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IMPROVED EXACT DISTRIBUTED RLS ALGORITHM FOR DECENTRALIZED ESTIMATION OVER SENSOR NETWORKS

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ABSTRACT

In this paper we propose improved exact distributed recursive least-squares algorithm, (dRLS) for decentralized estimation over sensor networks. We consider a situation where there are a number of sensors with high observation noise ('noisy sensors') in the network. To deal with noisy sensors, first we show that when there are such sensors in the network, the performance of incremental dRLS algorithm drastically decreases; In addition, by detecting and ignoring these sensors better performance in a sense of estimation can be achieved. To address the problem of noisy sensors, we propose a new algorithm which consists of noisy sensors detection method and modified dRLS algorithm. As our simulation results show, the proposed method outperforms the dRLS algorithm in the same condition.

Key words: *sensor networks, distributed estimation, adaptive filter.*

1. INTRODUCTION

The ability to detect events of interest is essential to the success of emerging sensor network technology. Detection often serves as the initial goal of a sensing system [1]. The distribution of the nodes in a sensor network yields spatial diversity, which should be exploited alongside the temporal dimension in order to enhance the robustness of the processing tasks [2]. Recently, distributed adaptive estimation algorithms that enable a network of nodes to function as an adaptive entity are proposed [3]–[11].

In [3]-[6] distributed adaptive algorithm using incremental optimization techniques such as IDLMS algorithm [3-5] and dRLS [6] are developed. The resulting algorithms are distributed, cooperative, and able to respond in real time to changes in the environment. In these algorithms each node is allowed to communicate with its immediate neighbor in order to exploit the spatial dimension while limiting the communications burden at the same time. In [7-9] diffusion implementation for distributed adaptive estimation algorithms are proposed. These algorithms are possible when more communication resources are available so that each node can communicate with all its neighbors as dictated by the network topology. Both LMS based diffusion and RLS based diffusion algorithm are available and their formulation and performance can be found in [10] and [11].

In the existing distributed adaptive estimation algorithms such as IDLMS and dRLS, either equal observation noise is assumed for all the nodes (sensors) in the network or same strategy is used for different observation noise condition. In

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practice, this assumption fails due to the many physical conditions. A situation where there are a number of sensors with high observation noise variance (which hereafter we will call them "noisy sensors") is considered in this paper. As it is shown in this paper, using these noisy sensors in the dRLS algorithm will cause a severe decrease in the algorithm's performance. To address this problem, in this paper consists of a noisy sensors detect method and modified dRLS algorithm.

2. BACKGROUND

2.1 problem statement

We consider a sensor network which consists of N nodes as shown in Fig. 1. Each node k has access to a $1 \times M$ regressor vector $u_{k,i}$ and measurement data $d_{k,i}$, .

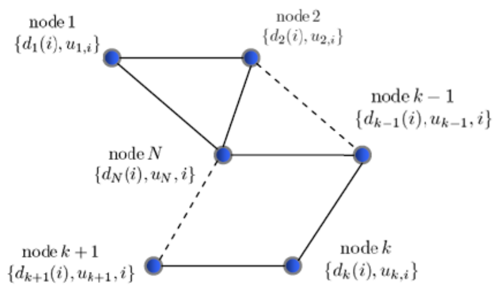

Fig. 1 A distributed network with N active nodes

The objective is to obtain an estimate of unknown vector w^o as accurate as possible. The unknown vector w^o relates to $\left\{d_k(i), u_{k,i}\right\}$ as

$$d_k(i) = u_{k,i} w^o + v_k(i) \tag{1}$$

At each time instant i, the network has access to space-time data

$$y_i = \begin{bmatrix} d_1(i) \\ d_2(i) \\ \vdots \\ d_N(i) \end{bmatrix} \quad \text{and} \quad H_i = \begin{bmatrix} u_{1,i} \\ u_{2,i} \\ \vdots \\ u_{N,i} \end{bmatrix} \tag{2}$$

