ISTANBUL UNIVERSITY –
JOURNAL OF ELECTRICAL & ELECTRONICS ENGINEERING

YEAR : 2005 (1391-1394)
VOLUME : 5
NUMBER : 2

A MODIFIED DIPOLE-ENHANCED APPROXIMATION FOR A DIELECTRIC SPHERE

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ABSTRACT

In this paper, a modification of the previously developed dipole-enhanced approximation for merging spheres in a dielectric mixture has been carried out. In this modified dipole-enhanced approximation, the equivalent enhanced field experienced by an arbitrarily selected sphere is related not only to the difference in permittivity and the ratio of the sphere's radius to the average distance between neighbor spheres, but also to the position of the field point. It is acceptable for the calculation of local electric field strength in a dielectric mixture with a large dielectric mismatch and a high-volume fraction of spheres or long cylinders. The electric field distributions calculated by this model on the surface of a sphere or the circumference of a circle are compared with those obtained with the dipole model, the previous dipole-enhanced model and the finite-element method, which shows that this kind of approximation is reasonable and much better than the previous dipole-enhanced model and the dipole model.

Keywords: *Dielectric mixture; Electric field; Finite element method; Modified dipole-enhanced approximation.*

1. INTRODUCTION

There are many areas of science and technology where the response of merging spheres to applied fields is of interest. Among many approaches to solve this problem, a widely adopted one is to treat the polarized sphere as a dipole. Such as the dipole approximation, it assumes that the distance between spheres is so far that the interactions among spheres is negligible which is successfully applied for the calculation of electrical charge in industrial electrostatic precipitation [1-5]. However, the dipole model is invalid for a dielectric mixture with a high-volume fraction of spheres or circles and a large dielectric mismatch ($\varepsilon_i / \varepsilon_e$) between the dielectrics (ε_i) and the environment (ε_e). In the case of merging small particles, and also larger bodies or surfaces to an electrostatic field, a general solution of calculating local electric field strength is not available [6-10].

A dipole-enhanced approximation considering the interactions among spheres was presented in our last paper [1]. Its calculated results, compared with those obtained by the finite-

Received Date : 09.02.2005
Accepted Date: 07.05.2005

element method, are fairly good at two poles sphere but not good at the other points on the surface of the sphere. Based on that, this paper presents a modified dipole-enhanced approximation whose calculated results agree with those obtained by the finite-element method (FEM) almost on the entire surface of one sphere.

2. THE DIPOLE MODEL FOR A DIELECTRIC MIXTURE

For a dielectric mixture in a uniform external field E_0, the dipole approximation rests on the assumption that the interactions among spheres is negligible and the field experienced by an arbitrarily selected central sphere is still uniform, and the magnitude of this external field is still E_0. That means the sum of the field from all the other spheres is zero and we just need to consider one sphere in a uniform external field E_0. Let's consider a sphere of radius R and permittivity ε_i in a uniform field E_0. The permittivity of environmental media is ε_e. The construction is shown in Fig. 1.

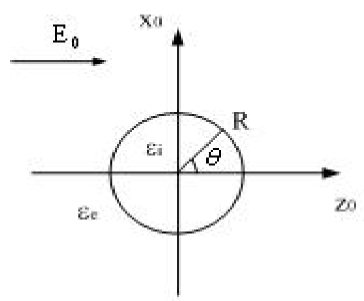

Fig. 1. One sphere subjected to a local field E_0.

Solving Laplace's equation in the spherical coordinate frame for one sphere and the cylindrical coordinate frame for one circle respectively, we can get.

$$\boldsymbol{E}_{\text{out}} = \mathbf{r_0}\left(E_0\cos\theta + k\cdot\frac{2E_0R^3\cos\theta}{r^3}\right) + \boldsymbol{\theta_0}\left(-E_0\sin\theta + k\cdot\frac{E_0R^3\sin\theta}{r^3}\right) \quad (1)$$

$$\boldsymbol{E}_{\text{in}} = \mathbf{z_0}(1-k)E_0 \quad (2)$$

where $k = (\varepsilon_i - \varepsilon_e)/(\varepsilon_i + 2\varepsilon_e)$, $\boldsymbol{E}_{\text{out}}$ is the outside field of the sphere, $\boldsymbol{E}_{\text{in}}$ is the inside field of the sphere. In terms of equation (1), the field amplitude at any point (θ) on the sphere surface with the dipole approximation is

