ISTANBUL UNIVERSITY ENGINEERING FACULTY
JOURNAL OF ELECTRICAL & ELECTRONICS

YEAR : 2002 **(521-525)**
VOLUME : 2
NUMBER : 2

COMPARISON OF THE ECONOMIC DISPATCH SOLUTIONS WITH AND WITHOUT TRANSMISSION LOSSES

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ABSTRACT

The short-term optimization problem is how to schelude generation to minimize the total fuel cost or to maximize the total profit over a study period of typically a day, subject to a large number of constraints that must be satisfied. The most important constraint is that the total generation must equal the half-hourly forecast demands for electricity. There are two related short-term optimization problems, "unit commitment" and "economic dispatch". This paper includes knowledge of economic dispatch and solution techniques with or without transmission losses. In the optimal economic dispatch solution, losses have very important roles. Choosing generator units is one of them.

ÖZET

Kýsa dönem optimizasyon problemlerinde amaç maliyeti minimum ve karý maksimum yapmaktýr. Bu gerçekle°tirilirken birçok ölçüt gözönünde bulundurulur. Ekonomik dispatch bu ölçütlerden bir tanesidir. Bu çalýþmada, ekonomik dispatch problemlerinin hatlardaki kayýplar olmaksýzýn ve kayýplar gözönünde bulundurulmuþ çözümü hakkýnda bilgi verilmiþtir.

Keywords: *O*ptimal economic dispatch, Lagrange function, cost function.

1. INTRODUCTION

Unit commitment that is one of the two short-term optimization problems is the process of deciding when and which generating units at each power station to start-up and shut-down. The other is economic dispatch and it is the process of deciding what the individual power outputs should be of the scheduled generating units at each time-point [1].

Before the solution of economic dispatch problem, we have to get the cost function of each

Received Date : 12.5.2001
Accepted Date: 05.5.2002

subset. In order to do, it could be used many mathematical techniques such as least-squares method, regression analysis, fuzzy logic etc. Then, by using the optimization methods like the Kuhn-Tucker, Lambda-Iteration, Gradient Methods etc. solutions can be reached. In this process; neglecting or considering transmission losses may be important, because power generating units may be in their the limites [2].

II. ECONOMIC DISPATCH PROBLEM

The objective of economic dispatch problem is to operate our power system in a manner that minimizes the costs of generator. It assumes that there are N units already connected to the system. The purpose of the economic dispatch problem is to find the optimum policy for these N units.

A. System Dispatching Without Losses

N thermal-generating units connected to a sinle bus-bar are illustrated in Figure 1. The input to each unit, shown as C_i, is the cost rate of the unit. The output of each unit, shown as P_i, represents the electrical power output of units [3].

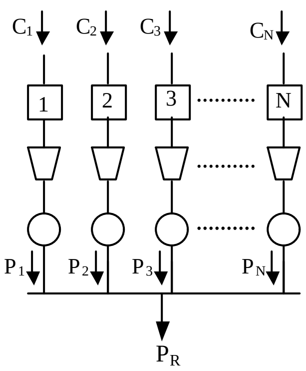

Fig 1. Simplified network supplied by several generator

In this system, the essential condition is to balance the sum of the generated powers and the load demand. Assume that C_T is equal to the total cost for supplying the indicated load. The problem is to minimize C_T subject to the constraints that are the sum of the power generated in each unit. In this solution process, operating limits of each generation unit must be known. That is,

$$C_T = C_1 + C_2 + C_3 + \dots\dots\dots + C_N$$

$$= \sum_{i=1}^{N} C_i(P_i) \tag{1}$$

$$P_R = \sum_{i=1}^{N} P_i \tag{2}$$

This constrained optimization problem is solved using advanced calculus methods that involve the Lagrange function.

Lagrange function is obtained by adding the constraint function to the objective function after the constraint function has been multiplied by an unknown multiplier (λ). It is shown in Eq.3.

$$L(P_i, \lambda) = C_T + \lambda(P_R - \sum_{i=1}^{N} P_i) \tag{3}$$

Derivative of the *Lagrange function* with respect to each of the independent variable and setting them equals to zero gives us $N+1$ equation. The N are values of power output, P_i, and plus one is the undetermined *Lagrange* multiplier, λ.

$$\frac{\partial L}{\partial P_i} = \frac{dC_i(P_i)}{dP_i} - \lambda = 0$$

$$\frac{dC_i(P_i)}{dP_i} - \lambda = 0 \tag{4}$$

$$P_R - \sum_{i=1}^{N} P_i = 0 \tag{5}$$

In addition, there are two inequalities that must be satisfied. Generated power of each unit must be greater than or equal to the minimum power permitted and must also be less than or equal to the maximum power permitted on that particular unit. This inequality is given by Eq. 6.

