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Research Article

Challenges and Possibilities in Teaching and Learning of Calculus : A Case Study of India

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Abstract
Introducing, and thereafter teaching calculus to senior secondary, early college and university students, at the expense of algebra and geometry, is causing half-baked calculus being served to relatively under-prepared students. In line with this proposition, the current research aims to identify how cognition of calculus takes place among learners, what teaching methodologies are used by Indian teachers, what pedagogical techniques are most efficient in calculus teaching, and what prerequisites are called for before commencement of the course on calculus? For this extensive study, data was gathered from school teachers and assistant/associate professors of colleges and universities, having more than 6 years of calculus teaching experience, drawn from 26 schools, 19 colleges and 7 university departments, spanning across 23 different states and union territories of India. A total of 142 teachers took part in this study. Data was collected using schedules, classroom observations, focus group interviews, and informal discussions that were carried out both before and after the classroom teaching. NVivo and Concordance softwares were used for analysis of the emerging content and classroom discourses. The study traversing between February 2016 to April 2019, is qualitative in its framework and lies purely within the interpretivist paradigm. The findings of this research shall mellow the understanding of calculus cognition operational among school, college and university going students.
Keywords :
curriculum, evaluation, mathematics education, pedagogical content knowledge, teaching-learning, teaching of calculus

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Introduction

It has been several years that researchers have been discussing and debating about the very nature and purpose of making school and college students learn mathematics (Dossey, 1992; Orton & Wain, 1994). In most of the countries, be it developed or developing, it is seen that mathematics in school has a position that is privileged over other subjects, and that the status which it enjoys is because of its usefulness and application which is in stark contrast to others' beliefs who view mathematics as the highest form of knowledge which emphasizes abstractness having formal proof and that it focusses inside of itself (Gardiner, 1995; Neumark, 1995).

The recurrent problem of introducing integration and differentiation to newbies are the frequent reinforcement of certain typical questions that involves asking them to solve, graph, calculate, plot, compute, differentiate, sketch, determine, etc. (Ferrini-Mundy & Graham, 1991). Students' learning a concept or a construct without knowledge and comprehension of its meaning has been the issue of research for several decades (Hiebert & Carpenter, 1992). In 1980s in the USA, because of the visible crisis pertaining to learning and teaching of calculus, US witnessed a movement that inspired changes in the manner in which calculus was taught to students (Tucker & Leitzel, 1995). There has been at attempt by several authors to expand the "Rule of Three" to incorporate enactive and formal representation (David Tall, 1996); representations using animations (Bowers, 1999; Leinbach, 1997); representation of real data (Kaput, 1998) wherein learners experiencing states of affairs that are close to reality and natural phenomenon and implanting the usage of functions in data that are real and representations that are verbal (Kennedy, 2000).

Some researchers talked about what existing literature says with regards to learner's understanding of functions and reports that students have a conception/mental representation of functions that is pretty infirm and that they show a tendency of relying on algebraic formulas while evolving and formulating their conception of functions (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997). (Koirala, 1997) sets the students' conceptual understanding while teaching calculus in the theoretical account that (Skemp, 1976) gave about the relational and instrumental cognition involving giving rules and formulas to students in solving problems on calculus and the application of those formulas by students in solving calculus problems that are of routine-nature which does not require students to put any brain to delve into its basics.

When student learns a certain construct, a concept image as well as a concept definition is built in their mind (David Tall & Vinner, 1981). It is mostly by accident that the process of concept construction occurs among learners because they learn it using identical textbook problems which lead to naïve or intuitive structures that are immune to transformation and the belief of abstractions that are reflective (Piaget, 1985), which (Dubinsky, 2002) had conceptualized for summarization of traits that is present in constructively reflective abstractions emanating from the viewpoint of mathematical thinking of higher order, constitutes co-ordination, encapsulation, internalizing, reversibility and generalization. Using Dubinsky's model, (Repo, 1994) has come up with an explanation of reflective abstraction in construction of cognitive structures on constructs pertaining to derivatives.

It is very much possible that if learners could internalize concepts that are specific to derivatives, and work towards development of capabilities in conceiving a function of derivatives as the nodal unit for processing, they can in subsequence easily fabricate a novel inverse process for delving in operations of differentials, thus making it pretty alike to determination of integration of the originally chosen function. (David Tall, 1996) delineates the proposition given by (Sfard, 1992) that viewing of mathematics operationally is preceded by structurally viewing it, considering objects and the formal definitions and this could have prominent implications with regards to theories of teaching. The difference which is there among "*concept definition*" and the holistic impression of "*concept image*" is reprised by (David Tall, 1996) and this distinguishing difference made some mathematicians (David Tall & Vinner, 1981; Vinner & Dreyfus, 1989) explicate certain lack of successes in students' understanding of it. (Gray & Tall, 1994) propounded and used the notional belief of "*procept*", describing it as an amalgamation of process and concept, thereby laying claim that it is, in particular, conformed to the contemplation of calculus learning and its initial analysis. They opined that functions, integrals, derivatives and the notions of fundamental limits are all examples of *procepts*.

When concepts are viewed in more than one setting and from diverse viewpoints, it becomes an essential noetic state of cognition that is visualized as the facet of "general idea of flexibility" talked about by (Dreyfus & Eisenberg, 2012). In their study, (De Guzmán, Hodgson, Robert, & Villani, 1998) shows that at different stages of an individual's learning and education, learners show varied levels of maturity while proving a theorem. It is specifically expected from tertiary level learners to showcase correct formalism while engaging with non-trivial proofs. An attempt was made, applicable to diverse mathematical areas to teach analysis. And was showcased in the study carried out by Legrand and the approach under consideration, often referred to as "scientific debate" (Artigue, 2001; Legrand, 1993) has its roots in a particular type of discourse among learners with regards to theorems' validity. If students encounter arguments that are structured with regards to mathematical content, deeper development of cognition of fundamental concepts are seen.

