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SOME CONSIDERATIONS OF DESIGNING A HIGH PERFORMANCE ROGOWSKI COIL FOR PULSED CURRENT MEASUREMENT

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ABSTRACT

Rogowski coil is made having nonmagnetic material in a core, which results in that Rogowski coil has typically been used to measure large pulsed currents. Therefore, studying the high-frequency performance of the coil is important to design an appropriate sensor for measuring heavy currents. The paper presents some analysis on the influences of the dimension and electromagnetic parameters of the coil. Some considerations and methods of optimizing these parameters have been proposed. Issues related to the geometrical effects and error characters of Rogowski coil are discussed. The prototype equipment embodies a differentiating Rogowski coil, specifically designed to obtain the reliability and scalability to meet the needs of measurement applications. Experimental results verify the validity of the proposed considerations for measuring pulsed currents.

Keywords: *Performance of Rogowski coil, dimension-and electromagnetic- parameters, pulsed current measurement.*

1. INTRODUCTION

The iron coil sensors constructed with oil filled porcelain insulators and thus are very large and heavy and may lead to measuring problems during transient events. To overcome such defects of the conventional iron coil sensor, some novel instruments had been implemented. The currents in the region of thousands of amps such as pulsed current, transient current, and AC current and so on are sensed by Rogowski coil (discussed by references [1~3]).

The principle of the pulsed currents measurement that uses a uniformly wound coil on a closed loop, nonmagnetic former was proposed by Rogowski back in 1912 [4]. The paper will introduce the influences between dimension parameters (i.e. the outer (b/m) diameter, the inner (a/m) diameter, average diameter (D/m)) and electromagnetic parameters (i.e. terminal resistor (R_S/Ω), self-inductance (L_C/H), resistor (R_C/Ω) and mutual inductance (M/H)) of the coil have been analyzed including their influences to the dynamic properties of the sensor. Putting strong emphasis on these fields is valuable and important to optimize the dimension and electromagnetic parameters of the coil. The performance of the Rogowski coil and experimental results of measuring the pulsed current have been presented.

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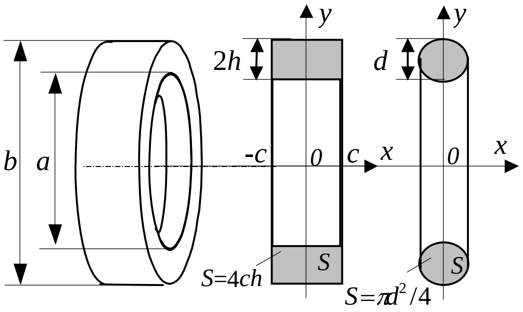
2. THEORY OF ROGOWSKI COIL

The dimensions of the coil (for rectangular or circular cross-section) are illustrated in Fig.1. The voltage induced in the coil, $e\ (t)$/V, is proportional to the coil's mutual inductance and the time derivative of the measured current, d $I_1(t)$/dt, as given by the following:

$$e(t) = -MdI_1(t)/dt \tag{1}$$



(a) (b) (c)

Fig.1 Dimensions of the Rogowski coil

Based on equivalent circuit of the Rogowski coil, induced voltage $e\ (t)$ is calculated by

$$e(t) = L_C(di_2/dt) + (R_C + R_S)i_2 \tag{2}$$

where i_2/A is the induced current along the coil. The coil output voltage, u_S/V, is determined by

$$u_S = R_S i_2 \tag{3}$$

Provided that $L_C(di_2/dt) << (R_S+R_C)i_2$, the measured current $I_1(t)$/A can be approximately calculated as following:

$$I_1(t) = -\frac{R_C + R_S}{MR_S}\int u_S(t)dt \tag{4}$$

To obtain a voltage proportional to the measured pulse current, the coil output voltage must be integrated [5~8]. According to reference [6], the mutual inductance for the rectangular cross-section (whose dimension is illustrated in Fig.1 (b)), M_R/H, is obtained as

$$M_R = \frac{n\mu_0 S}{2\pi r_c}\cdot\frac{1}{h/r_c}\cdot\ln\sqrt{\frac{1+h/r_c}{1-h/r_c}} \tag{5}$$

where μ_0 is the permeability of air ($=4\pi\times10^{-7}$H/m); n is the number of turns of the coil; S/m^2 is area of the rectangular cross section (given S=4ch, c/m and h/m are thickness and broadness); and r_C/m is average radius given by

$$r_C = \sqrt{(3a^2+3b^2+2ab)/32} \tag{6}$$

Hence, the relative error of M_R, δ_{MR}, is obtained from

$$\delta_{MR} = \frac{1}{h/r_c}\cdot\ln\sqrt{\frac{1+h/r_c}{1-h/r_c}} - 1 \tag{7}$$

