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POWER SYSTEM SERIES HARMONIC RESONANCE ASSESSMENT BASED ON IMPROVED MODAL ANALYSIS

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ABSTRACT

Harmonic resonances in the grid network, both the parallel resonance and the series resonance, have been given more and more attention in modern power system operation and study. The modal analysis approach that based on the node impedance matrix has shown a promising way for the parallel resonance assessment in recent years. With regard to the series resonance, a novel approach, combining the modal analysis and the dummy branch method, is proposed in this paper to compute the series resonance frequency and the corresponding branch information. It is found here that the loop impedance matrix should be used in assessing the series resonance problem rather than the node impedance matrix that we used in assessing the parallel resonance. It is also illuminated that since the network topology changes constantly when analyzing the series resonance phenomenon, the dummy branch method should be embedded into this approach. The tests results and practical application show the correctness and effectiveness of this method. The results of this paper can serve as a parallel approach which can provide more sufficient information of harmonic resonance than the conventional widely used frequency scan analysis do, and this can also be used for the checkout of resonance frequency of filters.

Keywords: *Power system, Harmonic resonance, Series resonance, Modal analysis, Dummy branch method*

1. INTRODUCTION

Most components and devices in power system are inductive. But due to the charging capacitance of high voltage transmission lines and the application of shunt capacitors for voltage support and power factor correction, the system appears to be an extremely complicated circuit consisted of numerous series or parallel connected inductive and capacitive elements.

Thus, different impedance characteristics and values are shown according to various frequencies, and even seriously, series and parallel harmonic resonances will happen under the excitements of certain harmonic resources. With the proliferation of harmonic-producing loads and the increasing awareness of harmonic effects, the possibility of system harmonic resonance has become a routine concern [1-2].

Although the cause of harmonic resonance is well understood, tools available to analyze the phenomenon are very limited. Frequency scan analysis is probably the only practical applicable method at present to identify the existence of resonance and to determine the resonance frequency [3-4]. Unfortunately, the tool cannot

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offer adequate information needed to solve the problem effectively. Reference [5-6] presented a novel harmonic resonance assessment method based on modal analysis. The Resonance Modal Analysis (RMA) technique obtains the information of resonance mechanics and degrees through analysis on the eigenvalues of network admittance matrix. RMA brings new approach for the analysis of harmonic resonance phenomenon. But, the RMA proposed in [5] and [6] are mainly concerned on parallel resonance problem with little consideration of the analysis of series resonance. Both series resonance and parallel resonance problem are the two important branches of power system harmonic resonance phenomenon. The RMA, as a novel method for resonance analysis, can be an integrated methodology only when it can explain both the two resonance problems effectively. Although parallel resonance occupies a large proportion in power system, the bad consequences brought by series cannot be neglected [7]. Especially in the stage of planning, it is important for electrical engineers to grasp the possibility and extent of series resonance.

In this paper, a novel method combined modal analysis and dummy branch method is proposed for obtaining series resonance frequency and corresponding branch information through analyzing loop impedance matrix. Firstly, simple RMA is pointed out not to be applicable for solving series resonance since the reason lies in the close relationship between loop impedance and the occurrence of series resonance. Furthermore, causation why the results are incomplete when the simple loop impedance matrix and RMA are applied for series resonance is analyzed. The truth is illuminated that the network topological structure varies while analyzing series resonance problem. A new means called “Dummy Branch Method” which traverses all possible network structures is presented to get accurate and integrated result of resonance problem when it is combined with RMA. Simulation example and practical application results have confirmed the validity and practicability of proposed approach. This method inherits modal analysis and perfects it, making which as the coordinate method with frequency scan that can solve harmonic resonance problem completely. The results of this paper can be used as the basic method for harmonic resonance problems and also for the checkout of resonance frequency of filters.

2. CONCEPT OF RESONANCE MODES

Imagine a system experiencing a sharp parallel resonance at frequency f according to the frequency scan analysis. This means that some elements of the voltage vector calculated from the following equation have large values at f.

$$\boldsymbol{U}_f = \boldsymbol{Y}_f^{-1}\boldsymbol{I}_f \qquad (1)$$

where $\boldsymbol{Y}_f$ is the network admittance matrix at frequency f. $\boldsymbol{U}_f$ is the nodal voltage and $\boldsymbol{I}_f$ the nodal current injection respectively. To simplify notation, the subscript f will be omitted hereinafter.

