ISTANBUL UNIVERSITY –
JOURNAL OF ELECTRICAL & ELECTRONICS ENGINEERING

YEAR : 2009 (769-779)
VOLUME : 9
NUMBER : 1

NEAR FIELD LOCALIZATION OF MOVING SOURCES WITH REM ALGORITHMS

Serap ÇEKLİ[1] Erdinç ÇEKLİ[2] Nihat KABAOĞLU[3]

[1]İstanbul University, Engineering Faculty, Department of Electrical and Electronics Engineering 34850, Avcılar, Istanbul, Turkey

serapc@istanbul.edu.tr

[2] Tübitak Marmara Research Center, Gebze, 41470, Kocaeli, Turkey

[3] The Department of Electronics Engineering, Maltepe University, Maltepe, 34857, Istanbul, Turkey

ABSTRACT

In this paper, we present the problem of joint tracking of the direction of arrival (DOA) and range parameters of moving sources in the near field of an antenna array with two Expectation-Maximization (EM) based recursive algorithms. The main characteristic of the first Recursive EM (REM) approach is to include computation of the gradient of the log-likelihood function and some form of the complete data Fisher information matrix. Compared to first REM approach, the second one utilizes the stochastic approximation of approximate conditional expectation of the complete data sufficient statistics. The proposed recursive algorithms in this work assume that the parameters of interest are described by a linear polynomial model. This paper concludes by presenting the simulation results of the suggested algorithms in order to illustrate the computational effectiveness of the both algorithms.

Keywords: *REM (Recursive Expectation Maximization), near field source localization*

1. INTRODUCTION

In recent years, the source localization problem using passive sensor arrays has become very important for many applications ranging from sonar, navigation, seismology, geophysics to surveillance. However, the majority of the localization techniques deals with the case in which the source is assumed to be in the far field of the array. This assumption considers that the curvature of the waves can be neglected hence, the waves can be considered as plane waves. Thus each source location can be characterized by only the azimuth in that case. In contrast, if the sources are located close to the array (i.e. near field), the inherent curvature of the waveforms can no longer be neglected. Therefore, the location of each source has to be parameterized in terms of the direction of arrival (DOA) and range parameters. The localization of the near field sources is more involved than the far field case since the parameter vector to be estimated is extended to include the source ranges. It is therefore necessary to derive more sophisticated localization algorithms for estimating the azimuth together with the range. In recent years, there has been lots of research effort on the near field source localization techniques. A total least squares ESPIRIT like algorithm, based on fourth order cumulants was proposed in [1]. Moreover, a high resolution

Received Date: *10.11.2009*
Accepted Date:5.1.2009

algorithm that uses only second order statistics of the array outputs was introduced in [2]. However, due to many attractive features suitable to the near-field scenario such as consistency, asymptotic unbiasedness, and asymptotic minimum variance, ML approaches have been proposed, recently [3]. The ML approach is efficient and gives superior performance in case of little sample numbers and the presence of coherent signals. On the other hand, ML estimation algorithm requires high computational cost. Hence, several techniques have been studied to reduce the complexity of the ML estimator including Expectation Maximization (EM) iterative technique [4], [5]. EM algorithm converts the multidimensional search problem to the less dimensional parallel search problems in order to prevent the computational complexity. EM algorithms for estimating constant DOA parameters were discussed in [6], [7], [8], [9]. Furthermore, recursive EM approaches are maintained because of the time consuming and massy calculation characteristic of the EM algorithm. After gathering only a little observation data, the recursive EM algorithm is appropriate for online processes i.e. tracking, while the conventional EM algorithm is more suitable for offline processes. In addition, the recursive version of the EM algorithms was also applied to the time varying DOA estimation problem [10], [11], [12].

In this paper, we primarily propose two recursive approaches to perform ML estimation of the time varying parameters of moving sources in near-field of the antenna array. These approaches are based on the recursive form of the EM algorithm. Although several Recursive EM (REM) implementations have been proposed for the far field case in the literature, we consider the extension of two EM based recursive approaches previously proposed only for DOA estimation of far field parameters to estimation of time varying near field parameters in this paper. Both algorithms provide the DOA parameters as well as the range parameters of the radiating sources. The first approach is based on the stochastic approximation procedure applied directly on the parameters of interest. It involves computation of the gradient of the log likelihood function and some form of the complete data Fisher information matrix [11]. The second sequential approach utilizes the stochastic approximation to approximate conditional expectation of the complete data sufficient statistics [12]. In contrast to first approach, the second one replaces the Expectation step by a recursive stochastic approximation, while keeping the Maximization step unchanged.

The rest of the paper is organized as follows; in section 2 the signal model constructed and the assumptions have been made related with the problem at hand is presented. The EM formulation and the motion of the sources are discussed in section 3. Two different REM algorithms are described in section 4 as two subsections. The simulations and results are provided and the performance of the algorithms is compared in section 5. The section 6 gives the conclusion and discuss the results.

