ISTANBUL UNIVERSITY –
JOURNAL OF ELECTRICAL & ELECTRONICS ENGINEERING

YEAR : 2003 (813-818)
VOLUME : 3
NUMBER : 1

A NOVEL BI-CHARACTERISTIC FDTD METHOD FOR WAVE PROPAGATION

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ABSTRACT

In this paper, the construction of time-dependent solution of electromagnetic fields in space-time by exploiting Taylor Series and Geometrical Optics' (GO) tools, i.e. wavefronts and rays, is introduced. Discontinuities in the fields and their successive time derivatives may only exist on the wavefronts and propagate along the rays. These discontinuities are transported via higher order transport equations. The proposed method is implemented by two different procedures on a sample problem of Hertzian dipole at origin in isotropic, homogeneous medium. In the first one, discontinuities themselves are transported directly by differential-type equations but in the latter by auxiliary vectors using both differential and integral type equations and conservation of energy is taken into account. Forward differences in time and central differences in space are applied. Simulation studies, when compared with analytical results, show that consistent and accurate results are obtained.

Keywords: *Geometrical Optics, characteristics, bi-characteristics, wavefronts, rays, numerical methods, finite differences, time domain, computational electromagnetics (CEM).*

1. INTRODUCTION

Maxwell's equations, which are of hyperbolic nature and partial differential type, define initial-boundary value problems in electromagnetics [1]. Although it was established more than a century ago, very few time-dependent solutions, either by analytical or by other means, are known. On the other hand, frequency-domain analytical/numerical research has been conducted extensively. However numerical studies in space-time have recently been quite popular due to many advantages of time-domain.

As for the solution of Maxwell's equations in space-time domain by numerical methods, the first milestone is the Yee's paper in 1966 [2]. There have been an increasing number of papers in this field following Taflove's paper in 1975 [3]. Although developed methods of the past provide good solutions for electromagnetic problems, they still need to be improved in some respects. Maximum time-step (Δt) depends on spatial grid intervals ($\Delta x, \Delta y, \Delta z$) and is limited by Courant-Friedrich-Levy (CFL) stability condition. Therefore, in general, conditionally stable algorithms have been developed [4]. The size of spatial grid intervals with respect to wavelength has also influence on the numerical accuracy. There exists numerical dispersion due to the discretization of computational domain. Absorbing boundary conditions (ABC) cannot be

Received Date : 08.10.2002
Accepted Date: 26.12.2002

implemented perfectly at the outer boundary of the computational domain. This leads to undesired reflected waves, which in turn propagate to the interior region resulting in unwanted modulation of signal and energy deposition. These reflected waves also deteriorate the numerical accuracy. This is a critical issue in Radar Cross Section (RCS) simulations, which typically require a dynamic range on the order of 60dB. Moreover these methods may also suffer from some other drawbacks such as computation of very large number of variables, storage of them, and inversion of huge matrices etc.

Apart from the studies in CEM, there has also been extensive research for the numerical solution of Euler equation in computational fluid dynamics and some researchers have tried the characteristic-based methods developed for Euler equation on Maxwell's equations since they are both hyperbolic, partial differential equations [5], [6], [7]. The gist of the method is that Maxwell's equations are reduced to three approximate 1-dimensional Riemann problems in each spatial direction and then each is solved successively one at a time using FDTD, FVTD etc. The eigenvalues of Riemann problem are real and gives us the direction and speed of propagating wave along the characteristics [8]. Discontinuities propagate along these characteristics. There exists a domain of influence and domain of dependence bounded by characteristic curves from the point of view of fields and sources that create them. Characteristic-based methods provide us good mathematical formulation, which is also consistent with the physics of wave propagation. They honor the physical domain of dependence and mimic the directional-signal propagation. Hence we have some advantages in numerical applications. More stable (conditional/unconditional), explicit/implicit, and efficient algorithms can be developed [5], [6], [7]. ABCs can be implemented perfectly at the outer boundary of the computational domain [5]. Variables can be decoupled in some cases and computational burden can be alleviated. Numerical dispersion is expected to be much less. The only disadvantage of characteristic-based methods is that the coefficient matrices can be diagonalised only in one dimension at a time. Diagonalisation in 2 and 3 dimensions has not been achieved yet to the knowledge of the authors.

In this study, contrary to the mentioned methods, we do not attempt to solve the Maxwell's equations directly in space-time, but rather, the time-dependent total electromagnetic fields' solutions are established by first finding the GO field and then improving it by addition of higher order discontinuities on the wavefronts and along the rays in time.