The mutual inductance of the circular (M_C/H) cross-section (illustrated in Fig.1 (c)) is analogously analyzed by

$$M_C = \frac{n\mu_0 S}{\pi D}\cdot\frac{2}{1+\sqrt{1-(d/D)^2}} \tag{8}$$

where d/m and S/m^2 are diameter and area of the circular cross section (given $S=\pi d^2/4$); D/m is the average diameter (given $D=2r_C$). The relative error of M_C, δ_{MC}, is given by

$$\delta_{MC} = \frac{2}{1+\sqrt{1-(d/D)^2}} - 1 \tag{9}$$

If the stray capacitance between the wires is neglected (i.e. C_c=0), self-inductance, L_C/H, could be deduced as below.

$$L_C = Mn \tag{10}$$

Hence, the relative error of L_C, δ_L, is calculated by

$$\delta_L = \delta_M + \delta_n \tag{11}$$

To obtain high precision Rogowski coils sensor, the following design criteria must obey: constant winding density (n=constant, hence, δ_n=0); constant coil cross-section (S=constant); winding cross-section is perpendicular to the middle line. Therefore, the relative error of L_C (δ_L) is determined by δ_M (i.e. δ_{MR} or δ_{MC}). Equations (7), (9) and (11), indicates that the circular cross-section has an advantage over rectangular to reduce δ_L in the same ratio (for rectangular $x=h/r_C$, for circular $x=d/D$). As usual, when the ratio is less than 0.04 (i.e. x<0.04), the choice of rectangular cross-section over circular one is due to manufacturing constraints [6].

The paper takes the rectangular cross-section as an example for illustrating the influences of the parameters of the coil. Given diameter ratio $x=b/a$, the self-inductance (L_C/H) and resistor (R_C/Ω) of the coil can be calculated respectively by

$$L_C = nM_R = \frac{n^2\mu_0 c}{\pi}\cdot 1nx \tag{12}$$

$$\mathrm{R_C} = 16\rho a^2\frac{(x+1)[(x-1)/2+\frac{c}{a}]}{\lambda^3} \tag{13}$$

where ρ (Ω m) and λ (m) are the resistivity and diameter of the copper leads. Let parameter

c=0.010m, only changes in the diameter ratio x and n, the graph of L_C (y-axis denotes L_C) vs. x (x-axis denotes diameter ratio) can be shown in Fig.2, which indicates that L_C will obviously increase with increasing of x or n.

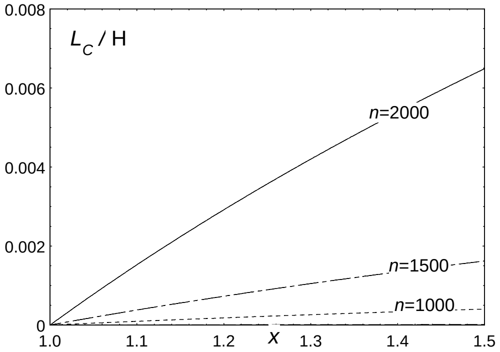


Fig.2 Relation curves of L_C-x (c=0.010m)

When the stray capacitance of the coil is omitted, by definition, the transfer function of the Rogowski coil sensor without an integrator can be deduced as following:

$$H(s)=\frac{u_S(s)}{I_1(s)}=-\frac{R_S}{n}\cdot\frac{s}{s+\alpha} \qquad 14)$$

where s is the Laplace variable, $\alpha=(R_S+R_C)/nM$. Hence, the time responses for unit step input is given by

$$u_S(t)=-\frac{MR_S}{R_S+R_0}\cdot(1-e^{-\alpha t}) \qquad (15)$$

According to geometrical property of the coil, some important equations can be obtained from

$$\lambda=\frac{\pi D}{n} \qquad (16)$$

$$R_C=\frac{16\rho(c+h)n^3}{\pi^3 D^2} \qquad (17)$$

$$\alpha=\frac{16\rho n^3(c+h)+\pi^3 D^2 R_s}{2\mu_0\pi^2 n^2 cD^2\ln\sqrt{(1+\frac{2h}{D})/(1-\frac{2h}{D})}} \qquad (18)$$