One study has examined the excogitation of derivatives as a case study of non-continuities in transitioning from the school to university and the findings attest to the attempts that students make in adjusting their mathematically sound learning aids to situations that are baffling and complicated leading these attempts towards oversimplifications that are consequences of their limited field of experience (Praslon, 1999). Most of the researches carried out till date are towards reforming calculus, emphasizing majorly on opinions made or descriptions given either on software programmes that ease calculus learning, or of the contemporary curricula which eventually lead to mere informing the readers of what and how of calculus learning and teaching, relevant examples to which can be found in (A. H. Schoenfeld, 1995), (DAVID, 2014), (Douglas, 1995), and (Solow, 1994).

In many instances of research in calculus education, the evaluations have made use of experimental and quasi-experimental designs that are straightaway borrowed from physical or natural sciences. They have used two groups, one where exposure is given and other where no exposure is given and comparison is drawn between the scores obtained from pre- and post-tests, where randomization techniques were used for selection of samples in a relatively controlled environment making use of the reform approach (Dada & Akpan). Tests sometimes were specifically designed for assessment of certain specific kinds of students' performance like their proficiency in handling traditional algorithms or in determination of their ability in solving problems that are conceptually driven. Relevant examples to it can be found in (Armstrong, Garner, & Wynn, 1994) and (Bookman & Friedman, 1994). Comparative studies of this kind have certain long-familiar limitations, see e.g., (A. Schoenfeld, 1994) prominent one is its poor suitability in studying phenomenon that are as complicated as *students' learning* and *teaching*.

Prima facie it is found that, in many cases of researches done towards improvement of pedagogy of calculus, what is identified turns out to be an interesting phenomenon but the findings, results and conclusions has less of bearing when it comes to its contribution on our cognition of effective techniques to be implanted in teaching and learning of calculus. Identification of interesting occurrences, say for instance, determination of differential performance of students from different courses on exam questions wherein the study is using diverse designs of experimental research or observations of learners' problem-solving behaviour while they are in calculus class, have the potential for generation of results possessing explanatory power for uncovering differences in performances of learners. In few of the researches, such methods are used in examining research questions that are non-comparative in nature, to read more on non-comparative studies and examples of it (Selden, Selden, & Mason, 1994; Palmiter, 1991; Park & Travers, 1996; Meadows, 2016) and (Bonsangue & Drew, 1995).

It is unfortunate though that studies yielding results possessing impregnable explanatory power are scarcely available and studies that are comparative in nature undertaken on teaching of calculus and reform in its curriculum have not been able to effectively advance our cognition about students' calculus learning and on how diverse pedagogical circumstances can positively impact their understanding. It has been documented in several researches that what appears to be pupils' impuissance in cognition of concepts of calculus might not really be the case; in fact it may just be the manifestations of their pre-existent comprehension of associated concepts (Pardimin, Arcana, & Supriadi). Learners may, for example, comprehend the conceptions of functions in a certain way that serves them considerably in few particular situations but are not in consonance with, or are not supportive of the developments of a sturdy inference of derivatives, illustrations related to it are witnessed in (David Tall, 1992), (Monk & Nemirovsky, 1994), (Ferrini-Mundy & Graham, 1994), (Williams, 1991), and (White & Mitchelmore, 1996). Study conducted by (Kuh, Kinzie, Schuh, & Whitt, 2011) ascertained that to accelerate pupil learning and collegiate excellence; thought-provoking and intellectually stimulating creative work is fundamental.

There are certain beliefs that are commonly held by secondary school students of mathematics that was outlined by (Garofalo, 1989) based on observations and conversations he engaged with in during his long career as a mathematics teacher. These beliefs include, teachers and textbooks are the sole authority of knowledge and that most of the mathematical problems could be unriddled by directly employing the formulas, facts, theorems, rules, and procedures emanating directly from teachers and textbooks, (p. 502); formulas rather than their derivations are important (p. 503); and that the teachers/textbooks are the sole dispensers of knowledge (p.503), concomitantly the way students engage with mathematics, the approach with which they solve math problems and their expectations with the nature of a mathematics classroom are directly affected by the belief systems students hold (Bookman, 1993; Schoenfeld, 1995).

It has been recommended by (Smith & Moore, 1990, 1991) that school/college teachers shall involve less in delivering lectures and more on encouraging them engaging in group tasks/activities [see also (Bookman & Blake, 1996)]. Most of the prominent contemporary researches in mathematics education that focusses on reformation leading to enhancement of teaching efficiency are works that are either on mathematics pedagogy at elementary or secondary level or on calculus learning at school and university level (Douglas, 1986; NCTM, 1989; NCTM, 1991; M. Tucker, 1990), mathematical fraternity thus are now recognizing the immense importance of calculus and are thus getting all the more connected in the process (Young, 1987). A constructivist mathematician would state that learning math is a process in which students engage in reorganization of their activities for resolving problem areas that are found excessively difficult to them (Cobb et al., 1991) and in alignment with this take, constructivists agree on construction of mathematical knowledge in the classrooms via reflective abstraction, and that there is continuous development of cognitive structures (Noddings, Maher, & Davis, 1990).

In the field of mathematics education, several researchers have showcased interest in facets of learners' cognition of functions at the senior-secondary schools and colleges (Buck, 1970; Dreyfus & Eisenberg, 1983, 1984; D. H. Monk, 1987). Findings of these researches have shown that most of the learners at this level are stuck to a single definition of functions that is being staged by the correspondence rule whose domain is unvarying in its entirety (Ferrini-Mundy & Graham, 1991; Markovits, Eylon, & Bruckheimer, 1986; Vinner & Dreyfus, 1989). When there is movement in thought process from graphical mode to algebraic mode, piecewise definition of functions bring about massive difficulty for the learners (Ferrini-Mundy & Graham, 1991; Markovits et al., 1986; Vinner & Dreyfus, 1989), and additionally, learners for most of the time, ascertain whether or not graphs represent functions by its measure of acquaintance (Ferrini-Mundy & Graham, 1991). In the settings where students have both graphical and algebraic data, they often view them independently and oftentimes find comfortability using methods whose reasoning is contradictory (Ferrini-Mundy & Graham, 1991).