A sharp harmonic resonance means that some nodal voltages are very high. This will occur when the $\boldsymbol{Y}$ matrix approaches singularity. The well-established theory of eigen-analysis can be applied for investigating of how the $\boldsymbol{Y}$ matrix approaches singularity [8]. According to the theory, the $\boldsymbol{Y}$ matrix can be decomposed into the following form:

$$\boldsymbol{Y} = \boldsymbol{L\Lambda T} \qquad (2)$$

where $\boldsymbol{\Lambda}$ is the diagonal eigenvalue matrix, $\boldsymbol{L}$ and $\boldsymbol{T}$ are the left and right eigenvector matrices respectively. $\boldsymbol{L} = \boldsymbol{T}^{-1}$.

Substituting (2) into (1) yields

$$\boldsymbol{U} = \boldsymbol{L\Lambda}^{-1}\boldsymbol{TI}$$

or

$$\boldsymbol{TU} = \boldsymbol{\Lambda}^{-1}\boldsymbol{TI} \qquad (3)$$

Defining $\boldsymbol{V} = \boldsymbol{TU}$ as the modal voltage vector and $\boldsymbol{J} = \boldsymbol{TI}$ as the modal current vector respectively, the above equation can be simplified as

$$\boldsymbol{V} = \boldsymbol{\Lambda}^{-1}\boldsymbol{J}$$

or

$$\begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \\ \vdots \\ \boldsymbol{V}_n \end{bmatrix} = \begin{bmatrix} \lambda_1^{-1} & 0 & 0 & 0 \\ 0 & \lambda_2^{-1} & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \lambda_n^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{J}_1 \\ \boldsymbol{J}_2 \\ \vdots \\ \boldsymbol{J}_n \end{bmatrix} \qquad (4)$$

The inverse of the eigenvalue has the unit of impedance and is named modal impedance Z_m. From (4), it is clear that if λ_1=0 or is very small, a small injection of modal 1 current $\boldsymbol{J}_1$ will lead to a large modal 1 voltage $\boldsymbol{V}_1$. On the other hand, the other modal voltages will not be affected since they have no ‘coupling’ with the mode 1 current. In other words, one can easily identify the ‘location’ of resonance in the modal domain. The implication is that the resonance actually takes place for a specific mode. It is not related to or caused by a particular bus injection. Therefore, the smallest eigenvalue is called the critical mode of harmonic resonance and its left and right eigenvectors the critical eigenvectors. The have the following two characteristics:

1) The bus that has the highest observability level (i.e., largest left eigenvector entry) for a mode is also the one that has the highest excitability level (i.e., largest right eigenvector entry). It implies that if a harmonic current matching the resonance frequency is injected into this bus, the bus will see the highest harmonic voltage level. If the current is injected into a different bus, the distortion level is likely to be amplified in the system.

2) The participation factors are equal to the square of the eigenvectors. As a result, one index, eigenvector or participation factor is sufficient for resonance modal analysis. The magnitude of the index characterizes how far the resonance will propagate. The bus with the highest participation factor can be considered as *the center of resonance*.

3. LOOP IMEDANCE MATRIX BASED MODAL ANALYSIS

A small injection of harmonic current will lead to a large harmonic voltage in the system while parallel resonance, where there is a large eigenvalue in corresponding $\boldsymbol{Y}^{-1}$matrix. With a simple test system shown in Fig. 1, the difference between series resonance and parallel resonance solved by RMA is analyzed in this section. The per-unit frequency is based on the fundamental frequency and is equal to harmonic number. There are three buses in the system where harmonic resonance could be excited or observed. The impedance frequency scan results and modal impedance curve of the test system are shown in Fig. 2 and Fig. 3 respectively, which obviously show the equality of harmonic resonance frequencies obtained by the two methods. This validates the RMA for parallel resonance problem.

