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A NOVEL COOPERATIVE COMMUNICATION SCHEME BASED ON MULTILEVEL CODES

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ABSTRACT

In wireless networks, coded cooperation is a way of realizing uplink transmit diversity at the mobile user with single antenna. In this work, in order to increase the power and bandwidth efficiency, a multilevel coded cooperative system using orthogonal signalling is proposed. Orthogonal signalling not only enables transmission of both users data in single channel symbol but also provides independent detection at the receiver. We investigate the performance of the system by derivation of pairwise and bit error probabilities together with Monte-Carlo simulations.

Keywords: Coded cooperation, multilevel coding, orthogonal signalling

I. INTRODUCTION:

In wireless channels, one possible solution to intense the detrimental effects of multipath fading is diversity. In transmit diversity. technique, the spatial locations of the antennas are chosen so that the paths from each antenna to the destination can be modelled as independent fading channels. But, since mobile station size, system complexity and transmitted signal power are all limiting factors for wireless systems, transmit diversity methods may not be applied. A recent approach that enables transmit diversity technique in these limited systems is cooperative communication, in which two or more single antenna users send their information sharing their antennas [1],[2].

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Coded cooperation combines cooperative diversity with channel coding [3]. Instead of a simple repetition relay, the code words of each mobile is partitioned and portions of each codeword is transmitted through independent fading channels which also means each user tries to transmit incremental redundancy for its partner. In related literature, convolutional codes have been chosen as component codes generally. Convolutional codes designed for large minimum Euclidean distance performs poor for fading channels, because for fading channels, minimum symbol (Hamming) distance is the design criteria together with minimum product distance. Multilevel coding technique, originally proposed by Imai and Hirakawa [4], combines multilevel binary codes with modulation using Ungerboeck's set partitioning rule [5], and has performance and bandwidth advantages over trellis coded modulation even with

an suboptimal multistage decoder. There is a one to one correspondence between the minimum Hamming distance of component codes and the number of branches of time diversity in multilevel coding and multilevel codes were shown to
outperform Ungerboeck's codes for fully interleaved Rayleigh fading channel [6]. This performance promises the use of multilevel codes as component codes for coded cooperation.

Bandwidth efficiency is a key parameter in designing coded cooperative systems. Most of the works assumes an orthogonal transmission (different frequency bands or different time slots) in order to uniquely detect the information. In [7], authors proposed a simple cooperative scheme which has the same power and bandwidth requirements with that of noncooperative scheme. similarly , we form orthogonal signaling via assigning the in-phase and quadrature components of the modulated signal to different users and thus transmit in the same frequency band and at the same time without sacrifying extra bandwidth for cooperation. Since we assign each dimension of the signal constellation to different users, it's also possible to detect the informations of each user independently at the corresponding receivers.

In this paper, we proposed a coded cooperative system using multilevel coding technique. The performance of the proposed system is compared with the multilevel coded system which has the same multilevel encoder but with two transmit antennas. In our scheme, each bit associated with corrresponding level belongs to individual users, i.e. the first level is partitioned with user's own data and the remaining levels are with that of cooperating users'. Simulation results shows that the multilevel coded cooperative system with perfect interuser channel achieves almost the same performance as the non-cooperative single user employing multilevel coding for a wide signal to noise ratio (SNR) region. Also the theoretical bit error rate upper bound obtained by union bounding technique supports the simulation results.

The paper is organized as follows. The second part gives the system model with the transmitter and receiver diagrams. The transmission scheme and the channel model is also given in this part. The theoretical pairwise and bit error probabilities are derived at the third part. Finally the simulation results are given together with conclusions and discussions at the last two parts.

2 SYSTEM AND CHANNEL MODEL

The multilevel coded cooperative (MCC) system considered here consists of two cooperating users equipped with single antenna and transmitting to the same destination. We assume frame based transmission with frame length N. We define the uncoded and coded data vectors of the i^{th} ($i \in \{1,2\}$) user at frame f as $\boldsymbol{d}_f^{(i)}$ and $\boldsymbol{c}_f^{(i)}$ respectively. The transmitted channel signal from the i^{th} user at frame f is $\boldsymbol{x}_f^{(i)}$. The multilevel encoding scheme employs two convolutional codes with rates $R_1 = k_1 / n_1$ and $R_2 = k_2 / n_2$ as component codes with two-level set partitioning. The cooperation between the users are enabled via assigning each level of encoder to individual users; encoder 1 with rate R_1 outputs the user's own codewords while the second encoder with rate R_2 produces the codewords of the partner. The two element signal vector formed by taking the outputs of the first and second encoders in order is mapped to one of the four phase shift keying (PSK) symbols based on set partitioning rule. Based on the given notation, Fig. 1 shows the encoding and modulation process at one of the user.

