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FOUR WAVE MIXING EFFECT IN FIBER AND EDFA OF AN OPTICAL WDM SYSTEM HAVING DIFFERENT CHANNEL SPACING AND CHROMATIC DISPERSION SCHEMES

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ABSTRACT

The performance of an optical wavelength division multiplexed communication system is analyzed taking the four wave mixing (FWM) effect into account. Also it is found that the efficiency of FWM generation can be greatly reduced by making the spacing between channels unequal. Taking into account the various channel spacing schemes and various distribution of different dispersion type optical fibers a comparative picture has been drawn to suggest the most efficient system with maximum FWM reduction. During the evaluation the FWM process in EDFA is also taken into consideration.

Keywords: *chromatic dispersion, erbium doped fiber amplifier, four wave mixing, power penalty, wavelength division multiplexing.*

INTRODUCTION

Optical wavelength division multiplexing (WDM) system is very much preferred now-a-days because of its high bit rate capability and of successful implementation of optical amplifiers like erbium doped fiber amplifiers (EDFA). A number of fiber nonlinear effects, like four wave mixing (FWM), stimulated Raman scattering (SRS), stimulated Brillouin scattering (SBS), self- and cross-phase modulation, cause performance degradation of the optical WDM system [1]. FWM effect generates new frequency signals, some which may interfere with the transmitted signals [2]. Periodic distribution of EDFAs in multiple fiber system further accumulates the FWM signals [3]. Unequal channel spacing may be employed to reduce the effect of FWM, however, at the expense of increased bandwidth [4]. Even with unequal channel spacing, the system may suffer from power depletion of the transmitted channels because of the generation of FWM signals [5]. FWM generation is affected by fiber chromatic dispersion because the phase matching pre-conditions are met when the transmitted channels are positioned around the zero-dispersion wavelength of the fiber [6]. Therefore, a solution to the FWM problem is to utilize a number of fiber sections with nonuniform chromatic dispersion so that the phase matching conditions for FWM generated waves are not satisfied [7]. However, all these schemes have limitations that they can not solve the problem of FWM

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degradation completely. In addition, no such analysis so far incorporated the FWM effect in EDFA.

The objective of this paper is to study the performance of a long distance optical WDM communication system in presence of FWM with equal, unequal and repeated unequal channel spacing. The effect of chromatic dispersion on the FWM efficiency is evaluated and hence the allowable maximum input power for fibers with uniform chromatic dispersion is determined. Then a system with multiple-fiber sections having different chromatic dispersion, however, maintaining a minimum overall dispersion, is studied. Various combinations of different channel spacing schemes and different chromatic dispersion schemes are investigated with a view to find the best solution to the FWM problem. During the performance evaluation under different schemes the FWM process in EDFA is also taken into consideration to observe the complete effect of FWM in an optical WDM system.

THEORETICAL ANALYSIS

In an optical transmission link, each repeater span is composed of a number of short-length fibers as shown in figure 1. The total transmission link consists of M sections, where each section has N number of fibers of equal length. There is $(M-1)$ number of erbium-doped fiber amplifiers (EDFA) with equal repeater span throughout the system to compensate for the power loss of the section just before the amplifier. Thus the power input at the beginning of each section is essentially equal. It is assumed that the polarization states of all optical sources are matched throughout the transmission system. For simplicity polarization mode dispersion is ignored.

When three lights of frequencies, f_1, f_2 and f_3, co-propagate through a fiber, due to fiber nonlinear effect, new light signals with frequencies f_F are generated according to the following relation [5].

$$f_F = f_{ijk} = f_i + f_j - f_k, \qquad i, j \neq k \quad (1)$$

The total number of FWM lights, N_F, generated in an optical WDM system of N channels is given by

$$N_F = \frac{1}{2}(N^3 - N^2) \qquad (2)$$

The generation of FWM lights causes performance degradation in two ways, namely, by depleting the power of transmitted signal lights and by interfering with the lights, which have the same frequencies as the FWM light.

