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PERFORMANCE OF THE SYSTEMATIC DISTANCE-4 CODES OVER FADING CHANNELS

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ABSTRACT

A new binary systematic linear block code construction technique, called as Systematic Distance-4 (SD-4) codes that generates all the optimal size Hamming distance-4 codes, is recently proposed. In this paper, we evaluate the error performance of some of the SD-4 codes to measure the coding gain of the codes over Rician and Rayleigh fading channels. We also compared the results with the error performance of the same size extended Hamming codes, which is the subset of SD-4 codes.

Keywords: *Error Performance, Linear block codes, Fading channels.*

I. INTRODUCTION

Single-error correction and double-error detection (SEC-DED), for example extended Hamming codes, have been widely used for error control in digital communication over the years owing to their high rates and decoding simplicity [1]. Encoder and decoder implementation of long linear block codes is difficult in practice and also requires large capacity of memory. Low Density Parity Check (LDPC) codes [2] receive enormous interest from the coding community as they hold higher Bit Error Rate (BER) performances [3]. Whereas, they have the disadvantage for encoding part as they have large, high-density and irregular generator matrices to encode information data, which cause difficulty in encoder implementations in practice. A generator matrix with low density of ones is considered as Low Density Generator Matrix (LDGM) and it was shown that it has practical advantage of decreased encoding complexity and increased decoder architecture flexibility [4] and also achieves a performance close to the Shannon theoretical limit [5]. Block Turbo Codes (BTC) or Turbo Product Codes (TPC) [6] and Product Accumulate (PA) codes [7] have been shown to provide near-capacity performance and low encoding and decoding complexity. They use simpler components codes to produce very long block codes. Recently [8], a similar approach to PA codes have been implemented using extended Hamming codes that achieves near Shannon limit performance for very high rate coding, which is useful for bandwidth efficient transmission. On the other hand, it has some difficulty for adjusting code rates by using extended Hamming codes as there are only limited numbers of code rates available with extended Hamming codes. This difficulty can be overcome by using the recently introduced Geometric Construction (GC) codes [9] as they are able to generate all the Hamming distance-4 even codes with full information rate (optimal). On the other hand, GC codes are not systematic and complexity of decoding process stands as a disadvantage.

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Recently, a new binary linear block code construction technique, called as Systematic Distance-4 (SD-4) codes, is proposed that generates all the optimal Hamming distance-4 codes greater than 7 [10]. SD-4 codes have a systematic structure and therefore it provides great simplicity in decoding process. By optimal Hamming distance-4 codes, we mean a code C = (n, k, d), where n, k and d are length, dimension and minimum Hamming distance, respectively, has the full information rate (k/n) for Hamming distance-4 with respect to the table of best known codes in [11]. Considering the properties like Hamming distance-4 and optimal and also any code length greater than 7, it is apparent that SD-4 codes, as a code family, includes the Extended Hamming codes and distance-4 Reed Muller (RM) codes [1] and also the GC codes of distance-4. This is because, Extended Hamming codes and distance-4 RM codes are the power of 2 in lengths and also the GC codes of distance-4 are the multiple of 2 in lengths. On the other hand, the proposed SD-4 codes have any length that is greater than 7. Another superiority of SD-4 codes is that their generator matrices are systematic, which in turn provides great flexibility and simplicity in encoding and decoding processes. Therefore, the SD-4 codes can be a good alternative for the component codes used in [7] and [8] as they give much greater flexibility and thus adoptability with respect to length of a code. Moreover, the constructed generator matrices of SD-4 codes contain low density of ones and cyclic property in the parity part of the generator matrices. A code with cyclic property can be encoded with less complexity [12], [13] in a similar way of cyclic codes. Therefore, GC codes also provide the advantages of LDGM codes and cyclic codes.

In this paper, we exhaustively evaluated the bit error performance of various code length SD-4 codes over Rician and Rayleigh fading channels. We performed computer simulations to obtain the bit error rates (BER) of SD-4 codes. In the simulations we utilised the sum-product algorithm (SPA) [14] for decoding processes and also we employed the binary phase shift keying (BPSK) for the modulation process. We compared our results with the well known extended Hamming codes. Regarding our results, we concluded that SD-4 codes can be considered as useful high rate and flexible codes that may find applications in satellite channels.