$$E_{DIP} = E_0\sqrt{(1+2k)^2\cos^2\theta + (k-1)^2\sin^2\theta}\,. \quad (3)$$

3. THE FINITE-ELEMENT METHOD FOR A DIELECTRIC MIXTURE

For a dielectric mixture in a uniform external field E_0, we can obtain the local electric field strength at any field point accurately by the finite-element method (FEM). Let's set up the boundary conditions. Outside one arbitrarily selected sphere, we construct an accessorial cylinder whose length and diameter equal to l which is the average distance between two neighbor spheres' centers as shown in Fig. 2. The potential difference between both ends of the cylinder is U_i ($U_i = E_0 l$) because the spheres are evenly distributed in the parallel direction to the external field. The potential at the side face satisfies with the natural boundary condition ($\partial\varphi/\partial n = 0$, the outward surface-normal component of electric field strength is zero) because the spheres are evenly and randomly distributed in the direction perpendicular to the external field. Only the electric field around a single sphere needs to be analyzed since this boundary condition implicitly accounts for the presence of the surrounding spheres.

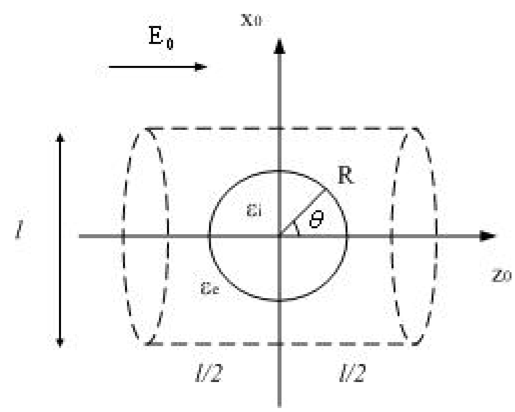

Fig. 2. A finite element model of one sphere in a cylindrical volume.

4. A MODIFIED DIPOLE-ENHANCED MODEL

For a dielectric mixture in a uniform external field E_0, we proposed a modified dipole-enhanced approximation. The modified dipole-enhanced approximation rests on the assumption that the field experienced by an arbitrarily selected central sphere is still uniform, but the magnitude is not the primary external field E_0 because we have to consider the perturbations due to all the surrounding spheres. Since the perturbations at different points on the central sphere surface are different, we adopt different external field magnitudes to calculate the electric field strengths at different points. When we calculate the electric field at point P (θ), we regard that the central sphere is subjected to a uniform external field E'_θ which means the perturbation (sum of the dipoles fields due to all the outside spheres) is not zero, as shown in Fig 3. We just need to consider one sphere in a uniform external field E'_θ. If the calculated results agree well with the FEM results, then we consider this approach is acceptable and expedient.

Using equation (1) and equation (2), we can get the potential difference between points 1 and 2 in the lines of $z = \pm(l/2)$, where l is the average distance between the centers of two neighboring spheres in the unit of sphere radius R.

$$\begin{aligned} U_i &= 2\int_0^R E_{in} dr + 2\int_R^{l/2\cos\theta} E_{out} dr \\ &= 2\int_0^R (1-k)E'_\theta \cos\theta dr + 2\int_R^{l/2\cos\theta} [E'_\theta \cos\theta + 2kE'_\theta \cos\theta (R/r)^3] dr \\ &= E'_\theta l[1-(2R/l)^3 \cos^3\theta] \end{aligned} \quad (4)$$

Because $E_0 = U_i / l$, we can obtain

$$E'_\theta = E_0 / [1 - k(2R/l)^3 \cos^3\theta], \quad (5)$$

Substituting equation (5) into equation (3) yields

$$\begin{aligned} E_{DIP-EN} &= E'_\theta \sqrt{(1+2k)^2 \cos^2\theta + (k-1)^2 \sin^2\theta} \\ &= \frac{E_0 \sqrt{(1+2k)^2 \cos^2\theta + (k-1)^2 \sin^2\theta}}{1 - k(2R/l)^3 \cos^3\theta}, \end{aligned} \quad (6)$$

which is the analytical formula calculating the electric field at any point on the central sphere surface. Clearly it considers the perturbations due to all the other spheres.