Each user has two convolutional encoders as component codes of the two level system; the input of the first encoder is the data of the user itself and the code words at the output this encoder

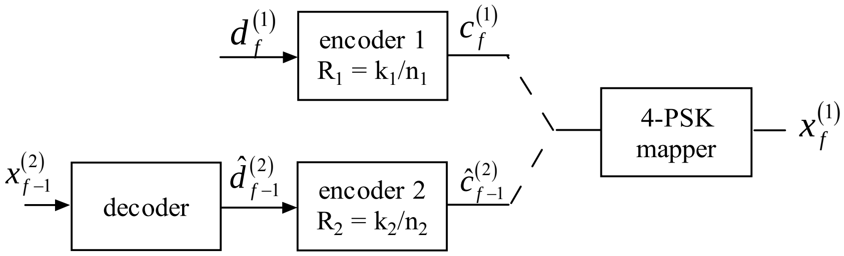

Figure 1. Transmitter Diagram

constitutes the first level of the multilevel system. Since the input of the second encoder is the single frame delayed partner data, the user first detects the partner's encoded data by demodulating the received signal from the partner in the previous frame. If we define the noisy received signal at user1 transmitted from user2, $\boldsymbol{y}_{f-1}^{(21)}$, as,

$$\boldsymbol{y}_{f-1}^{(21)} = \boldsymbol{h}_{f-1}^{(21)}.\boldsymbol{x}_{f-1}^{(2)} + \boldsymbol{n}_{f-1}^{(21)} \quad (1)$$

where $\boldsymbol{h}_{f-1}^{(21)}$ is the fading coefficient between the user2 and user1 at frame (*f-1*) and $\boldsymbol{n}_{f-1}^{(21)}$ is the complex Gaussian distributed additive noise at frame $(f\text{-}1)$ with variance $N_0/2$ per dimension. Then user2 data at frame (*f-1*) can be detected at user1 as,

$$\overline{\boldsymbol{c}}_{f-1}^{(2)} = \Im\left\{conj\left(\boldsymbol{h}_{f-1}^{(21)}\right).\boldsymbol{y}_{f-1}^{(21)}\right\} \quad (2)$$

where $\Im(.)$ denotes the imaginary part and *conj*() stands for complex conjugate. User1 then decodes this signal, and obtains an estimate of the partners codeword $\hat{\boldsymbol{c}}_{f-1}^{2}$ by re-encoding the decoded data. The outputs of the first and second encoder constitutes the in-phase and quadrature components, respectively in the signal constellation. The baseband representation of the transmitted channel signal for the user1 at frame f is,

$$\boldsymbol{x}_f^{(1)} = \sqrt{E_b/2}\left(\boldsymbol{c}_f^{(1)} + j\hat{\boldsymbol{c}}_{f-1}^{(2)}\right) \quad (3)$$

All this process is repeated at user2 with a slight difference. User2 detects the user1 data from the real part of the received signal as;

$$\overline{\boldsymbol{c}}_{f-1}^{(1)} = \Re\left\{conj\left(\boldsymbol{h}_{f-1}^{(12)}\right).\boldsymbol{y}_{f-1}^{(12)}\right\} \quad (4)$$

where $\boldsymbol{h}_{f-1}^{(12)}$ is the fading coefficient between the user1 and user2 at frame (*f-1*), $\Re(\ )$ denotes the real part and $\boldsymbol{y}_{f-1}^{(12)}$ is the noisy received signal at user2 transmitted from user1 defined as,

$$\boldsymbol{y}_{f-1}^{(12)} = \boldsymbol{h}_{f-1}^{(12)}.\boldsymbol{x}_{f-1}^{(1)} + \boldsymbol{n}_{f-1}^{(12)} \quad (5)$$

