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DESIGN TABLE FORMATION OF STEPPED IMPEDANCE PROTOTYPE FILTERS

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ABSTRACT

In literature, there exist some tables containing normalized element values, to design low-pass lumped-element prototype filters with several different responses like maximally flat or equal-ripple. In this work, an algorithm is presented, to form such tables for stepped impedance prototype filters. By using the proposed algorithm, a table for 0.5 dB equal-ripple stepped impedance prototype filter is formed. Then a design example is given, to show the utilization of the formed table.

***Keywords**: Filters, Stepped impedance, Maximally flat, Equal ripple, Distributed elements*

1. INTRODUCTION

Lumped-element filters can be designed via insertion loss method, which is a well-known technique in the literature [1]. But it is not practical, if the number of elements in the filter is large; lots of complex calculations must be made to obtain the desired filter element values. To overcome this problem, for a normalized low-pass design where the generator impedance is $R_G = 1$ and the cutoff frequency is $\omega_c = 1$, the element values for the ladder-type circuits have been tabulated [1,2], figure 1. So by using these tables, it is extremely easy to design lumped-element prototype filters and there is no need to make any complex calculation. On the other hand, distributed-element filters can be designed via the proposed techniques in [2,3]. But the calculations in these methods are very complex, if we compare them with the lumped-element design via the tables. So in this paper, an algorithm to form such tables for stepped impedance prototype filters has been presented. In the algorithm, Simplified Real Frequency Technique (SRFT) has been utilized. As an example, a table for 0.5 dB equal-ripple stepped impedance prototype filter has been formed. Similar tables can be obtained via the same procedure, to design other kind of responses like maximally flat.

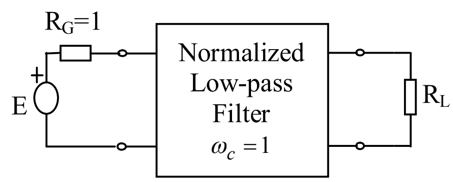

Figure 1. Normalized low-pass prototype filter with the generator impedance $R_G = 1$, cutoff frequency $\omega_c = 1$.

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In the following section, first distributed-element networks and the Simplified Real Frequency Technique (SRFT) are explained briefly. Then the algorithm to form the tables is presented. Finally, a 0.5 dB stepped impedance equal-ripple prototype filter design example is given, to illustrate the utilization of the formed table.

2. DISTRIBUTED-ELEMENT NETWORK CHARACTERIZATION

In figure 2, Darlington representation of a distributed-element network is given. Here, $S_{11}(\lambda)$ represents the bounded real (BR) input reflectance function, $\lambda = \Sigma + j\Omega$ is the conventional Richards variable associated with the equal-length transmission lines (Unit elements, UEs), or so-called commensurate transmission lines [4]. In detail, $\lambda = \tanh p\tau$, where $p = \sigma + j\omega$ is the complex frequency and τ is the commensurate delay of the transmission line. Specifically on the imaginary axis ($\Sigma = 0$), the transformation takes the form $(\lambda = j\Omega = j\tan\omega\tau)$.

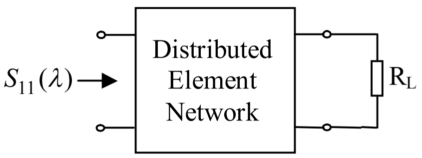

Figure 2. Darlington representation of a distributed-element network.

Let $\{S_{kl}(\lambda); k,l = 1,2\}$ designate the scattering parameters of the distributed-element network that defines a reciprocal, lossless two-port (i.e., the Darlington two-port). For such a two-port, the scattering parameters may be expressed in the Belevitch form as follows [5,6]:

$$S(\lambda) = \begin{bmatrix} S_{11}(\lambda) & S_{12}(\lambda) \\ S_{21}(\lambda) & S_{22}(\lambda) \end{bmatrix} = \frac{1}{g(\lambda)} \begin{bmatrix} h(\lambda) & \mu f(-\lambda) \\ f(\lambda) & -\mu h(-\lambda) \end{bmatrix}, \quad (1)$$

where $\mu = f(-\lambda)/f(\lambda) = \pm 1$. For a lossless two-port with resistive termination, energy conversation requires that

$$S(\lambda)S^T(-\lambda) = I, \quad (2a)$$

where I is the identity matrix and "T" designated the transpose of the matrix. The explicit form of (2a) is known as the Feldtkeller equation and given as

$$g(\lambda)g(-\lambda) = h(\lambda)h(-\lambda) + f(\lambda)f(-\lambda). \quad (2b)$$