Let D=0.100m, R_S=1500Ω, h=0.005m, c=0.010m, based on Eqs. (15) and (18), the unit step response and bode diagram of the sensor are illustrated in Fig.3 and 4 by means of MATLAB software (discussed by references in [6~8]), only changes in the number of turns (n). Figures 2, 3 and 4 indicate that to increase the number of turns increases the output inductance of the coil, which limits the high-frequency performance of the sensor. Figures 3 and 4 indicate that the output inductance of the coil forms a low-pass filter, affecting the dynamic properties of the sensor. Thus to achieve good high-frequency performance, there is a limit on the acceptable coil output inductance, and hence on the number of turns used.

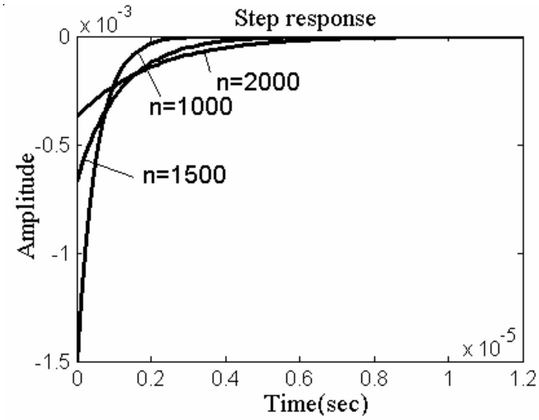


Fig.3Unit step response of Rogowski coil (C_C=0,only changes of n)

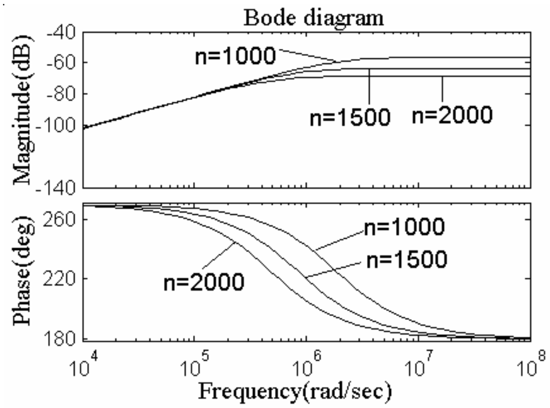


Fig.4 Bode diagram of Rogowski coil (C_C=0, only changes of n)

For a densely wound coil, the stray capacitance (C_C/F) between the shield and the coil windings can be approximately calculated by the capacitance of the cylindrical capacitor:

$$C_C\approx\frac{5\pi^2\varepsilon(c+h)D}{\lambda\ln x} \qquad (19)$$

where ε is the permittivity of the dielectric between two plates (F/m), diameter ratio $x=b/a$. The stray capacitance can be effectively minimized if the diameter ratio is more than one (i.e. $x>1$) (discussed by reference [1]). Based on Eq. (19), it is concluded that the stray capacitance can obviously decrease by increasing λ and x, or decreasing c, h and D.

Taken into account the stray capacitance, the transfer function of the coil without integrator is

$$H_C(s)=\frac{u_S(s)}{I_1(s)}=-\frac{Ms}{L_C C_C(s-s_1)(s-s_2)} \qquad (20)$$

where $\omega_n^2=(R_C+R_S)/(L_C R_S C_C)$, ξ is damper ratio, and given $\xi\omega_n=(L_C+R_S R_C C_C)/(2L_C R_S C_C)$, s_1 and s_2 are two characteristic roots, and given $s_{1,2}=-\xi\omega_n\pm\sqrt{(\xi\omega_n)^2-\omega_n^2}$. Hence, the unit step responses of the coil can be given by