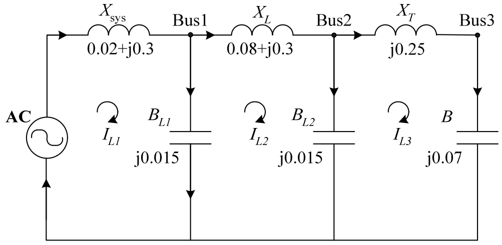

Fig. 1: *3-bus test system*

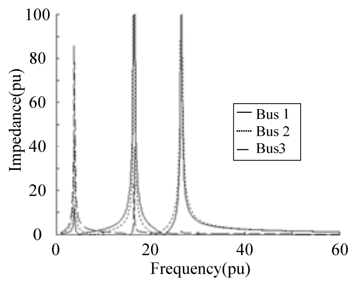

Fig. 2: *Impedance frequency scan results of the test system*

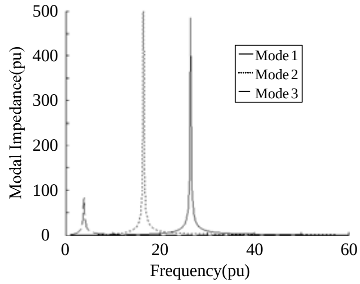

Fig. 3: *Modal impedance curve of the test system*

The emphasis of this paper is on an implementation of RMA approach to solve harmonic series resonance problems. While series resonance, a small applied harmonic voltage on a bus will lead to a large current. In frequency scan, it performs as a small value of harmonic impedance or a large value of harmonic admittance. As shown in Fig. 4, peaks in the admittance frequency curve mean that there are 6 frequencies may have the possibility of series resonance.

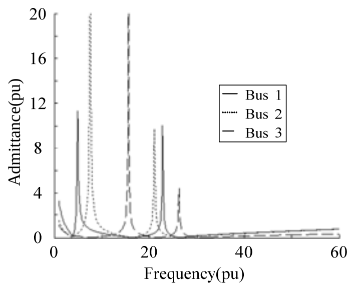

Fig. 4: *Admittance frequency scan results of the test system*

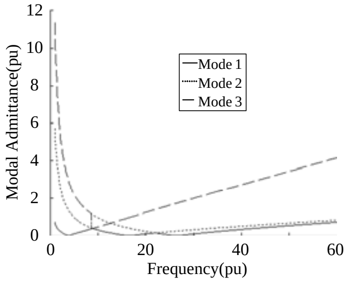

Fig. 5: *Series resonance analysis results by using harmonic node impedance matrix*

Applying RMA for series resonance problem, the first thing come to mind is the inverse of harmonic admittance matrix $\boldsymbol{Z}=\boldsymbol{Y}^{-1}$. The eigenvalue λ_i of $\boldsymbol{Z}$ matrix (where i=1, 2, …n, n is the total number of buses) can be obtained by eigen-analysis technology, and then it is easy to determine which bus can experience a particular series resonance more easily through analysis on the large values in $1/\lambda_i$. According to the above clew, the discussion of the test system is made and the result is shown in Fig. 5. However, the inverse of the eigenvalues of the $\boldsymbol{Z}$ matrix shown in the figure doesn't have large values at all. This is a surprising finding. If one analyzes the problem further, however, the phenomenon becomes understandable. A series resonance means that the circuit has a loop with very small loop impedance. If the loop is applied with a voltage, a large loop current will be produced. The occurrence of series resonance is closely related to loops, rather than simple nodes and branches. The correct formulation to identify series resonance should, therefore, be the $\boldsymbol{Z}_{\text{loop}}$ matrix defined in loop equation.

$$\boldsymbol{Z}_{\text{loop}}\boldsymbol{I}_{\text{loop}} = \boldsymbol{E} \tag{5}$$

where $\boldsymbol{Z}_{\text{loop}}$ is the loop impedance matrix. $\boldsymbol{I}_{\text{loop}}$ is the loop current matrix and $\boldsymbol{E}$ is the loop voltage matrix respectively. The subscript loop will be omitted hereinafter.

According to RMA method, similar equations can be defined for series resonance problem. The $\boldsymbol{Z}$ matrix can be decomposed into the following form:

$$\boldsymbol{Z} = \boldsymbol{L}\boldsymbol{\Lambda}\boldsymbol{T} \tag{6}$$

where $\boldsymbol{\Lambda}$ is the diagonal eigenvalue matrix, $\boldsymbol{L}$ and $\boldsymbol{T}$ are the left and right eigenvector matrices respectively. $\boldsymbol{L} = \boldsymbol{T}^{-1}$.