Tendency of students have been noted by researchers and while examining their behaviour about a graph both locally and at any one point and also algebraically, students have shown the tendency of evaluating formulas for one domain value, while on the contrary, a holistic rendition is usually of utmost importance in cognizing concepts in calculus (Bell & Janvier, 1981; D. H. Monk, 1987). It has been observed that researches on pupils' cognition of limits are not very extensive and learners often encounter conflicts among precise/formal and informal definitions that use interpretations in simple language and in natural discourses (Confrey, 1981; Graham & Ferrini-Mundy, 1989; D Tall & Schwarzenberger, 1978; Williams, 1991) and pupils often have this feeling that a limit can never be reached and are mostly anxious for the fear of encountering a mismatch between their instinctual knowledge and the solutions they come up with via mathematical processing (Ferrini-Mundy & Graham, 1991) and in connection to it (Davis & Vinner, 1986) figured out that learners keep holding a very similar visceral perception regarding limits of sequences. Easy accessibility and inexpensiveness of graphics calculators make students of pre-calculus and calculus studying in formal educational institutions, use this technology extensively thereby showcasing substantial impact on classroom instruction.

It is widely claimed that usage of graphic calculators make room for enhanced conceptual approaches to problem solving, refined understanding of bond between graphic representation and symbolic algebra, and sharpened ability among students in solving mathematical problems(Kök & Davaslıgil, 2014). Students now take all the benefits emanating from it, earlier they solved questions in the way traditional and formal mathematics required (Demana, Waits, & Clemens, 1993; Mathematics, 1989; Davld Tall & Blackett, 1986). In line with the issues emerging from the literature, and considering the aforementioned in mind, the researcher aims to investigate the challenges and possibilities of learning and teaching of elementary calculus in India vis-à-vis the process through which calculus cognition takes place among calculus learners in India's educational milieu.

Research Problem

Introducing, and thereafter teaching calculus to senior secondary, early college and university students, at the expense of algebra and geometry, is causing half-baked calculus being served to relatively under-prepared students. In line with this proposition, the current research aims to identify :

- How cognition of calculus takes place among learners?
- What teaching methodologies are used by Indian teachers?
- What pedagogical techniques are most efficient in calculus teaching?
- What prerequisites are called for, before commencement of the course on calculus?; and
- Does calculus background at the senior-secondary level, enhance student's interest in going forward to study disciplines that have intensive mathematical rigour?

Method

Research Model

The study traversing between February 2016 to April 2019, is qualitative in its framework and lies purely within the interpretivist paradigm.

Participants

For the stated objectives, data was collected from post graduate teachers and assistant/associate professors, having more than 5 years of calculus teaching experience, drawn from 26 schools, 19 colleges and 7 university departments spanning over 23 different states and union territories of India. It was specifically taken care of that the 52 educational institutions chosen for study were geographically separated from one another. This was done to ensure that the sample was a true representation of India. A total of 142 teachers took part in this study. They were communicated about the goals and design of research, and were mandatorily made aware about the recording of their interviews. They were also made cognizant about their right of dissociating themselves from research, at any stage during the entire research process. The participating faculty members represented a mix of both genders, and their age varied from 31 to 63 years. In order to prevent identification of affiliating institutions, teachers were advised to make anonymous the details of the educational institutes and hide any such materials that could lead to the recognition of their schools/universities.

Data Collection Tools

After attending to these ethical and methodical concerns, interviews were conducted through a pre-designed schedule, incorporating the objectives of study. Apart from interview, data was amassed using focus group interactions and informal discussions, mostly carried out after classroom sessions. The researcher was always an integral part of such interactions, and ahead of such meetings, the agenda and mission of the research, was explicated to the participating faculties. It was also categorically stated beforehand that anonymity of each and every participant, shall be maintained without fail. Approval for conducting this research was obtained from local ethics committee, and in order to shield the identity of the teachers as well as of their employing institutions, pseudonyms were rendered to them.

With this pre-requisite, the faculty members were asked to maintain prudence of what they utter, and to be as frank, forthright and honest, as possible. Probing questions were asked in the focus group interviews so that they easily open up and that no elements remained untouched. To establish rapport and to make them feel at ease, informal chit chat was done to attract their interest. Semi-structured, open ended and adept enquiry approaches were followed in such sessions, as there were several entwined dimensions related to the research objectives. Due to this, very often, the length of information-exchange prolonged between 90 to 120 minutes. Audio-recording of interviews were done. Simultaneously, the investigator also took note of calculus related scribblings done by faculty members for explaining their class room teaching techniques and experiences. All interviews, focus group interactions and informal discussions, were transcribed thereafter. After transcription, contact was re-established with the participants and their respective transcripts were shared for seeking their consent. Upon objection or dis-agreement, suitable modifications were carried out, before accepting their final testimony.

Data Analysis

Separate analyses were carried out for each interview/meetings. The instructional planning of calculus teachers used in classroom condition, and factors impacting the cognition and belief systems about the nature and learning of calculus, across the country, were excerpted and clubbed together for identical conditions. Those themes that kept egressing and echoing repeatedly, were keyed out and indexed differently. NVivo and Concordance softwares were brought in use for analyzing the emerging contents and classroom discourses, making usage of plain enumeration method. The entire research work, since inception, till completion, traversed from February 2016 to April 2019.

With its framework confluenting within an interpretative paradigm, the conducted study was purely qualitative in nature. The said research design was deliberately applied as it abetted the researcher in comprehending the cognition of calculus among mathematics learners, in a more persuasive manner. Parallelly, grounded theory approach coupled with inductive methodology and constant comparative method (Bogdan & Biklen, 2003), assisted in thematic categorization of empirically derived data, and development of theoretical and explanatory principles for the transcribed verbatim interviews.