II. CODE CONSTRUCTION

In this section we briefly describe the construction of SD-4 codes as follows:

The systematic generator matrix of an SD-4 code is constructed as in (1).

$$\mathbf{G'} = [\mathbf{G} \quad \mathbf{I}] \tag{1}$$

Here, the matrices **G** and **I** are appended to form a systematic matrix, where **G** is the generator matrix that we construct and **I** is the corresponding identity matrix of the row size of **G**. We now define the rule of constructing the matrix **G** as follows.

In order to construct the matrix **G**, we first define the *generator codewords* and by cyclically shifting them we obtain the whole matrix **G**. To give a simple definition of the construction we continue by explaining the rule with an example. Since our SD-4 codes are full information we look at the table of best-known codes [11] and pick up a full information rate code size of distance-4. Let's now construct the SD-4 code of C = (*n, k, d*) = (128, 120, 4), where *n, k, d* are the code length, dimension and Hamming distance, respectively (R = 120/128 is a full information rate). Obviously, the necessary identity matrix of this code is in the size of (120x120). That means the size of **G** is (120x8), which are 120 rows and 8 columns. Therefore the length of *generator codewords* that will be formed is 8 and each *generator codeword* is able to produce another 7 codewords by cyclically shifting it. The first *generator codeword* is formed by placing 3 of the binary ones sequentially as g^1 = [1 1 1 0 0 0 0 0]. We consider this *generator codeword* as (3+0) case regarding the placement of binary ones. This is useful to observe the cyclic versions of the codeword. Then, we form the variations of the *generator codewords* that contains three of binary ones in the form of (2+1), which are g^2 = [1 1 0 1 0 0 0 0], g^3 = [1 1 0 0 1 0 0 0], g^4 = [1 1 0 0 0 1 0 0], g^5 = [1 1 0 0 0 0 1 0]. It is important to emphasize that we do not form the *generator codewords* in the form of (1+2) since they are already the cyclic shifted versions of (2+1) generator codewords. Then we form the other possible variations of the generator codewords that contain three of binary ones like (1+1+1), which are g^6 = [1 0 1 0 1 0 0 0] and g^7 = [1 0 1 0 0 1 0 0]. Since we have completed all the possible versions of the *generator codewords* with three of binary ones, we proceed by forming another type of *generator codewords* that contain five of binary ones like g^8 = [1 1 1 1 1 0 0 0] which is in the form of (5+0). The rest of the process is similar to the previous type of *generator codewords* that means we will form all the variations in the form of (4+1), (3+2), (3+1+1) and then (2+2+1) as follows: g^9 = [1 1 1 1 0 1 0 0], g^{10} = [1 1 1 1 0 0 1 0], g^{11} = [1 1 1 0 1 1 0 0], g^{12} = [1 1 1 0 0 1 1 0], g^{13} = [1 1 1 0 1 0 1 0] and g^{14} = [1 1 0 1 1 0 1 0]. The last type of *generator codewords* is the one that contains seven of binary ones since the length of *generator codewords* is 8 for this example. Therefore the only possible generator codeword is g^{15} = [1 1 1 1 1 1 1 0]. Since each of the generator codewords produce another 7 of codewords, including themselves, they ultimately form the generator matrix **G** that contains 15x8=120 rows and 8 columns. Then the process of constructing **G** is complete and it can be placed in (1) to obtain the final systematic generator matrix of (128, 120, 4) SD-4 code.

We always place the number of 3, 5, 7, 9, … of binary ones to form the generator codewords. We care not to reproduce a codeword while cyclically shifting the generator codewords. Thus, for instance, we do not form (2+1+2) since it is a cyclic version of the formed generator codeword (2+2+1).

Using these generator codewords of the above example, we can construct the generator matrices of binary linear block codes with length from 65 to128. It straight forward to see that, for example, to obtain an SD-4 code of length 125, we erase the last three row of G and obtain the **G** in (1) that results the generator matrix of (125, 117, 4) SD-4 code.

