matrix $Q\in R^{n\times n}$, $h(t)$ a discrete time-varying delay with $0<h_1\leq h(t)\leq h_2$, the vector function $w:[-h_2,0]\to R^n$ such that the integrations concerned are well defined, we have

$$-[h_2-h_1]\int_{-h_2}^{-h_1}w^T(s)Qw(s)ds$$

$$\leq -\int_{-h(t)}^{-h_1}w^T(s)dsQ\int_{-h(t)}^{-h_1}w(s)ds$$

$$-\int_{-h_2}^{-h(t)}w^T(s)dsQ\int_{-h_2}^{-h(t)}w(s)ds.$$

where $h_1,h_2\in R.$

We now investigate the exponential stability of zero solution of NIDE (1.2). It is clear that

$$0=x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds \qquad (1.3)$$

and

$$0=x(t)-x(t-\gamma\tau(t))-\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds, \qquad (1.4)$$

where γ is a given positive real constant. Then, we have

$$0=r_1x(t)-r_1x(t-\tau(t))-r_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds \qquad (1.5)$$

and

$$0=r_2x(t)-r_2x(t-\gamma\tau(t))-r_2\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds, \qquad (1.6)$$

where $r_1,\ r_2\in R$ will be chosen later. By equalities (1.3)-(1.6), NIDE (1.2) can be written as the following form

$$\frac{d}{dt}[p_1x(t)+p_2x(t-\tau(t))+x(t-\gamma\tau(t))$$
$$+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds]$$
$$=-(a_1-r_1-r_2)h(x(t))-(a_2+r_1)h(x(t-\tau(t))$$
$$-(a_2+r_1)\int_{t-\tau(t)}^{t}(h(x(s)))'ds-r_2h(x(t-\gamma\tau(t)))$$
$$-r_2\int_{t-\gamma\tau(t)}^{t}(h(x(s)))'ds+b(t)\tanh x(t-\sigma(t))$$
$$+\int_{t-p(t)}^{t}C(t,s)x(s)ds. \qquad (1.7)$$

Let

$$D(t)=p_1x(t)+p_2x(t-\tau(t))+x(t-\gamma\tau(t))$$
$$+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds].$$

Hence, it follows from (1.7) that

$$\dot{D}(t)=-(a_1-r_1-r_2)h(x(t))-(a_2+r_1)h(x(t-\tau(t))$$
$$-(a_2+r_1)\int_{t-\tau(t)}^{t}(h(x(s)))'ds]$$
$$-r_2h(x(t-\gamma\tau(t)))$$
$$-r_2\int_{t-\gamma\tau(t)}^{t}(h(x(s)))'ds+b(t)\tanh x(t-\sigma(t))$$
$$+\int_{t-p(t)}^{t}C(t,s)x(s)ds. \qquad (1.8)$$

2. Material and Method

We first present some notations, which are needed later.