To overcome the problem of FWM generation, several ways have been investigated. Unequal channel spacing (US) between intelligence channels may be employed so that the FWM lights produced do not coincide with any of the intelligence channels. However, unequal channel spacing requires a very high bandwidth, which may be even impractical for large number of channels. To reduce the bandwidth requirement, repeated unequal spacing (RUS) scheme is proposed [8]. Here a scheme of unequal spacing for a group of channels is repeated for other groups of channels.

To design a channel scheme for repeated unequal spacing, first, a base scheme of unequal spacing is fixed for a number of channels. Let such a base scheme be $\tilde{n}_{us}$, with $\tilde{N}_{us}$ number of channels and spacing between adjacent channels $\tilde{d}_{us} = \{d1, d2, d3 \ldots\ldots.d_{N_{US}-1}\}$. Now this base allocation will be repeated as many times as it requires to complete the allocation for all the channels in the WDM system [8]. Let the desired RUS spacing vector be $\tilde{d}_{rus} = \{d1, d2, \ldots..d_{N_{US}-1}, d1, d2, \ldots..d_{N_{US}-1}, \ldots\}$. Then a number of spacing independent of each other by extracting consecutive $(N_{US}-1)$ elements from $\tilde{d}_{rus}$ will allocated as follows.

$$\tilde{d}^{[1]} = \{d1, d2, d3 .... d_{N_{US}-1}\}$$

$$\tilde{d}^{[2]} = \{d2, d3 .... d_{N_{US}-1}, d1\}$$

. . . .

$$\tilde{d}^{[N_{US}-1]} = \{d_{N_{US}-1}, d1, d2, d3 ...., d_{N_{US}-2}\} \quad (3)$$

Then these allocations are to be checked whether all of them are US. If so, $\tilde{d}_{us}$ may be considered as a base unit for the RUS and the required complete RUS allocation can be obtained by repeating $\tilde{d}_{us}$ as $\tilde{d}_{rus}$. If all of these allocations are not US then other US allocations for the same number of base channels are to be investigated until a suitable one as above may be found.

FWM generation is affected by fiber chromatic dispersion because the phase matching pre-conditions for FWM generation are met when the transmitted channels are positioned around the zero-dispersion wavelength of the fiber [6]. Therefore, a solution to the FWM problem is to utilize highly chromatic dispersive optical fibers, which will introduce phase delay among different propagating channels [7]. However, the signal-to-noise ratio (SNR) will be worst degraded at the output due to pulse broadening in highly dispersive fibers. A good solution is to use a number of fibers with different chromatic dispersion along the transmission length in such a way so that the signal phases in individual fibers are hardly matched to generate FWM signal, while the overall chromatic dispersion is maintained at a very low value.

The probability of error or the bit error rate (BER) can be determined as [9]

$$P_e = \frac{1}{\sqrt{2\pi}} \int_Q^{\infty} \exp[-\frac{t^2}{2}] dt \quad (4)$$

where, Q is the quality factor and is given by [3], [5]

$$Q = \frac{bPs}{\sqrt{N_{th} + N_{sh} + N_{FWM}} + \sqrt{N_{th}}} \quad (5)$$

Here, $b = \frac{\eta e}{h\nu}$, η is the coupling efficiency of the detector, e is the electron charge, h is the Planck's constant, ν is the light frequency, P_s is the signal power, $N_{sh} = k_s b P_s$ is the shot noise power, k_s is a proportional constant, N_{th} is the thermal noise power and N_{FWM} is the FWM noise power. The signal power can be found as

$$P_s = P_o \exp(-\alpha L) L_r \quad (6)$$

where, L_r is the insertion loss before receiver, P_o is the input power of each channel, L is the section length, α is the attenuation constant. The power of the FWM signal for the general case of chromatic dispersion can be found as [10]

