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A COMPARATIVE STUDY ON ENTROPIC THRESHOLDING METHODS

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ABSTRACT

Image thresholding is an important task both for digital image processing applications and for pattern recognition. Image segmentation by thresholding is the simplest technique. In this study, we intent to carry out a comparative study of entropic thresholding methods. We examine several entropic thresholding methods which are the most popular in the literature. These methods are minimum cross entropy, maximum entropy sum method, Renyi's entropy and Havrda&Charvat entropy. We perform experiments for demonstrating the effectiveness of the examined methods on image thresholding.
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Keywords: *Image thresholding, minimum cross entropy, Renyi's entropy, maximum entropy sum method, Havrda-Charvat entropy*

1. INTRODUCTION

Image thresholding is an important task both for digital image processing applications and for pattern recognition. It is also regarded as the first step for image understanding. Generally, a well segmented image will yield good visualization and increase the accuracy and efficiency of the subsequent processing. Image thresholding is the simplest technique and involves the assumption that the object and the background in the image have distinct gray level distributions. Segmentation is than performed by assigning the pixels having gray levels below the threshold to the background and other pixels having gray levels above the threshold to the object, or vice versa.

Entropy is the measure of information content in probability distribution. On the other hand, Shannon defined the entropy of a system as the measure of uncertainty about its actual structure [1]. In recent years, information theoretic approaches based on Shannon's entropy concept have received considerable interest. Moreover, several different entropy definitions have been proposed over the last two decades [2-12]. The first entropy based method was proposed by Pun [2]. Kapur et al. proposed a new entropic thresholding method [3]. This method is similar to Pun's method. Both of the methods maximize the a priori entropies of the object and the background classes. Li and Lee proposed a cross entropy thresholding method [4]. This method provides an unbiased estimate of a binarized version of the image in an information theoretic sense. N. R. Pal than proposed another cross

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entropy thresholding method where image histogram is modeled by a mixture of Poisson distributions [5]. Brink et al. showed the relationship of the minimum cross entropy to Otsu's measure, cross correlation and Kapur's chi-square method [6]. Aforesaid methods are one-dimensional (1D) approach based on the entropy of the image gray level histogram. Two-dimensional (2D) entropic techniques using local neighborhood as well as point pixel information have been proposed by Abutaleb [7]. Pal and Pal have used a gray level co-occurrence matrix [8]. Brink also used two-dimensional entropic thresholding for digital images [9]. Recently, Sahoo et al. proposed a thresholding method based on two-dimensional Renyi's entropy [10]. Sezgin et al. [13] conduct an exhaustive survey of image thresholding methods. They give an extensive classification of the entropic thresholding methods.

In this paper, we intent to carry out a comparative study of entropic thresholding methods. For the purpose of comparison, we examine several entropic thresholding methods that are given at the next section. We use several synthetically generated histograms on the normal distributions and we also consider the histogram of real world images. The comparison results are represented. The organization of this paper is as follows, in section 2, we give a brief description of the digital image model and 1-D entropy definitions. Several different entropy thresholding methods are examined in this section as well. In section 3, experimental studies and the results are presented. In section 4, we present the conclusions and discussions.

2. IMAGE MODEL AND 1-D ENTROPY DEFINITIONS

Let $f(x,y)$ be the gray value of the pixel located at the point (x,y). In a digital image $\{f(x,y) \mid x \in \{1,2,3,...,M\}, y \in \{1,2,3,...,N\}\}$ of size *MxN,* let the histogram be *h(m)* for $m \in \{0,1,2,3,...,255\}$. We denote the set of all gray levels $\{0,1,2,3,...,255\}$ as *G*. Suppose *t* is an assumed threshold. t partition the image into two regions, namely the object and the background. Let $p_i = n_i / NM$ be the estimate of the probability gray-level value, where n_i represents the number of pixels with gray level value i. Thus the a priori entropy of the entry image is defined by $H = -\sum_{i=0}^{255} p_i \log(p_i)$. Here, p_i is the probability of occurrence of pixel i, and the condition of $\sum_{i=1}^{n} p_i = 1$ must be satisfied.