So far, we described the construction method of SD-4 codes by an example. Using this approach, all the distance-4 optimal systematic binary linear block codes can be obtained. Whereas, we will provide Table 1 to give a quick reference for generator codewords of length up to 10 that allows generating SD-4 code with maximum length of 512. The rest of the generator codewords can be formed regarding our description and the smaller size examples as presented in Table 1. Additionally, there are some specific cases that need careful consideration. When a generator codeword is formed like [1 0 0 1 0 0 1 0 0], we can shift it only two times since it repeats itself at the third phase of shifting. This means, the generator codeword can only produce another two codeword and while calculating the total number of rows of **G** the matter is counted into account. In the Table 1, we point out this sort of small number of shifting by appending for instance *x3*.

Table 1. Some of the *generator codewords* of SD-4 codes, where *t* is the row length.

t=4	Type	t=7	Type	t=8	Type
1110	(3+0)	1110000	(3+0)	11100000	(3+0)
		1101000	(2+1)	11010000	(2+1)
t=5		1100100	(2+1)	11001000	(2+1)
11100	(3+0)	1100010	(2+1)	11000100	(2+1)
11010	(2+1)	1010100	(1+1+1)	11000010	(2+1)
11111	(5+0)	1111100	(5+0)	10101000	(1+1+1)
		1111010	(4+1)	10100100	(1+1+1)
t=6		1110110	(3+2)	11111000	(5+0)
111000	(3+0)	1111111 x1	(7+0)	11110100	(4+1)
110100	(2+1)			11110010	(4+1)
110010	(2+1)			11101100	(3+2)
101010 x2	(1+1+1)			11100110	(3+2)
111110	(5+0)			11101010	(3+1+1)
				11011010	(2+2+1)
				11111110	(7+0)
t=9		t=10		t=10 (continues)	
111000000	(3+0)	1110000000	(3+0)	1101100010	(2+2+1)
110100000	(2+1)	1101000000	(2+1)	1100110100	(2+2+1)
110010000	(2+1)	1100100000	(2+1)	1100110010	(2+2+1)
110001000	(2+1)	1100010000	(2+1)	1100011010	(2+2+1)
110000100	(2+1)	1100001000	(2+1)	1101010100	(2+1+1+1)
110000010	(2+1)	1100000100	(2+1)	1101010010	(2+1+1+1)

101010000	(1+1+1)	1100000010	(2+1)	1101001010	(2+1+1+1)
101001000	(1+1+1)	1010100000	(1+1+1)	1100101010	(2+1+1+1)
101000100	(1+1+1)	1010010000	(1+1+1)	1111111000	(7+0)
100100100 x3	(1+1+1)	1010001000	(1+1+1)	1111110100	(6+1)
111110000	(5+0)	1010000100	(1+1+1)	1111110010	(6+1)
111101000	(4+1)	1001001000	(1+1+1)	1111101100	(5+2)
111100100	(4+1)	1111100000	(5+0)	1111100110	(5+2)
111100010	(4+1)	1111010000	(4+1)	1111101010	(5+1+1)
111011000	(3+2)	1111001000	(4+1)	1111011100	(4+3)
111001100	(3+2)	1111000100	(4+1)	1111001110	(4+3)
111000110	(3+2)	1111000010	(4+1)	1111011010	(4+2+1)
111010100	(3+1+1)	1110110000	(3+2)	1111010110	(4+1+2)
111010010	(3+1+1)	1110011000	(3+2)	1110111010	(3+3+1)
111001010	(3+1+1)	1110001100	(3+2)	1110110110	(3+2+2)
110110100	(2+2+1)	1110000110	(3+2)	1111111110	(9+0)
110110010	(2+2+1)	1110101000	(3+1+1)	1010101010 x2	(1+1+1+1+1)
110011010	(2+2+1)	1110100100	(3+1+1)		
110101010	(2+1+1+1)	1110100010	(3+1+1)		
111111100	(7+0)	1110010100	(3+1+1)		
111111010	(6+1)	1110010010	(3+1+1)		
111110110	(5+2)	1110001010	(3+1+1)		
111101110	(4+3)	1101101000	(2+2+1)		
111111111 x1	(9+0)	1101100100	(2+2+1)		

There are some important properties of SD-4 codes that should be emphasized. First they are optimal and full information rate codes (for example (512, 502, 4)) since we select the code size from the table of best-known codes [11]. As proved in Theorem 1 in [10], SD-4 code family has the Hamming distcance-4 and namely they include the code families of Extended Hamming codes, distance-4 RM codes and also the recently introduced coding scheme, the GC codes. SD-4 codes are binary systematic linear block codes and therefore easy to decode using well known decoding techniques. At the next section we demonstrate the error performance of some of the SD-4 codes using Sum-Product Algorithm (SPA), which is the popular decoding technique used for decoding LDPC codes. We also compare the error performance of SD-4 codes with some other codes.

