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FAST CALCULATION OF ALL STABILIZING GAINS FOR DISCRETE-TIME SYSTEMS

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ABSTRACT

In this paper, two methods for calculating all stabilizing gains for discrete-time systems are given. The first method focuses on converting the problem using a bilinear transformation and then applying a previously developed theorem for continuous time systems. Unlike previous results, the method introduced here does not use the Generalised Hermite-Biehler Theorem and therefore provides a computational advantage. The second method demonstrates the use of Chebyshev Polynomials in the solution of the problem.

Keywords: *Bilinear Transformation, Nyquist's Criterion, Chebyshev Polynomials*

1. INTRODUCTION

Calculation of all stabilizing low-order controllers for linear time invariant systems has attracted a lot of attention in the recent years [1-9]. Recently in [3], a generalized version of the Hermite-Biehler theorem to find all stabilizing P, PI and PID compensators for continuous time systems has been used. This result has then been extended to discrete-time systems to provide a general formulation of stabilizing low-order controllers [2, 4, 5, 9]. An inherent problem with the application of the Generalised Hermite-Biehler theorem is that it requires a search in an exponentially growing set.

Therefore a possible alternative to this approach, where a generalised Nyquist criterion is used, was presented for continuous time systems in [7,8]. This paper extends these results to discrete time systems and provides an alternative to the Generalised Hermite-Biehler theorem based approaches [2, 4, 5, 9].

Two methods are introduced to find the entire set of stabilizing gains. The first method requires the bilinear transformation to solve the problem of constant gain stabilization of a discrete time control system. Furthermore, this method suggests to determine the number of the unstable poles for gain intervals obtained by calculating the location and direction of the crossing of the Nyquist plot with the real axis. The second method requires to calculate the Chebyshev representation of a given system by the use of the

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first and second kinds of Chebyshev polynomials. With all such calculations, we obtain ordinary real polynomials. Since Nyquist stability criterion is used, imaginary part of the Chebyshev representation of a given system is equalized to zero to find the intersections of the Nyquist plot with the real axis and then, the values found are substituted into real part of the Chebyshev representation of the system. Thus, the entire set of stabilizing gains are determined by the use of Nyquist stability criterion. Furthermore, when Chebyshev polynomials are used, the solution of the problem of stabilization is reduced to a set of linear equalities.

2. STABILITY ANALYSIS USING THE BILINEAR TRANSFORMATION

The bilinear transformation is a method that can be used to determine the Schur stability of discrete-time control systems. This method requires transformation from the z plane to another complex w plane. The problem of calculating the Schur stability of discrete-time control systems can be converted to an equivalent Hurwitz stability problem via the bilinear transformation [9]. Several different bilinear transformations can be used for this purpose. The bilinear transformation that is used in this paper is given by

$$z = \frac{w+1}{w-1} \qquad (1)$$

When equation (1) is solved for w, it gives

$$w = \frac{z+1}{z-1} \qquad (2)$$

and maps the inside of the unit circle in the z plane into the left half of the w plane. Let

$$\delta(z) = a_n z^n + a_{n-1} z^{n-1} + ... + a_1 z + a_0 \qquad (3)$$

be a polynomial of degree n with real coefficients. Then, the bilinear transformation of $\delta(z)$ is given by

$$W\{\delta(z)\} = \frac{\delta(w)}{(w-1)^n} \qquad (4)$$

where $\delta(w) = b_m w^m + b_{m-1} w^{m-1} + ... + b_1 w + b_0$ is a polynomial of degree *m*.

Recently, in continuous time control systems a new and simple computational method has been developed for the construction of the set of low-order compensators [6,7]. This compensators ensure the closed-loop system poles lie in a given stability region. In this paper, the method developed in [6] is modified for discrete-time control systems via the bilinear transformation and the results are given in Theorem 1. Thus, a simple method is suggested to compute all of the stabilizing set of low-order compensators for a given discrete time system using Theorem 1. This method has several advantages over the technique of [3]. The most important advantage is that the suggested method is computationally much faster than that of [3], particularly for high order systems. In Theorem 1, a search in an exponentially growing set does not require to find $k \in K_i$ intervals. These are advantages over the technique of [3].