In (1) and (2b), $g(\lambda)$ is the strictly Hurwitz polynomial of n^{th} degree with real coefficients, and $h(\lambda)$ is a polynomial of n^{th} degree with real coefficients. The polynomial function $f(\lambda)$ includes all transmission zeros of the two-port; its general form is given by

$$f(\lambda) = f_0(\lambda)(1-\lambda^2)^{n_\lambda/2}, \quad (3)$$

where n_λ specifies the number of equal-length transmission lines (Unit elements, UEs) contained in the two-port, and $f_0(\lambda)$ is an arbitrary real polynomial. According to (3), there may be a finite number of transmission zeros in the right half of the $\lambda-$plane. Realization of distributed-element network functions containing this kind of factors requires more complex structures like coupled lines, Ikeno loops et cetera, which are difficult to realize and, therefore, undesirable [7,8].

A powerful class of networks contains simple, series or shunt, stubs and equal-length transmission lines only. Series-short stubs and shunt-open stubs produce transmission zeros for $\lambda = \infty$, corresponding to the frequency $\omega = \pi/2\tau$ and odd multiples thereof. Series-open stubs and shunt-short stubs produce transmission zeros for $\lambda = 0$ (i.e., $\omega = 0$). For such networks, the polynomial function $f(\lambda)$ takes the more practical form

$$f(\lambda) = \lambda^k (1-\lambda^2)^{n_\lambda/2} \quad (4)$$

where n_λ is the number of equal-length transmission lines in cascade, k is the total number of series-open and shunt-short stubs, and the difference $n-(n_\lambda + k)$ gives the number of series-short and shunt-open stubs. Here, n denotes the degree of the two-port, which is also the degree of $g(\lambda)$. The synthesis of the input impedance, $Z_{in}(\lambda) = (1+S_{11}(\lambda))/(1-S_{11}(\lambda))$, for this case, is accomplished by extracting poles at 0 and ∞, corresponding to stubs, while equal-length transmission lines are extracted by employing Richards extraction method [3]. Alternatively, the synthesis can be carried out in a more general fashion using the cascade decomposition technique by Fettweis, which is based on the factorization of transfer matrices [9]. Also, the algorithm proposed in [10,11] can be used to synthesize the cascaded commensurate transmission lines.

2.1. Simplified Real Frequency Technique (SRFT)

In the Simplified Real Frequency Technique, the canonic polynomial representation of the scattering matrix is used to describe the lossless network [12,13].

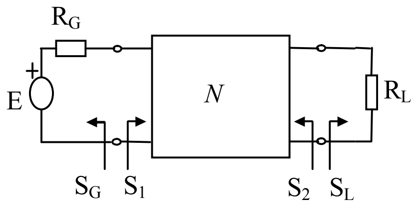

Figure 3. Lossless network referring to scattering approach.

If the designer specifies the complexity of the network N by the degrees $\{n, n_\lambda, k\}$, decides the form of $f(\lambda)$ and initializes the coefficients of the polynomial $h(\lambda)$, then $g(\lambda)$ can be generated by explicit factorization of (2b), since it is a strictly Hurwitz polynomial. At the end of this process, the optimum coefficients of the polynomial $h(\lambda)$ are determined. So the scattering parameters describing the network are obtained. Then the corresponding transducer power gain (TPG) of the entire network agrees with the given TPG data.

The transducer power gain of the network can be expressed via the unit normalized scattering parameters of the two-port, the load and source resistances: So TPG can be written as

$$TPG(\lambda) = \frac{\left(1-|S_G|^2\right)\left(1-|S_1|^2\right)}{|1-S_G S_1|^2} \qquad (5)$$

where S_1 is the unit normalized input reflection coefficient when the network output is loaded by R_L, (figure 3). Also S_1 can be written in terms of the scattering parameters of the network and the load resistance reflection coefficient S_L as

$$S_1 = S_{11} + \frac{S_{12}^2 S_L}{1-S_{22}S_L}. \qquad (6)$$

Substituting (6) in (5) and after some manipulations, the following TPG expression can be obtained

$$TPG(\lambda) = \frac{\left(1-|S_G|^2\right)|S_{21}|^2\left(1-|S_L|^2\right)}{|1-S_{11}S_G|^2 |1-S_2 S_L|^2} \qquad (7)$$

where

$$S_G = \frac{R_G-1}{R_G+1},\; S_L = \frac{R_L-1}{R_L+1},$$
$$S_2 = S_{22} + \frac{S_{21}^2 S_G}{1-S_{11}S_G} \qquad (8)$$