Finally, the transmitted channel signal from the user2 at frame f is

$$\boldsymbol{x}_f^{(2)} = \sqrt{E_b/2}\left(\hat{\boldsymbol{c}}_{f-1}^{(1)} + j\boldsymbol{c}_f^{(2)}\right) \quad (6)$$

For additive white Gaussian noise (AWGN) channels, the set partitioning method used in multilevel codes follows the Ungerboeck's set partitioning rule which maximizes the minimum intra-subset Euclidean distance. But for Rayleigh fading channels, multilevel coding scheme with Gray labeling outperforms the most known trellis codes because of increased diversity. Also in order to apply the orthogonal signaling, instead of Ungerboeck's set partitioning rule, the set partitioning shown in Figure 2 was applied. The minimum Euclidian

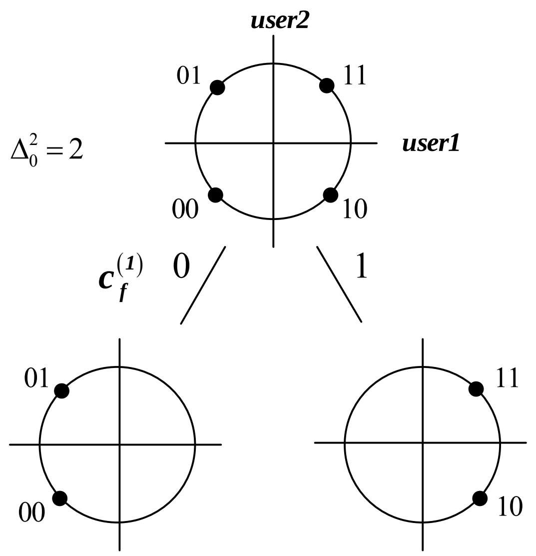

Figure 2. Set Partitioning of 4-PSK

distance is the same for two levels and is equal to $\Delta_0^2 = 2$.

The block diagram of the receiver will be as given Fig.3. Cooperative transmission introduces a transmit diversity to the system, hence the receiver at the destination applies optimal combining method maximum ratio combining (MRC). Since the cooperation takes place in consecutive frames, the inputs of the combiner are the current symbol and the previous symbol obtained by delay element. The faded received signal of user1 at the destination is,

$$\boldsymbol{y}_f^{(1d)} = \boldsymbol{h}_f^{(1d)} \boldsymbol{x}_f^{(1)} + \boldsymbol{n}_f^{(1d)} \tag{7}$$

where the elements of fading coefficients vector for frame *f*, $\boldsymbol{h}_f^{(1d)}$, are zero mean complex Gaussian random variables with unit variance and the additive noise terms $\boldsymbol{n}_f^{(1d)}$ are also zero mean complex Gaussian random variables with variance of $N_0/2$. Similarly,

$$\boldsymbol{y}_f^{(2d)} = \boldsymbol{h}_f^{(2d)} \boldsymbol{x}_f^{(2)} + \boldsymbol{n}_f^{(2d)} \tag{8}$$

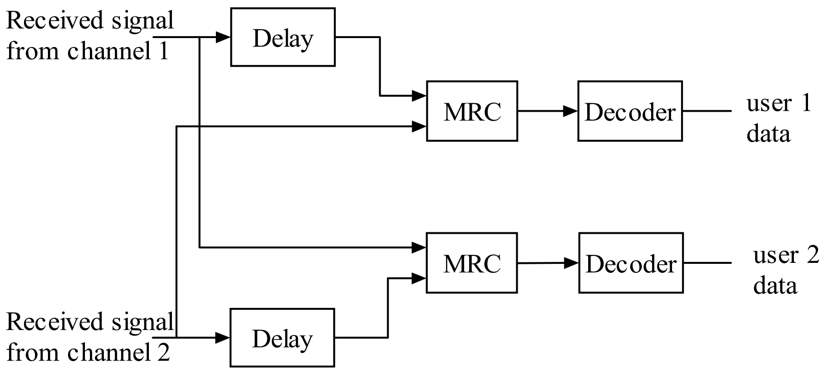

Figure 3. The Block Diagram of The Receiver at the Destination

To detect the user1 data, the upper MRC in Fig.3 calculates,

$$\tilde{\boldsymbol{c}}_{f-1}^{(1)} = \Re\left\{conj\left(\boldsymbol{h}_{f-1}^{(1d)}\right).\boldsymbol{y}_{f-1}^{(1d)} + conj\left(\boldsymbol{h}_{f}^{(2d)}\right).\boldsymbol{y}_{f}^{(2d)}\right\} \tag{9}$$

