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IMPROVED EXPRESSION FOR THE AVERAGE BEP OF MC-CDMA SYSTEMS WITH MRC DIVERSITY RECEPTION

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Abstract: *Multi-carrier code division multiple access (MC-CDMA) which is a combination of multicarrier modulation and code division multiple access has gained wide acceptance in recent works because of its many advanced features. This paper gives an evaluation of the bit error probability (BEP) of an uplink MC-CDMA system using maximal-ratio combining (MRC) receivers over a Nakagami-m fading channel. An improved and accurate BEP expression for uplink with MRC receivers is derived. Numerical results in term of average BEP show that the proposed expression has excellent accuracy in different conditions, and could serve as an attractive alternative to previously derived expression for evaluating the average BEP of an uplink MC-CDMA system.*

Keywords: *High capacity wireless communications, MC-CDMA uplink system, diversity reception, maximal-ratio combining, BEP evaluation.*

1. Introduction

Multi-carrier code division multiple access (MC-CDMA) transmission system introduced in [1] is a multiple access scheme which combines orthogonal frequency division multiplex (OFDM) and code division multiple access (CDMA). As MC-CDMA is based on OFDM modulation, its bandwidth efficiency is much better than that of the traditional direct sequence CDMA schemes. In MC-CDMA transmission, each input data stream is partitioned into parallel substreams that are modulated at different subcarriers, which are orthogonal, each other. Due to the orthogonality between different subcarriers, the spectrum belonging to different subcarriers can be overlapped without inter-symbol interference. In addition, MC-CDMA systems are known to have good spectral properties and are potentially robust to channel frequency selectivity, which makes equalization very simple [2]-[7]. It is well known that the performance and system capacity belonging to mobile communication systems are limited by the multipath fading channel. In order to minimize signal fading in mobile communication systems, the most commonly used methods are diversity techniques, in which the maximal-ratio combining (MRC) and equal-gain combining (EGC) are two most popular and robust techniques [8], [9]. It is useful to note that MRC is the optimum spatial diversity technique to reduce the signal fluctuations caused by the multipath propagation. The Nakagami-*m* distribution [10] has gained widespread application in the modeling of physical fading radio channels. Through the fading parameter, the Nakagami-*m* can model signal fading conditions that range from severe to moderate, to light fading or no fading. The primary justification for the use of the Nakagami-*m* fading model is its good fit to empirical fading data [11]-[15]. In order to evaluate the average bit error probability (BEP) of MRC receiver for uplink MC-CDMA over Nakagami-*m* fading channels, an approximated expression is proposed in [1] and [16]. It should be noted that the approximated expression obtained by applying the Law of Large Numbers (LLN) is very simple and does not require complicated functions. However, it should be noted that the uplink average BEP evaluation is not efficient enough for the small values of fading parameter.

In this paper, we propose an accurate expression, which can be used as an attractive alternative to evaluate the average BEP of MC-CDMA signals with MRC over Nakagami-*m* fading channels. The simulated annealing algorithm, which is a heuristic optimization procedure, is used in the process of determining the optimum value of the coefficient in the proposed expression. Numerical results show that a more accurate expression can be obtained and it improves the accuracy of the average BEP evaluation previously studied in the literature.

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2. System Description

We consider the multiple access system originally described in [16] where *M* active users are transmitting simultaneously and synchronously. Assume that binary phase-shift keying (BPSK) modulation and deterministic binary spreading codes are employed. The MC-CDMA signal corresponding to the *k*th data bit transmitted by the *m*th user may be expressed as

$$s_m(t)=\sum_{i=0}^{N-1} a_m(k)c_m(i)\cos\left(2\pi f_c t+2\pi i\frac{F}{T_b}t\right)\cdot P_{T_b}(t-kT_b) \quad (1)$$

where $c_m(0)$, $c_m(1)$, . . . , $c_m(N-1)$ represents the spreading code of the *m*th user, *N* is the processing gain, $P_{T_b}(t)$ is the rectangular pulse defined in [0, T_b] with T_b denoting the symbol duration, f_c is the carrier frequency, and *F* is an positive integer that can be chosen to ensure independent fading for different subcarriers. $a_m(k)$ is the input data symbols to be transmitted and assumed to takes on values of -1 and 1 with equal probability. In this study, a frequency-selective channel with $1 / T_b \ll BW_c \ll F / T_b$ where BW_c is the coherence bandwidth, is considered. The scaling of the amplitudes are assumed to be independent and identically distributed Nakagami random variables of the form [10]