III. ERROR PERFORMANCE RESULTS

In the below figures, we demonstrate some simulation results in order to demonstrate the BER performances of SD-4 codes. We also compare the results with the extended Hamming codes of same size. The simulations are performed in Rician, and Rayleigh channels. Rician probability density function (pdf) can be written as,

$$P(\rho)=2\rho(1+K)\ e^{(-\rho^2(1+K)-K)} I_0\left[2\rho\sqrt{K(1+K)}\right] \tag{2}$$

where ρ is fading effect, I_0 is the modified Bessel function of the first kind, order zero and K is fading parameter. Rician pdf turns into Rayleigh pdf if parameter K is chosen as 0.

In our simulations, Rician fading parameter K is chosen as 10dB. Sum-Product Algorithm (SPA) [14] is used with maximum of 100 iteration using Binary Phase Shift Keying (BPSK) modulation. We also plot the uncoded BPSK and Shannon limit for comparison purposes. In all the considered codes, BPSK modulation is used and perfect bit and block synchronizations are assumed.

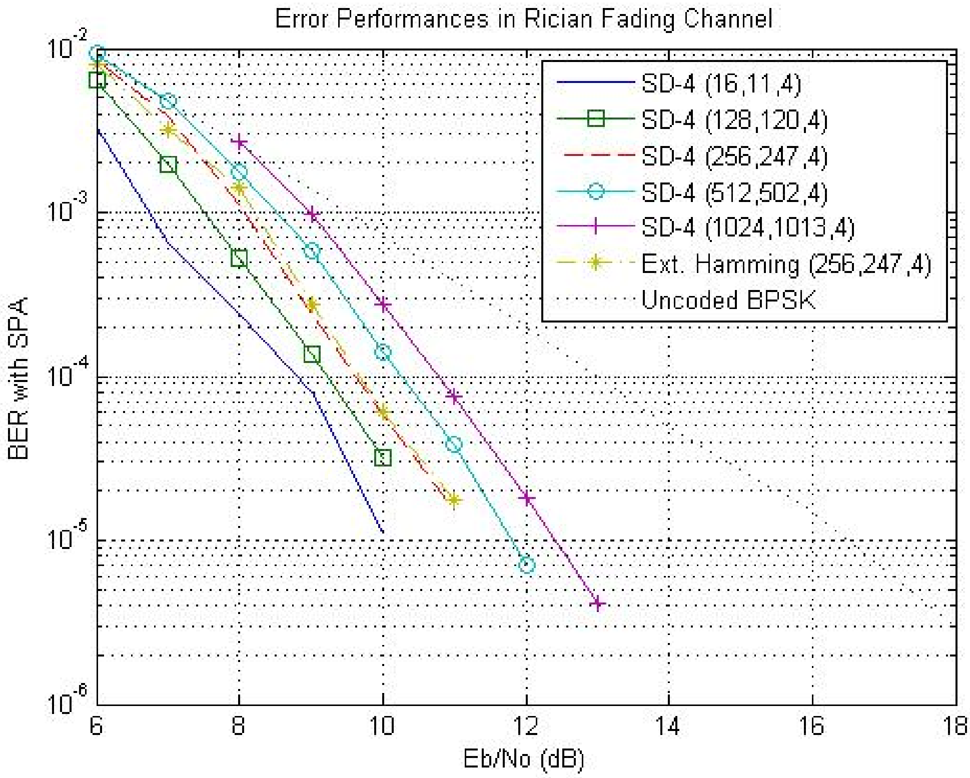

Fig.1 BER of some SD-4 codes and an Extended Hamming code over Rician channel.