$$\begin{aligned} P_{abc}\left(N_{amp}(l+L)\right) &= \left(\frac{D_{abc}}{3}\right) P_{a0} P_{b0} P_{c0} \\ &\times \left[\frac{\sin\left(N_{amp}\frac{(\Delta\beta l + \Delta BL)}{2}\right)}{\sin\left(\frac{(\Delta\beta l + \Delta BL)}{2}\right)}\right]^2 \\ &\times \left\{\left|\gamma_{EDF}\sqrt{G(l)} \int_0^l G(z) e^{i0\beta z} dz\right|^2\right. \\ &+ \gamma^2{}_{TF} G(l)^2 \frac{\left(1-e^{-\alpha L}\right)^2 + 4e^{-\alpha L}\sin^2\left(\frac{\Delta BL}{2}\right)}{\alpha^2 + \Delta B^2} \\ &+ 2\gamma_{EDF}\gamma_{TL} G(l)^{3/2} \times \left[\int_0^l G(z)\cos(\Delta\beta z) dz\right. \\ &\times \int_0^L e^{-\alpha z}\cos(\Delta\beta z + \Delta Bz) dz + \int_0^l G(z)\sin(\Delta\beta z dz) \\ &\left.\left.\times \int_0^L e^{-\alpha z}\sin(\Delta\beta l + \Delta Bz) dz\right\}\right] \end{aligned} \quad (7)$$

where, D_{abc} is the degeneracy factor, γ_{tf} and γ_{EDF} are the nonlinearity factors for the transmission fiber and the EDFA, respectively, $G(z)$ is the gain function of the EDFA, $\Delta\beta$ and ΔB are difference in propagation constants in EDFA and transmission fiber, respectively, L is the length of transmission fiber span and l is the length of EDFA, α is the attenuation constant of the transmission fiber, P_{a0}, P_{b0} and P_{c0} are the input powers of channels a, b and c and N_{amp} is the number of amplifiers in the system. The analysis of EDFA gain is quite complicated [11]. However, for simplicity a particular EDFA with a fixed pump power is considered in this study and the expression for the gain function $G(z)$ by a polynomial fit.

The difference in propagation constants of the FWM signals due to chromatic dispersion is given by [6]

$$\begin{aligned}\Delta\beta^{mn} &= \beta_a^{mn} + \beta_b^{mn} - \beta_c^{mn} - \beta_F^{mn} \\ &= -\frac{\pi\lambda^4}{c^2}\frac{dD_c}{d\lambda}\begin{Bmatrix}\left(f_a - f_o^{mn}\right) \\ +\left(f_b - f_o^{mn}\right)\end{Bmatrix} \\ &\times\left(f_a - f_c\right)\left(f_b - f_c\right)\end{aligned} \qquad (8)$$

where, β_a^{mn}, β_b^{mn}, β_c^{mn} and β_F^{mn} are the propagation constants for channels a, b, c and FWM, respectively, of the nth fiber in the mth section, f_o^{mn} is the zero-dispersion frequency of nth fiber in mth section and D_c is the chromatic dispersion coefficient.

Finally, N_{FWM} can be estimated as [5]

$$N_{FWM} = \frac{1}{4}b^2 P_s P_{FWM} \qquad (9)$$

Putting the equation (9) in equation (5) we get

$$k_s + \frac{1}{4}bP_{FWM} = \frac{bP_{SF}}{Q^2} - \frac{2}{Q}\sqrt{N_{th}} \qquad (10)$$

and in absence of FWM

$$k_s = \frac{bP_{s0}}{Q^2} - \frac{2}{Q}\sqrt{N_{th}} \qquad (11)$$

where, P_{SF} , P_{S0} are the received signal powers with and without FWM, respectively, at a certain value of Q. Thus the power penalty, $P_p = P_{SF}/P_{S0}$, at a given value of Q_o can estimated as

$$P_p = \frac{1}{1 - \frac{1}{4}\frac{P_{FWM}}{P_{SF}}Q_0^2} \qquad (12)$$

Finally, the allowable input power at a given penalty can be evaluated from the above equation.