Coding of emanating themes was done consistently and robustly following the rubrics of grounded theory approach, which directly supported the verbatim data coming from interviews. After initial coding, the database of excerptions underneath every factor, was re-perused to ascertain coherent applications of excerpts, and the factors were systematically framed thereafter. Validity checks on similar themes, allowed cogency of authors' experiential accounts, assaying elucidation and illustration of central ideas. Simultaneously, devotion of concentration to aberrant illustrations, cases and examples were also executed with extreme detail and caution.

Considering the varied context of interviews, it was less feasible to develop inter-rater reliability scores. In view of this, several codes were identified during initial stages of analysis, but later on, only those displaying strong bearing (assorted with more than 50% of interview samples), were aggrouped and expended. However, complemental codes and constructs, which uniquely contributed to analysis, were also accommodated. Marking and linking of segments, followed by inter-thematic comparisons and re-shuffling eventually resulted in finalization of eleven broad themes in the current research for capturing teachers' understanding about learners' processes of calculus cognition. They were as follows :

- What instructional planning is developed by calculus teachers for the enactment in their classroom practice?
- What potential impacts do student's calculus cognition and belief systems have over the nature of calculus?
- What difficulties do calculus students face when they are having an underdeveloped conception of *variables* ?
- What are the excogitation difficulties of calculus while traversing across several disciplines?
- Students often commit several types of errors while solving a calculus problem. How are these errors expended by educators as a pedagogic aid in effective teaching of elementary calculus?
- How *blended learning* helps in enhancing the cognition of primary constructs used in elemental calculus and how effective are *flipped classrooms*?

- Significance of calculus and its need to be learnt by school students?
- What is the scope of calculus learning?
- What prominent issues emanate while teaching and learning of calculus?
- What are the dominant methodologies that are used in teaching of calculus by Indian teachers?
- What is the current status of calculus education in India?

Research Assumptions

This work is based around some of the very prominent key research assumptions. Development of formal procedural operations' cognitive acquisition is the product of conceptual learning. Vygotsky has contended that cognitive development of higher order skills and understanding of advanced concepts are something which starts developing very early in life. It is just the matter of having the right kind of environment for learning to take place. From learner's conceptual cognition of advanced calculus principles and ideas, the development and understanding of learner's skills in procedural calculus is derived. It is thus implied that the development of conceptual understanding among students with regards to principles of calculus should start as early as when the learners are in Std-9 or Std-10. Acquisition happens moving from abstract to concrete, understanding complex ideas before moving to particularized procedures, also the concepts pertaining to single variable principles in calculus be made understood to students before they learn multi-variable calculus.

In this research, it is also assumed that making use of modelling and technology in the cognitive-visual conceptualization will act as a pivotal function in initial cognition of precepts of calculus. A learner when teaches a specific topic to her class, s/he learns through this way more thoroughly as compared to, when asked to simply learn it on their own. This approach is often referred to as learning through teaching. It has a prominent relevance in teacher preparation. There's often seen a lack of time with pre-service teachers in teaching calculus in actual classrooms, and is especially visible in traditional teacher training programs. Their limited exposure to actual classroom teaching inhibits their confidence and enforces low moral level in conceptual understanding of calculus and its pedagogy.

Calculus is one of very few areas of study in mathematics which very few pupil teachers have learnt in depth before. The learning experience which pupil teachers gain during their field-based experience wherein they get the opportunity to immediately apply their pedagogic techniques in actual classroom setup. Researches by (David Tall, 1996) and (Ferrini-Mundy & Lauten, 1993) have revealed that when students enter a calculus course, their cognition of the concepts of *functions* and *continuity* are primitive and pretty naïve. (Schnepp & Nemirovsky, 2001) in their research have noted that to develop the foundational understanding of fundamental calculus ideas, it is imperative to coordinate and correlate the concept of functions in algebra and their graphical representations and sadly it has been observed that students face cognitive difficulties in doing the same.

Results and Discussion

Theme 1. Significance and Scope of Calculus and Reasons for it to be Taught

Professor of calculus (teacher-59), when asked about the placement of calculus in school syllabus, enunciates that:

> *"…there has been a downward movement in concepts of school mathematics curriculum and has consequently led to enhanced requirement of people who are mathematically competent, in almost all of the emerging occupations and results of this trend led to the incorporation of calculus in the mathematics curriculum of senior secondary schools…..today in senior-secondary schools calculus has been commonly offered to students and a large numbers of mathematics educators have backed and advocated for this development…."*

Interview with calculus professor, Teacher-79 accentuated calculus to be that branch of mathematics that goes far beyond what is learnt in geometry and in algebra and is the mathematical study of change and consequently many of the colleges and universities now require students to take up not just an introductory course but an advanced course in calculus to complete graduation or post-graduation degree, considering its wide usage in the fields of engineering, economics, technology, architecture, and sciences. He avers:

> *"…a solid foundation is required in limits, functions and other building blocks of calculus to learn mathematical sciences, natural sciences and physical sciences and is consequently a prerequisite for students wishing to pursue engineering and management degrees. Also, calculus has emerged as a significant foundational course as economics, operational research, business analytics and accounting students move further up the ladder in academia to major in these areas of study…."*

Participants (calculus teachers) also emphasized that there is always a scope for students to take up a course that are more suited to what they wish to pursue further in academics, e.g., business calculus or applied calculus which tailors the abstractness of calculus to real life circumstances like measurement of loan interest, marginal revenues and costs incurred in setting up of a business. Teacher-136 pronounces that:

> *"….the abstractness in solving problems and the differing techniques of calculation employed in vocations are derived from these majors, for example, from statisticians, accountants, economists and even business owners….calculus is required in aeronautical, chemical, electrical, civil, automobile, mechanical and other types of programs in engineering and technology as a foundation course for*

> *solving problems efficiently and essential changes linked to careers emanating from it….”*

Professor, (teacher-46), on significance of learning calculus, opines and enunciates that :

> *“…..methods from calculus are often required in defining and comprehending relationships between observations and data efficiency maximization for items of everyday usage, for example, in construction of roads and dams and a basic introduction to calculus is essential for students who major in physics, biology, and chemistry and also for those preparing to attend medical, dental or veterinary school…. the toughest of entrance tests like the admission tests to medical, dental and veterinary colleges, as well as the coursework that are an integral part of these professional schools require strong mathematical skills and thereby understanding of calculus becomes all the more essential…..”*