Collecting all the data available up to time i into global matrices $\mathcal{Y}_i$ and $\mathcal{H}_i$ yields

$$\mathcal{Y}_i = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_i \end{bmatrix} \quad \text{and} \quad \mathcal{H}_i = \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_i \end{bmatrix} \tag{3}$$

Now, using (3), the estimation problem can be written as a regularized weighted least-square (LS) problem as follows

$$\min_w \triangleq \left[\lambda^{i+1} w^* \prod w + \left\|\mathcal{Y}_i - \mathcal{H}_i w\right\|^2_{\mathcal{W}_i}\right] \tag{4}$$

where for a vector x and a Hermitian matrix $A > 0$ the notation $\|x\|^2_A$ is defined as

$$\|x\|^2_A = x^* A x \tag{5}$$

The weight matrix in (4) is chosen as

$$\mathcal{W}_i \triangleq \operatorname{diag}\left\{\lambda^i D, \lambda^{i-1} D, \cdots, \lambda D, D\right\} \tag{6}$$

with a spatial weighting factor as

$$D = \operatorname{diag}\left\{\gamma_1, \gamma_2, \cdots, \gamma_N\right\}, \gamma_i \geq 0 \tag{7}$$

and time forgetting factor as $0 \ll \lambda \leq 1$. Moreover, we have $\prod > 0$. The solution of problem (4) is given by

$$w_i = P_i \mathcal{H}_i^* \mathcal{W}_i \mathcal{H}_i \tag{8}$$

where

$$P_i = \left(\lambda^{i+1} \prod + \mathcal{H}_i^* \mathcal{W}_i \mathcal{H}_i\right)^{-1} \tag{9}$$

2.2 Exact dRLS implementation

It is desired to have a distributed recursive equation to update w_i. To this aim, it is necessary to have a distributed recursive equation for P_i. In [6] such an equation is given as

$$\left\{\begin{array}{l} P_{o,i} \leftarrow \lambda^{-1} P_{i-1} \\ \text{for } k = 1 : N \\ \quad P_{k,i} = P_{k-1,i} - \dfrac{P_{k-1,i} u^*_{k,i} u_{k,i} P_{k-1,i}}{\gamma_k^{-1} + u_{k,i} P_{k-1,i} u^*_{k,i}} \\ \text{end} \\ P_i \leftarrow P_{N,i} \end{array}\right. \tag{10}$$

To obtain a distributed recursive equation for w_i, an incremental path is defined across the network cycling from node 1, to node 2, and so forth, until node N as shown in Fig. 2.

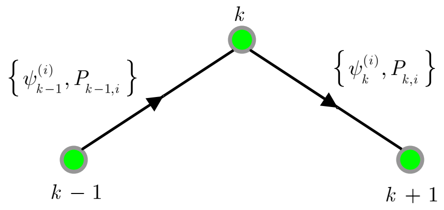

Fig. 2 The cooperation strategy of the exact distributed RLS algorithm (dRLS).

Define the intermediate global matrices $\mathcal{Y}_{i-1}^{k}$ and $\mathcal{H}_{i-1}^{k}$ that collect the data blocks $\left\{\mathcal{Y}_{i-1}, \mathcal{H}_{i-1}\right\}$ in addition to the data collected along the network at time i up to node k

$$\mathcal{Y}_i^k = \begin{bmatrix} \mathcal{Y}_{i-1} \\ d_1(i) \\ d_2(i) \\ \vdots \\ d_k(i) \end{bmatrix} \quad \text{and} \quad \mathcal{H}_i = \begin{bmatrix} \mathcal{H}_{i-1} \\ u_{1,i} \\ u_{2,i} \\ \vdots \\ u_{N,i} \end{bmatrix} \tag{11}$$

Let $\psi_k^{(i)}$ be the solution of following LS problem

$$\min_{\psi} \left[\lambda^{i+1} \psi^* \prod \psi + \left\| \mathcal{Y}_i^k - \mathcal{H}_i^k \psi \right\|_{\mathcal{W}_i^k}^2 \right] \tag{12}$$

where

$$\mathcal{W}_i^k = \begin{bmatrix} \lambda \mathcal{W}_{i-1} & \mathbf{0} \\ \mathbf{0} & D_k \end{bmatrix}, \; D_k = \operatorname{diag}\left\{ \gamma_1, \cdots, \gamma_N \right\} \tag{13}$$