2.1. Minimum Cross Entropy Method (MCE)

Li and Lee proposed a cross entropy method for gray level image thresholding [4]. This method uses the Kullback's information theoretic distance *D* between two probability distributions. *D* is also known as directed divergence or cross entropy. Let $P = \{p_1, p_2, ..., p_n\}$ and $Q = \{q_1, q_2, ..., q_n\}$ be two probability distributions then the cross entropy is defined as follows;

$$D(P,Q) = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i} \qquad (1)$$

D(P,Q) is not symmetric. The symmetric version of the cross entropy (S-MCE) was proposed by Pal [5]. The symmetric cross entropy is defined as follows;

$$D_t(P,Q) = \sum p_i \log \frac{p_i}{q_i} + \sum q_i \log \frac{q_i}{p_i} \qquad (2)$$

In Ref. [5], Pal gave the details of the method. He used Poisson distribution for modeling the gray level histogram. In Ref. [6], Brink et al. also used a symmetric cross-entropy for thresholding.

2.2. Maximums Entropy Sum Method

The maximum entropy sum method proposed by Kapur et al. [3] is based on the maximization of the information measure between object and background. The a priori entropy of the entry image is calculated as follows;

$$H_M = -\sum p_i \ln(p_i) \qquad (3)$$

If we assume C_1 and C_2 for the object and background classes respectively, the maximum entropy can be calculated as follows;

$$\begin{aligned} H_{C_1}(t) &= -\sum_{i=0}^{t} \frac{p_i}{p(C_1)} \ln \frac{p_i}{p(C_1)} \\ H_{C_2}(t) &= -\sum_{t+1}^{255} \frac{p_i}{p(C_2)} \ln \frac{p_i}{p(C_2)} \end{aligned} \qquad (4)$$

Where $p(C_1)=\sum_{i=0}^{t} p_i \ and \ p(C_2)=\sum_{i=t+1}^{255} p_i$ and $p(C_1) + p(C_2) = 1$ must be satisfied. Thus, the threshold value t is selected that maximizes the sum of $H_{C1}(t)$ and $H_{C2}(t)$

2.3. Renyi's Entropy

The Renyi's entropy [14] is a one parameter generalization of the Shannon's entropy since $\lim_{\alpha \to 1} H_T^{\alpha} = H_T$ is defined as follows;

$$H_T^{\alpha} = \frac{1}{1-\alpha} \ln \sum (p_i)^{\alpha} \quad (5)$$

Here H_T^{α} denotes Renyi's entropy and H_T denotes the Shannon's entropy. The Renyi's entropies associated with the object (C_1) and background (C_2) classes are calculated by;

$$\begin{aligned} H_{C1}^{\alpha}(t) &= \frac{1}{1-\alpha} \ln \sum_{i=0}^{t} \left(\frac{p_i}{p(C_1)} \right)^{\alpha} \\ H_{C2}^{\alpha}(t) &= \frac{1}{1-\alpha} \ln \sum_{i=t+1}^{255} \left(\frac{p_i}{p(C_2)} \right)^{\alpha} \end{aligned} \quad (6)$$

$p(C_1) \ and \ p(C_2)$ can be calculated as described at previous section. The optimum threshold value t is chosen which maximizes $H_{C1}^{\alpha}(t) + H_{C2}^{\alpha}(t)$.

2.4. The Havrda and Charvat Entropy

Let $P = (p_1, p_2, p_3, ..., p_n) \in \Delta_n$, where

$$\begin{aligned} \Delta_n &= \{P = (p_1, p_2, p_3, ..., p_n) : p_i \geq 0, \\ i &= 1,2,3,..., n, n \geq 2, \sum_{i=1}^{n} p_i = 1\} \end{aligned} \quad (7)$$

is a set of discrete finite probability distributions. Havrda and Charvat [15] defined entropy of a discrete finite probability distribution P as;

$$H_T^r = \left(\frac{1}{2^{1-r} - 1} \right) \left[\sum_{i=1}^{n} p_i^r - 1 \right] r \neq 1, r > 0 \quad (8)$$

where r is a real-valued parameter.

approach that were taken by Li et al. on the normal distributions as given in reference (4).