3. TABLE FORMATION ALGORITHM VIA SRFT FOR STEPPED IMPEDANCE PROTOTYPE FILTERS

Inputs:

- ω_i; $i = 1,2,..,N_\omega$: Sample frequencies.
- N_ω : Total number of sample frequencies.
- $TPG(\omega_i)$; $i = 1,2,..,N_\omega$: Sample points measured or calculated from the transducer power gain of the lumped-element prototype.
- n_λ : Total number of equal-length transmission lines (For stepped impedance filters n : Total number of distributed-elements $= n_\lambda$, k=0:Total number of series-open and shunt-short stubs ($n-(n_\lambda+k)$ gives the number of series-short and shunt-open stubs)).
- $f(\lambda)$: A monic polynomial constructed on the transmission zeros.
- $h_0^{(0)}, h_1^{(0)}, h_2^{(0)}, ..., h_n^{(0)}$: Initialized coefficients of the polynomials $h^{(0)}(\lambda)$.
- δ : The stopping criteria for the sum of the squared errors. For many practical problems, it is sufficient to choose $\delta = 10^{-3}$.

Computational Steps:

Step 1: Set $r=1$ and start the iterations.

Step 2: By using the $(r-1)^{th}$ initial coefficients $h_0^{(r-1)}, h_1^{(r-1)}, h_2^{(r-1)}, ..., h_n^{(r-1)}$, compute the strictly Hurwitz polynomial $g^{(r-1)}(\lambda)$ employing (2b).

Step 3: Calculate $TPG_C(\tan(\omega_i\tau))$ of the filter via (9).

Step 7: Compute the error $\varepsilon^{(r-1)}(\omega_i) = TPG(\omega_i) - TPG_C(\tan(\omega_i\tau))$ over the sample frequencies.

Step 8: Compute the sum of the squared errors $\delta^{(r-1)} = \sum_{i=1}^{N_\omega} \left|\varepsilon^{(r-1)}(\omega_i)\right|^2$. If $\delta^{(r-1)} \le \delta$, set $S_{11}(\lambda) = \frac{h^{(r-1)}(\lambda)}{g^{(r-1)}(\lambda)}$ and stop. Otherwise go to the next step.

Step 9: Change the coefficients of the polynomial $h(\lambda)$ via any optimization algorithm.

Step 10: Set $r = r+1$ and go to Step 2.

In the following section, a 0.5 dB equal-ripple stepped impedance prototype filter is designed via the table obtained by using this algorithm.

4. EXAMPLE: 0.5 dB EQUAL-RIPPLE STEPPED IMPEDANCE PROTOTYPE FILTER TABLE AND DESIGN

The following 0.5 dB equal-ripple stepped impedance prototype filter table was formed via the proposed algorithm above.

Table 1. Element values for 0.5 dB equal-ripple stepped impedance prototype filter ($g_0 = 1$, $\omega_c = 1$, $k = 0$, $n = n_\lambda$ =1 to 7, E : Electrical length).

n	1	2	3	4	5	6	7
g_1	16.7697	8.4613	6.4522	3.4970	2.8175	2.7582	2.1976
g_2	1	0.2379	0.2354	0.4491	0.5681	0.5501	0.7401
g_3		1.9841	6.4522	4.3669	3.7790	3.5811	2.7407
g_4			1	0.6122	0.5681	0.4754	0.6654
g_5				1.9841	2.8175	3.5751	2.7407
g_6					1	0.7061	0.7401
g_7						1.9841	2.1976
g_8							1
E	3^o	12^o	18^o	36^o	45^o	45^o	60^o
τ	0.6285	0.6285	0.6285	0.6285	0.6285	0.6285	0.6285

For comparison, element values for 0.5 dB equal-ripple lumped-element prototype filter table are given in Table 2. It can be seen that if n is odd, the termination resistor is unity, and if n is even, the termination resistor is 1.9841 in the both tables. Also element values are symmetrical in both case, when n is odd.