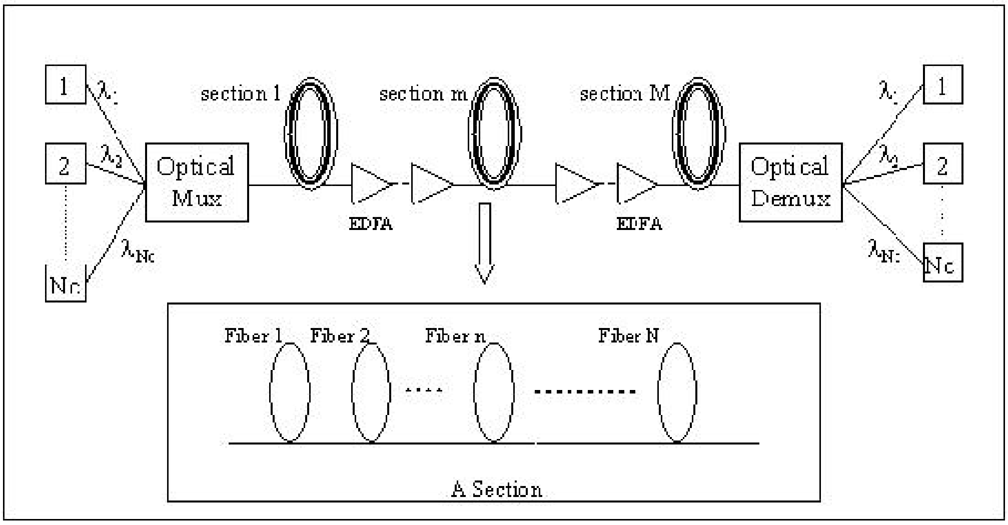

***Fig. 1**: Configuration of the optical transmission system*

RESULTS AND DISCUSSION

The performance of an optical WDM system is evaluated considering the effect of FWM using the theoretical formulation developed in the previous section. A multi-amplifier optical system with sixteen sections each consisting twenty fibers is studied. The typical system parameters assumed in calculation are n_o = 1.45, operating guide wavelength λ = 1.55 μm, A_{eff} = 50 μm^2, χ = 4 X 10^{-15} esu, α = 0.21 db/km, L_o = 2.73 km, number of fibers per section = 25 and the dispersion slope $dDc/d\lambda$ = 0.07 ps/(km-nm^2). The total haul of the system considered is 1090 km.

While evaluating the FWM power produced in the unequal channel spacing scheme, the zero dispersion wavelength λ_0 has been set up at the middle of the two central channels. These two central channels will suffer the most from power depletion, as they are the only ones to contribute power to phase-matched mixing waves [4]. It is found that a signal to FWM power ratio of 7.5 dB is required to maintain a power penalty of 0.7 dB. The minimum channel spacing is considered to be 100 GHz for equal channel spacing scheme and 125 GHz for unequal channel spacing scheme such that there is no interference by FWM lights. We have ignored the power depletion in case of equal and repeated unequal spacing schemes.

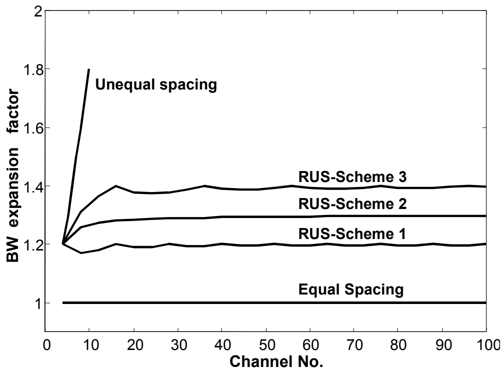

Fig. 2: Bandwidth expansion factor with increasing number of channels

Unequally spaced channels require much higher bandwidth, which is practically not feasible for large number of channels. The bandwidth required by the US scheme compared to that required by the ES scheme, named as the bandwidth expansion factor, is determined and shown in figure 2.

The bandwidth expansion factor is found to increase sharply with the number of channels in the system. The bandwidth requirement for the RUS scheme is also estimated considering three base schemes, scheme 1 having 4 base channels, scheme 2 having 5 base channels and scheme 3 having 6 base channels. The bandwidth requirement is, though higher than the ES scheme, much less than the US scheme. In addition, the bandwidth expansion factor remains almost flat with the number of channel, which means that the scheme will not cause much problem in designing for large channels. It also appears from the figure that if the number of base channels is increased, it will improve the system performance, however, at the expense of increased bandwidth.