$$f_{\rho_{m,i}}(\rho_{m,i})=\begin{cases} 2\left(\dfrac{g_{m,i}}{\Omega_{m,i}}\right)^{g_{m,i}}\dfrac{1}{\Gamma(g_{m,i})}(\rho_{m,i})^{2g_{m,i}-1}e^{-\left(\frac{g_{m,i}}{\Omega_{m,i}}\rho_{m,i}^2\right)}, & \rho_{m,i}\geq 0 \\ 0 & \rho_{m,i}<0 \end{cases} \quad (2)$$

where $\rho_{m,i}$ is the channel amplitude. The indexes *m* and *i* have been added to differentiate between the *m*th user and the *i*th subcarrier [16]. Γ(·) is the standard Gamma function and *g* is the fading parameter defined as

$$g_{m,i}=\frac{\Omega_{m,i}^2}{E[(\rho_{m,i}^2-\Omega_{m,i})^2]}\geq\frac{1}{2} \quad (3)$$

(*E*[·] denotes expectation), $\Omega_{m,i}$ controls the spread of the distribution. The distribution has a mean value defined by

$$E(\rho_{m,i})=\left(\frac{\Omega_{m,i}}{g_{m,i}}\right)^{0.5}\frac{\Gamma(g_{m,i}+0.5)}{\Gamma(g_{m,i})} \quad (4)$$

and a mean square value defined by

$$E(\rho_{m,i}^2)=\frac{\Gamma(g_{m,i}+1)}{\Gamma(g_{m,i})}\left(\frac{\Omega_{m,i}}{g_{m,i}}\right)=\Omega_{m,i} \quad (5)$$

The received signal corresponding to the *k*th data bit transmitted can be written as

$$r(t)=\sum_{m=0}^{M-1}\sum_{i=0}^{N-1}\rho_{m,i}c_{m,i}a_m(k)\cos(\varphi_i)+n(t) \quad (6)$$

with $\varphi_i=2\pi f_c t+2\pi i(F/T_b)t+\theta_{m,i}$. The effects of the channel have been included in $\rho_{m,i}$ and $\theta_{m,i}$, and $n(t)$ is additive white Gaussian noise (AWGN) with one sided power spectral density N_0. The implementation of the MC-CDMA receiver is shown in Figure 1, where it has been assumed that $m = 0$ corresponds to the desired signal.

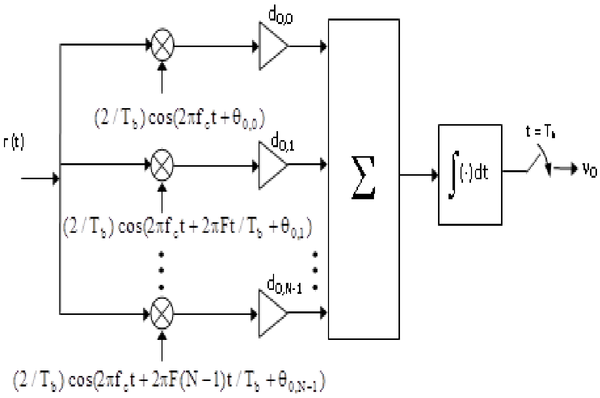


Figure 1. MC-CDMA receiver model

Applying the receiver model of Figure 1 to the received signal given in Eqn (6) and assuming the users are synchronized in time and a perfect phase correction (negligible difference between estimated phase $\hat{\theta}_{0,i}$ and the true phase $\theta_{0,i}$) yields the following decision variable for the *k*th data symbol

$$v_0=a_0(k)\sum_{i=0}^{N-1}\rho_{0,i}d_{0,i}+\beta_{\text{int}}+\eta \quad (7)$$