Substituting (6) into (5) yields

$$\boldsymbol{I} = \boldsymbol{L}\boldsymbol{\Lambda}^{-1}\boldsymbol{T}\boldsymbol{E}$$

or

$$\boldsymbol{T}\boldsymbol{I} = \boldsymbol{\Lambda}^{-1}\boldsymbol{T}\boldsymbol{E} \tag{7}$$

Defining $\boldsymbol{J} = \boldsymbol{T}\boldsymbol{I}$ as the modal current vector and $\boldsymbol{V} = \boldsymbol{T}\boldsymbol{E}$ as the modal voltage vector respectively, (7) can be simplified as

$$\boldsymbol{J} = \boldsymbol{\Lambda}^{-1}\boldsymbol{V}$$

or

$$\begin{bmatrix} \boldsymbol{J}_1 \\ \boldsymbol{J}_2 \\ \vdots \\ \boldsymbol{J}_n \end{bmatrix} = \begin{bmatrix} \lambda_1^{-1} & 0 & 0 & 0 \\ 0 & \lambda_2^{-1} & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \lambda_n^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \\ \vdots \\ \boldsymbol{V}_n \end{bmatrix} \tag{8}$$

The inverse of the eigenvalue, $1/\lambda$, has the unit of admittance and is named modal admittance. Form (8), it can be easily found that if λ_1=0 or is very small, a small applied modal 1 voltage $\boldsymbol{V}_1$ will lead to a large modal 1 current $\boldsymbol{J}_1$. On the other hand, the other modal currents will not be affected since they have no 'coupling' with the mode 1 voltage. By now, one can implement the above method to solve series resonance problem. The loop reference direction is clockwise, as marked in Fig. 1. Analysis result is shown in Fig. 6. The number of peaks in the modal admittance curve is only 3, which is 3 less than the result of frequency scan. This phenomenon indicates that there is still something wrong with the application of RMA for series resonance.

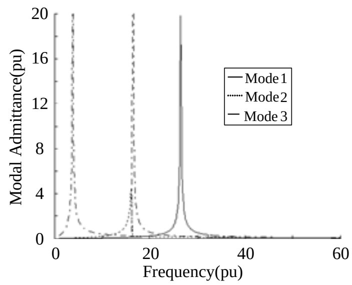

Fig. 6: *Series resonance problem results by using basic modal analysis*

4. DUMMY BRANCH METHOD

By theoretical analysis and experimental investigation, the cause and solution of above phenomenon is discussed in this section. Analyzing parallel resonance, the frequently used method is to inject an ideal unit harmonic current resource to obtain the frequency response of corresponding node. The branch impedance of an ideal current resource is infinite and the current resource branch is open circuit. Thus, the introduction of injection current branch doesn't alter the structure of studied network. Things are different with series resonance problem. Investigating a series resonance case, an ideal unit harmonic voltage resource is commonly applied to obtain the frequency response of a certain bus. The branch impedance of an ideal voltage resource is considered to be zero, hence it is equivalent to add a short circuited branch between the reference node and the bus that ideal voltage resource added. As a result, the loop impedance matrix is modified. This is the causation why only partial solutions are obtained when the single fixed loop impedance matrix is applied to solve series resonance problem. Therefore, all the scenarios with the different loop impedance matrixes when there is a dummy short circuited branch between each bus and the reference node respectively must be analyzed and integrated to get the accurate solution.

According to above analysis, a new method called dummy branch method is proposed in this paper to simulate the scenarios when the studied bus and reference node is short-circuited. The detailed steps are listed as follows. Assume that a branch is added between each bus and reference node. The impedance of added branches is generally set to be infinite, and these branches can be considered to be open circuit and have no effect on the network structure. When the *i*th scenario is being analyzed, the impedance of corresponding dummy branch *i* is set to be zero for short circuit simulation. After the *i*th scenario is finished and the (*i*+1)th one is being studied, the impedance of dummy branch *i* is set back to be infinite and the (*i*+1)th branch impedance is set to be zero. By this means, every loop impedance matrix of the scenarios can be generated. The eigenvalues characteristics of all these harmonic loop impedance matrixes are analyzed by modal approach. Finally, synthesis of all the analysis results will reflect all the positions and frequencies of series resonance points. The technique that combines RMA and dummy branch method can be called as the Improved Resonance Modal Analysis (IRMA). The detailed flowchart of IRMA is shown in Fig. 7.

Suppose the total number of studied network nodes and branches are n+1 and b respectively, the number of independent nodes and tree branches are both n, and the number of fundamental loops and are both b-n. If n dummy branches are added into the network, the total number of branches increases to $n+b$ and fundamental loops to b, which indicates that the dimension of the new loop impedance matrix is b. Thanks to the existing well developed algorithms for eigenvalue analysis technology, the CPU time consumed for achieving final results is few, even for those large networks, though the dimension of loop impedance matrix rises. This is also confirmed by the practical experience of program tests.