3. EXPERIMENTAL RESULTS

For the purpose of comparison, we use several synthetically generated histograms that the same we also consider the histogram of real world images. Figure 1 shows the synthetic histograms and Table 1 depicts the threshold values obtained by examined methods. Figure 2 also shows the real world images and their histograms. The images are *rice.tif*, *fingerprint .tif*, *blood1.tif*. Image thresholding is an optimization process of valley seeking on the histogram of the given gray level image. This valley value can be detected heuristically from the histogram. When we consider the first synthetically generated histogram, the optimum threshold value is 56. Both of the minimum cross entropy methods perform the best results. On the other hand Renyi's entropy measure method also yields considerable result on the first synthetic histogram. At the second and third synthetic histograms Renyi's entropy gives the optimum threshold values. Actually when we consider the third histogram, all examined methods generate reasonable results. On the other hand, at the rice image, Pal's S-MCE method gives the appropriate result. The better results are also generated by MCE and S-MCE methods at the fingerprint image. The optimum threshold value for the Blood1 image is performed by Li and Lee's method.

4. CONCLUSIONS

In this paper, we have carried out a comparative study on entropic thresholding methods. This methods use the gray level histogram of given image for seeking an optimum threshold value. This valley seeking process is an optimization problem. Several gray level image thresholding methods are proposed in the literature at the past decades. Here, we try to answer the question of which of the entropic thresholding methods perform better result on automatically thresholding the gray level images. The experimental results can be seen at Table 1. From the view of the experimental results, none of the examined entropy thresholding methods yield better performance for all kind of histogram type.

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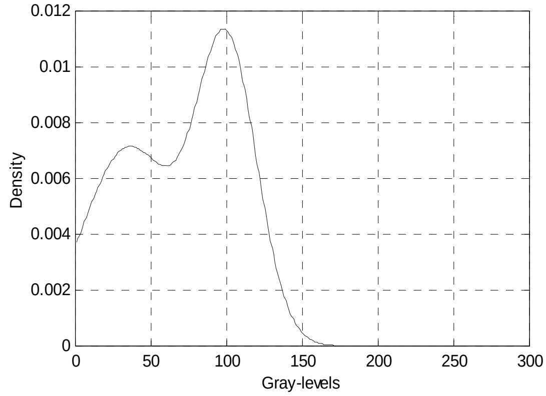
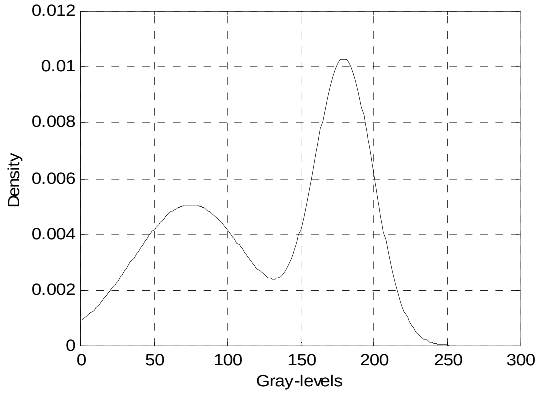
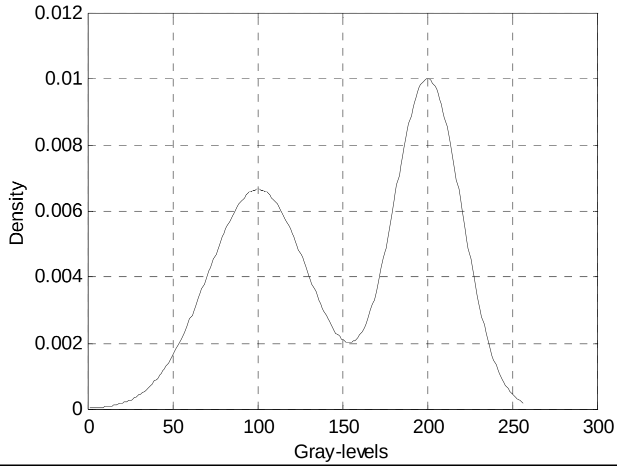


Histogram (1)

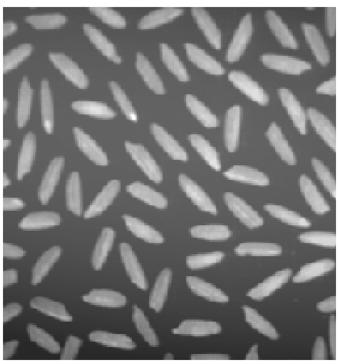


Histogram (2)