$$u_S(s)=\frac{H_C(s)}{s}=-\frac{M}{L_C C_C(s-s_1)(s-s_2)} \qquad (21)$$

Therefore, the time responses for unit step input is

$$u_S(t)=\frac{M}{L_C C_C}\frac{\exp(s_2 t)-\exp(s_1 t)}{s_1-s_2} \qquad (22)$$

Based on analysis above, it is concluded that the coil without an integrator is stable owing to the product of ξ and ω_n is positive (i.e. $\xi\omega_n>0$). For guaranteeing good performances to the sensor, such electromagnetic parameters: R_C, R_S, L_C, M, C_C must be selected carefully. However, these parameters have a direct relationship with dimension parameters of the coil. In the case of damper ratio $\xi=0.707$, the optimal terminal resistance R_S/Ω is

$$R_S=\sqrt{\frac{L_C^2}{2L_C C_C-C_C^2 R_C^2}} \qquad (23)$$

Let D=0.100m, R_S=1500Ω, h=0.005m, c=0.010m, the unit step response and bode diagram of the sensor are illustrated in Fig.5 and 6 respectively, only changes in the number of turns (n).

Compared figures 5 and 6 with figures 3 and 4, it can be shown that the stray capacitance will give direct influence to the dynamic properties of the sensor. Based on Figures 3~6, it is shown that to achieve satisfactory high-frequency performance of the sensor, there are limits on the acceptable turns number and stray capacitance, and hence on the dimensions of the coil used.

Let D=0.100m, n=1000, c=0.010m, and h=0.010m, the unit step response and bode diagram of the sensor are illustrated in Fig.7 and 8 respectively, only changes in the terminal resistance (R_S). It can be concluded that the terminal resistance R_S with output inductance of the coil forms a low-pass filter, affecting the dynamic properties of the sensor including its phase and magnitude. Thus to prevent the coil inductance from resonating with the integrator amplifier input, the coil needs to terminate with an appropriate terminal resistance R_S. Other researches indicate that with increasing c and h will give bad influence to the high-frequency performance of the sensor.

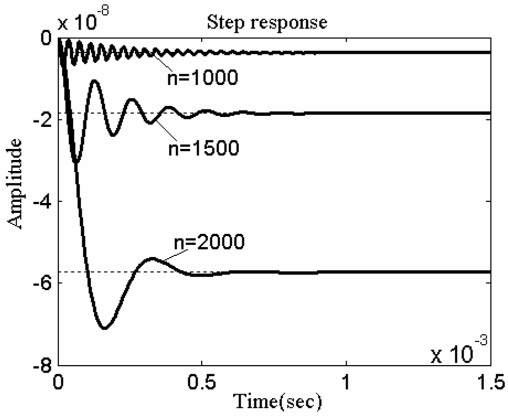


Fig.5 Unit step response of Rogowski coil ($C_C\neq 0$, only change of n)

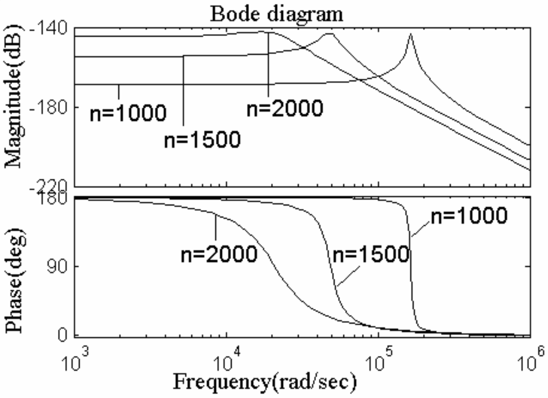


Fig.6 Bode diagram of Rogowski coil ($C_C\neq 0$, only change of n)

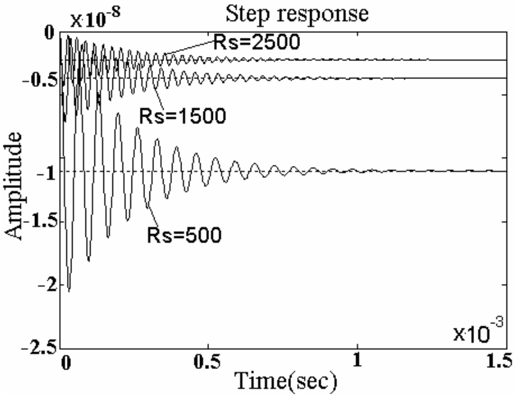


Fig.7 Unit step response of Rogowski coil (only change of R_S)

Analysis above indicates that the dynamic performance of the sensor will be directly influenced by electromagnetic parameters (i.e. R_C, R_S, C_S, and L_C), and hence influenced by dimension parameters (i.e. *h*, *c*, *a*, *b* and *D*). Therefore, it is indispensable to carefully choose the dimension parameters of the coil in designing a proper Rogowski coil for the heavy pulse currents measurement application.