Theme 2. Dominant Methodologies in Calculus Teaching by Indian Teachers

Participants (calculus teachers) also emphasized that it is never enough simply to introduce integration as a limit of sums. This limit, taken over all pairs consisting of a partition of the interval in question and a set of tags or points from subintervals, is conceptually complex. Professor, Teacher-07, about the importance of understanding of calculus construct avers that:

> *“….if teachers want students to understand integration as a limit, then they need experience working with these sums in contexts that lead them to appreciate the importance of this definition….working with Riemann sums does not mean forcing students to evaluate the area under a parabola using the sum of squares formula…..”*

Students' cognition of definite integrals and Riemann sums and the constructs inherent therein, require attention of learners to cognize, broadly for two important rationalities. First, several practical applications that we encounter in our day-to-day lives necessitate those *functions* that have no anti-derivatives which could possibly be explicated as elemental functions. Taking an example, the anti-derivative of the function $f(x) = e^{x^2}$, expressing which in elemental function form is not possible. Consequently, the "*Fundamental Theorem of Calculus*" cannot be utilized, and other methodologies for solving the definite integral, such as Riemann sums will be required. This takes us to the second argument that states that learners require to possess a deeper cognition of the concepts inherent in Riemann sums. Although to approximate a definite integral, Riemann sums cannot be considered as the most effective methodology, yet different methods including the trapezoid rule, or for that matter midpoint rule, or in several other cases Simpson's method have a strong basing on the structure of Riemann sums. Teacher-36 while sharing her experience with teaching of calculus, says that:

> *"……there is a common belief that students who have struggled through this will then appreciate the computational ease of invoking the Fundamental Theorem of Integral Calculus (FTIC)…..calculus students are more likely to resent being forced to do calculus problem the hard way when there is already a much easier route, and the resentment is only compounded when many or most of the students arrive in college having already seen enough integration to know there is an easier approach…….and for calculus teachers to see in students an intellectual need for evaluating limits of Riemann sums, teachers give students extensive experience with good and unfamiliar problems that involve accumulation, problems that require thought in constructing approximations to something that is accumulating continuously and for which the evaluation of the limit can be accomplished via the Fundamental Theorem of Integral Calculus……"*

We find many researches that emphasizes on those themes of math that help in building definite integrals of the form: $\lim_{n\to\infty} \sum_{i=1}^{n} f\ (x_i)\ \Delta x$. Mathematical applications including multiplication, limits, sequences and series, rates of change, and functions are all contained in the broader framework of definite integrals, and an enormously large number of researches have been devoted to the understanding of aforementioned areas of mathematical concepts. About defining integration to students, professor, Teacher-114 animadverts:

> *"….these problems are tailored to the students' needs in the class so that they are challenging but not overwhelming….and despite teachers' efforts to define integration as a limit of sums, the working definition of integration for most students continues to be anti-differentiation…this tendency is aggravated by the fact that the majority of students in college calculus course arrive with the anti-differentiation definition deeply embedded from their high school experience with calculus...."*

With regards to student's belief of assessment of their knowledge, one professor, Teacher-91, speaks out:

> *"….even those students who arrive as 'tabula rasa' vis-à-vis calculus, quickly learn from their peers and their own experience, the way teachers assess their knowledge of integration thereby believing that it is most efficient to think of integration as anti-differentiation…..and….because of this, when presenting this theorem, teachers remind students that the definite integral is shorthand for the limit of Riemann sums…it points out that the equation* $\frac{d}{dx}\int_a^x f(t)dt = f(x)$*, for a* $< x <$ *b, conveys that teachers can use this limit to bring forth the anti-derivative for any function that is continuous, and emphasizes that the equation* $\int_a^b f(t)dt = F(b) - F(a)$ *conveys that, when an anti-derivative is known, the limit can be easily calculated in terms of that anti-derivative…."*

The community of mathematicians and math educators has previously delineated the notion of the systems of beliefs among learners and teachers, whereby a belief system guarantees and adds value to pedagogic engagements. Pragmatic reasonableness is consanguine to belief system where both of them make attempts to explicate the reason that regularizes the actions. Use of pragmatic reasonableness and of tendencies is in no way an alternative to say it to be equivalent to belief systems. Beliefs are commonly attributed to people. Alternatively, pragmatic reasonableness is presented for consideration, as restricted, according to rules and regulations that are of definitive contexts and to be equivalent in circumstances to people those who perform a role that are marked by close resemblances. Pragmatic reasonableness showcases a jointly clenched arrangement of explicit approvals for certain characteristic engagements in a unique general state of things. Non-representationally, pragmatic reasonableness is the arrangement of tendencies that accords stakeholders in a certain state of affairs to manage the effrontery that they should or shouldn't conform to the existing standard criterion. Accentuating on functions as descriptions of co-varying relationships, professor, Teacher-143, recounts :

> *"…..calculus teachers re-introduce the adjective 'integral' into the name of this theorem, emphasizing the fact that it tells about the dual nature of integration, which can be viewed either as a limit of sums or as anti-differentiation…..on calculus learning, the very first step is to understand functions as descriptions of co-varying relationships…."*

Systems of belief are usually assumed anterior to the true didactic occurrence, and bound through several decades of practicing of the art of teaching. Talking about student's belief system, Teacher-117, animadverts:

> *"….most students think of functions as static objects, either algebraic expressions—in Thompson's words, as a short expression on the left hand side separated by an equal to sign followed by a long expression on the right—or as the geometric object that is the graph of the function…….teachers expect students to comprehend the dynamic view of the FTIC even though they are unable to see functions as describing a dynamic relationship between co-varying quantities…."*

Because the primary function of the common ownership of inquiry is to increase the levels of noetic, behavioral and motivational regularization of the self, the educator's primary goal thus is the dissemination of their own understanding to the system of educates, a possibility of future success that inclines to counteract the conventional expression of didactic competence. Analogously, the secondary purpose is to kick-start pupils into the cognitive projects, and to furnish them with opportunities for apportioning the sense of ownership for cognitive tasks, assuming that sense of ownership is an essential condition for achievement or even the power to withstand hardships or stress of the system of didactics. On discussing about traditional practices, Professor, Teacher-68, pronounces that:

> *"….understanding co-variation is only the first step toward the FTIC….next comes, accumulation…..teachers would do well to require their students to formulate their own explanations of the Mertonian rule…..the next step calculus*

teachers do is to consider change in the rate of the accumulation function…..now that rate of change, being a difficult concept, compounding it with an accumulator makes it all the more difficult…but once all of these pieces are put in place and are understood, the FTIC is virtually self-evident…."