Note that the decision variable consists of three terms. The first term corresponds to the desired signal component and the second corresponds to the interferences

$$\beta_{\text{int}}=\sum_{m=1}^{M-1}\sum_{i=0}^{N-1}a_m(k)c_m(i)c_0(i)\rho_{m,i}d_{0,i}\cos(\tilde{\theta}_{m,i}) \quad (8)$$

where $\tilde{\theta}_{m,i}=\theta_{m,i}-\theta_{0,i}$. It may be noticed that the interference terms vanish for an ideal transmission channel ($\rho_{m,i}=1$ and $\theta_{m,i}=0$). The last term corresponds to a noise η given by

$$\eta = \sum_{i=0}^{N-1} \int_{kT_b}^{(k+1)T_b} \frac{2}{T_b} n(t) c_0(i) d_{0,i} \cdot \cos(2\pi f_c t + 2\pi i (F/T_b) t + \hat{\theta}_{0,i}) dt \qquad (9)$$

3. BEP Analysis for MRC Reception

In this paper, we will consider MRC diversity reception technique which is mentioned in [16]. The gain factor at the ith subcarrier, $d_{0,i}$, is defined as

$$d_{0,i} = \rho_{0,i} c_0(i) \qquad (10)$$

The variance of the interference components β_{int} can be computed as

$$\sigma^2_{\beta_{\text{int}}} = 2\frac{(M-1)}{N}\overline{p_0}\,\overline{p_m} \qquad (11)$$

where $\overline{p_0}$ and $\overline{p_m}$ represent the local mean power for the 0th user and the local mean power for the mth user, respectively. The noise term given by Eqn (9) can be approximated to a zero-mean Gaussian random variable with variance

$$\sigma^2_{\eta} = 2\frac{N_0}{T_b}\overline{p_0} \qquad (12)$$

Using Eqns (11) and (12), the conditional instantaneous BEP for MRC can be calculated to be

$$\Pr\left(e \middle| \{\rho_{0,i}\}_{i=0}^{N-1}, \overline{p_0}, \overline{p_m}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{\frac{1}{2}\rho_1^2}{\sigma^2}}\right) \qquad (13)$$

where $\rho_1 = \sum_{i=0}^{N-1} \rho_{0,i}^2$. In order to obtain an exact expression for the average BEP, it is considered that the quantity $r = (1/2)\rho_1$ has the following distribution

$$p(r) = \frac{1}{(Ng-1)!}\left(\frac{g}{\overline{p_{0,i}}}\right)^{Ng} r^{Ng-1} e^{-\left(\frac{g}{\overline{p_{0,i}}}\right)r}, \quad r \geq 0 \qquad (14)$$

Averaging Eqn (13) over (14) results in the following exact expression for the average BEP using MRC

$$\Pr_{exact} = \Pr\left(e \middle| \overline{p_0}, \overline{p_m}\right) \cong \int_0^{+\infty} p(r) \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{r^2}{\sigma^2}}\right) dr \qquad (15)$$

In the limiting case of large N, an approximation for the average BEP, obtained by applying the LLN, is given as [16]

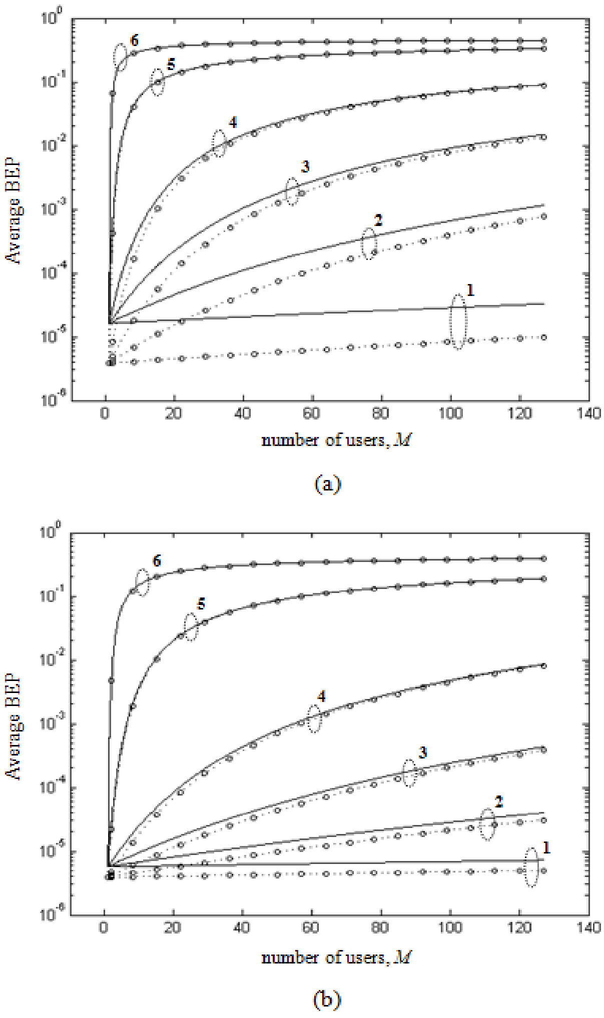
$$\Pr_a = \Pr\left(e \middle| \overline{p_0}, \overline{p_m}\right) \cong \frac{1}{2}\,\text{erfc}\left(\sqrt{\gamma}\right) \qquad (16)$$