Let

$$\Sigma=[\Omega_{(i,j)}]_{25\times 25,}\ (i,\ j=1,2,...,25), \qquad (2.1)$$

where Ω is a symmetric matrix, that is,

$$\Omega_{(i,j)}=\Omega_{(j,i)},$$

and

$\Omega_{(1,1)} = 2k_1\alpha - 2q_1,$
$\Omega_{(1,2)} = q_1 p_1 - q_2 + k_1(r_1 + r_2 - a_1)h(x(t)),$
$\Omega_{(1,3)} = q_1 p_2 - q_3 - k_1(r_1 + a_2)h(x(t-\tau(t))),$
$\Omega_{(1,4)} = -q_1 p_1 - q_4 - k_1(r_1 + a_2),$
$\Omega_{(1,5)} = q_1 - q_5 - k_1 r_2 h_1(x(t-\gamma\tau(t))),$
$\Omega_{(1,6)} = q_1 - q_6 - k_1 r_2,$
$\Omega_{(1,7)} = \Omega_{(1,8)} = \Omega_{(1,9)} = \Omega_{(1,10)} = 0,$
$\Omega_{(1,11)} = k_1 b(t),\ \Omega_{(1,12)} = \Omega_{(1,13)} = \Omega_{(1,14)} = 0,$
$\Omega_{(1,15)} = \Omega_{(1,16)} = \Omega_{(1,17)} = \Omega_{(1,18)} = 0,$
$\Omega_{(1,19)} = \Omega_{(1,20)} = \Omega_{(1,21)} = 0,$
$\Omega_{(1,22)} = \Omega_{(1,23)} = \Omega_{(1,24)} = 0,\ \Omega_{(1,25)} = k_1,$
$\Omega_{(2,2)} = 2q_2 p_1 + k_2 + k_3 + k_4 + k_5 + k_6\tau_2^2,$
$+ k_7\gamma^2\tau_2^2 + k_8 + k_9\sigma_2^2$
$+ w_5\tau_1^2 + w_6\gamma^2\tau_1^2 + w_7(\tau_2 - \tau_1)^2$
$+ w_8\gamma^2(\tau_2 - \tau_1)^2$
$+ w_{10}(\sigma_2 - \sigma_1)^2 + k_{10} + wp_2^2 + w_9\sigma_1^2,$
$\Omega_{(2,3)} = q_2 p_2 + q_3 p_1,\ \Omega_{(2,4)} = -q_2 p_1 + q_4 p_1,$
$\Omega_{(2,5)} = q_2 + q_5 p_1,\ \Omega_{(2,6)} = q_2 + q_6 p_1,$
$\Omega_{(2,7)} = \Omega_{(2,8)} = \Omega_{(2,9)} = 0,$
$\Omega_{(2,10)} = \Omega_{(2,11)} = \Omega_{(2,12)} = 0,\ \Omega_{(2,13)} = -q_7 a_1 h_1(x(t)),$
$\Omega_{(2,14)} = \Omega_{(2,15)} = \Omega_{(2,16)} = 0,$
$\Omega_{(2,17)} = \Omega_{(2,18)} = \Omega_{(2,19)} = 0,$
$\Omega_{(2,20)} = \Omega_{(2,21)} = \Omega_{(2,22)} = 0,$
$\Omega_{(2,23)} = \Omega_{(2,24)} = \Omega_{(2,25)} = 0,$
$\Omega_{(3,3)} = 2q_3 p_2 - k_3 e^{-2\alpha\tau_2} + k_3\tau_d,$
$\Omega_{(3,4)} = -q_3 p_1 + q_4 p_2,\ \Omega_{(3,5)} = q_3 + q_5 p_2,$
$\Omega_{(3,6)} = q_3 + q_6 p_2,\ \Omega_{(3,7)} = \Omega_{(3,8)} = \Omega_{(3,9)} = 0,$
$\Omega_{(3,10)} = \Omega_{(3,11)} = \Omega_{(3,12)} = 0,$
$\Omega_{(3,13)} = -q_7 a_2 h_1(x(t-\tau(t))),$
$\Omega_{(3,14)} = \Omega_{(3,15)} = \Omega_{(3,16)} = 0,$
$\Omega_{(3,17)} = \Omega_{(3,18)} = \Omega_{(3,19)} = 0,$
$\Omega_{(3,20)} = \Omega_{(3,21)} = \Omega_{(3,22)} = 0,$
$\Omega_{(3,23)} = \Omega_{(3,24)} = \Omega_{(3,25)} = 0,\ \Omega_{(4,4)} = -2q_4 p_1,$
$\Omega_{(4,5)} = q_4 - q_5 p_1,\ \Omega_{(4,6)} = q_4 - q_6 p_1,$
$\Omega_{(4,7)} = \Omega_{(4,8)} = \Omega_{(4,9)} = 0,$
$\Omega_{(4,10)} = \Omega_{(4,11)} = \Omega_{(4,12)} = 0,\ \Omega_{(4,13)} = -q_7 a_2,$
$\Omega_{(4,14)} = \Omega_{(4,15)} = \Omega_{(4,16)} = 0,$
$\Omega_{(4,17)} = \Omega_{(4,18)} = \Omega_{(4,19)} = 0,$
$\Omega_{(4,20)} = \Omega_{(4,21)} = \Omega_{(4,22)} = 0,$
$\Omega_{(4,23)} = \Omega_{(4,24)} = \Omega_{(4,25)} = 0,$
$\Omega_{(5,5)} = 2q_5 - k_5 e^{-2\alpha\gamma\tau_2} + k_5\gamma\tau_d,\ \Omega_{(5,6)} = q_5 + q_6,$
$\Omega_{(5,7)} = \Omega_{(5,8)} = \Omega_{(5,9)} = 0,$
$\Omega_{(5,10)} = \Omega_{(5,11)} = \Omega_{(5,12)} = 0,$
$\Omega_{(5,13)} = \Omega_{(5,14)} = \Omega_{(5,15)} = \Omega_{(5,16)} = 0,$
$\Omega_{(5,17)} = \Omega_{(5,18)} = \Omega_{(5,19)} = 0,$
$\Omega_{(5,20)} = \Omega_{(5,21)} = \Omega_{(5,22)} = 0,$
$\Omega_{(5,23)} = \Omega_{(5,24)} = \Omega_{(5,25)} = 0,\ \Omega_{(6,6)} = 2q_6,$
$\Omega_{(6,7)} = \Omega_{(6,8)} = \Omega_{(6,9)} = 0,$
$\Omega_{(6,10)} = \Omega_{(6,11)} = \Omega_{(6,12)} = 0,$
$\Omega_{(6,13)} = \Omega_{(6,14)} = \Omega_{(6,15)} = \Omega_{(6,16)} = 0,$
$\Omega_{(6,17)} = \Omega_{(6,18)} = \Omega_{(6,19)} = 0,$
$\Omega_{(6,20)} = \Omega_{(6,21)} = \Omega_{(6,22)} = 0,$
$\Omega_{(6,23)} = \Omega_{(6,24)} = \Omega_{(6,25)} = 0,$
$\Omega_{(7,7)} = -(k_2 + w_3)e^{-2\alpha\tau_2},\ \Omega_{(7,8)} = \Omega_{(7,9)} = 0,$
$\Omega_{(7,10)} = \Omega_{(7,11)} = \Omega_{(7,12)} = 0,$
$\Omega_{(7,13)} = \Omega_{(7,14)} = \Omega_{(7,15)} = \Omega_{(7,16)} = 0,$
$\Omega_{(7,17)} = \Omega_{(7,18)} = \Omega_{(7,19)} = 0,$
$\Omega_{(7,20)} = \Omega_{(7,21)} = \Omega_{(7,22)} = 0,$
$\Omega_{(7,23)} = \Omega_{(7,24)} = \Omega_{(7,25)} = 0,\ \Omega_{(8,8)} = -k_6 e^{-2\alpha\tau_2},$
$\Omega_{(8,9)} = \Omega_{(8,10)} = \Omega_{(8,11)} = 0,$
$\Omega_{(8,12)} = \Omega_{(8,13)} = \Omega_{(8,14)} = 0,$
$\Omega_{(8,15)} = \Omega_{(8,16)} = \Omega_{(8,17)} = 0,$
$\Omega_{(8,18)} = \Omega_{(8,19)} = \Omega_{(8,20)} = 0,$
$\Omega_{(8,21)} = \Omega_{(8,22)} = \Omega_{(8,23)} = \Omega_{(8,24)} = \Omega_{(8,25)} = 0,$
$\Omega_{(9,9)} = -(k_4 + w_4)e^{-2\alpha\gamma\tau_2},$
$\Omega_{(9,10)} = \Omega_{(9,11)} = \Omega_{(9,12)} = 0,$
$\Omega_{(9,13)} = \Omega_{(9,14)} = \Omega_{(9,15)} = 0,$
$\Omega_{(9,16)} = \Omega_{(9,17)} = \Omega_{(9,18)} = 0,$
$\Omega_{(9,19)} = \Omega_{(9,20)} = \Omega_{(9,21)} = 0,$
$\Omega_{(9,22)} = \Omega_{(9,23)} = \Omega_{(9,24)} = \Omega_{(9,25)} = 0,$
$\Omega_{(10,10)} = -k_7 e^{-2\alpha\gamma\tau_2},$
$\Omega_{(10,11)} = \Omega_{(10,12)} = \Omega_{(10,13)} = 0,$