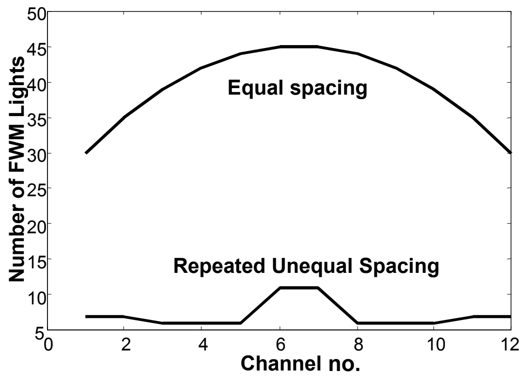

Fig. 3: Number of FWM lights produced in different channel positions for US and RUS schemes

The repeated unequal channel spacing is not strictly an unequal channel spacing scheme. Here the FWM lights are semi-suppressed, that is, they are not completely filterable from the intelligence lights. However, it provides a very small interaction between the FWM lights and the intelligence lights as compared to ES system. This fact is illustrated in figure 3, which shows the number of FWM lights generated at various channel positions. An ES frequency allocation with 12 channels and a channel spacing of 140 GHz is considered in this calculation. The total bandwidth occupied by the signal lights is 1540 GHz. For the RUS system, a base unit of 6 channels and a minimum channel spacing of 100 GHz with B_b = 700 GHz are assumed. The RUS frequency allocation also has 12 channels and hence the total bandwidth is 1500 GHz. It can be observed from the figure that the number of FWM lights produced on the channel frequencies for RUS is very small as compared to that of ES.

The generation of FWM signals in EDFA is investigated along with that generated in the fiber and shown in figure 4. It can be observed that a significant amount of FWM power is generated in the EDFA, which must be accounted for in the study of FWM effect in WDM system.

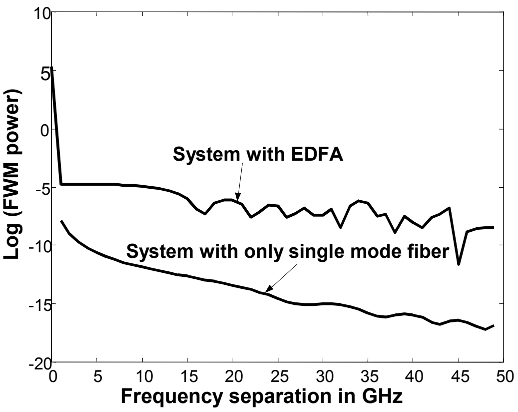

Fig. 4: Generation of FWM power in EDFA

The generated FWM power in the EDFA strongly depends on its length as can be seen from the figure 5.

The FWM power increases with the EDFA length, which signifies the importance of studying the effect of FWM generation in EDFA for a long-haul optical communication system, where the number of EDFAs is very large and hence is the total length of EDFA. It can also be noted that the FWM power varies periodically with the EDFA length, which indicates that the EDFA length should be carefully selected to reduce the FWM power to a minimum level.

The effect of input power on the bit error rate is illustrated in figure 6 where the power penalty is considered to be 0.7 dB. The figure lists the BER curves for various systems under consideration.