2. THEORETICAL STUDIES

Time-dependent Maxwell's equations in source-free, isotropic medium are given in (1). In the solution of these equations in space-time, discontinuities in the fields and their successive time derivatives exist on discontinuity hypersurfaces (characteristics) and propagate along the bi-characteristic curves [8], [9]. The projections of characteristics and bi-characteristics into the space domain are called as wavefronts φ and rays respectively. Figure-1 illustrates a typical hypersurface (a hypercone in 2-Dimension), bi-characteristic, wavefronts and rays. According to Geometrical Optics, electromagnetic fields propagate along the rays and wavefronts satisfy the well-known Eikonal equation.

$$\nabla\times\vec{E} = -\frac{\partial(\mu\vec{H})}{\partial t} \quad , \quad \nabla\times\vec{H} = \frac{\partial(\varepsilon\vec{E})}{\partial t} \qquad (1)$$

Discontinuities are defined as, [9]:

$$\vec{A}_v(\vec{R},t) = \left[\frac{\partial^v\vec{E}(\vec{R},t)}{\partial t^v}\right] = \frac{\partial^v\vec{E}_1(\vec{R},t)}{\partial t^v} - \frac{\partial^v\vec{E}_2(\vec{R},t)}{\partial t^v}, \quad t=\frac{\varphi}{c}, \quad v\geq 0$$

$$\vec{B}_v(\vec{R},t) = \left[\frac{\partial^v\vec{H}(\vec{R},t)}{\partial t^v}\right] = \frac{\partial^v\vec{H}_1(\vec{R},t)}{\partial t^v} - \frac{\partial^v\vec{H}_2(\vec{R},t)}{\partial t^v}, \quad t=\frac{\varphi}{c}, \quad v\geq 0 \qquad (2)$$

where $\left[\vec{E}\right] = \vec{E}_1 - \vec{E}_2$ and $\vec{E}_1$ and $\vec{E}_2$ denote the field vectors on two sides of a discontinuity hypersurface.

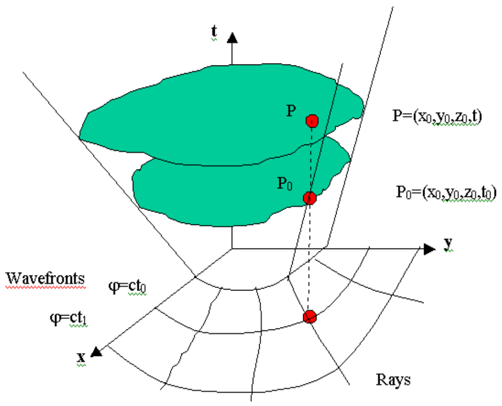

Figure 1: Illustration of hypersurface, bi-characteristics, wavefronts and rays.

Procedure 1: Discontinuities of the fields which correspond to ν=0 are the GO electric and magnetic fields. These fields propagate in the normal direction to the wavefront. Rays are curves normal to the wavefront in isotropic medium. They turn into straight lines in homogeneous medium. Discontinuities for electric field are transported along rays by higher order transport equations given in (3). Similar formulas can also be written for magnetic field [9]. Here, τ is a parameter along a ray and is related to s (arc length) and t (time) by $ds=nd\tau$ and $\tau=ct/n^2$ respectively where c is speed of light and n is the index of refraction of the medium. One outstanding property of these higher order transport equations is that they are ordinary differential equations (ODE). The other one is that they are recursive. It is also apparent that electric and magnetic fields are decoupled. One should also note that we have homogeneous equations for ν=0 and inhomogeneous equations for ν>0 which also implies the recursive nature of them. In homogeneous medium, equations (3) get simpler form and field components themselves also become decoupled.

$$
\begin{aligned}
&2\frac{d\vec{A}_\nu}{d\tau}+\vec{A}_\nu\Delta_\mu\varphi+\frac{2}{n}\left(\vec{A}_\nu.\vec{\nabla}n\right)\vec{\nabla}\varphi=-\vec{C}_\nu \\
&\Delta_\mu\varphi=\mu\left(\left(\varphi_x/\mu\right)_x+\left(\varphi_y/\mu\right)_y+\left(\varphi_z/\mu\right)_z\right) \\
&\vec{C}_\nu=\mu\vec{\nabla}\times\left(\left(c/\mu\right)\vec{\nabla}\times\vec{A}_{\nu-1}\right)-\vec{\nabla}\left(\left(c/\varepsilon\right)\vec{\nabla}.(\varepsilon\vec{A}_{\nu-1})\right)
\end{aligned} \tag{3}
$$