$\Omega_{(10,14)}=\Omega_{(10,15)}=\Omega_{(10,16)}=0,$
$\Omega_{(10,17)}=\Omega_{(10,18)}=\Omega_{(10,19)}=0,$
$\Omega_{(10,20)}=\Omega_{(10,21)}=\Omega_{(10,22)}=0,$
$\Omega_{(10,23)}=\Omega_{(10,24)}=\Omega_{(10,25)}=0,$
$\Omega_{(11,11)}=-k_8e^{-2\alpha\sigma_2}+k_8\sigma_d-k_{10},\ \Omega_{(11,12)}=0,$
$\Omega_{(11,13)}=q_7b(t),\ \Omega_{(11,14)}=\Omega_{(11,15)}=\Omega_{(11,16)}=0,$
$\Omega_{(11,17)}=\Omega_{(11,18)}=\Omega_{(11,19)}=0,$
$\Omega_{(11,20)}=\Omega_{(11,21)}=\Omega_{(11,22)}=0,$
$\Omega_{(11,23)}=\Omega_{(11,24)}=\Omega_{(11,25)}=0,$
$\Omega_{(12,12)}=-k_9e^{-2\alpha\sigma_2},$
$\Omega_{(12,13)}=\Omega_{(12,14)}=\Omega_{(12,15)}=\Omega_{(12,16)}=0,$
$\Omega_{(12,17)}=\Omega_{(12,18)}=\Omega_{(12,19)}=0,$
$\Omega_{(12,20)}=\Omega_{(12,21)}=\Omega_{(12,22)}=\Omega_{(12,23)}=0,$
$\Omega_{(12,24)}=\Omega_{(12,25)}=0,\ \Omega_{(13,13)}=-2q_7,$
$\Omega_{(13,14)}=\Omega_{(13,15)}=\Omega_{(13,16)}=0,$
$\Omega_{(13,17)}=\Omega_{(13,18)}=\Omega_{(13,19)}=0,$
$\Omega_{(13,20)}=\Omega_{(13,21)}=\Omega_{(13,22)}=0,$
$\Omega_{(13,23)}=\Omega_{(13,24)}=0,\ \Omega_{(13,25)}=q_7,$
$\Omega_{(14,14)}=(w_3-w_1)e^{-2\alpha\tau_1},\ \Omega_{(14,15)}=\Omega_{(14,16)}=0,$
$\Omega_{(14,17)}=\Omega_{(14,18)}=\Omega_{(14,19)}=\Omega_{(14,20)}=\Omega_{(14,21)}=0,$
$\Omega_{(14,22)}=\Omega_{(14,23)}=\Omega_{(14,24)}=\Omega_{(14,25)}=0,$
$\Omega_{(15,15)}=(w_4-w_2)e^{-2\alpha\gamma\tau_1},\ \Omega_{(15,16)}=0,$
$\Omega_{(15,17)}=\Omega_{(15,18)}=\Omega_{(15,19)}=\Omega_{(15,20)}=0,$
$\Omega_{(15,21)}=\Omega_{(15,22)}=\Omega_{(15,23)}=\Omega_{(15,24)}=\Omega_{(15,25)}=0,$
$\Omega_{(16,16)}=-w_5e^{-2\alpha\tau_1},$
$\Omega_{(16,17)}=\Omega_{(16,18)}=\Omega_{(16,19)}=\Omega_{(16,20)}=\Omega_{(16,21)}=0,$
$\Omega_{(16,22)}=\Omega_{(16,23)}=\Omega_{(16,24)}=\Omega_{(16,25)}=0,$
$\Omega_{(17,17)}=-w_6e^{-2\alpha\gamma\tau_1},$
$\Omega_{(17,18)}=\Omega_{(17,19)}=\Omega_{(17,20)}=\Omega_{(17,21)}=0,$
$\Omega_{(17,22)}=\Omega_{(17,23)}=\Omega_{(17,24)}=\Omega_{(17,25)}=0,$
$\Omega_{(18,18)}=-w_7e^{-2\alpha\tau_2},$
$\Omega_{(18,19)}=\Omega_{(18,20)}=\Omega_{(18,21)}=\Omega_{(18,22)}=0,$
$\Omega_{(18,23)}=\Omega_{(18,24)}=\Omega_{(18,25)}=0,$
$\Omega_{(19,19)}=-w_7e^{-2\alpha\tau_2},$
$\Omega_{(19,20)}=\Omega_{(19,21)}=\Omega_{(19,22)}=0,$
$\Omega_{(19,23)}=\Omega_{(19,24)}=\Omega_{(19,25)}=0,$
$\Omega_{(20,20)}=-w_8e^{-2\alpha\gamma\tau_2},\ \Omega_{(20,21)}=\Omega_{(20,22)}=0,$
$\Omega_{(20,23)}=\Omega_{(20,24)}=\Omega_{(20,25)}=0,$
$\Omega_{(21,21)}=-w_8e^{-2\alpha\gamma\tau_2},\ \Omega_{(21,22)}=0,$
$\Omega_{(21,23)}=\Omega_{(21,24)}=\Omega_{(21,25)}=0,$
$\Omega_{(22,22)}=-w_9e^{-2\alpha\sigma_1},$
$\Omega_{(22,23)}=\Omega_{(22,24)}=\Omega_{(22,25)}=0,$
$\Omega_{(23,23)}=-w_{10}e^{-2\alpha\sigma_2},\ \Omega_{(23,24)}=\Omega_{(23,25)}=0,$
$\Omega_{(24,24)}=-w_{10}e^{-2\alpha\sigma_2},\ \Omega_{(24,25)}=0,$
$\Omega_{(25,25)}=-we^{-2\alpha p_2}$, $\sigma_1,\sigma_2,\sigma_d,\tau_1,\tau_2,\tau_d,p_1,p_2,\alpha$ and γ are positive some real constants.