$$P_{i(\min)} \le P_i \le P_{i(\max)} \tag{6}$$

Eq.4, 5 and 6 are necessary conditions for minimizing the cost function without losses. These conditions may be expanded as shown in the set of equations making up Eq 7.

$$\begin{aligned} \frac{dF_i(P_i)}{dP_i} &= \lambda \quad \text{for} \quad P_{i(\min)} < P_i < P_{i(\max)} \\ \frac{dF_i(P_i)}{dP_i} &\le \lambda \quad \text{for} \quad P_i = P_{i(\max)} \\ \frac{dF_i(P_i)}{dP_i} &\ge \lambda \quad \text{for} \quad P_i = P_{i(\min)} \end{aligned} \tag{7}$$

B. System Dispatching With Losses

The economic dispatching problem shown in Fig. 2. is more complicated to set up than the without losses case. The objective function, C_T, is the same as that defined for Eq. 1. However transmission losses must be added to constraint equation in Eq. 2. That is,

$$P_R + P_L - \sum_{i=1}^{N} P_i = 0 \qquad (8)$$

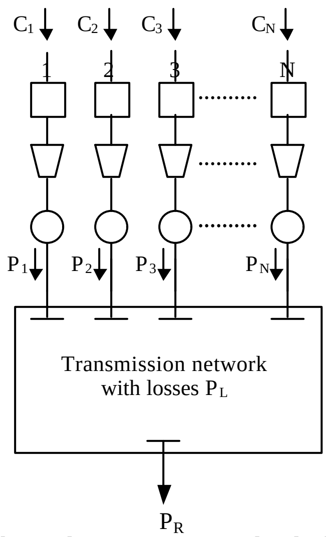

Fig 2. N thermal units serving load through transmission network

In order to obtain the *Lagrange function* for the new case, the same procedure as in the lossless is followed. The Lagrange function is shown in Eq. 9.

$$L(P_i, \lambda) = C_T + \lambda (P_R + P_L - \sum_{i=1}^{N} P_i) \qquad (9)$$

Taking the derivative of the *Lagrange function* with respect to the independent variables give Eq. 10. and 11. that are necessary conditions to solve economic dispatch problem with losses.

$$\frac{\partial L}{\partial P_i} = \frac{dC_i(P_i)}{dP_i} - \lambda\left(1 - \frac{\partial P_L}{\partial P_i}\right) = 0$$

$$\frac{dC_i(P_i)}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \qquad (10)$$

$$P_R + P_L - \sum_{i=1}^{N} P_i = 0 \qquad (11)$$

Because Eq. 10. involves the computation of the network loss, it is more difficult to solve this problem. There are two general approaches to the solution of the problem. The first is the development of a mathematical expression for the losses as a function of the power output of each unit. The other approach to the solution of the problem is to obtain constraints using the load-flow equations. This general approach is called *optimal load flow* [5,6].

III. APPLICATION

In the sample unit that will be used in this paper there have been three generator units. Unit 1 is oil-fired steam unit whose minimum and maximum outputs are 122 MW and 400 MW respectively. Unit 2 is coal-fired steam unit and its minimum output power is 260 MW and its maximum output power is 600 MW. The last unit is also oil-fired steam unit and minimum-maximum output limits are 50 MW and 445 MW.

TABLE I.
COST VALUES FOR THE UNITS CORRESPONDING DIFFERENT POWER OUTPUT

UNIT 1		UNIT 2		UNIT 3	
P_1[MW]	C_1 [$]	P_2[MW]	C_2 [$]	P_3[MW]	C_3 [$]
130	1353.6	260	2567.0	50	944.5
150	1520.0	285	2783.5	80	1158.4
170	1689.6	310	3003.2	110	1377.7
190	1862.4	335	3226.0	140	1602.4
210	2038.4	360	3452.0	170	1832.5
230	2217.6	385	3681.0	200	2068.0
250	2400.0	410	3913.2	230	2308.9
270	2585.6	435	4148.5	260	2555.9
290	2774.4	460	4387.0	290	2806.9
310	2966.4	485	4628.5	320	3064.0
330	3161.6	510	4873.2	350	3326.5
350	3360.0	535	5121.0	380	3594.4
370	3661.6	560	5372.0	410	3867.7
390	3766.4	585	5626.0	440	4146.4

For the units, the cost values of different power output are given in Table I. Before the solution of dispatching, it must be satisfied cost function would be obtained by using any technique. In this application Least-Squares Method is used.