Theme 3. Current State of Calculus Education in India

Results of this research furnish the basis for belief, showcasing that educators involve themselves in proffers, which bring about palpable results in learners' achievement, which is an assuring substitute we wish to enhance the content and didactic understanding of in-service educators. Professor, Teacher-62, when asked about poor performance of students in calculus course, devoices:

> *"….calculus students in colleges and universities of India fail to perform well in calculus not because they find it tough to learn it or that they lack the potential owing to its complexity but because they lack the skills and fundamentals of pre-calculus including a deficiency of skillfulness, knowledge, and command over trigonometry, algebra, logarithms, and exponentials, and an understanding of the role of functions in linking co-varying quantities that are the essential ingredients required to learn calculus and consequently there is a requirement for reliable assessment tools to measure the readiness of students for learning calculus and a strong alternative to calculus who wish to opt out of it…."*

Talking about the pre-requisite to learning of calculus and possible need for revamping of calculus curriculum, professor, Teacher-145, explains:

> *"….algebraic and geometric thinking skills are a better parameter to judge whether or not the learners are prepared for college level calculus and the ones who "memorize" the techniques in solving integration, differentiation and differential equations are obviously the ones not prepared to learn advanced calculus…..students those who study calculus in school have no additional advantage over the rest when they enter college to study calculus and usually the former category of students join college calculus course with an inflated sense of their ability to tackle advanced calculus and consequently it is incumbent on the curriculum developers and educational policy makers to revamp calculus curriculum to acknowledge the ubiquitousness of calculus in secondary school curriculum and accordingly shape the college calculus curriculum for appropriateness of pupils having already experienced introductory calculus in all its thoroughness in senior-secondary school, and offer better alternatives to calculus…."*

About the relevance/obsoleteness of contemporary calculus curriculum, keeping in view the growing number of millennial kids taking up the calculus course, Teacher-69, asseverates:

> *"….it is unacceptable for colleges to pretend that it is okay to teach calculus in the same fashion as was taught during the 2000s or in first decade of 21st century for now it is important to recognize that the tech-savvy millennial kids who enter college today are already familiar with the latest, most advanced and the standard*

> *techniques and procedures used in effective learning of calculus and that they come equipped with strong preconception of the nature of calculus course they have opted to learn and what all is required to succeed in the course…..contrarily…also…at the same time…students are primed to ignore calculus' conceptual development and the push that is visible in acceleration of senior-secondary school students is indicative of the fact that many enter college calculus with a weaker mathematical foundation landing in revisiting the materials they already studied in school which is by no means an effective engagement of students in classroom and in ramping up their desire to learn calculus…."*

On instructional planning and impact of belief systems of students-and-teachers on the effectiveness of learning and teaching of calculus, less has been shared by calculus teachers. However, professor, Teacher-148, expresses:

> *"….the instructional planning a calculus teacher uses for enactment in her classroom practice and that what potentially impacts her students' calculus cognition and belief systems about the nature of calculus could very well be understood because the belief system of the teacher which is intertwined in her experience with teaching and learning of calculus, and that…belief systems of individual calculus teachers might be very contradictory, considering different calculus teachers or one calculus teacher teaching different dimensions of calculus....and in my long standing career I have observed that when calculus is introduced into the senior secondary school mathematics program the quality of the other courses picks up…...the teachers are thence stimulated and so are students in other courses….. causing the calculus teacher to do a better job in his other courses too…."*

Variables play a crucial role in learning of calculus. For example, as was explained by Felix Klein in 1908 that "one may well declare that real mathematics begins with operations with letters" and Alfred Tarski wrote in 1941 that "the invention of variables constitutes a turning point in the history of mathematics". When asked about the conception of variables among calculus learners, professor, Teacher-99, verbalizes:

> *"….most of calculus students' difficulties lies in the underdeveloped conception of variables and particularly in that case when students treat variables as some sort of symbol that are to be manipulated and not when they have to draw relations between quantities…."*

However, Teacher-39, expresses:

> *"….there are students who are able to confidently use defined variables but when it comes to identifying and defining their own variables they cannot do it, such students are essentially using symbols to showcase the process of relationship that one variable have with another and these students are still at the condensation phase of developing their concept of a variable…."*

Algebraic symbols don't take stand for themselves. What mathematicians view in algebraic symbols reckons on the necessaries of the questions where they are to be used. Also, it being contingent upon what the mathematician is capable to comprehend and are geared up to observe. Regarding manipulation and modeling of arithmetic expressions by learners, professor, Teacher-137, pronounced that:

> *"….students who create variables to solve complex problems have reified variables and reification can only occur after extensive successful experience using variables in the operational mode and the amount of experience that is required in manipulating and modelling arithmetical expressions, the same amount must precede reification of algebraic expressions and a considerable amount of time is required by the students of calculus to spend on the usage of algebra to manipulate relations in order for them to gain maturity over the concept of variables…."*

Attention to "*rote learning"* is coherent with the assumption that learning is for acquirement of knowledge wherein learners attempt to make sense of novel experiences to their brains (Mayer, 1999). "*Meaningful learning"* is regarded as a significant long-term goal of educational instruction. It commands teaching to go farther from mere plain introduction of "*Factual Knowledge"* and that evaluation demands more from learners than plainly retrieving or discerning "*Factual Knowledge"* (Bransford, Brown, & Cocking, 1999; Lambert & McCombs, 1998). Talking about rote learning of formulas in calculus, professor, Teacher-131, maintains that:

> *"….students' calculus cognition can be known by examining their ability to describe and explain the inherent concepts, how they apply them in specific contexts, and how they reflect it on their learning; and contrarily many calculus students who are excellently gifted in logic and math and yet they rely on memorization of formulas and often tend to apply them in rote manner…."*

When professors were asked about the excogitation of calculus while traversing across several disciplines, professor, Teacher-19, stresses that:

> *"….when teachers, for better understanding, make connections for their students between such concepts that traverse across disciplines, e.g., between calculus and concepts in physics like momentum, it help students develop richer understanding of semantics of calculus as well as that of its procedural knowledge and because of that students construct understandings to varied degrees of sophistication which consequently help deepen and broaden their ability for explanation, application, and self-assessment of their learning of calculus making this kind of interdisciplinary calculus teaching time-consuming that requires from both teachers and students of calculus…commitment, patience, and flexibility…."*

On errors as a pedagogic tool in effective calculus teaching, not much was posited by professors but was explicated only by one professor, Teacher-117, who says that:

> *"….in the process of learning calculus, students often encounter several types of errors and the traditional methods that are in use for teaching of calculus are not sufficient to prepare excellent students who could apply calculus creatively and mere*

> *accumulation of additional factual knowledge in calculus is not likely to improve the non-routine problem-solving ability of calculus learners…."*

When asked from professor, Teacher-98, about her use of blended learning in calculus teaching and about the use of flipped classrooms in her teaching, she proclaims:

> *"... the combination of algebraic skills and long-term perseverance and competitiveness play a significant role in the prediction of achievement in the school calculus tests and that blended learning and flipped classrooms help in enhancing the understanding of key concepts used in elementary calculus."*

Her views are in agreement with other researches done pertaining to effectiveness of "*flipped classrooms*" such as those by (Ruddick, 2012)(Herreid & Schiller, 2013); (Brunsell & Horejsi, 2013); (Herreid & Schiller, 2013) and (Kay & Kletskin, 2012).

About flipped classrooms, professor, Teacher-83 asseverates:

> *"...students of calculus prefer to watch flipped classroom videos over reading from calculus textbooks thus revealing that flipped classroom model ensues increased achievement of students in calculus learning leading to increased student achievement, better preparation, and lesser anxiety among today's tech savvy learners..."*

Her experiences are in sync with findings of earlier researches in different disciplinary domains, such as those by (Fulton, 2012), (Herreid & Schiller, 2013), (Brunsell & Horejsi, 2013), (Ruddick, 2012), and (Herreid & Schiller, 2013). Concerning the effectiveness and preparatory habits of learners in a flipped classroom environment, professor, Teacher-03, explains that:

> *"….there are several diverse advantages of the model of "flipped classroom", including a student's changed preparatory habits before she attends the class on calculus, levels of conceptual understanding among learners of calculus are seen to be improved, and enhanced efficacy at the time calculus lectures are delivered are seen both in students as well as in teachers of calculus…."*

It is observed that her experiences are in congruence with results of earlier researches, namely by (Kay & Kletskin, 2012), (Herreid & Schiller, 2013), and (Zappe, Leicht, Messner, Litzinger, & Lee, 2009).

Professor, Teacher-105, talking about the role of videos in flipped classroom model, enunciates that:

> *"….calculus learners are very keen to get an educational module of their choice that is having videos to study and understand them, rather than a calculus course having no videos under the model where classrooms are flipped….such behavior in learners are viewed to be emanating from the observations that pupils in flipped classrooms, in choosing a convenient method to prepare for the class, get greater freedom and flexibility, and that in this model they do not get anxious on missing*

out a class or even when they find it difficult to understand the course materials, ……because now they have all the freedom to watch the video content in their free time and have the liberty to pose questions to their teachers or peers during class meetings, face-to-face….”

(Zappe et al., 2009) and (Brunsell & Horejsi, 2013) have found similar observations in their respective researches.

When asked about the evaluation and assessment of learners, professor, Teacher-144, states that:

“….regarding the criteria used by different school teachers to arrive at student grades for the calculus course, focusing on methodological issues in the investigation of teacher grading practices, on the importance of such investigation, and on steps that might be taken to reduce discrepancies, there are substantial differences in content emphasis, as well as differences in the match between content coverage and testing emphasis…..”

Theme 4. Prominent Issues in Teaching and Learning of Calculus

There is a high probability that learning calculus may not at all be a goal of the senior-secondary school students and more so for those students who have no plans to study mathematics further. When enquired about it, following are the questions that emerged from the interviews with calculus teachers that attempts to throw light on the importance of learning calculus while transitioning from senior-secondary school level to college level mathematics.

Table 1.
Questions that Emerged from the Interviews with Calculus Teachers

Sl.	Questions
01	Who takes calculus in senior-secondary school?
02	What makes them to take it?
03	What result one expect of, on their taking up of a calculus course?
04	What would be the short-term and long term results of taking up a calculus course?
05	Which student population in the senior-secondary school is expected to take up calculus?
06	What measures shall be taken to ensure that all the students who take up the calculus course get a quality calculus education?
07	Should senior-secondary school calculus be available to all or at least to majority of the students?
08	Is completion of calculus course by a student a sign of intellectual achievement?
09	Is achievement in calculus course a yardstick to judge one's intellectual ability?
10	Is positive correlation between student's general academic achievement and student's calculus achievement a sufficient rationale for directing all the students to take up calculus course in senior-secondary school?
11	To what extent having calculus written in one's senior-secondary school transcript enhances one's chances of getting into a good college for higher studies?
12	Does having studied calculus in senior-secondary school enhance a students' interest in going forward to study disciplines that have intensive mathematical rigour?
13	How will it be known as to when a student is ready for academic acceleration into a sequence that includes senior-secondary school calculus?
14	What shall be done to prepare the students for their academic acceleration?
15	Do policy-makers have any mechanism or any specific policy or practice in place that can help check inappropriate acceleration?
16	How can curriculum framers build alternative pathways that can enable the pupils to back off the track to senior-secondary school without damaging their prospects for post-secondary studies that are mathematically rigorous?
16	What can be done to ensure that a senior-secondary school curriculum that incorporates calculus provides to students both a breadth of understanding about the nature of mathematics that is sufficient for students' mathematization and at the same time has the ability to enable students to use all the standard tools of advanced mathematics?
17	What obstacles do students face in transitioning from senior-secondary school calculus to college mathematics course?