We now state the exponentially stability result.

Theorem. If there exist positive real constants $w,k_i,w_i,\ (i=1,2,...,10)$, such that the following matrix inequality

$$\Sigma=[\Omega_{(i,j)}]_{25\times 25}<0 \tag{2.2}$$

holds, then zero solution of NIDE (1.2) is exponentially stable with a decay rate α.

Proof. We define Lyapunov-Krasovskii functional candidate for NIDE (1.8) of the form

$$V(t,x_t)=\sum_{i=1}^{5}V_i(t,x_t),$$

where

$$V_1(t,x_t)=k_1D^2(t),$$

$$V_2(t,x_t)=k_2\int_{t-\tau_2}^{t}e^{2\alpha(s-t)}x^2(s)ds+k_3\int_{t-\tau(t)}^{t}e^{2\alpha(s-t)}x^2(s)ds$$
$$+k_4\int_{t-\gamma\tau_2}^{t}e^{2\alpha(s-t)}x^2(s)ds$$
$$+k_5\int_{t-\gamma\tau(t)}^{t}e^{2\alpha(s-t)}x^2(s)ds$$
$$+w_1\int_{t-\tau_1}^{t}e^{2\alpha(s-t)}x^2(s)ds$$

$$+w_2\int_{t-\gamma\tau_1}^{t}e^{2\alpha(s-t)}x^2(s)ds$$

$$+w_3\int_{t-\tau_2}^{t-\tau_1}e^{2\alpha(s-t)}x^2(s)ds$$

$$+w_4\int_{t-\gamma\tau_2}^{t-\gamma\tau_1}e^{2\alpha(s-t)}x^2(s)ds,$$

$$V_3(t,x_t)=k_6\tau_2\int_{-\tau_2}^{0}\int_{t+s}^{t}e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds$$

$$+k_7\gamma\tau_2\int_{-\gamma\tau_2}^{0}\int_{t+s}^{t}e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds$$

$$+w_5\tau_1\int_{-\tau_1}^{0}\int_{t+s}^{t}e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds$$

$$+w_6\gamma\tau_1\int_{-\gamma\tau_1}^{0}\int_{t+s}^{t}e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds$$

$$+w_7(\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+s}^{t}e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds$$

$$+w_8\gamma(\tau_2-\tau_1)\int_{-\gamma\tau_2}^{-\gamma\tau_1}\int_{t+s}^{t}e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds,$$

$$V_4(t,x_t)=k_8\int_{t-\sigma(t)}^{t}e^{2\alpha(s-t)}\tanh^2 x(s)ds$$

$$+k_9\sigma_2\int_{-\sigma_2}^{0}\int_{t+s}^{t}e^{2\alpha(\theta-t)}\tanh^2 x(\theta)d\theta ds$$

$$+w_9\sigma_1\int_{-\sigma_1}^{0}\int_{t+s}^{t}e^{2\alpha(\theta-t)}\tanh^2 x(\theta)d\theta ds$$

$$+w_{10}(\sigma_2-\sigma_1)$$

$$\times\int_{-\sigma_2}^{-\sigma_1}\int_{t+s}^{t}e^{2\alpha(\theta-t)}\tanh^2 x(\theta)d\theta ds,$$

$$V_5(t,x_t)=wp_2\int_{-p_2}^{0}\int_{t+s}^{t}e^{2\alpha(\theta-t)}x^2(\theta)d\theta ds.$$