Table 2. Element values for 0.5 dB equal-ripple lumped-element prototype filter ($g_0 = 1$, $\omega_c = 1$, $n = 1$ to 7) [1].

n	1	2	3	4	5	6	7
g_1	0.6986	1.4029	1.5963	1.6703	1.7058	1.7254	1.7372
g_2	1	0.7071	1.0967	1.1926	1.2296	1.2479	1.2583
g_3		1.9841	1.5963	2.3661	2.5408	2.6064	2.6381
g_4			1	0.8419	1.2296	1.3137	1.3444
g_5				1.9841	1.7058	2.4758	2.6381
g_6					1	0.8696	1.2583
g_7						1.9841	1.7372
g_8							1

Let us now design a five-lumped-element and a five-distributed-element prototype filter by using the tables above. From table 1, normalized characteristic impedances, electrical length, delay of the commensurate lines, generator and load resistors are $Z_1 = Z_5 = 2.8175$, $Z_2 = Z_4 = 0.5681$, $Z_3 = 3.7790$, $E = 45^o$ (half of the quarter wavelength), $\tau = 0.6285$, $R_G = R_L = 1$, respectively. Normalized lumped-element values from Table 2 are $L_1 = L_3 = 1.7058$, $L_2 = 2.5408$, $C_1 = C_2 = 1.2296$, $R_G = R_L = 1$. These filters and their transducer power gain curves are given in figure 4, figure 5 and figure 6, respectively. The simulations are performed via Microwave Office of Applied Wave Research Inc. [14].

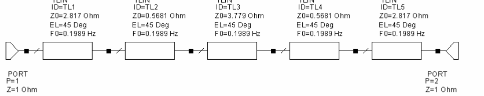

Figure 4. Stepped-impedance prototype filter, (normalized element values: $Z_1 = Z_5 = 2.8175$, $Z_2 = Z_4 = 0.5681$, $Z_3 = 3.7790$, $E = 45^o$, $\tau = 0.6285$ ($F0 = 1/8\tau = 0.1989$), $R_G = R_L = 1$).

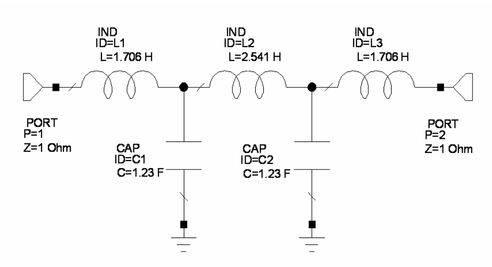

Figure 5. Lumped-element prototype filter, (normalized element values: $L_1 = L_3 = 1.7058$, $L_2 = 2.5408$, $C_1 = C_2 = 1.2296$, $R_G = R_L = 1$).

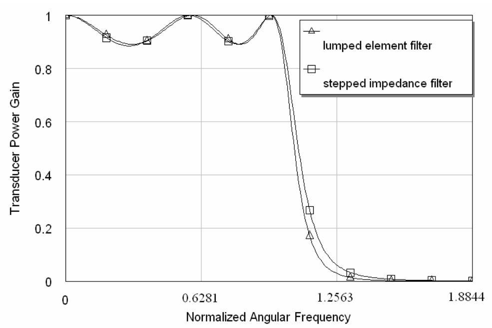

Figure 6. Transducer power gain curves of the lumped- and stepped impedance prototype filters.

As can be seen from figure 6, transducer power gain curves of the filters are nearly the same. So once the tables for different filter responses are formed via the proposed algorithm, there is no need to make lots of complex calculations during the filter design process.

4. CONCLUSION

In this paper, an element value table to design 0.5 dB equal-ripple stepped impedance prototype filters is presented. To be able to prepare the table, an SRFT-based algorithm is proposed. Then, an example prototype filter was designed using this table. The performance of the stepped impedance prototype filter was compared with that of the lumped-element prototype filter. It was shown that the design of stepped impedance prototype filter is extremely simple via the proposed table. By using the proposed algorithm, similar tables for other kind of filter responses can be obtained easily. So the designers would not have to make complex calculations during the stepped impedance filter designs. It would be extremely easy, to design stepped impedance filters using the tables.

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