Procedure 2: In other way [9], discontinuities can also be transported as in (4), (5) and (6). In (4), W_ν is the electromagnetic energy density for the ν^{th} discontinuity and K is the Gaussian curvature belonging to the wavefront. To start, discontinuities for ν=0 and corresponding electromagnetic energy densities are known at an initial point τ_0 on the ray and auxiliary vectors $\vec{P}_0$ and $\vec{Q}_0$ are calculated. These vectors are transported to the next point $\tau(x,y,z)$ on the ray by using (5). Electromagnetic energy densities are also calculated at $\tau(x,y,z)$ by using the GO ray tube technique. Then discontinuities for ν=0 are found by using (4) at this new position again.

$$
\sqrt{\varepsilon}\vec{A}_\nu=\sqrt{W_\nu}\vec{P}_\nu,\quad \sqrt{\mu}\vec{B}_\nu=\sqrt{W_\nu}\vec{Q}_\nu,
$$

$$
\frac{W_\nu(\tau)}{n(\tau)K(\tau)}=\frac{W_\nu(\tau_0)}{n(\tau_0)K(\tau_0)} \tag{4}
$$

$$
\begin{aligned}
&2\frac{d\vec{P}_0}{d\tau}+\frac{2}{n}\left(\vec{P}_0.\vec{\nabla}n\right)\vec{\nabla}\varphi=0 \quad , \\
&2\frac{d\vec{Q}_0}{d\tau}+\frac{2}{n}\left(\vec{Q}_0.\vec{\nabla}n\right)\vec{\nabla}\varphi=0
\end{aligned} \tag{5}
$$

Then auxiliary vectors $\vec{P}_\nu$ and $\vec{Q}_\nu$ for ν>0 are calculated by (4) and (6) in a fashion similar to $\vec{P}_0$ and $\vec{Q}_0$. Equation (6) gives the procedure for $\vec{P}_\nu$. Similar integral-type equations can be written for $\vec{Q}_\nu$ [9]. We also define $\hat{p}$ as a unit vector along the ray and normal to the wavefront. $\vec{P}_0$ and $\vec{Q}_0$ auxiliary vectors are transverse vectors perpendicular to each other and to $\hat{p}$. Hence $\left(\hat{p},\vec{P}_0,\vec{Q}_0\right)$ is a triple orthogonal vector set propagating along ray. Projections of other auxiliary vectors onto these triple vector set are found by using (6).

$$
\begin{aligned}
&\vec{P}_\nu.\hat{p}=-\frac{1}{2}\int_{\tau_0}^{\tau}\frac{\sqrt{\varepsilon}}{\sqrt{W_\nu}}\hat{p}.\vec{C}_\nu d\tau+\left(\vec{P}_\nu.\hat{p}\right)\Big|_{\tau_0} \\
&\vec{P}_\nu.\vec{P}_0=-\int_{\tau_0}^{\tau}\vec{P}_0.\left\{\frac{\sqrt{\varepsilon}}{2\sqrt{W_\nu}}\vec{C}_\nu+\left(\vec{P}_\nu.\hat{p}\right)\vec{\nabla}n\right\}d\tau+\left(\vec{P}_\nu.\vec{P}_0\right)\Big|_{\tau_0} \\
&\vec{P}_\nu.\vec{Q}_0=-\int_{\tau_0}^{\tau}\vec{Q}_0.\left\{\frac{\sqrt{\varepsilon}}{2\sqrt{W_\nu}}\vec{C}_\nu+\left(\vec{P}_\nu.\hat{p}\right)\vec{\nabla}n\right\}d\tau+\left(\vec{P}_\nu.\vec{Q}_0\right)\Big|_{\tau_0}
\end{aligned} \tag{6}
$$

Construction of Total Field: After transporting the discontinuities along the rays by using either of the above methods, total time-dependent electromagnetic fields at an arbitrary point in space ($\vec{R}_0$) and at time t ($t>t_0$), e.g. for electric field, can be constructed by exploiting the Taylor series as in (7). Similar Taylor series expression can be written for magnetic fields. It should be noted that Taylor series expansion is convergent under the assumption of "finite" discontinuities. This series is also one-sided convergent, that is for $t>t_0$. It may converge for $t<t_0$ but does not represent true fields.