Calculating the time derivatives of $V(t)=V(t,x_t)$ along solutions of NIDE (1.8), we obtain

$$\dot{V}(t,x_t)=\sum_{i=1}^{5}\dot{V}_i(t,x_t). \tag{2.3}$$

It can be easily shown from $V_1(t,x_t)$ and NIDE (1.2) that

$$\dot{V}_1(t,x_t)=2k_1D(t)\dot{D}(t)$$

$$=2k_1D(t)[-(a_1-r_1-r_2)h(x(t))$$

$$-(a_2+r_1)h(x(t-\tau(t))$$

$$-(a_2+r_1)\int_{t-\tau(t)}^{t}(h(x(s)))'ds$$

$$-r_2h(x(t-\gamma\tau(t)))$$

$$-r_2\int_{t-\gamma\tau(t)}^{t}(h(x(s)))'ds$$

$$+b(t)\tanh x(t-\sigma(t))$$

$$+\int_{t-p(t)}^{t}C(t,s)x(s)ds]$$

$$+2q_1D(t)[-D(t)+p_1x(t)$$

$$+p_2x(t-\tau(t))+x(t-\gamma\tau(t))$$

$$+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds]$$

$$+2q_2x(t)[-D(t)+p_1x(t)$$

$$+p_2x(t-\tau(t))+x(t-\gamma\tau(t))$$

$$+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds]$$

$$+2q_3x(t-\tau(t))[-D(t)+p_1x(t)$$

$$+ p_2 x(t-\tau(t)) + x(t-\gamma\tau(t))$$

$$+ \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_4 \int_{t-\tau(t)}^{t} \dot{x}(s)ds[-D(t) + p_1 x(t)$$

$$+ p_2 x(t-\tau(t)) + x(t-\gamma\tau(t))$$

$$+ \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_5 x(t-\gamma\tau(t))[-D(t) + p_1 x(t) + p_2 x(t-\tau(t))$$

$$+ x(t-\gamma\tau(t)) + \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_6 \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds[-D(t) + p_1 x(t) + p_2 x(t-\tau(t))$$

$$+ x(t-\gamma\tau(t)) + \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_7 \dot{D}(t)[-\dot{D}(t) - a_1 h(x(t)) - a_2 h(x(t-\tau(t))$$

$$- a_2 \int_{t-\tau(t)}^{t} (h(x(s)))' ds + b(t)\tanh x(t-\sigma(t))$$

$$+ \int_{t-p(t)}^{t} C(t,s)x(s)ds] + 2\alpha k_1 D^2(t) - 2\alpha V_1(t).$$

Hence, in view of the assumptions of the given theorem and the condition $h'(x) \geq 1,\ x \neq 0,$ we can derive that

$$\dot{V}_1(t,x_t) \leq 2k_1 D(t)[-(a_1 - r_1 - r_2)h(x(t))$$

$$-(a_2 + r_1)h_1(x(t-\tau(t))x(t-\tau(t))$$

$$-(a_2 + r_1)\int_{t-\tau(t)}^{t} \dot{x}(s)ds$$

$$- r_2 h_1(x(t-\gamma\tau(t)))x(t-\gamma\tau(t))$$

$$- r_2 \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds$$

$$+ b(t)\tanh x(t-\sigma(t))$$

$$+ \int_{t-p(t)}^{t} C(t,s)x(s)ds]$$

$$+ 2q_1 D(t)[-D(t) + p_1 x(t)$$

$$+ p_2 x(t-\tau(t)) + x(t-\gamma\tau(t))$$

$$+ \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_2 x(t)[-D(t) + p_1 x(t)$$

$$+ p_2 x(t-\tau(t)) + x(t-\gamma\tau(t))$$

$$+ \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_3 x(t-\tau(t))[-D(t) + p_1 x(t) + p_2 x(t-\tau(t))$$

$$+ x(t-\gamma\tau(t)) + \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_4 \int_{t-\tau(t)}^{t} \dot{x}(s)ds[-D(t) + p_1 x(t) + p_2 x(t-\tau(t))$$

$$+ x(t-\gamma\tau(t)) + \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_5 x(t-\gamma\tau(t))[-D(t) + p_1 x(t) + p_2 x(t-\tau(t))$$

$$+ x(t-\gamma\tau(t)) + \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_6 \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds[-D(t) + p_1 x(t) + p_2 x(t-\tau(t))$$

$$+ x(t-\gamma\tau(t)) + \int_{t-\gamma\tau(t)}^{t} \dot{x}(s)ds - p_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds]$$

$$+ 2q_7 \dot{D}(t)[-\dot{D}(t) - a_1 h(x(t)) - a_2 h(x(t-\tau(t))$$

$$- a_2 \int_{t-\tau(t)}^{t} (h(x(s)))' ds + b(t)\tanh x(t-\sigma(t))$$

$$+ \int_{t-p(t)}^{t} C(t,s)x(s)ds] + 2\alpha k_1 D^2(t) - 2\alpha V_1(t)$$

and

$$\dot{V}_2(t,x_t) = (k_2 + k_3 + k_4 + k_5 + w_1 + w_2)x^2(t)$$
$$-(k_2 + w_3)e^{-2\alpha\tau_2} x^2(t-\tau_2)$$