3 PAIRWISE ERROR PROBABILITY ANALYSIS

For the first case, we take into account the Rayleigh fast fading in which the fading coefficients in a single frame are Rayleigh distributed and independent from each other. We assume perfect knowledge of fading coefficients at the respective receivers and an error free transmission between the users. Receiver applies maximum likelihood decoding with MRC, so in order to decode the user1 data, the received sequences $\boldsymbol{y}_f^{(1)}$ and $\boldsymbol{y}_{f+1}^{(2)}$ are jointly processed. Since the pairwise error probability (PEP), defined as the probability that the transmitted sequence $\boldsymbol{x}^{(1)}$ is incorrectly decoded as $\hat{\boldsymbol{x}}^{(1)}$, depends on the fading coefficients, the conditional PEP at the receiver can be expressed as

$$P\left(\boldsymbol{x}^{(1)} \rightarrow \hat{\boldsymbol{x}}^{(1)} \middle| \boldsymbol{h}\right) = P\left(\sum_{j\in\eta}\left(M\left(x_f^{(1)}(j), x_{f+1}^{(2)}(j); h\right) - M\left(\hat{x}_f^{(1)}(j), \hat{x}_{f+1}^{(2)}(j); \hat{h}\right)\right) \geq 0 \middle| \boldsymbol{h}\right) \tag{10}$$

where η is the index set with cardinality *w* for which $x_f^{(1)}(j) \neq \hat{x}_f^{(1)}(j)$, *w* is the Hamming distance between $\boldsymbol{x}^{(1)}$ and $\hat{\boldsymbol{x}}^{(1)}$ and $\boldsymbol{h}=\left[\boldsymbol{h}_f^{(1d)}, \boldsymbol{h}_{f+1}^{(2d)}\right]$. Also

$$M\left(\boldsymbol{x}_f^{(1)}(j), \boldsymbol{x}_{f+1}^{(2)}(j); \boldsymbol{h}\right) = -\min_{x\in C_b}\left(\left|\boldsymbol{y}_f^{(1)}(j) - \boldsymbol{h}_f^{(1)}\boldsymbol{x}_f^{(1)}(j)\right| + \left|\boldsymbol{y}_{f+1}^{(2)}(j) - \boldsymbol{h}_{f+1}^{(2)}\boldsymbol{x}_{f+1}^{(2)}(j)\right|\right) \tag{11}$$

Without loss of generality, we can assume that the all-zero sequence is transmitted. Also we assume that the uplink SNR's are equal. Following the technique in [5] which evaluates the pairwise error probabilities for a three level coded 8-PSK modulation with block partitioning, the union bound yields the following expression for the first decoding stage for all the code sequences associated with **x** of weight ,

$$P\left(\boldsymbol{x}_f^{(1)} \to \hat{\boldsymbol{x}}_f^{(1)} \middle| \boldsymbol{h}\right) = 2^{-w} Q\left(\sqrt{\frac{2RE_b}{N_0}\nabla_0^2\left(\sum_{j\in\eta}\left(h_f^{(1)}\right)^2 + \sum_{j\in\eta}\left(h_{f+1}^{(2)}\right)^2\right)}\right) \tag{12}$$

where

$$Q(x) = \frac{1}{2\pi}\int_x^{\infty} e^{-t^2/2}dt$$

and E_b / N_0 is the energy-per-bit-to-noise ratio. Since for Rayleigh fading channel, the fading coefficients are i.i.d. Rayleigh distributed random variables with $E(\boldsymbol{h}) = 1$, the random variables $r_1 = \sum_{j\in\eta}\left(h_f^{(1d)}\right)^2$ and $r_2 = \sum_{j\in\eta}\left(h_{f+1}^{(2d)}\right)^2$ will have an w-Erlang distribution with parameter one, hence their pdf are

$$f_R(r) = \frac{1}{(w-1)!} r^{(w-1)} e^{-r}, \quad r \geq 0. \tag{14}$$

Thus, the unconditional pairwise error probability is,

$$P\left(\mathbf{x}_f^{(1)} \to \hat{\mathbf{x}}_f^{(1)}\right) = \int_0^{\infty}\int_0^{\infty} 2^{-w} Q\left(\sqrt{\frac{2RE_b}{N_0}\nabla_0^2(r_1 + r_2)}\right) f_{R_1}(r_1)\, f_{R_2}(r_2)\, dr_1 dr_2 \tag{15}$$