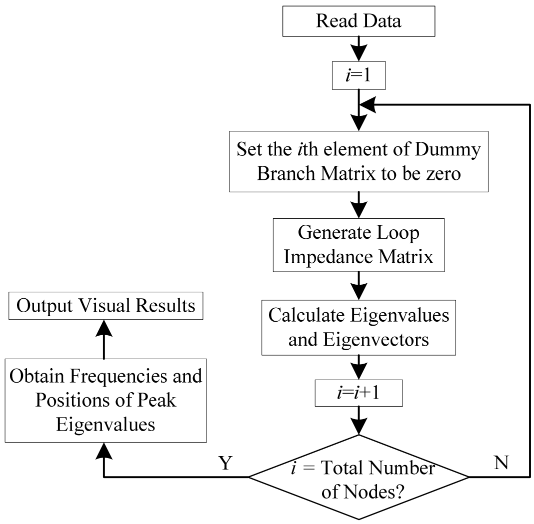

Fig. 7: *Flowchart of modal analysis combined with dummy branch method*

The test system in Fig. 1 is used for experiment. Loop reference direction is clockwise. Three dummy branches *Ri* (where i=1,2,3), whose impedances are set to be 0Ω or 1×10^6Ω, are added into the system while the other parameters of the circuit are unaltered. Due to the added dummy branches, the total number of fundamental loops increases to 6. Applying IRMA for the system, test results can be obtained, which are shown in Fig.8. All of the six series resonance frequencies are found out. In order to make the contrast more obviously, Fig. 9 gives the comparison of results from frequency scan and IRMA method, which are indicated by dotted lines and solid lines respectively. As shown in Fig. 9, it is clear that the peak positions obtained by the two methods match well. Tab. 1 documents the information, such as resonance frequencies, peak values of modal admittances, and corresponding branches that induce resonance, of the 6 series resonance points. All the results are identical to the results of frequency scan analysis. For examples, the capacitor B is in series with X_T. The approximate series resonance frequency is $f = \sqrt{1/(BX_T)} = 7.559$ p.u., which coincides the resonance frequency of 7.6p.u. obtained from IRMA. Capacitor B is also in series with the X_{sys}, X_L and X_T, the approximate series resonance frequency is $f = \sqrt{1/B(X_{sys} + X_L + X_T)} = 4.099$ p.u., which is very close to with the resonance frequency of 3.9p.u. gotten by IRMA. The results therefore confirm that the information provided by the IRMA can indeed reveal the locations easiest to excite harmonic series resonance.

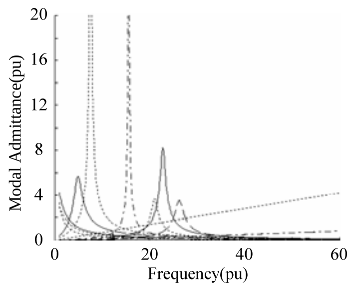

Fig. 8: *Series resonance problem results by using IMRA*

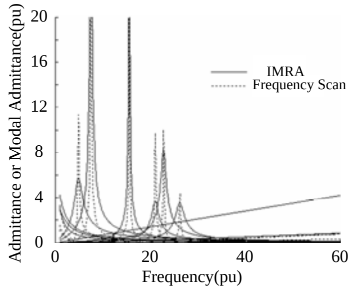

Fig. 9: *Results contrast of Frequency Scan and IMRA*

Table 1: *Modal analysis results of the test system*

Resonance frequency(pu)	Peak value of modal admittance	Corresponding capacitor branches	Corresponding reactive branches
3.9	5.7252	B	$X_{sys}+X_L+X_T$
7.6	147.7398	B	X_T
15.6	117.2630	B_{L2}	X_T
21	3.6555	B_{L1}	$X_{sys}//X_L$
22.8	8.2946	B_{L2}	$X_L//X_T$
26.2	3.5338	B_{L1}	$X_{sys}//X_L//X_T$

5. PRACTICAL APPLICATION

A real power network is used to show the validity of the proposed IRMA approach for series resonance analysis. This network is part of a city's network in Hunan Province, China, and the topology of this network is shown in Figure 10.