Now, consider the control system shown in Figure 1 where the plant is represented by its discrete-time transfer function $P(z)$

$$P(z) = \frac{N(z)}{D(z)} \qquad (5)$$

with $N(z)$ and $D(z)$ are polynomials with real coefficients. Constant gain controller is given by

$$C(z) = K \qquad (6)$$

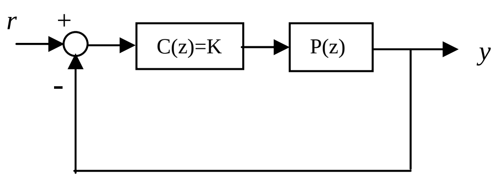

Figure 1. Closed loop system with constant gain

The characteristic polynomial of the closed loop system is obtained by the following equation.

$$\delta(z) = D(z) + KN(z) \qquad (7)$$

Applying the bilinear transformation to $P(z)$, we have

$$P(w) = \frac{N(w)}{D(w)} \qquad (8)$$

where $P(w)$ represents the new plant in the w-domain. Similarly, applying the bilinear transformation to $\delta(z)$, we have $\delta(w) = D(w) + KN(w)$. If we substitute $w \hat{=} j\omega$ into equation (8), we have

$$P(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \frac{N_{re} + jN_{im}}{D_{re} + jD_{im}} \tag{9}$$

where $D_{re} \hat{=} \mathrm{Re}\{D(j\omega)\}$, $D_{im} \hat{=} \mathrm{Im}\{D(j\omega)\}$ and N_{re} and N_{im} are defined similarly. By noting that

$$D_{re} = D_e(-\omega^2) \qquad D_{im} = D_o(-\omega^2)\omega \tag{10}$$
$$N_{re} = N_e(-\omega^2) \qquad N_{im} = N_o(-\omega^2)\omega \tag{11}$$

It is possible to write

$$\begin{aligned} P(j\omega) &= \frac{N_e + j\omega N_o}{D_e + j\omega D_o} \\ &= \frac{D_e N_e + D_o N_o \omega^2}{D_e^2 + D_o^2 \omega^2} + j\omega \left[\frac{D_e N_o - D_o N_e}{D_e^2 + D_o^2 \omega^2} \right] \\ &= \frac{X(\omega^2)}{Z(\omega^2)} + j\omega \frac{Y(\omega^2)}{Z(\omega^2)} \end{aligned} \tag{12}$$

where,

$$X(\omega^2) \hat{=} D_e N_e + D_o N_o \omega^2 \tag{13}$$
$$Y(\omega^2) \hat{=} D_e N_o - D_o N_e \tag{14}$$
$$Z(\omega^2) \hat{=} D_e^2 + D_o^2 \omega^2 \tag{15}$$

and where for notation purposes D_e, D_o, N_e, and N_o are used instead of $D_e(-\omega^2)$, $D_o(-\omega^2)$, $N_e(-\omega^2)$, and $N_o(-\omega^2)$, respectively. The imaginary part of $P(j\omega)$ is given by $\mathrm{Im}\{P(j\omega)\} = \omega \frac{Y(\omega^2)}{Z(\omega^2)}$. By denoting $v \hat{=} \omega^2$, and the positive real roots of $Y(v)$ as $v_1^*, v_2^*, \ldots, v_\gamma^*$ it is obvious that the Nyquist plot of $P(j\omega)$ crosses the real axis only if $\omega = 0$, $\omega = \infty$, or $\omega = \pm\sqrt{v_i^*}$ for $i = 1,2,\ldots,\gamma$. Therefore, denoting $v_{\gamma+1}^* = 0$ and $v_{\gamma+2}^* = \infty$, the real axis crossing points are found as $x_i = X(v_i^*)/Z(v_i^*)$ for $i = 1,2,\ldots,\gamma+2$. Relabeling the pairs (x_i, v_i^*) (for $i = 1,2,\ldots,\gamma+2$) as $(x_i, v_{i,j}^*)$ (for $i = 1,2,\ldots,q$) such that $x_i < x_{i+1}$ and $x_i = X(v_{i,j}^*)/Z(v_{i,j}^*)$ (for all $j = 1,2,\ldots,p_i$), it is possible to state the following theorem.