Note that

$$\mathcal{W}_i^k = \begin{bmatrix} \mathcal{W}_i^{k-1} & \mathbf{0} \\ \mathbf{0} & \gamma_k \end{bmatrix} \tag{14}$$

By using (9) and (14), a recursion equation to update $\psi_k^{(i)}$ in a distributed fashion can be found as

$$\psi_k^{(i)} = P_{k,i} \mathcal{H}_i^{k*} \mathcal{W}_i^k \mathcal{Y}_i^k \tag{15}$$

After some computation, update equation for $\psi_k^{(i)}$ is given by

$$\psi_k^{(i)} = \psi_{k-1}^{(i)} + \frac{P_{k-1,i}}{\gamma_k^{-1} + u_{k,i} P_{k-1,i} u_{k,i}^*} u_{k,i}^* \left(d_k(i) - u_{k,i} \psi_{k-1}^{(i)} \right) \tag{16}$$

Finally the incremental distributed exact RLS (dRLS) algorithm can be summarized as follows

$$\left\{ \begin{array}{l} \psi_0^{(i)} \leftarrow w_{i-1}; \quad P_{0,i} = \lambda^{-1} P_{i-1} \\ \text{for } k = 1:N \\ \quad e_k(i) = d_k(i) - u_{k,i} \psi_{k-1}^{(i)} \\ \quad \psi_k^{(i)} = \psi_{k-1}^{(i)} + \dfrac{P_{k-1,i}}{\gamma_k^{-1} + u_{k,i} P_{k-1,i} u_{k,i}^*} u_{k,i}^* \left(d_k(i) - u_{k,i} \psi_{k-1}^{(i)} \right) \\ \quad P_{k,i} = P_{k-1,i} - \dfrac{P_{k-1,i} u_{k,i}^* u_{k,i} P_{k-1,i}}{\gamma_k^{-1} + u_{k,i} P_{k-1,i} u_{k,i}^*} \\ \text{end} \\ w_i \leftarrow \psi_k^{(i)}; \; P_i \leftarrow P_{N,i} \end{array} \right. \tag{17}$$

3. PROPOSED ALGORITHM

3.1 Motivation

The motivation to propose a new algorithm stems from the two following reasons:

1. Equal observation noise assumption for all nodes could not comply with real scenario; however, considering some noisy sensors in the network is a better assumption for sensor network.
2. When there are some noisy sensors in the network the performance of dRLS algorithm drastically decreases.

To show the poor performance of dRLS algorithm in the presence of noisy sensors, let's consider a network with $N = 40$ nodes and assume Gaussian repressors with $R_{u,k} = I$. we further assume that there are $N_s = 10$ noisy sensors in the network and background white noise for these sensors has a variance of $\sigma_v^2 = 10^{-1}$ whereas for other sensors this quantity is equal to $\sigma_v^2 = 10^{-3}$.

In Fig. 3 the mean square error (MSE) performance of dRLS algorithm for two different cases is plotted. The MSE criteria can be calculated as follows

$$\mathrm{MSE} \triangleq E\left|e_k(\infty)\right|^2 \qquad (18)$$

These cases are: 1) There are some noisy sensors in the network 2) The noisy sensors are ignored. As it is clear from Fig. 3, if the noisy sensors in the network are perfectly known and ignored in dRLS update equation, a better result can be achieved.