where

$$\gamma = \frac{\overline{p_0} T_b}{\left(\left(\frac{M-1}{N}\right)\overline{p_m} T_b + N_0\right)} \qquad (17)$$

4. Proposed Expression for BEP

Figure 2 (a) and (b) show the average BEP, obtained by using Eqns (15) and (16), as a function of M for different values of system parameters when uplink transmission in Nakagami-m fading channel is considered.



(a)

(b)

—— Exact BEP for g =1

· · · · Exact BEP for g =170

○ BEP obtained by the approximated expression

Figure 2. Exact and approximated results of BEP versus the number of users, $M \in \{0, 1, 2, \cdot\cdot, 127\}$, for uplink transmission in

Nakagami-*m* fading channel. α_m(dB) = -20, -10, -5, 0, 10, 20 dB for curves labeled 1 to 6: (a) $N = 127$; (b) $N = 255$.

In this point, we can draw three important observations from Figure 2: (*i*) The average BEP estimations produced by the approximated expression are in very good agreement with the exact BEP results computed by Eqn (15) for high values of $\alpha_m = \overline{p_m} / \overline{p_0}$ (typically $\alpha_m > 0$ dB) (*ii*) The average BEP estimations produced by the approxi-mated expression cannot follow the exact BEP curves closely enough for small values of *g*. (*iii*) The difference between the estimated and exact BEP values tends to decrease with the increasing values of *N* for the whole values of α_m and *g*. Note that Eqn (16) does not explicitly include a term representing the knowledge of the fading parameter *g*. The observations reported above highlighted that the BEP evaluation should include the *g* to improve the accuracy of the evaluation, especially for the low values of *N*. As a result, Eqn (16) can be modified to include a term x/Ng that takes into consideration of *g* with the knowledge of *N* as given below:

$$\Pr_{proposed} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\gamma^*}\right) \tag{18}$$

where

$$\gamma^* = \frac{\overline{p_0}T_b}{\left(\left(\frac{M-1}{N}\right)\overline{p_m}T_b + N_0\right)} + \frac{x}{Ng} \tag{19}$$

It is useful to note that the proposed modification realized on Eqn (16) will improve the analyzing accuracy of the performance of an uplink MC-CDMA system using MRC receivers. In order to find an appropriate value for *x* in Eqn (19), the simulated annealing algorithm, which is one of the heuristic optimization procedures, is employed.

4.1. Simulated Annealing Algorithm

Simulated annealing (SA) algorithm was initially proposed by Kirkpatrick, Gelatt and Vecchi [17]. This algorithm is one of the heuristic optimization procedures, which is particularly effective for the optimization of nonlinear and multimodal functions.

The idea of SA was derived from Metropolis algorithm, which is modeled the physical annealing process and based on Boltzman's probability [18]. The algorithm employs a random search, which not only accepts changes that decrease the objective function (assuming a minimization problem) but also some changes that increase it, which is accepted according to Metropolis's criterion [18, 19]. The Metropolis criterion always accepts the perturbed solutions as the next current solution if its cost is lower than that of the current solution. The criterion also allows for the probabilistic acceptance of higher cost perturbed solutions as the next current solution. The probabilistic acceptance enables the SA algorithm to avoid becoming trapped in local minima.