Theorem 1

Consider a linear time-invariant system given by a proper rational transfer function $P(w) = N(w)/D(w)$ given as in (8), and assume that $D(w)$ has no roots on the imaginary axis. Let $X(\omega^2)$, $Y(\omega^2)$, and $Z(\omega^2)$ be polynomials as defined equations (13)-(15), and the pairs $(x_i, v_{i,j}^*)$ $(i = 1,2,\ldots,q)$ be as defined above. Furthermore, denote the first coefficient of $Y(v)$ as y_1, and the last nonzero coefficient of $Y(v)$ as y_0. Then, for a given gain $k \in K_i \hat{=} (-1/x_{i-1}, -1/x_i)$, the number of unstable poles of the closed-loop system is given by

$$u_i = u_0 + \sum_{t=1}^{i-1} r_t \tag{16}$$

where u_0 is the number of unstable poles of $P(w)$,

$$r = \sum_{j=1}^{p_i} d_{i,j} \tag{17}$$

and

$$d_{i,j} = \begin{cases} (1-(-1)^l)Sgn(Y^{(l)}(v_{i,j}^*)) & if \quad 0 < v_{i,j}^* < \infty \\ Sgn(y_0) & if \quad v_{i,j}^* = 0 \\ -Sgn(y_1) & if \quad v_{i,j}^* = \infty \end{cases} \tag{18}$$

in which $Y^{(l)}(v_{i,j}^*)$ is the first nonzero derivative of $Y(v)$ at the point $v_{i,j}^*$. Theorem 1 can easily be extended to cover systems with imaginary axis poles [8].

3. STABILITY ANALYSIS USING THE CHEBYSHEV POLYNOMIALS

The first and second kinds of Chebyshev polynomials can be used to solve Schur stabilization problem of discrete-time control systems. Thus, Chebyshev representation of a discrete-time system is obtained by using the first and second kinds of Chebyshev polynomials. This problem results in a determination of the entire set of stabilizing gains as a solution of sets of linear inequalities. For this aim, a procedure has been developed to determine the phase unwrapping of real polynomial or rational function along the unit circle in [4], but this procedure requires analysis of exponentially growing signs. In this section, Nyquist stability criterion is applied to Chebyshev representation of the system. The set of stabilizing gains are determined by computation of the intersections of the Nyquist plot with the real axis. For the stability analysis with respect to the unit circle, it will be necessary to determine the unit circle image of $\delta(z)$ given by equation (3). It is possible to write

$$\{\delta(z): \quad z = e^{j\theta} \quad 0 \le \theta \le 2\pi \} \tag{19}$$

As the coefficients a_i are real, $\delta(e^{j\theta})$ and $\delta(e^{-j\theta})$ are conjugate complex numbers, and so it suffices to determine the image of upper half of the unit circle

$$\{\delta(z): \quad z = e^{j\theta} \quad 0 \le \theta \le \pi \} \tag{20}$$

Since

$$z^k \Big|_{z=e^{j\theta}} = \cos k\theta + j \sin k\theta \tag{21}$$

it is possible to write

$$\begin{aligned} \delta(e^{j\theta}) &= (a_n e^{j\theta n} + a_{n-1} e^{j\theta n-1} + ... + a_1 e^{j\theta_1} + a_0) \\ &= (\underbrace{a_n \cos n\theta + ... + a_1 \cos\theta + a_0}_{\bar{R}(\theta)}) \\ &\quad + j(\underbrace{a_n \sin n\theta + ... + a_1 \sin\theta}_{\bar{I}(\theta)}) \\ \delta(e^{j\theta}) &= \bar{R}(\theta) + j\bar{I}(\theta) \end{aligned} \tag{22}$$

$\cos k\theta$ and $\sin k\theta / \sin\theta$ can be written as polynomials in $\cos\theta$ using Chebyshev polynomials. Write $t = -\cos\theta$. Then as θ runs from $0 \to \pi$, t runs from –1 to +1. From $\sin^2\theta + \cos^2\theta = 1$, we have

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-t^2} \tag{23}$$

From (21) and (23), we obtain

$$z \hat{=} e^{j\theta} = -t + j\sqrt{1-t^2} \tag{24}$$

3.1. The First Kind Of Chebyshev Polynomials

The first kind of Chebyshev polynomials is defined for $t \in [-1,+1]$ and they have the following form [5, 10].

$$t=-\cos\theta \qquad c_k(t)=\cos k\theta \qquad k=1,2,3,.... \tag{25}$$

Using (25), we find the first six of the first kind of Chebyshev polynomials as follows.