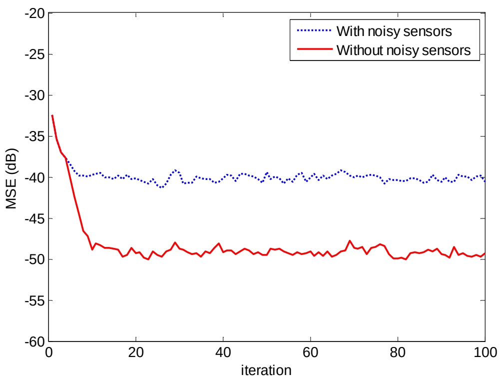

Fig. 3 The performance of dRLS algorithm in two different cases: 1) with noisey sensors, 2) ignoring the noisy senosrs

3.2 Proposed Method

The proposed method to enhance the poor performance of dRLS algorithm consisting of following steps:

1. Obtaining a primitive estimate of w^o.
2. Estimating the observation noise in each node.
3. Detecting the noisy sensors and using modified dRLS algorithm.

To obtain an estimate of observation noise at each node, we consider again the equation (1). Using (16) and repeating it for Ls times (where Ls is a suitably chosen integer), it is possible to have an primitive estimate of w^o. It must be noted that the primitive estimate of w^o is used just to obtain an primitive estimate of observation noise at each sensor this estimate is not the final estimate of w^o. Denoting the $\psi_k^{(Ls)}$ as the estimate of w^o in the Ls^{th} iteration in N^{th} node we will have:

$$\psi_N^{(Ls)} = \psi_k^{(i)}\Big|_{i=Ls,k=N} \qquad (19)$$

Using (1) and (17) the observation noise at each sensor can be estimated as

$$n_k(i) = d_k(i) - u_{k,i}\psi_N^{(Ls)},\ i = 1,\cdots,Ls \qquad (20)$$

Having $n_k(i)$, we define the following vectors for each sensor

$$\underset{k=1,2,\cdots,N}{B_k} = n_k(i),\ i = 1,2,\cdots,Ls \qquad (21)$$

The observation noise for noisy sensors have is bigger than other sensors, so we compute the following parameters

$$m_k = \frac{1}{Ls}\sum_{i=1}^{Ls} B_k(i), \qquad (22)$$

$$\tilde{\sigma}_k = \sum_{i=1}^{Ls}\left(B_k(i) - m_k\right)^2, \qquad (23)$$

Now by using the vectors $\{\tilde{\sigma}_k\}_{k=1}^{N}$ and a suitable threshold it is possible to recognize the noisy sensors in the network. The main drawback of this method to detect the noisy sensors is that for any specific network a new threshold must be chosen. To address this problem, we do as follows: First we define the following parameters

$$V = \frac{1}{N}\sum_{k=1}^{N}\tilde{\sigma}_k, \qquad (24)$$

$$M_k = \tilde{\sigma}_k - V, \quad k = 1,2\ldots,N \qquad (25)$$

In Fig. 4 the $\tilde{\sigma}_k$ and M_k, $k = 1,2,..,N$ are plotted.

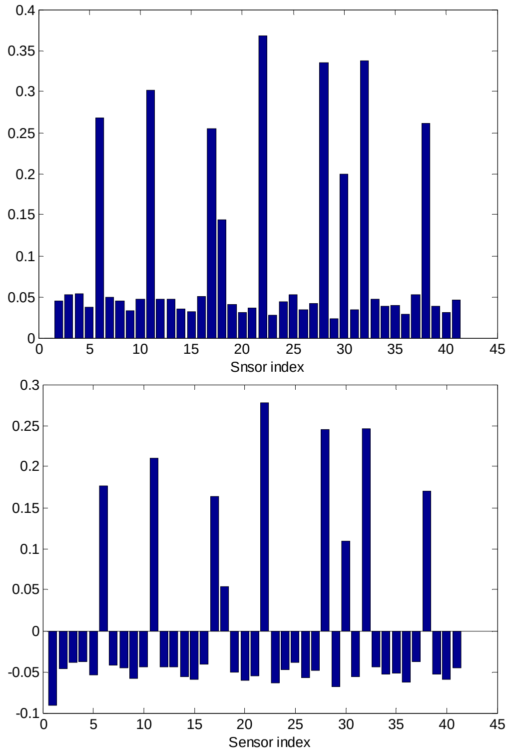

Fig. 4 The $\{\tilde{\sigma}_k\}_{k=1}^{N}$ (up) and $\{M_k\}_{k=1}^{N}$ (down).