$$-k_3(1-\dot{\tau}(t))e^{-2\alpha\tau(t)}x^2(t-\tau(t))$$
$$-(k_4+w_4)e^{-2\alpha\gamma\tau_2}x^2(t-\gamma\tau_2)$$
$$-k_5(1-\gamma\dot{\tau}(t))e^{-2\alpha\gamma\tau(t)}x^2(t-\gamma\tau(t))$$
$$+(w_3-w_1)e^{-2\alpha\tau_1}x^2(t-\tau_1)$$
$$+(w_4-w_2)e^{-2\alpha\tau_1}x^2(t-\gamma\tau_1)-2\alpha V_2(t)$$
$$\leq(k_2+k_3+k_4+k_5+w_1+w_2)x^2(t)$$
$$-(k_2+w_3)e^{-2\alpha\tau_2}x^2(t-\tau_2)-k_3e^{-2\alpha\tau_2}x^2(t-\tau(t))$$
$$+k_3\tau_d x^2(t-\tau(t))-(k_4+w_4)e^{-2\alpha\gamma\tau_2}x^2(t-\gamma\tau_2)$$
$$-k_5e^{-2\alpha\gamma\tau_2}x^2(t-\gamma\tau(t))+k_5\gamma\tau_d x^2(t-\gamma\tau(t))$$
$$+(w_3-w_1)e^{-2\alpha\tau_1}x^2(t-\tau_1)$$
$$+(w_4-w_2)e^{-2\alpha\tau_1}x^2(t-\gamma\tau_1)-2\alpha V_2(t).$$

Obviously, for any scalars $s\in[t-\tau_2,t]$ and $s\in[t-\gamma\tau_2,t]$, we can get $e^{-2\alpha\tau_2}\leq e^{2\alpha(s-t)}\leq 1$ and $e^{-2\alpha\gamma\tau_2}\leq e^{2\alpha(s-t)}\leq 1$, respectively. In view of Lemma 1.1 and Lemma 1.2, the given assumptions, from $V_3(t,x_t)$ and NIDE (1.2), it follows that

$$\dot{V}_3(t,x_t)=k_6\tau_2\int_{-\tau_2}^{0}x^2(t)ds-k_6\tau_2\int_{-\tau_2}^{0}e^{2s\alpha}x^2(t+s)ds$$
$$+k_7\gamma\tau_2\int_{-\gamma\tau_2}^{0}x^2(t)ds$$
$$-k_7\gamma\tau_2\int_{-\gamma\tau_2}^{0}e^{2s\alpha}x^2(t+s)ds$$
$$+w_5\tau_1\int_{-\tau_1}^{0}x^2(t)ds$$
$$-w_5\tau_1\int_{-\tau_1}^{0}e^{2s\alpha}x^2(t+s)ds$$
$$+w_6\gamma\tau_1\int_{-\gamma\tau_1}^{0}x^2(t)ds$$
$$-w_6\gamma\tau_1\int_{-\gamma\tau_1}^{0}e^{2s\alpha}x^2(t+s)ds$$
$$+w_7(\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}x^2(t)ds$$
$$-w_7(\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}e^{2s\alpha}x^2(t+s)ds$$
$$+w_8(\gamma\tau_2-\gamma\tau_1)\int_{-\gamma\tau_2}^{-\gamma\tau_1}x^2(t)ds$$
$$-w_8(\gamma\tau_2-\gamma\tau_1)\int_{-\gamma\tau_2}^{-\lambda\tau_1}e^{2s\alpha}x^2(t+s)ds-2\alpha V_3(t)$$
$$\leq k_6(\tau_2)^2x^2(t)-k_6e^{-2\alpha\tau_2}\left(\int_{t-\tau_2}^{t}x(s)ds\right)^2$$
$$+k_7(\gamma\tau_2)^2x^2(t)-k_7e^{-2\alpha\gamma\tau_2}\left(\int_{t-\gamma\tau_2}^{t}x(s)ds\right)^2$$
$$+w_5(\tau_1)^2x^2(t)-w_5e^{-2\alpha\tau_1}\left(\int_{t-\tau_1}^{t}x(s)ds\right)^2 \text{b}$$
$$+w_6(\gamma\tau_1)^2x^2(t)-w_6e^{-2\alpha\gamma\tau_1}\left(\int_{t-\gamma\tau_1}^{t}x(s)ds\right)^2$$
$$+w_7(\tau_2-\tau_1)^2x^2(t)-w_8(\gamma\tau_2-\gamma\tau_1)^2x^2(t)$$
$$-w_7e^{-2\alpha\tau_2}\left(\int_{t-\tau(t)}^{t-\tau_1}x(s)ds\right)^2-w_7e^{-2\alpha\tau_2}\left(\int_{t-\tau_2}^{t-\tau(t)}x(s)ds\right)^2$$
$$-w_8e^{-2\gamma\alpha\tau_2}\left(\int_{t-\gamma\tau(t)}^{t-\gamma\tau_1}x(s)ds\right)^2$$
$$-w_8e^{-2\gamma\alpha\tau_2}\left(\int_{t-\gamma\tau_2}^{t-\gamma\tau(t)}x(s)ds\right)^2-2\alpha V_3(t).$$

Similarly, by using Lemma 1.1, Lemma 1.2, the inequality $\tanh^2 x(t)\leq x^2(t)$, $V_4(t,x_t)$ and NIDE (1.2), we have

$$\dot{V}_4(t,x_t)=k_8\tanh^2 x(t)$$

$$-k_8(1-\dot{\sigma}(t))e^{-2\alpha\sigma(t)}\tanh^2 x(t-\sigma(t))$$

$$+k_8\sigma_2\int_{-\sigma_2}^{0}\tanh^2 x(t)ds$$

$$-k_9\sigma_2\int_{-\sigma_2}^{0}e^{2\alpha s}\tanh^2 x(t+s)ds$$

$$+w_9\sigma_1\int_{-\sigma_1}^{0}\tanh^2 x(t)ds$$

$$-w_9\sigma_1\int_{-\sigma_1}^{0}e^{2\alpha s}\tanh^2 x(t+s)ds$$

$$-w_{10}(\sigma_2-\sigma_1)\int_{-\sigma_2}^{-\sigma_1}e^{2\alpha s}\tanh^2 x(t+s)ds\ -2\alpha V_4(t)$$