5. RESULTS AND DISCUSSION

The electric field distributions on the surface of an arbitrary selected sphere are calculated using the dipole approximation, the FEM, the dipole-enhanced approximation and the modified dipole-enhanced approximation are all shown in Figs. 4 and 5. As seen in these two figures, we find that the calculated result by the modified dipole-enhanced approximation (curve 4) have a good agreement with the FEM result at almost the entire surface, but that calculated by previous dipole-enhanced approximation are good just at the two poles (0 and 1), which shows that the modified dipole-enhanced approximation has a great improvement.

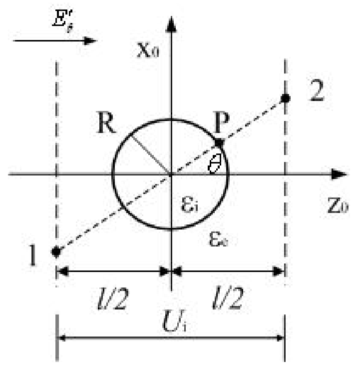

Fig. 3. A sphere subjected to a local field E'_θ.

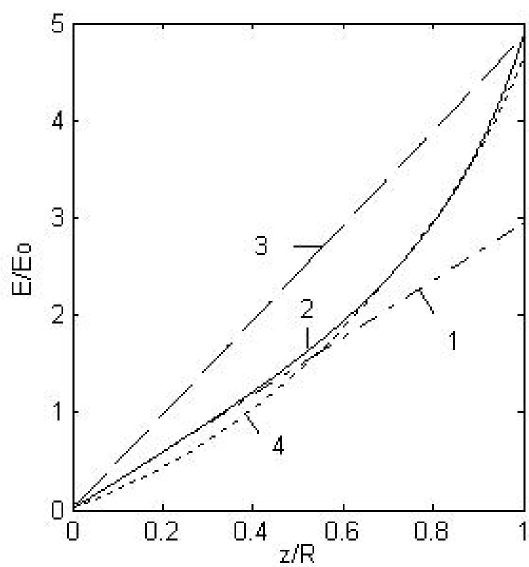

Fig. 4. The electric field distributions on the surface of one sphere for $\varepsilon_i / \varepsilon_e = 100$, $l/R = 2.7$. (1. The dipole approximation; 2. The FEM; 3. The dipole-enhanced approximation; 4. The modified dipole-enhanced approximation.)

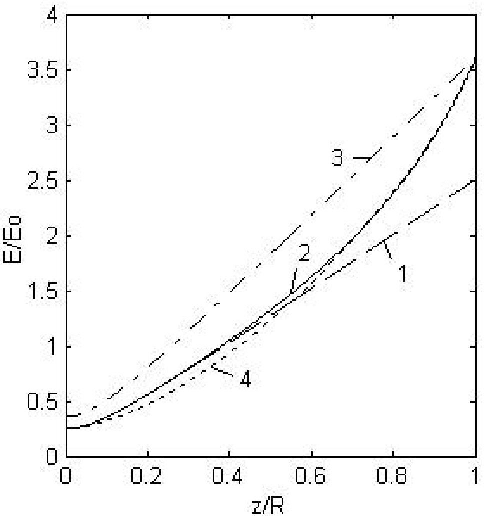

Fig. 5. The electric field distributions on the surface of one sphere for $\varepsilon_i/\varepsilon_e = 10$, $l/R = 2.7$. (1. The dipole approximation; 2. The FEM; 3. The dipole-enhanced approximation; 4. The modified dipole-enhanced approximation.)

6. CONCLUSION

For a dielectric mixture in a uniform external field E_0, a modified dipole-enhanced approximation, considering the interactions among spheres, is presented to calculate the electric field distributions on the surface of an arbitrarily selected sphere in this paper. Its calculated results agree well with the exactly the FEM results at almost all points. The modified dipole-enhanced approximation are suitable for intermediate or strong coupling mixtures with closely spaced particles of high permittivity since it fully considers the interactions of spheres with a uniformly equivalent electric field E'_θ other than the primary external field E_0. The modified dipole-enhanced approximation provides a great improvement over the previous dipole-enhanced approximation.

Acknowledgements

The research reported in this paper was supported by a Grant from the National Science Foundation of China, Gant No. 50237010.

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