The harmonic in this network is mainly contributed from three resources, including the metallurgic load, the chemical load, and the system on the 500 kV side. According to the released Harmonic Working Report from Hunan Power Grid Technical Committee [9], there are 4 groups of parallel compensating capacitors with capacity of 6000 kVar per group connected to the 10 kV bus of the 220 kV substation D (shown in Fig. 10). The commissioning test results show: the 3rd harmonic component is seriously amplified with a current of 550 A, which occupies 56% of the fundamental current; the maximum voltage distortion rate reaches 8.6%, which far exceeds the allowable value (4%) set in the national standards. Because there is no harmonic source directly connected to the 10kV bus of the substation D and the main transformer in this substation is three-winding, the compensator branches connected to the 10kV bus are easily looped in a series circuit together with the background harmonic sources in system. Thus, series harmonic resonance, rather than parallel resonance, is easily to take place if the impedances selecting for capacitors and reactors doesn't match well. Hence, the proposed IRMA approach is applicable to this case analysis.

By adopting the IRMA approach, the modal admittance-frequency characteristic of the compensator branches is obtained and illustrated in Figure 11. According to numerical results, the two calculated series harmonic resonance frequencies corresponding to the capacitors are 2.8p.u. and 4.0 p.u.. The 2.8 p.u. frequency component can be considered as the frequency of the 3rd harmonic resonance, and this is consistent with the commissioning test result. The presence of the 4.0 p.u. component shown in Fig. 11 represents that the physical parameters of the substation capacitor branches may lead to the occurrence of 4th series resonance. But generally, only 3rd, 5th, 7th harmonic resources are mainly concerned in actual public electric network, and hence the 4th series resonance can be considered not to be excited because there is no enough excitation of 4th harmonic resources in the system.

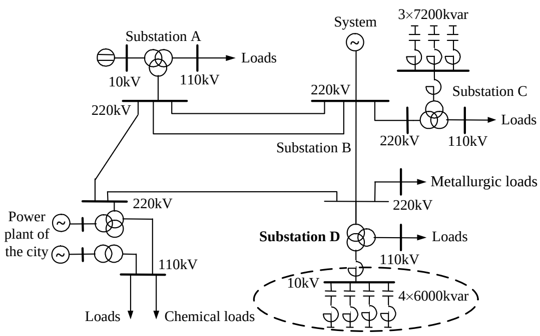

Fig. 10: Single line diagram of part of a city's network

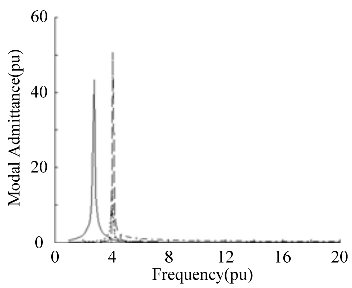

Fig. 11: Series resonance curves of capacitors in part of a city's network

6. CONCLUSIONS

The parallel resonance and the series resonance constitute the two aspects of harmonic resonances problem. As explained in this paper, the original RMA approach, which only analyzes the node admittance matrix, is not applicable for assessing the series resonance problem. This work presents an IRMA approach, which combines the modal analysis and the dummy branch method. The main findings of this paper are:

1) The series resonance is taking a loop form in the network. The characteristic of this kind of resonance is closely related with the loop impedance, rather than the node impedance. Thus the loop impedance matrix is adopted for analysis in this work.

2) With regard to the parallel resonance, the ideal harmonic current source branch is equivalent to an open circuit, and thus the original network topology won't change. However, when analyzing the series resonance, the ideal harmonic voltage source branch is equivalent to a short circuit, and thus both the network topology and the corresponding loop impedance matrix will change.

3) Purely analyzing the fixed loop impedance matrix can not reach an accurate solution for series harmonic resonance problem. All the different loop impedance matrixes obtained by adopting the dummy branch method should be used in analysis.

Both test system research and practical application validate the proposed method. Combining this IRMA approach with the work described in [5, 6], a complete solution scheme can be formed to assess harmonic resonance problems. The results of this paper can be used as the basic method for harmonic resonance problems and also for the checkout of resonance frequency of filters.

7. FUTURE WORK

The proposed modal analysis method is a new tool for harmonic resonance assessment. Thus, some cues for a further approach from IRMA application to theory background research are provided in this section. The physical meaning of modes in harmonic resonance is still needed to be discussed. What's more, the definitions of eigenvectors and participation factors for different modes should be established to describe the observability and excitability of different modes and the resonance propagation status. Unlike the parallel resonance problem, it is hard to determine the corresponding branch information, due to the complicated variety of impedance matrixes when analyzing harmonic series resonance problem, especially for largescale networks. In the future work, the above problems should be studied to perfect the modal analysis approach for power system harmonic resonance phenomenon.

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