A. PROBLEM 1

Let's determine the economic operating point for these three units when delivering a total of 745 MW. The fuel cost function of each unit is,

$$C_1 = 323.634 + 7.424 P_1 + 0.0035 P_1^2$$
$$C_2 = 565.230 + 6.972 P_2 + 0.0028 P_2^2$$
$$C_3 = 602.105 + 6.717 P_3 + 0.0030 P_3^2 \quad (12)$$

subject to $P_1 + P_2 + P_3 = 745$ [MW] (13)

Using Eq. 9 and 10, the conditions for an optimum dispatch are

$$\frac{dC_1}{dP_1} = 7.424 + 0.0070 P_1 = l \quad (14)$$

$$\frac{dC_2}{dP_2} = 6.972 + 0.0056 P_2 = l \quad (15)$$

$$\frac{dC_3}{dP_3} = 6.717 + 0.0060 P_3 = l \quad (16)$$

$$P_1 + P_2 + P_3 = 745 \text{ MW} \quad (17)$$

Then,

$$l = 8.5435 \text{ \$/MWh}$$
$$P_1 = 159.938 \text{ MW}$$
$$P_2 = 280.635 \text{ MW}$$
$$P_3 = 304.427 \text{ MW} \quad (18)$$

Note that all constraints are met, that is, each unit is within its high and low limit and the total output when summed over all three units meets the desired 745 MW.

B. PROBLEM 2

For the problem 1. with using the transmission losses, the loss formula is necessary. Then, we assume that the simplified loss expression is

$$P_L = 0.00003 P_1^2 + 0.00009 P_2^2 + 0.00012 P_3^2 \quad (19)$$

Applying Eq.9 and 10

$$\frac{dC_1}{dP_1} = l\left(1 - \frac{dP_L}{dP_1}\right) \quad (20)$$

becomes

$$7.424 + 0.0070 P_1 = l\left[1 - 2(0.00003) P_1\right] \quad (21)$$

Similarly for P_2 and P_3

$$6.972 + 0.0056 P_2 = l\left[1 - 2(0.00009) P_2\right] \quad (22)$$

$$6.717 + 0.0060 P_3 = l\left[1 - 2(0.00012) P_3\right] \quad (23)$$

and

$$P_1 + P_2 + P_3 - 745 - P_L = 0 \quad (24)$$

Firstly to begin the solution, it must be selected starting values for P_1, P_2 and P_3 that sum to the load. Selected values for this application are P_1 *=150 MW*, P_2 *=300 MW*, and P_3 *=295 MW.*

For the starting values;

$$7.424 + 0.0070 P_1 = l\, 0.9910$$
$$6.972 + 0.0056 P_2 = l\, 0.9460$$
$$6.717 + 0.0060 P_3 = l\, 0.9292$$
$$P_1 + P_2 + P_3 = 869.218 \quad (25)$$

These equaions are now linear, so it can be solved for l directly. These results are

$$l = 9.2277 \text{ \$/MWh}$$
$$P_1 = 245.81 \text{ MW}$$
$$P_2 = 313.83 \text{ MW}$$
$$P_3 = 309.57 \text{ MW} \quad (26)$$

These results must be compared with the initial power values. If there is no significant change in any one of the value, solution can be stopped. But in the other condition, iterations must be carried to the no significant change. Table II. shows that the iterative process used to solve this problem.

TABLE II.
ITERATIVE PROCESS USED TO SOLVE PROBLEM 2

Iteration	P_1 (MW)	P_2 (MW)	P_3 (MW)	Loss (MW)	l ($/MWh)
Start	150.00	300.00	295.00	19.218	9.0021
1	213.87	275.71	274.62	17.263	8.9781
2	205.46	278.55	278.24	17.539	8.9819
3	206.65	278.39	277.48	17.495	8.9811
4	206.54	278.27	277.67	17.500	8.9810

IV. CONCLUSION

This paper presents the comparison of the economic dispatch solution with and without transmission losses. In the solution with losses, power output of Unit 1 increases according to the solution of lossless. Fig. 3. shows the power output of the units.

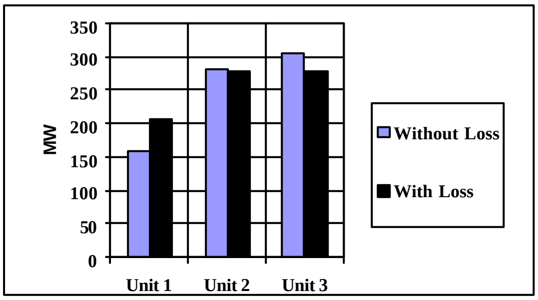

Fig. 3. The output power of the units

Increasing of the Unit 1 responses to the network losses. Because; for this application, loss expression shown in Eq. 19. becomes minimum for increasing of the output power of Unit 1. Economic dispatch solution of the power system with network losses represents an important point of view. Before choosing the power generator units for any system, it must be prefered in appropriate power range for units.

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