As it is clear from Fig. 4, noisy sensors in general have $M_k > 0$ so finding the sensors with this feature is equivalent to (with high probability) noisy sensor detection. After noisy sensors detection, for the rest of w^o estimation process, (i.e. $i = Ls+1, Ls+2, \ldots$) the update equation (16) is modified as follows

- For noisy sensors
$$\psi_k^{(i)} = \psi_{k-1}^{(i)}, \quad i > Ls, k = 1,2,\ldots,N$$

- For other sensors

$$\psi_k^{(i)} \underset{k=1,2,\ldots,N}{=} \psi_{k-1}^{(i)} - \mu u_{k,i}^* \left[d_k(i) - u_{k,i}\psi_{k-1}^{(i)}\right] \tag{26}$$

After i^{th} iteration $i \to \infty$ the nth node contains the appropriate estimate of w^o i.e.

$$\lim_{i\to\infty} \psi_N^{(i)} \to w^o \tag{27}$$

4. SIMULATION RESULTS

In this section the result of simulation results are presented. We consider a network with $N = 40$ nodes and Gaussian repressors with $R_{u,k} = I$. we further assume that there are $N_s = 10$ noisy sensors in the network and background white noise for these sensors has a variance of $\sigma_v^2 = 10^{-1}$ and other sensors $\sigma_v^2 = 10^{-3}$. The curves are obtained by averaging over 200 experiments.

In the proposed algorithm it is necessary to obtain a suitable primitive estimate of w^o which is strongly depends on the value of Ls. Fig. 5 shows this fact where the performance of the dRLS algorithm and proposed algorithm for $Ls = 2$ and $Ls = 10$ is plotted.

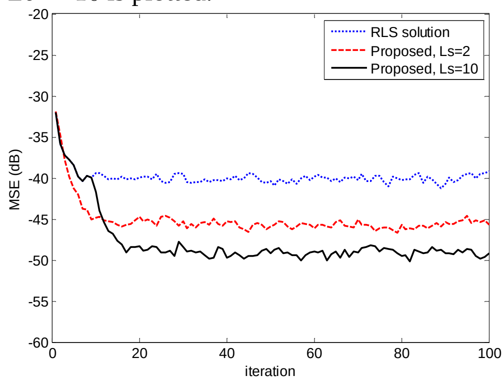

Fig. 5 Performance of proposed algorithm for $Ls = 2$ and $Ls = 10$.

As it is clear from Fig. 5, as Ls increases, better primary estimate of w^o and as a result, a better final estimate of w^o is obtained. On the other hand, the proposed algorithm outperforms the dRLS algorithm. It must be noticed that by increasing the Ls, no better primary of w^o is obtained. This situation occurs when dRLS algorithm is in its steady-state position. Fig. 6 depicts the performance of the proposed algorithm for different Ls in comparison with dRLS algorithm.

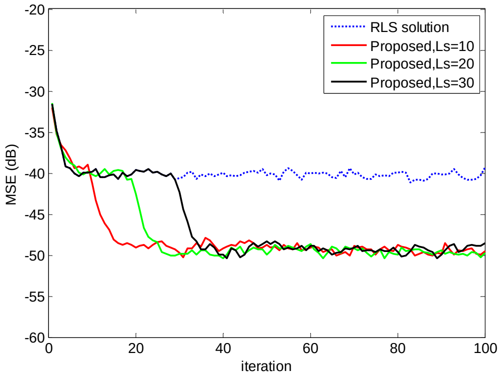

Fig. 6 The performance of the proposed algorithm for different Ls

The performance of proposed algorithm for different number of noisy sensors (i.e. Ns) and different number of sensors (i.e. K) are plotted in Figs. 7 and 8 respectively.