$$\begin{aligned} c_1(t) &= -t \\ c_2(t) &= 2t^2 - 1 \\ c_3(t) &= -4t^3 + 3t \\ c_4(t) &= 8t^4 - 8t^2 + 1 \\ c_5(t) &= -16t^5 + 20t^3 - 5t \\ c_6(t) &= 32t^6 - 48t^4 + 18t^2 - 1 \end{aligned} \tag{26}$$

3.2. The Second Kind Of Chebyshev Polynomials

The second kind of Chebyshev polynomials has the following form for $t \in [-1,+1]$

$$s_k(t) = \frac{\sin[k \ \cos^{-1} t]}{(1-t^2)^{1/2}} = \frac{\sin k\theta}{\sin\theta} \tag{27}$$

Using (27), we find the first six of the second kind of Chebyshev polynomials as follows

$$\begin{aligned} s_1(t) &= +1 \\ s_2(t) &= -2t \\ s_3(t) &= 4t^2 - 1 \\ s_4(t) &= -8t^3 + 4t \\ s_5(t) &= 16t^4 - 12t^2 + 1 \\ s_6(t) &= -32t^5 + 32t^3 - 6t \end{aligned} \tag{28}$$

Substituting equations (25) and (27) into equation (22), we have

$$\delta(e^{j\theta})\big|_{t=-\cos\theta} = \underbrace{[a_n c_n(t) + a_{n-1}c_{n-1}(t) + ... + a_1 c_1(t) + a_0]}_{R(t)} + j\sqrt{1-t^2}\underbrace{[a_n s_n(t) + a_{n-1}s_{n-1}(t)... + a_1 s_1(t)]}_{T(t)} \quad (29)$$

$$\delta(e^{j\theta})\big|_{t=-\cos\theta} = R(t) + j\sqrt{1-t^2}\; T(t) \hat{=} \delta_c(t)$$
(30)

$\delta_c(t)$ refers to Chebyshev representation of $\delta(t)$. $R(t)$ and T(t) are real polynomials with leading coefficients of opposite sign and equal magnitude. Respectively, $R(t)$ and T(t) have the following form.

$$R(t) = a_n c_n(t) + a_{n-1}c_{n-1}(t) + ... + a_1 c_1(t) + a_0 \quad (31)$$

$$T(t) = a_n s_n(t) + a_{n-1}s_{n-1}(t)... + a_1 s_1(t) \quad (32)$$

3.3.Stability Of Closed-Loop System With Constant Gain

In this section, we will apply the results given in Section 3.1 and Section 3.2 to the problem of constant gain stabilization of a discrete-time control system. The control system in Figure 1 is considered again. Determining the entire set of constant gains to stabilize $\delta(z)$ given in (7) is our aim. Therefore, Chebyshev representations of $N(z)$ and $D(z)$ are found. From equation (30), respectively Chebyshev representations of $N(z)$ and $D(z)$ are given below.

$$N(e^{j\theta})\big|_{t=-\cos\theta} = R_N(t) + j\sqrt{1-t^2}\,T_N(t) \quad (33)$$

$$D(e^{j\theta})\big|_{t=-\cos\theta} = R_D(t) + j\sqrt{1-t^2}\,T_D(t) \quad (34)$$

Substituting equations (33) and (34) into equation (5), we have

$$P(t) = \frac{N(t)}{D(t)} = \frac{R_N(t) + j\sqrt{1-t^2}\,T_N(t)}{R_D(t) + j\sqrt{1-t^2}\,T_D(t)} \quad (35)$$

When we multiply the numerator and denominator of the last equation by the complex conjugate of the denominator, equation (35) becomes

$$P(t) = \frac{\lfloor R_N(t)R_D(t) + (1-t^2)T_N(t)T_D(t)\rfloor}{R_D{}^2(t) + (1-t^2)T_D{}^2(t)} + j\sqrt{1-t^2}\,\frac{[R_D(t)T_N(t) - R_N(t)T_D(t)]}{R_D{}^2(t) + (1-t^2)T_D{}^2(t)} \quad (36)$$

Respectively, the real and imaginary parts of $P(t)$ are given below.