$$\le k_8\tanh^2 x(t)\ \ -k_8e^{-2\alpha\sigma_2}\tanh^2 x(t-\sigma(t))$$

$$-k_8\sigma_d\tanh^2 x(t-\sigma(t))$$

$$+k_9{\sigma_2}^2x^2(t)-k_9e^{-2\alpha\sigma_2}\left(\int_{t-\sigma_2}^{t}\tanh x(s)ds\right)^2$$

$$+w_9{\sigma_1}^2x^2(t)-w_9e^{-2\alpha\sigma_1}\left(\int_{t-\sigma_1}^{t}\tanh x(s)ds\right)^2$$

$$+w_{10}(\sigma_2-\sigma_1)^2x^2(t)$$

$$-w_{10}e^{-2\alpha\sigma_2}\left(\int_{t-\sigma(t)}^{t-\tau_1}\tanh x(s)ds\right)^2,$$

$$\dot{V}_5(t,x_t)=wp_2\int_{-p_2}^{0}[x^2(t)-e^{-2\alpha s}x^2(t+s)]ds-2\alpha V_5(t)$$

$$=wp_2^2x^2(t)-wp_2\int_{-p_2}^{0}e^{2\alpha s}x^2(t+s)ds$$

$$-2\alpha V_5(t)$$

$$\le wp_2^2x^2(t)-wp_2e^{-2\alpha p_2}\int_{t-p_2}^{t}x^2(s)ds$$

$$-2\alpha V_5(t)$$

$$\le wp_2^2x^2(t)-we^{-2\alpha p_2}\left(\int_{t-p_2}^{t}x(s)ds\right)^2$$

$$-2\alpha V_5(t)$$

$$\le wp_2^2x^2(t)-we^{-2\alpha p_2}\left(\int_{t-p(t)}^{t}x(s)ds\right)^2-2\alpha V_5(t)$$

$$\le wp_2^2x^2(t)-we^{-2\alpha p_2}\left(\int_{t-p(t)}^{t}C(t,s)x(s)ds\right)^2$$

$$-2\alpha V_5(t).$$

On gathering the above discussion in (2.3), we arrive at

$$\dot{V}(t,x_t)+2\alpha V(t,x_t)$$
$$\le 2k_1D(t)[-(a_1-r_1-r_2)h(x(t))$$
$$-(a_2+r_1)h_1(x(t-\tau(t))x(t-\tau(t))$$
$$-(a_2+r_1)\int_{t-\tau(t)}^{t}\dot{x}(s)ds$$
$$-r_2h_1(x(t-\gamma\tau(t)))x(t-\gamma\tau(t))-r_2\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds$$
$$+b(t)\tanh x(t-\sigma(t))+\int_{t-p(t)}^{t}C(t,s)x(s)ds]$$
$$+2q_1D(t)[-D(t)+p_1x(t)+p_2x(t-\tau(t))$$
$$+x(t-\gamma\tau(t))+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds]$$
$$+2q_2x(t)[-D(t)+p_1x(t)+p_2x(t-\tau(t))$$
$$+x(t-\gamma\tau(t))+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds]$$
$$+2q_3x(t-\tau(t))[-D(t)+p_1x(t)+p_2x(t-\tau(t))$$
$$+x(t-\gamma\tau(t))+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds]$$