To evaluate the double integral given in (15) in closed form, we use the alternative form of *Q* function given by,

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-x^2/2\sin^2\theta} d\theta \tag{16}$$

and the pre-calculated integral result

$$\frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)^n d\theta = \frac{1}{2}\left(1 - \sqrt{\frac{c}{1+c}}\right)^n \sum_{k=1}^{n-1}\binom{n-1+k}{k}\left(\frac{1}{2}\left(1 + \sqrt{\frac{c}{1+c}}\right)\right)^k \tag{17}$$

Using (16) and (17) in (15), we find the pairwise error probability as,

$$P\left(\boldsymbol{x}_f^{(1)} \to \hat{\boldsymbol{x}}_f^{(1)}\right) = 2^{-w}\left(\frac{1-\delta}{2}\right)^{2w} \sum_{k=0}^{2w-1}\binom{2w-1+k}{k}\left(\frac{1+\delta}{2}\right)^k \tag{18}$$

where

$$\delta = \frac{R_1 E_b / N_0}{1 + R_1 E_b / N_0}$$

and the bit error rate (BER) is upper bounded as,

$$P_b \le \sum_{w=w_{\min}}^{N} \frac{w}{N} A_w^{(1)} P\left(\boldsymbol{x}_f^{(1)} \to \hat{\boldsymbol{x}}_f^{(1)}\right) \tag{19}$$

where $A_w^{(1)}$ denotes the number of the codewords with weight w obtained from the transfer function of the corresponding encoder.

If quasi-static Rayleigh fading is assumed,

$$P\left(\boldsymbol{x}_f^{(1)} \to \hat{\boldsymbol{x}}_f^{(1)} \left|\boldsymbol{h}\right.\right) = 2^{-w} Q\left(\sqrt{\frac{R_1 E_b}{N_0} w\left(\sum_{j\in\eta}\left(h_f^{(1)}\right)^2 + \sum_{j\in\eta}\left(h_{f+1}^{(2)}\right)^2\right)}\right) \tag{20}$$

Since the squares of the fading coefficients follows exponential distribution, taking the expected value of conditional PEP with respect to this exponential r.v. yields the unconditional PEP as,

$$P\left(\boldsymbol{x}_f^{(1)} \to \hat{\boldsymbol{x}}_f^{(1)}\right) \le 2^{-(w+1)} \left(\frac{1}{1 + 2w\dfrac{E_b}{N_0}}\right) \tag{21}$$

and the BER expression is the same with (19) with. $P\left(\boldsymbol{x}_f^{(1)} \to \hat{\boldsymbol{x}}_f^{(1)}\right)$ is given by (21).

4 SIMULATION RESULTS

For simulation purposes, we examined the performance of the two user multilevel cooperative system in Rayleigh flat fading environment for quasi-static and fast fading cases. 1/2 rated four state convolutional code with generator matrix (7,5) was chosen as component code in the two level encoder for both source and relay nodes. The outputs of the encoders then forms the 4-PSK symbols based on the set partitioning rule and orthogonal signaling scheme given in part 2. Simulations were performed for both perfect and imperfect inter-user channel conditions but as given in part 3, the theoretical union bound for bit error probability was evaluated for only perfect inter-user case. The analysis for erroneous inter-user transmission between the users has been left as a future work. The frame length was chosen as 256 for all cases. Since the aim of cooperation is to reach the performance of non-cooperative single user system with same diversity gain, we also simulated a multilevel coded system with two transmit antennas. Total power used for two cases is fixed for a fair comparison. The signal to noise ratios for inter-user transmission is named as iSNR. Also the SNR values for the user-destination links are assumed to be equal.