Conclusion and Recommendations

Calculus teachers have several ways to handle problems pertaining to calculus learning and teaching; one could be to take up a modelling approach focusing only on differential equations or by making this an introductory course in analysis or even by making students start with a non-calculus course such as linear algebra or discrete mathematics. Another way could be the introduction of stringent entrance requirements to study calculus in colleges, based solely on their understanding of variables, or by offering at the college or university a suitable 1-month pre-calculus course booster for effective cognition of calculus. There should be more summer institutes, academic year institutes and in-service training courses planned for potential calculus teachers. Calculus educators shall have an attitude that is positive, towards their learners' errors, and shall conceive learners' errors as a tool to adapt and correct their didactic methodologies; in addition, educators shall also showcase their learners how to forestall and preclude the errors, and view their errors as didactic appraises to bestow on ameliorating the adequacy of calculus teaching in colleges and schools thereby developing such a pedagogy that will provide learners a context that is socially meaningful for the acquisition of skills and knowledge necessary for succeeding the calculus course.

Belief system of teachers of calculus could be viewed as a hierarchical system of goals that is multi-layered and that which makes sense to each calculus teacher, individually and this sense-making could make us lean to think in the direction about the relationship between teachers' goals what they espouse and the goals that they eventually act upon, which though is pretty difficult, yet a very crucial relationship. There is a dire need for extensive modification in calculus syllabus in Indian schools and colleges so that it give lay people from the wider community who are outside the formal school/college/university system, a chance to learn some of the powerful ideas that have shaped the development of modern mathematics and to especially encourage and develop interest for advanced mathematics, especially among women, to help them build their confidence and thereby increase their learning opportunities.

To this end, there's a need for teachers of calculus to merge their theoretical, practical, and research understanding of calculus to develop interpretive tools for efficacious learning and teaching of calculus. 12th grade math course invest considerable time on calculus topics. This may not be the best use of time in such a course, and additional exploration of the topics addressed and the effectiveness of such brief introductions is warranted. In place of differentiation, integration, and differential equation, other topics that are more suitable for inclusion in the senior secondary mathematics course, include the firm anchoring on the cognition of functions and the conceptual underpinnings of calculus.

A large scale government funded calculus reform project needs to be initiated to move schools/college/university calculus in a direction that is more conceptual, applications-oriented, and technology intensive. The procedural facility necessary in integration and differentiation shall be provided by sophisticated hand-held calculators, while the technical skill previously necessary in areas of curve-sketching shall be deferred to accessible microcomputer graphics packages. These new approaches to calculus will encourage pedagogical changes, such as the use of small-group work, laboratory approaches, and more student-cantered activities. All of these curricular shifts will present enormous logistical challenges to school/colleges/universities that are currently organized only to offer calculus in large lecture settings.

A marathon discussion needs to be initiated for the greater good, for now it is viewed how our schools and colleges are troubled by the nature and approach to teach calculus to the learners. The entire argument is built around these three propositions. 1) There is no opposition to the belief that calculus is an integral part of mathematics curriculum, yet it is pretty disturbing to see its placement as the most supreme of all mathematical subjects specifically compared to algebra, geometry, and other such relevant mathematical subjects. 2) Teaching of calculus to students too early in their academic life does not serve much of the purpose. Instead of it, teaching other mathematical subjects in early school years would fetch better mathematical understanding among the young students. 3) The way calculus is taught and presented to students is flawed. In yesteryears, college mathematics students had a full-length course in analytic geometry in their 2nd year of 3-year undergraduate degree programme, followed by calculus towards the end of the year. In 3rd year more calculus and differential equations were taught.

Standard methods failed to work in understanding of situations including the end-point maxima. Rigour was given no attention. In contemporary terminology, it is called the "cookbook" calculus. This approach lasted for a couple of decades except for few institutions of mathematics that emphasized on the notion of calculus in 1st year of college or in senior school mathematics curriculum. In contemporary mathematics education arrangement, a rather diverse variety of approaches are being taken up for teaching of calculus.

Some still use the same old Granville way of teaching calculus whereas others stress on rigour that was never thought of a couple of decades ago. They emphasize neglecting the applications of calculus in their teaching. Complete disappearance of Geometry is seen. Whatever little mention of Geometry is there in the textbooks are conveniently neglected by the teachers while transacting the curriculum. The rush to teach calculus to students as early as one could, in their school life, is visible here.

This very idea is flawed. It is pretty much visible that after the course in algebra at the intermediate level, derivation of algebraic functions becomes the simplest of task. The big question here is then not of whether there is a possibility of teaching calculus at secondary school but whether it would be *wise* taking into account the viewpoint of whole curriculum. Curriculum framers already have gone too far in the pell-mell rush of introducing calculus as early as could possibly be done. The argument given for the introduction of calculus early is its overwhelming superiority/importance that it could displace any subject of mathematics to carve its place. Introducing calculus at the expense of algebra or geometry by quoting its use in elementary science and engineering courses is overstated. This is wrecking of the curriculum: a clear case of serving half-baked calculus by inefficient and incompetent teachers to unprepared high school students.

Future Research/Way Forward

Based on results and conclusion, it can be put forth that the relationship between teachers' espoused beliefs, their enacted beliefs as well as the kind of relationship between classroom practice and learning needs of students, needs further research. An endeavor in this domain shall surely assist and promote the understanding of challenges and possibilities in mathematics education in general, and calculus teaching in particular.

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