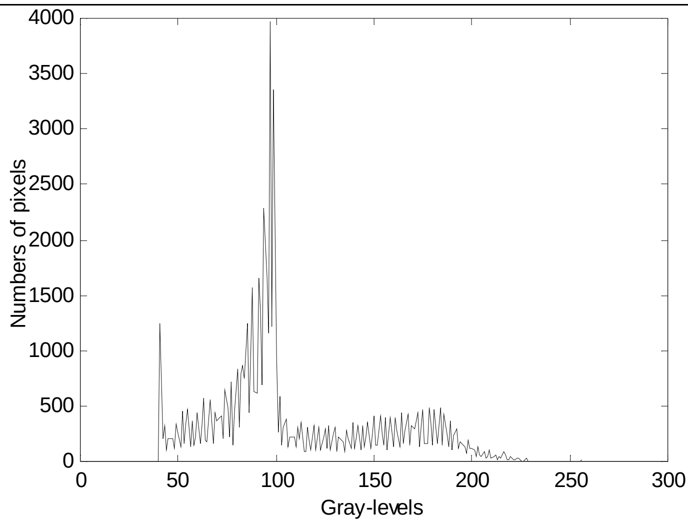


Histogram (3)

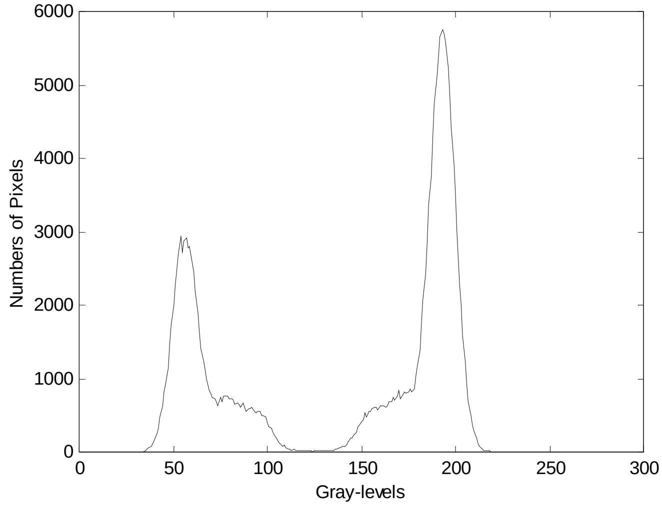
Fig. 1 Synthetically generated histograms

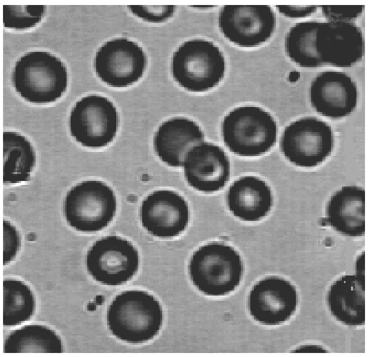


Rice image




Fingerprint image




Blood1 image

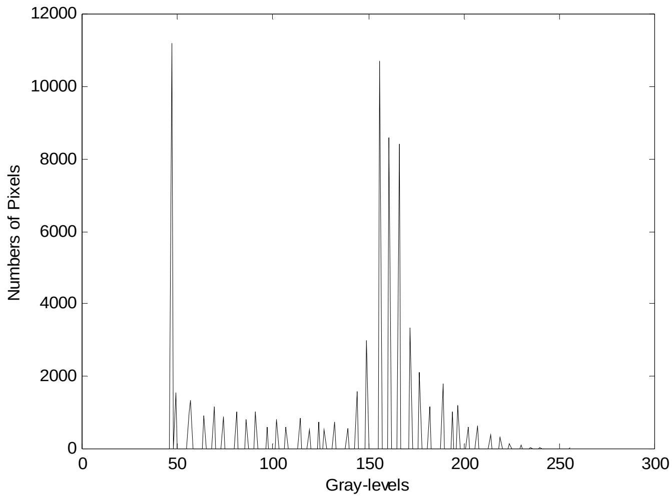


Fig 2. Real world images and their histograms

Table 1. The examined entropy thresholding methods and the obtained threshold values

	Hist. (1)	Hist. (2)	Hist (3)	Rice	Fingerprint	Blood1
Li & Lee MCE	55	107	138	120	117	107-113
Pal S-MCE	54	113	143	134	110	86-90
Renyi's Entropy	69	137-138	155-156	102	175-176	174-177
Kapur's Entropy	70	118-119	140-141	120	161	151-155
Havrda & Charvat	155	127	131-132	116	154-155	177-181