Summing up, the SA algorithm may be viewed as a randomization device that allows some ascent steps during the course of the optimization, through an adaptive acceptance / rejection criterion. In this paper, the SA algorithm is used for searching the optimum value of the coefficient *x*. Determination of the coefficient value for the proposed expression can be considered as an optimization problem of the cost function *J*(**x**) stated as the following:

$$\min_{\mathbf{x}\in \mathrm{X}} J(\mathbf{x}) \tag{20}$$

where $\mathbf{X} = [x]^T$ is the coefficient vector of the proposed expression. The aim is to minimize the cost function *J*(**x**) by adjusting **x**. The cost function is usually expressed as the time-averaged cost function can be given by

$$J(\mathbf{x}) = \sqrt{\left(\frac{1}{M}\sum_{k=1}^{U}\left(\Pr_{exact}(k) - \Pr_{proposed}(k)\right)^2\right)} \tag{21}$$

where $\Pr_{exact}$ (*k*) and $\Pr_{proposed}$ (*k*) are the results of the exact expression for the average probability of error using MRC and the proposed expression (which employs the values computed by γ^*) at time *k*, respectively. *U* is the number of samples used for the calculation of the cost function. The coefficients of the proposed expression are successively adjusted by SA algorithm until the error between the outputs of the exact expression and the proposed expression is minimized.

Based on the optimization procedure detailed above, the optimum value for *x* is found as 2. After the determined coefficient value, the new expression can be written as:

$$\Pr_{proposed} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\overline{p_0}T_b}{\left(\left(\frac{M-1}{N}\right)\overline{p_m}T_b + N_0\right)} + \frac{2}{Ng}}\right) \tag{22}$$

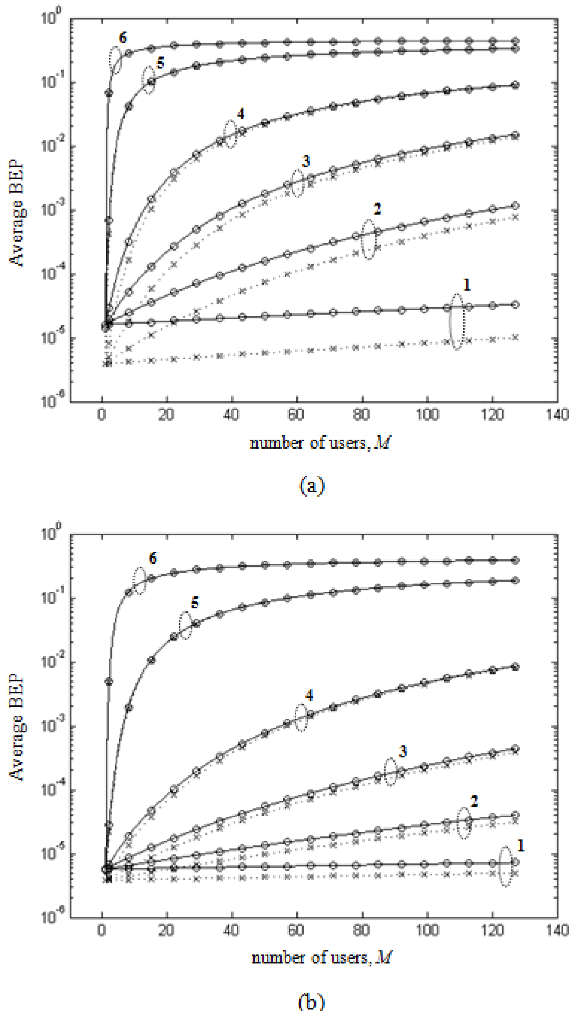
It is useful to note that the Q function is related to the erfc function as follows [20],

$$Q(x) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right) \tag{23}$$

5. Numerical Results and Conclusion

This section discusses the numerical results based on the new expression obtained in the previous section. To evaluate the effectiveness of the proposed expression, the average BEP for uplink transmission in Nakagami fading channel has been numerically evaluated for different values of system parameters. The plot of average BEP for MC-CDMA is obtained for $N = 127$ and $N = 255$, and assuming that the mean power of each interferer (user) is equal to the mean power of the desired user.