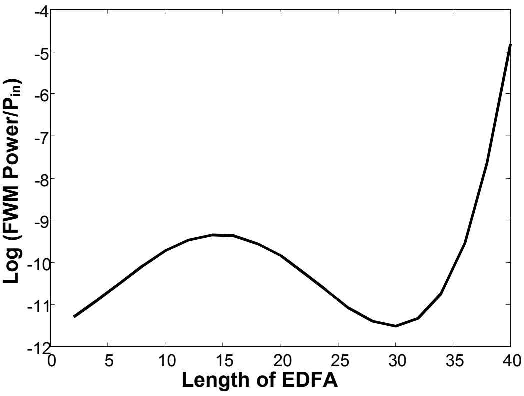

Fig 5: *Variation of FWM power with EDFA length*

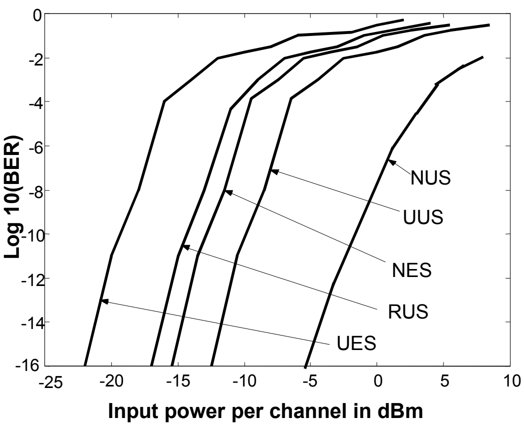

Fig. 6: Bit error rate characteristics for varying input power; UES: uniform dispersion and equal spacing, NES: nonuniform dispersion and equal spacing, UUS: uniform dispersion and unequal spacing, NUS: nonuniform dispersion and unequal spacing, RUS: repeated unequal spacing

It can be observed that repeated unequal channel spacing gives better performance compared to equal spacing. Similarly, use of nonunifrom chromatic dispersion helps in reducing the FWM effect significantly as compared to uniform dispersion. In the nonuniform chromatic dispersion scheme, each fiber in a section is assumed to have a different dispersion. The zero-dispersion wavelengths of the fibers are considered to be distributed over an 8 nm width around 1550 nm such that the average zero-dispersion wavelength is maintained at 1550 nm.

The penalty suffered by the systems to achieve a BER of 10^{-9} is determined and shown in figure 7. The worst penalty is found to be suffered by the system with equal channel spacing and uniform chromatic dispersion.

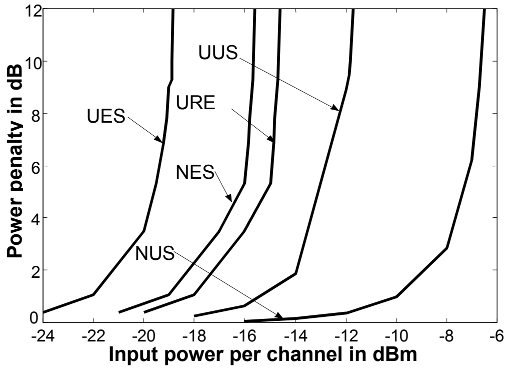

Fig. 7: Power penalty with varying input power

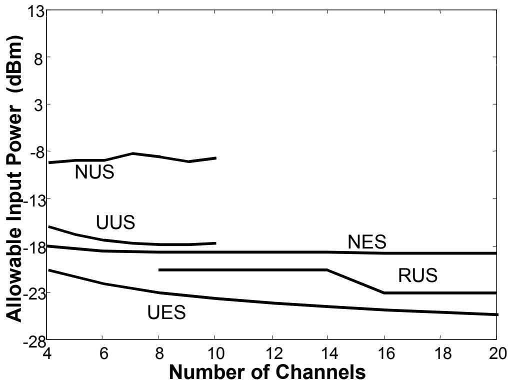

Fig. 8: Allowable input power for varying number of channels

For a given maximum power penalty of 0.7 dB at a BER of 10^{-9}, the maximum input power is found to be restricted due to the accumulation of FWM power. The allowable input power with varying number of channels for different systems under investigation are evaluated and shown in figure 8. It can be observed that the effect of FWM can well be managed by using nonuniform dispersion with unequal channel spacing.

CONCLUSION

The generation of FWM power seriously degrades the performance of an optical WDM system. It is observed that the performance of a conventional system becomes unsatisfactory when the number of channel or the input power per channel is large. The performance can be improved significantly by controlling the channel spacing or the chromatic dispersion. Unequal channel spacing can completely eliminate the interference of FWM light with signal lights, but at the expense of large bandwidth. Repeated unequal spacing scheme can improve the situation by requiring less bandwidth. Use of nonuniform dispersion along the transmission length can also help in reducing the FWM power significantly. The performance evaluation and comparison of various schemes are done by taking the FWM generation in the EDFA into account, in addition to that in the transmission fiber. The results of this work can be utilized effectively in designing a long-haul optical WDM system where the effect of FWM power will be minimum.

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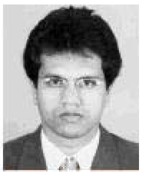

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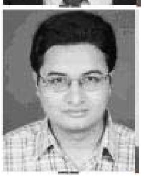

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