$$\mathrm{Re}\{P(t)\} = \frac{\lfloor R_N(t)R_D(t) + (1-t^2)T_N(t)T_D(t)\rfloor}{R_D{}^2(t) + (1-t^2)T_D{}^2(t)} \quad (37)$$

$$\mathrm{Im}\{P(t)\} = \sqrt{1-t^2}\,\frac{[R_D(t)T_N(t) - R_N(t)T_D(t)]}{R_D{}^2(t) + (1-t^2)T_D{}^2(t)}$$
(38)

According to Nyquist stability criterion, $\mathrm{Im}\{P(t)\}$ is equalized to zero to find the intersections of the Nyquist plot with the real axis and then, the values found are substituted into $\mathrm{Re}\{P(t)\}$. The result is

$$\mathrm{Im}\{P(t)\} = 0 \quad \text{and} \quad \mathrm{Re}\{P(t)\} = \sigma \quad (\sigma \in \Re) \quad (39)$$

Thus, the characteristic equation of the closed loop system is

$$\frac{1}{K} + P(t) = 0 \quad (40)$$

Due to the fact that K is equal to $-1/\sigma$, the Nyquist plot intersects the real axis at the $-1/K$ point. From eqs. (38) and (39), we can write

$$R_D(t)T_N(t) - R_N(t)T_D(t) = 0 \quad (41)$$

$$1 - t^2 = 0 \quad (42)$$

From equation (42), we compute that t is equal to ± 1. When $t = \pm 1$ and from equation (41), real values lying in $t \in [-1,+1]$ are substituted into equation (37), we find the intersections of the Nyquist plot with the real axis and gain intervals not changing the number of unstable poles. The gain interval being equal o to zero is interval stabilizing the closed loop system. Thus, the entire set of stabilizing gains are determined.

4. EXAMPLE

Consider the example given in [9].

$$P(z)=\frac{N(z)}{D(z)}=\frac{100z^3+2z^2+3z+11}{100z^5+2z^4+5z^3-41z^2+52z+70}$$

The set of Hurwitz stabilizing K's are to be calculated by both using of bilinear transformation and using of Chebyshev polynomials

4.1. Solution Of The Problem Using the Bilinear Transformation

Substituting (1) into the numerator and denominator polynomials, we have

$$N(w)=116w^5+34w^4-88w^3-300w^2+148w+90$$
$$D(w)=188w^5+46w^4+1880w^3+308w^2+652w+126$$

Applying the even-odd decompositions to the $N(w)$ and $D(w)$ polynomials, it is possible to write $(w \hat{=} j\omega)$

$$N_e(-\omega^2)=34\omega^4+300\omega^2+90$$
$$N_o(-\omega^2)=116\omega^4+88\omega^2+148$$
$$D_e(-\omega^2)=46\omega^4-308\omega^2+126$$
$$D_o(-\omega^2)=188\omega^4-1880\omega^2+652$$

from $v \hat{=} \omega^2$, we have

$$N_e(-v)=34v^2+300v+90$$
$$N_o(-v)=116v^2+88v+148$$
$$D_e(-v)=46v^2-308v+126$$
$$D_o(-v)=188v^2-1880v+652$$

and from eqs. (13)-(15), it is possible to write

$$X(v)=21808v^5-199972v^4-58656v^3-304840v^2+106576v+11340$$

$$Y(v)=-1056v^4-24160v^3+519232v^2-60896v-40032$$

$$Z(v)=35344v^5-704764v^4+3751216v^3-2345064v^2+347488v+15876$$

When we compute the roots of $D(w)$, we see that there are two unstable roots. Hence u_0 is equal to 2. The positive real roots of $Y(v)$ are $v_1^*=0.345853$ and $v_2^*=13.4212$. Adding $v_3^*=0$ and $v_4^*=\infty$, there exist four crossing frequencies for this problem. The crossing points corresponding to these frequencies are given by x_1=7.91936, x_2=2.39371, x_3=0.714286, and x_4=0.61702. Relabeling the pairs (x_i, v_i^*), and noting that $Y'(0.345853)=289416$, $Y'(13.4212)=-939088$, $y_0=-40032$, $y_1=1$, the net crossing counts are calculated as shown in Table 1.