Figure 3 (a) and (b) show the plot of average BEP computed by the new expression as a function of *M* for different values of system parameters. From Figure 3 (a) and (b), we can induce that the BEP curves produced by the new expression are in excellent agreement with the exact BEP curves in the case of small values of *g*. Moreover, it is useful to note that an excellent agreement between the calculated and exact BEP results is also maintained for high values of *g*.



(a)

(b)

—— Exact BEP for g =1

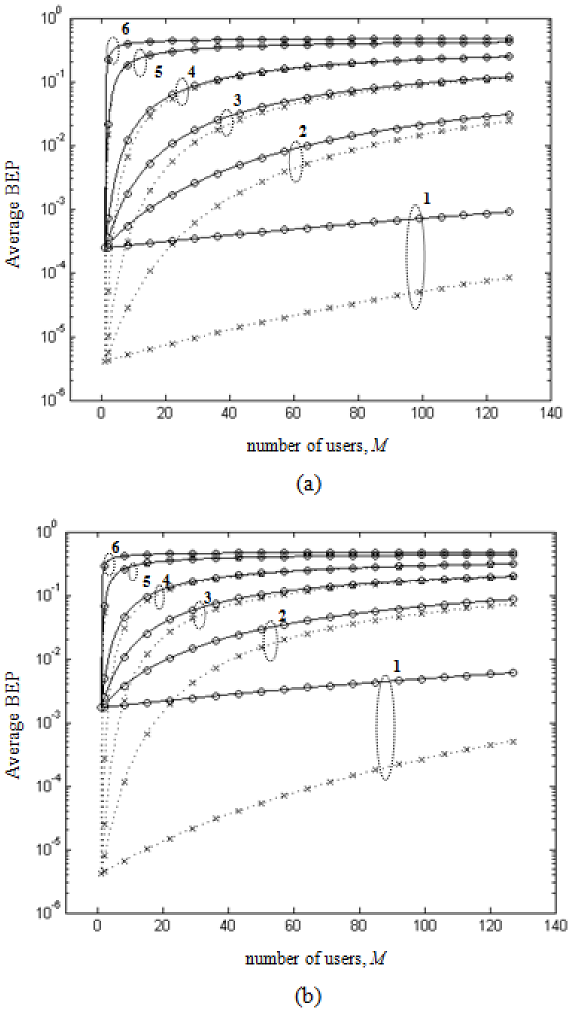
· · · · Exact BEP for g = 170

× BEP obtained by the proposed expression (g = 170)

○ BEP obtained by the proposed expression (g = 1)

Figure 3. Exact and proposed results of BEP against the number of users, $M \in \{0, 1, 2, \cdot\cdot, 127\}$, for uplink transmission in Nakagami-m fading channel. α_m(dB) = -20, -10, -5, 0, 10, 20 dB for curves labeled 1 to 6:(a) N = 127; (b) N = 255.

It is clear that the approximated expression which was obtained by LLN is originally available for large number of *N*. Within our proposed expression, it is also possible to evaluate the BEP performance of the uplink MC-CDMA system with MRC for low values of *N*. Figure 4 (a) and (b) show the plot of average BEP computed by the proposed expression, for *N* = 31 and *N* = 15, respectively. As seen, the proposed expression achieved the task in the highest accuracy especially when the processing gain *N* is low. In conclusion, the new expression proposed in this study enables researches to more accurately evaluate the BEP performance of MC-CDMA signals with MRC over Nakagami-*m* fading channels.



(a)

(b)

—— Exact BEP for g =1

· · · · Exact BEP for g = 170

× BEP obtained by the proposed expression (g = 170)

○ BEP obtained by the proposed expression (g = 1)

Figure 4. Exact and proposed results of BEP versus the number of users, $M \in \{0, 1, 2, \cdot\cdot, 127\}$, for uplink transmission in Nakagami-m fading channel. α_m(dB) = -20, -10, -5, 0, 10, 20 dB for curves labeled 1 to 6: (a) N = 31; (b) N = 15.

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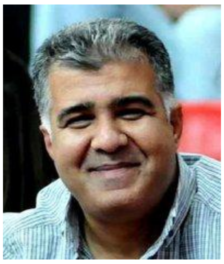
Note:



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