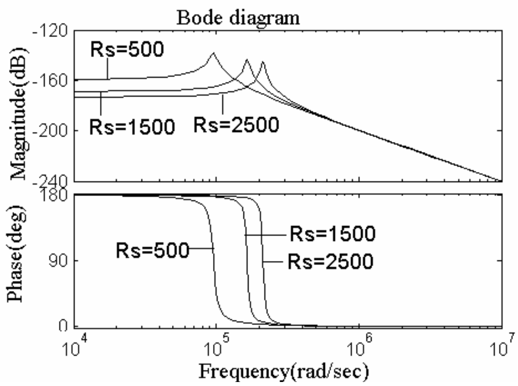


Fig.8 Bode diagram of Rogowski coil (only change of R_S)

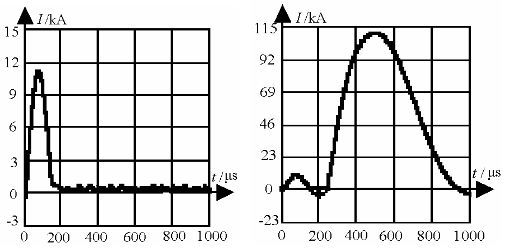
3. MEASUREMENT RESULTS

As usual, the high voltage and heavy current with high d*i*/d*t* is indispensable to PSM (power supply module) of high-power laser source. The main electric circuits of each PSM consist of two parts: preionization loop and primary discharge loop. Some specifications of the experimental equipment of PSM are listed in references [6~8]. Some optimized parameters (i.e. R_S, R_C, *a*, *b*, *h*, *c*, *D*, r_C, λ) of the prototype have been illustrated in Table 1.

Figures.9 represents the measurement results of applying the proposed coil to measure both preionization loop and primary discharge loop. Figures.9 (a) illustrates the pulse current waveform of preionization loop, whose duration and current peak value are 140μs and 11.5kA. Waveform of primary discharge loop is shown in Figures.9 (b), whose duration and current peak value are 600μs and 111.5kA respectively. And primary discharge loop is preceded by the preionization loop 250μs. The dynamic response of the sensor is much faster than the process. Consequently, its time constants and dead time can often be considered negligible. Comparing with the computing results, the measurement results are accurate and reliable. The current measurement instrument has been successfully used to measure the laser source in the Pulse Power Center (PPC) of HUST for more than three years.

TABLE 1
The typical parameters of the prototype Rogowski coil sensor

Definitions	*Symbols and value*
Average radius of the coil/m	$r_C=0.055$
Broadness of the coil/m	$h=2.5\times10^{-3}$
Thickness of the coil/m	$c=7.6\times10^{-3}$
Cross section area/m²	$S\approx80\times10^{-6}$
Total number of turns/ Turn	$n=1400$
Diameter of copper leads/m	$\lambda=2.5\times10^{-4}$
Permeability of the free space/ H/m	$\mu_0=4\pi\times10^{-7}$
Mutual inductance of coil/ μH	$M=0.399$
Stray capacitance of coil / μF	$C_C=0.031$
Terminating resistance of the coil/ kΩ	$R_S=1.5$
Resistance of coil/Ω	$R_C=18.2$
Self-inductance of coil/ μH	$L_C=559.7$
Damper ratio	ξ **=0.707**



(a) *(b)*

Fig.9 Measurement results of currents of both preionization loop and primary discharge loop

4. CONCLUSIONS

The dynamic properties of the Rogowski coil will be directly influenced not only by electromagnetic parameters: R_C, R_S, C_C, L_C, but also by dimension parameters: *h*, *c*, *a*, *b*, *D*. The paper presents some analyses and considerations for designing an appropriate Rogowski coil with high-frequency performance. These thoughts are employed in the design of the sensor for measuring heavy pulse currents of the laser source. And a prototype coil is implemented. Experimental results demonstrate the feasibility of the proposed approaches for measuring pulsed currents. It is shown that the methods provide considerable reliability and accuracy.

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