Table 1. Calculation of d_i, u_i and the stabilizing intervals

i	$v_i^*=0$	x_i	d_i	u_i	K_i
1	∞	0.61702	1	2	(0,∞)U(-∞,-1.6206)
2	0	0.71428	-1	3	(-1.6206, -1.3999)
3	13.4212	2.39371	-2	2	(-1.3999, -0.41776)
4	0.34585	7.91936	2	0	(-0.41776,-0.1263)
5	-	∞	-	2	(-0.1263, 0)

Forming K_i, and noting that u_0=0 , the number of unstable closed-loop system poles, u_i, are calculated from (16). An examination of Table 1 reveals that the closed-loop system is stable for gains $K\in(-0.41776,\ -0.1263)$, which agrees with the result of Xu in [9].

4.2. Solution Of The Problem Using Chebyshev Polynomials

Substituting the first five of the first kind of Chebyshev polynomials into $D(z)$, we obtain $R_D(t)$ as follows.

$$R_D(t)=-1600t^5+16t^4+1980t^3-98t^2-537t+113$$

Similarly, substituting the second kind of Chebyshev polynomials into $D(z)$ and $N(z)$, we have

$$T_D(t)=1600t^4-16t^3-1180t^2+90t+147$$
$$T_N(t)=400t^2-4t-97$$

From eqs. (37) and (38), respectively $\mathrm{Re}\{P(t)\}$ and $\mathrm{Im}\{P(t)\}$ are obtained as follows.

$$\mathrm{Re}\{P(t)\} = \frac{Num\{\mathrm{Re}\{P(t)\}\}}{Den\{\mathrm{Re}\{P(t)\}\}}$$

$$Num\{\mathrm{Re}\{P(t)\}\} = 8800t^5 - 1288t^4 + 3522t^3 - 13620t^2 - 9705t + 6621$$

$$Den\{\mathrm{Re}\{P(t)\}\} = 112000t^5 - 42720t^4 - 154584t^3 + 47104t^2 + 47451t - 17189$$

$$\mathrm{Im}\{P(t)\} = \frac{Num\{\mathrm{Im}\{P(t)\}\}}{Den\{\mathrm{Im}\{P(t)\}\}}$$

$$Num\{\mathrm{Im}\{P(t)\}\} = 2(\sqrt{1-t^2})\ (4400t^4 - 644t^3 - 10039t^2 - 1792t + 3071)$$

$$Den\{\mathrm{Im}\{P(t)\}\} = Den\{\mathrm{Re}\{P(t)\}\}$$

The roots of $\mathrm{Im}\{P(t)\}$ are obtained as follows.

t_1= -0.861315, t_2= 0.486046, t_3= +1, t_4= -1

t_5= -1.5646, t_6= 1.5781

the first four of this roots lie in [-1,+1] interval and are real. Substituting this roots lying in [-1,+1] into $\mathrm{Re}\{P(t)\}$, we obtain the intersections of the Nyquist plot with the real axis and gain intervals not changing the number of unstable poles. Thus, it is possible to show that

$$t_1=-0.861315 \Rightarrow \mathrm{Re}\{P(-0.861315)\} = 2.39372$$
$$t_2=0.486046 \Rightarrow \mathrm{Re}\{P(0.486046)\} = 7.91944$$
$$t_3=+1 \Rightarrow \mathrm{Re}\{P(+1)\} = 0.7142857$$
$$t_4=-1 \Rightarrow \mathrm{Re}\{P(-1)\} = 0.6170213$$

Gain intervals not changing the number of unstable poles of the closed-loop system are given below.

K < -1.606897	→ 2 unstable poles
-1.606897 < K < -1.4	→ 3 unstable poles
-1.4 < K < -0.4177598	→ 2 unstable poles
-0.4177598 < K <-0.12627	→ 0 unstable pole
-0.12627 < K	→ 2 unstable poles

It is clear that when we use Chebyshev polynomials, the solution of stabilization problem by constant gain is reduced the set of linear equalities. This is particularly advantageous for high order systems. As seen, the closed loop system is stable for $K \in (-0.4177598, -0.12627)$, which agrees with both the result obtained in Section 4.1 and the result of Xu in [9].

5. CONCLUSIONS

The problem of calculation of all stabilizing gains for discrete-time systems has been solved both using the bilinear transformation and Chebyshev polynomials in this paper. The methods show that the number of unstable closed loop system poles for a given constant gain can be calculated by the help of Nyquist stability criterion. We consider that the use of the bilinear transform or Chebyshev representation bring practical and theoretical advantages in the solution of the problem. Although it is not mandatory, the use of a symbolic algebra language can be helpful in applying the bilinear transformation and finding the Chebyshev representation.

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