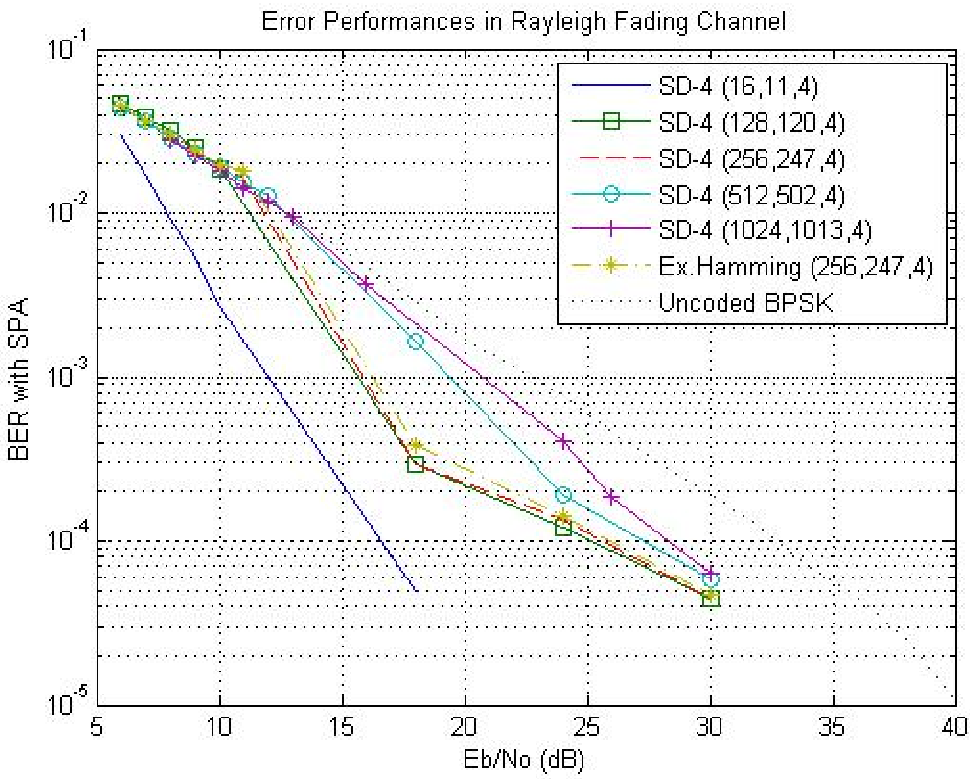

Fig.2 BER of some SD-4 codes and an Extended Hamming code over Rayleigh channel.

The simulations demonstrate that SD-4 codes have similar performance with extended Hamming codes. Although SD-4 codes are able to generate any length of full information rate code of Hamming distance-4, we only evaluated the SD-4 codes of length power of 2 since it allows us to compare the results with extended Hamming codes. We observed that SD-4 codes and extended Hamming codes have quite similar error performance.

We also compare the BER performances of SD-4 codes with uncoded BPSK error rates in both Rician and Rayleigh fading channels in order to measure the coding gain of SD-4 codes. For example, in Rician channel, (16,11,4) SD-4 code provides about 6.3 dB coding gain and (512, 502, 4) SD-4 code provides about 4.6 dB coding gain at the BER of 10^{-5}. Also, in Rayleigh channel, (16,11,4) SD-4 code provides about 17 dB coding gain and (512, 502, 4) SD-4 code provides about 7 dB coding gain at the BER of 10^{-4}.

As it can be seen from the figures, SD-4 codes have significant error correction capability, whereas, they have superior properties such as optimal code size, any code length greater than 7, and most importantly systematic, low density generator matrix. Having these superiorities with similar error performance of their counterparts; SD-4 codes stand as a very useful code family that may well find applications in digital communication applications such as satellite communications and computer communications.

IV. CONCLUSION

We presented the bit error performances of some of SD-4 codes in Rician and Rayleigh fading channels using sum-product algorithm for decoding process and also binary phase shift keying for modulation process.

We concluded that SD-4 codes have significant coding gain in these fading channel environments but they have similar error performances with the well known extended Hamming codes. They generate all the optimal size codes for Hamming distance–4 as a code family that contain the well known codes such as extended Hamming codes and also distance-4 RM codes. Generator matrices of SD-4 codes have some useful properties like low density and systematic structure; they are practical for encoder and decoder design of very long length block codes. Thus we state that SD-4 codes, with their significant coding gain and practical structure, will be an important code family of forthcoming telecommunication applications.

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