$$+2q_4\int_{t-\tau(t)}^{t}\dot{x}(s)ds[-D(t)+p_1x(t)+p_2x(t-\tau(t))$$
$$+x(t-\gamma\tau(t))+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds]$$
$$+2q_5x(t-\gamma\tau(t))[-D(t)+p_1x(t)+p_2x(t-\tau(t))$$
$$+x(t-\gamma\tau(t))+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds]$$
$$+2q_6\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds[-D(t)+p_1x(t)+p_2x(t-\tau(t))$$
$$+x(t-\gamma\tau(t))+\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds-p_1\int_{t-\tau(t)}^{t}\dot{x}(s)ds]$$
$$+2q_7\dot{D}(t)[-\dot{D}(t)-a_1h(x(t))-a_2h(x(t-\tau(t))$$
$$-a_2\int_{t-\tau(t)}^{t}(h(x(s)))'ds+b(t)\tanh x(t-\sigma(t))$$
$$+\int_{t-p(t)}^{t}C(t,s)x(s)ds]+2\alpha k_1D^2(t)-2\alpha V_1(t)$$
$$+(k_2+k_3+k_4+k_5+w_1+w_2)x^2(t)$$
$$-(k_2+w_3)e^{-2\alpha\tau_2}x^2(t-\tau_2)$$
$$-k_3e^{-2\alpha\tau_2}x^2(t-\tau(t))+k_3\tau_dx^2(t-\tau(t))$$
$$-(k_4+w_4)e^{-2\alpha\gamma\tau_2}x^2(t-\gamma\tau_2)$$
$$-k_5e^{-2\alpha\gamma\tau_2}x^2(t-\gamma\tau(t))+k_5\gamma\tau_dx^2(t-\gamma\tau(t))$$
$$+(w_3-w_1)e^{-2\alpha\tau_1}x^2(t-\tau_1)$$
$$+(w_4-w_2)e^{-2\alpha\tau_1}x^2(t-\gamma\tau_1)-2\alpha V_2(t)$$
$$+k_6(\tau_2)^2x^2(t)-k_6e^{-2\alpha\tau_2}\left(\int_{t-\tau_2}^{t}x(s)ds\right)^2$$
$$+k_7(\gamma\tau_2)^2x^2(t)-k_7e^{-2\alpha\gamma\tau_2}\left(\int_{t-\gamma\tau_2}^{t}x(s)ds\right)^2$$
$$+w_5(\tau_1)^2x^2(t)-w_5e^{-2\alpha\tau_1}\left(\int_{t-\tau_1}^{t}x(s)ds\right)^2$$
$$+w_6(\gamma\tau_1)^2x^2(t)-w_6e^{-2\alpha\gamma\tau_1}\left(\int_{t-\gamma\tau_1}^{t}x(s)ds\right)^2$$
$$+w_7(\tau_2-\tau_1)^2x^2(t)-w_8(\gamma\tau_2-\gamma\tau_1)^2x^2(t)$$
$$-w_7e^{-2\alpha\tau_2}\left(\int_{t-\tau(t)}^{t-\tau_1}x(s)ds\right)^2-w_7e^{-2\alpha\tau_2}\left(\int_{t-\tau_2}^{t-\tau(t)}x(s)ds\right)^2$$
$$-w_8e^{-2\gamma\alpha\tau_2}\left(\int_{t-\gamma\tau(t)}^{t-\gamma\tau_1}x(s)ds\right)^2$$
$$-w_8e^{-2\gamma\alpha\tau_2}\left(\int_{t-\gamma\tau_2}^{t-\gamma\tau(t)}x(s)ds\right)^2-2\alpha V_3(t)$$
$$+k_8\tanh^2x(t)-k_8e^{-2\alpha\sigma_2}\tanh^2x(t-\sigma(t))$$
$$-k_8\sigma_d\tanh^2x(t-\sigma(t))$$
$$+k_9{\sigma_2}^2x^2(t)-k_9e^{-2\alpha\sigma_2}\left(\int_{t-\sigma_2}^{t}\tanh x(s)ds\right)^2$$
$$+w_9{\sigma_1}^2x^2(t)-w_9e^{-2\alpha\sigma_1}\left(\int_{t-\sigma_1}^{t}\tanh x(s)ds\right)^2$$
$$+w_{10}(\sigma_2-\sigma_1)^2x^2(t)$$
$$-w_{10}e^{-2\alpha\sigma_2}\left(\int_{t-\sigma(t)}^{t-\tau_1}\tanh x(s)ds\right)^2$$
$$+k_{10}x^2(t)-k_{10}\tanh^2x(t)$$
$$-w_{10}e^{-2\alpha\sigma_2}\left(\int_{t-\sigma_2}^{t-\sigma(t)}\tanh x(s)ds\right)^2-2\alpha V_4(t)$$
$$+wp_2^2x^2(t)-we^{-2\alpha p_2}\left(\int_{t-p(t)}^{t}C(t,s)x(s)ds\right)^2$$
$$-2\alpha V_5(t).$$

Then, it be followed that

$$\dot{V}(t,x_t)+2\alpha V(t,x_t)\le\xi^T(t)\Sigma\xi(t),$$

where

$$\xi^T(t)=[D(t),x(t),x(t-\tau(t)),\int_{t-\tau(t)}^{t}\dot{x}(s)ds,x(t-\gamma\tau(t)),$$
$$\int_{t-\gamma\tau(t)}^{t}\dot{x}(s)ds,x(t-\tau_2),\int_{t-\tau_2}^{t}x(s)ds,x(t-\gamma\tau_2),$$
$$\int_{t-\gamma\tau_2}^{t}x(s)ds,\ \tanh x(t-\sigma(t)),$$

$$\int_{t-\sigma_2}^{t} \tanh x(s)ds, \dot{D}(t), x(t-\tau_1),$$

$$x(t-\gamma\tau_1), \int_{t-\tau_1}^{t} x(s)ds, \int_{t-\gamma\tau_1}^{t} x(s)ds,$$

$$\int_{t-\tau(t)}^{t-\tau_1} x(s)ds, \int_{t-\tau_2}^{t-\tau(t)} x(s)ds, \int_{t-\gamma\tau(t)}^{t-\gamma\tau_1} x(s)ds,$$

$$\int_{t-\gamma\tau_2}^{t-\gamma\tau(t)} x(s)ds, \int_{t-\sigma_1}^{t} \tanh x(s)ds,$$

$$\int_{t-\sigma_2}^{t-\sigma(t)} \tanh x(s)ds, \int_{t-p(t)}^{t} C(t,s)x(s)ds]$$

and Σ is defined by (2.1). If the condition (2.2) holds, then

$$\dot{V}(t,x_t) + 2\alpha V(t,x_t) \le 0, \forall t \in R^+. \qquad (2.4)$$

From (2.4), it is easy to see that

$$\|x(t,\phi)\| \le \beta \|\phi\| e^{-\alpha t}, t \in R^+.$$

This means that equation (1.2) is exponentially stable. The proof of the theorem is complete.

Remark

It is clear that the derivative given by (2.3) is a quadratic form with respect to the variables in (2.4). Then, this derivative can be arranged as a quadratic form. In that case, we can obtain the matrix (2.2).

3.Conclusion

The exponential stability of zero solution of a neutral integro-differential equation of first order (NIDE) with discrete and distributed time-varying delays was investigated. A theorem was proved on the exponential stability of the zero solution of the considered NIDE by means of a new defined Lyapunov-Krasovskii functional. Our result improves and includes some results that can be found in the literature.

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