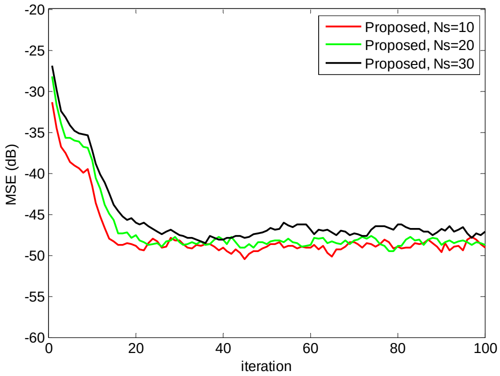

Fig. 7 The performance of proposed algorithm for different number of noisy sensors

As the number of noisy sensors in the network increases, the performance of the proposed algorithm decreases. On the other hand, by increasing the number of sensors, better performance can be achieved as Fig. 8 shows.

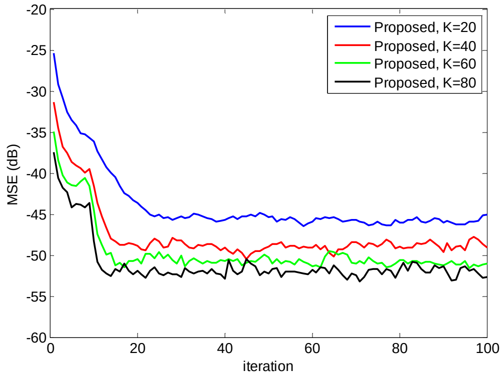

Fig. 8 The performance of proposed algorithm for different number of sensors

5. CONCLUSION

In this paper we considered the issue of distributed adaptive estimation over sensor when there are some sensors with high observation noise in the network. We showed that these sensors reduce the performance of dRLS algorithm. In addition, by detecting and ignoring these sensors better performance can be achieved. To deal with the mentioned problems, we proposed a new algorithm which is based on a primary estimation of each sensors observation noise. As our simulation results show, the proposed method has better performance than dRLS algorithm in the same condition.

REFERENC

[1] J. F. Chamberland and V. V. Veeravalli, “Wireless sensors in distributed detection applications,” *IEEE signal processing magazine*, vol. 24, pp, 16-25, May 2007.
[2] D. Bertsekas, “A new class of incremental gradient methods for least squares problems,” *SIAM J. Optim.*, vol.7, no. 4, pp. 913-926, Nov.1997.
[3] C. Lopes and A. H. Sayed, "Distributed adaptive incremental strategies: Formulation and performance analysis," *Proc. ICASSP*, Toulouse, France, vol. 3, pp. 584-587, May 2006.
[4] C. Lopes and A. H. Sayed, "Distributed processing over adaptive networks," *Proc. Adaptive Sensor Array Processing Workshop*, MIT Lincoln Laboratory, MA, June 2006.
[5] C. G. Lopes and A. H. Sayed, "Incremental adaptive strategies over distributed networks,"

IEEE Transactions on Signal Processing, vol. 55, no. 8, pp. 4064-4077, August 2007.

[6] A. H. Sayed and C. Lopes, "Distributed recursive least-squares strategies over adaptive networks," *Proc. 40th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, pp. 233-237, October-November, 2006.

[7] C. G. Lopes and A. H. Sayed, "Diffusion least-mean-squares over adaptive networks," *Proc. ICASSP*, Honolulu, Hawaii, vol. 3, pp. 917-920, April 2007.

[8] F. Cattivelli, C. G. Lopes, and A. H. Sayed, "A diffusion RLS scheme for distributed estimation over adaptive networks," *Proc. IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Helsinki, Finland, pp. 1-5, June 2007.

[9] C. G. Lopes, and A. H. Sayed, "Steady-state performance of adaptive diffusion least-mean squares," *Proc. IEEE Workshop on Statistical Signal Processing (SSP)*, pp. 136-140, Madison, WI, August 2007.

[10] C. G. Lopes and A. H. Sayed, "Diffusion least-mean squares over adaptive networks: Formulation and performance analysis," *to appear in IEEE Transactions on Signal Processing*, 2008.

[11] F. Cattivelli, C. G. Lopes, and A. H. Sayed, "Diffusion recursive least-squares for distributed estimation over adaptive networks," *IEEE Transactions on Signal Processing*, Vol: 56, Issue 5, pp.1865 – 1877, May 2008.

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