$$\vec{E}(\vec{R}_0,t) = \left[\vec{E}(\vec{R}_0,t_0)\right] + \left[\frac{\partial \vec{E}(\vec{R}_0,t_0)}{\partial t}\right](t-t_0) + \left[\frac{\partial^2 \vec{E}(\vec{R}_0,t_0)}{\partial t^2}\right]\frac{(t-t_0)^2}{2!} + \ldots \quad (7)$$

One remark is fruitful here. Comparing the Taylor series expansion (7) with Luneberg-Klein series (8), the first term in (7) is the GO electric field, which is the counter part of the first term of (8) except for a phase factor. Other terms in (7) provide the improvements beyond the GO term in time-domain. (8) is expanded in an asymptotic sense as frequency goes to infinity while the only restriction for (7) is "finiteness" of discontinuities for convergence.

$$\vec{E}(\vec{R},w) = e^{-jk\varphi(\vec{R})}\sum_{m=0}^{\infty}\frac{\vec{E}_m(\vec{R})}{(jw)^m} \quad (8)$$

3. SIMULATION STUDIES

Simulations have been conducted for a Hertzian dipole at origin in an isotropic, homogeneous medium ($\varepsilon=\varepsilon_0$, $\mu=\mu_0$). Frequency-domain field solutions corresponding to $e^{j\omega t}$ time input for this problem are given in spherical coordinates (R,θ,ϕ) as in (9), [1], where Idl and Z are strength of the source and wave impedance of the medium respectively.

$$E_\theta(\vec{R}) = \frac{ZIdl}{4\pi}\frac{\sin\theta}{R}\left[jk + \frac{1}{R} + \frac{1}{jkR^2}\right]e^{-jkR},$$

$$E_R(\vec{R}) = \frac{ZIdl}{4\pi}\frac{\cos\theta}{R}\left[\frac{2}{R} + \frac{2}{jkR^2}\right]e^{-jkR}$$

$$H_\varphi(R) = \frac{Idl}{4\pi}\frac{\sin\theta}{R}\left[jk + \frac{1}{R}\right]e^{-jkR} \quad (9)$$

Time-dependent field solutions corresponding to impulse, $\delta(t)$, time input are calculated by using inverse Fourier transform and found as in (10).

$$E_\theta(\vec{R},t) = \frac{ZIdl}{4\pi}\frac{\sin\theta}{R}\left\{\frac{\delta'(t-R/c)}{c} + \frac{1}{R}\delta(t-R/c) + \frac{c}{R^2}u(t-R/c)\right\}$$
$$E_R(\vec{R},t) = \frac{ZIdl}{4\pi}\frac{\cos\theta}{R}\left\{\frac{2}{R}\delta(t-R/c) + \frac{2c}{R^2}u(t-R/c)\right\},$$
$$H_\varphi(\vec{R},t) = \frac{Idl}{4\pi}\frac{\sin\theta}{R}\left\{\frac{\delta'(t-R/c)}{c} + \frac{1}{R}\delta(t-R/c)\right\} \quad (10)$$

Discontinuities in the fields and their successive time-derivatives stem from the doublet terms. Successive differentiation and determination of discontinuities yield the results given in (11). One can observe that discontinuities disappear after v>2 for electric field and after v>1 for magnetic field.

$$\vec{A}_0(\vec{R}) = \frac{ZIdl}{4\pi}\frac{\sin\theta}{cR}\hat{\theta},\quad \vec{A}_1(\vec{R}) = \frac{ZIdl}{4\pi}\left\{\frac{2\cos\theta}{R^2}\hat{R} + \frac{\sin\theta}{R^2}\hat{\theta}\right\},$$
$$\vec{A}_2(\vec{R}) = \frac{ZIdl}{4\pi}c\left\{\frac{2\cos\theta}{R^3}\hat{R} + \frac{\sin\theta}{R^3}\hat{\theta}\right\}$$
$$\vec{B}_0(\vec{R}) = \frac{Idl}{4\pi}\frac{\sin\theta}{cR}\hat{\varphi},\quad \vec{B}_1(\vec{R}) = \frac{Idl}{4\pi}\left\{\frac{\sin\theta}{R^2}\hat{\varphi}\right\} \quad (11)$$

In the simulations, a Gaussian pulse whose maximum effective frequency content is 1GHz is used. To this effect, the expressions given in (10) must be convolved with the Gaussian pulse. Since (11) are the discontinuities arising from the doublet term for impulse $\delta(t)$ input, discontinuities exhibit Gaussian derivative time behavior for Gaussian input after convolution. Time-step is selected to be $\Delta t=1.25\times10^{-10}$ which also imposes radial spacing along the rays to be $\Delta R=c\times\Delta t$. Angular spacing in elevation is also taken to be $\Delta\theta=\pi/90$. The nature of the problem is independent of the ϕ variable in azimuth plane. For this sample problem, wavefronts are spheres of $\varphi=nR$ and rays are straight radial lines passing through the origin. In numerical implementations, discontinuities are defined analytically on an initial wavefront and are treated as initial values. Forward differences in time and central differences in space are applied. Standing at a fixed point in space and letting the time vary, normalized values of E_θ are calculated and are presented in Figures 2-3 where the first one corresponds to Procedure-1 and the latter corresponds to Procedure-2.