In Fig.4, the BER versus SNR for fast fading case were plotted along with the upper bound calculated with union bounding technique. Since the theoretical error probability analysis for non-cooperative system with two transmit antennas is the same with cooperative system with perfect inter-user channel, it's no surprise to see that the performances of multilevel coded cooperative system with two users and the multilevel system with single user equipped with two transmit antennas remain almost the same for the whole SNR region. Also the upper bound obtained by union bounding technique coincides with the simulation results. Another observation from the simulation results is that with erroneous transmission between cooperating users, an error floor occurs for high SNR values. This error floor comes from the re-encoded symbols in error occurring at the partner.

The simulation results for quasi-static Rayleigh fading case is given in Fig. 5. Again, the performance of multilevel coded cooperative case with perfect inter-user channel performs almost the same as non-cooperative multilevel coded system with two transmit antennas. The upper bound calculated with the derived formula (21), is closer to the simulation results compared to fast fading case.

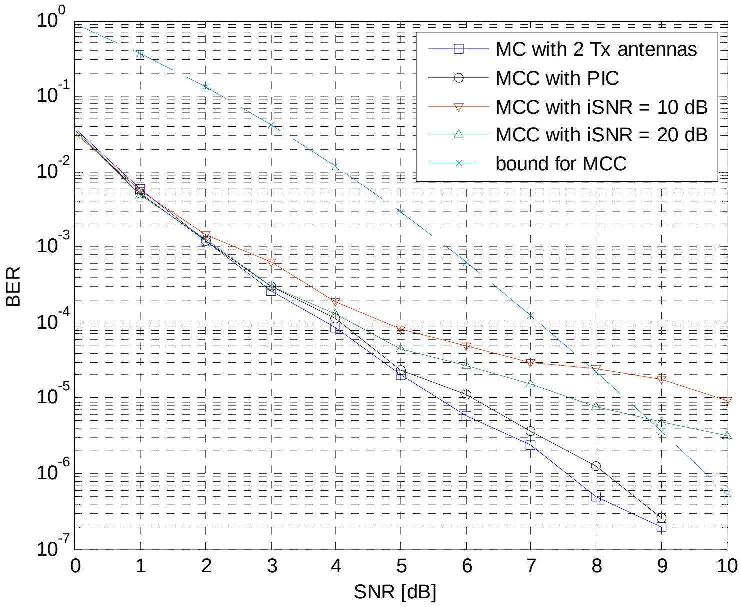

Figure 4. Performance of Multilevel Coded Cooperative and Noncooperative Systems for Rayleigh Fast Fading Channel

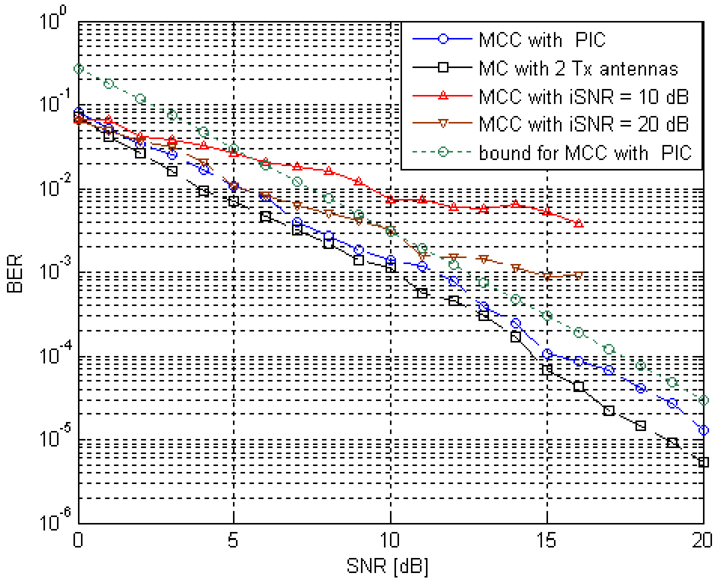

Figure 5. Performance of Multilevel Coded Cooperative and Noncooperative Systems for Quasi Static Rayleigh Fading Channel

5 CONCLUSIONS

Simulation results show that proposed multilevel coded cooperative system achieves the performance of the comparable non-cooperative multilevel coded system employing transmit diversity with same bandwidth requirement and slight increase in complexity for both quasi static and fast Rayleigh fading channels. Since the re-encoded symbols in error occurring at the partner result with an error floor, proper modifications at the multilevel encoder and optimization of transmitted signal power can decrease this effect. Also the analysis of PEP for erroneous interuser transmission has been left as a future work.

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