In simulation results, it is observed that GO term is very dominant and the rest in the series are negligible. Derivative of Gaussian pulse is obtained as the total field. These observations are consistent with the physics of wave propagation in free space with Gaussian input. Procedure-2 is superior to Procedure-1 from stability point of view. This is especially due to imposing the condition of energy conservation in the formulation.

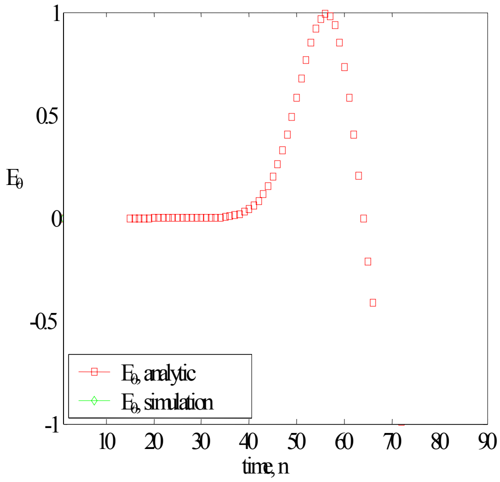

Figure 2: E_θ evaluated by Procedure-1.

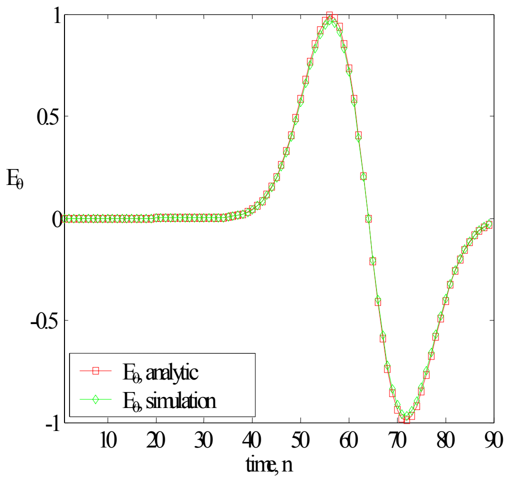

Figure 3: E_θ evaluated by Procedure-2.

4. CONCLUSION

Maxwell's equations have been solved successfully by using a novel method without attempting to diagonalise, discretize and solve them directly. The advantages of previous methods are kept but their deficiencies are eliminated. Proposed method transports the discontinuities along the rays in space-time, hence, in effect, reduces the problem to 1-Dimension along each ray. Electric and magnetic fields are calculated independently since they are uncoupled. Computational burden and storage requirement are not more than those of the other methods. Numerical stability and accuracy advantages of characteristic-based algorithms are still preserved. Sommerfeld radiation condition can be written in 1-Dimension along the ray and implemented perfectly at the outer boundary of computational domain. No unwanted reflected waves exist. Numerical dispersion due to discretization of computational domain is avoided since our grids where we perform our computations lie on the wavefronts and along the rays. All these improvements stem from the fact that the transport equations for the fields along a ray direction are a one-dimensional problem, i.e. one has to deal with ordinary differential equations instead of a partial differential equation.

With the proposed method, one can calculate the time-dependent electromagnetic fields at all points in space. Conversely, one can also calculate the time-dependent electromagnetic fields at a region of interest from rays that pass through it. These could be called as Time-Domain Ray Optics (TDRO) and Time-Domain Ray Tracing (TDRT) respectively.

Future studies shall include the detailed error, stability and numerical dispersion analyses of the proposed method. Application to isotropic, inhomogeneous medium shall be done for arbitrary shape wavefronts, rays and complex source geometries. The method shall also be extended to anisotropic media case. Electromagnetic phenomena of reflection, refraction, and diffraction shall be incorporated into the method. Discontinuities shall be related to source quantities at t=0 on initial discontinuity hypersurface. Our target is to solve/examine all electromagnetic problems/phenomena in 3-Dimension and in time-domain. The proposed method is also expected to be applicable for acoustics.

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