2. SIGNAL MODEL

Before introducing the two EM based recursive approaches for joint tracking of the time varying azimuth and range parameters of the near field sources, we will first describe the time varying near field signal model in the sequel. In the near field scenario under consideration, it is assumed that the source signals are collected by a uniform linear array. M narrow band signals from time varying $\boldsymbol{\theta}(t) = [\theta_1(t),...,\theta_M(t)]$ directions arrive at an array of N sensors. $\boldsymbol{r}(t) = [r_1(t),...,r_M(t)]$ vector represents the unknown range parameters of the mobile sources with nonlinear movement. Moreover, $\boldsymbol{\Theta}(t) = [\boldsymbol{\theta}^T(t), \boldsymbol{r}^T(t)]^T$ represents the parameter super vector to be estimated corresponding to the moving sources. Thus, the signal model for the data observed at the output of the sensors at time instant t is $\boldsymbol{x}(t) \in \mathbb{C}$;

$$\boldsymbol{x}(t) = \boldsymbol{H}(\boldsymbol{\Theta}(t))\boldsymbol{s}(t) + \boldsymbol{u}(t) \qquad (1)$$

where the steering matrix is

$$\boldsymbol{H}(\boldsymbol{\Theta}(t)) = [\boldsymbol{d}(\boldsymbol{\Theta}_1(t)),...,\boldsymbol{d}(\boldsymbol{\Theta}_M(t))] \in \mathbb{C}^{N\times M} \qquad (2)$$

Steering matrix consists of M steering vectors

$$\boldsymbol{d}(\boldsymbol{\Theta}_m(t)) \in \mathbb{C}^{N\times 1},\ m = 1,...,M \qquad (3)$$

which is a function of unknown parameter vector

$$\boldsymbol{\Theta}_m(t) = [\boldsymbol{\theta}_m^T(t), \boldsymbol{r}_m^T(t)]^T \tag{4}$$

For the m th source with an array of N sensors, the steering vector can be approximated as [8]

$$\boldsymbol{d}(\mu_m, \zeta_m) = \begin{bmatrix} e^{j(k_{\min}\mu_m(t)+k_{\min}^2\zeta_m(t))} \\ \vdots \\ 1 \\ e^{j(\mu_m(t)+\zeta_m(t))} \\ e^{j(2\mu_m(t)+4\zeta_m(t))} \\ \vdots \\ e^{j(k_{\max}\mu_m(t)+k_{\max}^2\zeta_m(t))} \end{bmatrix} \tag{5}$$

k_{min} and k_{max} denote $-N/2$ nd and $N/2$ nd sensors, respectively. The steering vector parameters $\mu_m(t)$ and $\zeta_m(t)$ are functions of the DOA parameter $\theta_m(t)$ and the range parameter $r_m(t)$ of the m th source as

$$\mu_m(t) = -\frac{2\pi\Delta}{\lambda}\sin\theta_m(t) \tag{6}$$

$$\zeta_m(t) = \frac{\pi\Delta^2}{\lambda r_m(t)}\cos^2\theta_m(t) \tag{7}$$

where λ is the wavelength of wavefronts, Δ is the distance between two successive sensors [7]. We also assume that $M < N$, and the waveforms of the M narrow band signals $\boldsymbol{s}(t) = [s_1(t),...,s_M(t)]^T \in C^{M\times 1}$ are unknown and deterministic. Noise process $\boldsymbol{u}(t) \in C^{N\times 1}$ is independent, identical white complex Gaussian distributed with zero mean and covariance matrix $\nu\boldsymbol{I}$, where ν represents an unknown noise spectral parameter and $\boldsymbol{I}$ is the identity matrix.

Before discussing the development of two REM approaches, it is helpful to introduce the common assumptions on the signal (1) for both approaches:

Assumption 1: Let $\boldsymbol{x}(1), \boldsymbol{x}(2),...$ be independent observations with $f(\boldsymbol{x};\boldsymbol{\vartheta})$ the probability density function, where $\boldsymbol{\vartheta}$ denotes an unknown parameter vector.

Assumption 2: The augmented data associated with the EM algorithm $\boldsymbol{y}(1), \boldsymbol{y}(2),...$ is characterized by the pdf $f(\boldsymbol{y};\boldsymbol{\vartheta})$ [11]. The augmented data $\boldsymbol{y}(t)$, $\mathcal{M}(\boldsymbol{y}(t)) = \boldsymbol{x}(t)$ is a many to one mapping [4]. Let $\boldsymbol{\vartheta}^t$ denote the estimate after t observations.

The problem taken into consideration is the estimation of the direction of arrivals $\boldsymbol{\theta}(t)$ and range parameters $\boldsymbol{r}(t)$ of the time-varying signals recursively from the observation $\boldsymbol{x}(t)$ for a known number of sources. With this problem at hand, we present a recursive ML solutions based on the EM algorithm in the sequel. However we will first introduce general EM framework.

3. EM FORMULATION

The EM algorithm provides ML estimation of parameters when maximization of the likelihood function may not be feasible directly. It is an iterative procedure which consists of expectation and maximization steps. Although, the EM algorithm is a batch oriented approach, it is desirable to process the received data in a recursive form in order to eliminate the delay, reduce storage requirements and increase the computational efficiency. We therefore consider tracking of near-field parameters via recursive form of the EM algorithm [10].

To be able to easily apply the EM algorithm, the signal model must be formed in terms of the observed data (incomplete data) and a hypothetical data set (complete data). The complete data must be chosen in such a way that the complete data log likelihood function is easily maximized and the complete data log likelihood function can be easily estimated from the incomplete data [14]. The complete data $\boldsymbol{y}(t)$ and the incomplete data are related by a linear transformation.

Moreover, even for the application of the REM algorithm, the augmented data would be chosen with the following relation between the augmented data $\boldsymbol{y}_m(t)$ and the incomplete data $\boldsymbol{x}(t)$

$$\boldsymbol{x}(t) = \sum_{m=1}^{M} \boldsymbol{y}_m(t) \tag{8}$$

The augmented data is obtained via separating the array output $\boldsymbol{y}(t)$ into its components as given below,

$$\boldsymbol{y}(t) = [\boldsymbol{y}_1^T(t), \ldots, \boldsymbol{y}_m^T(t), \ldots, \boldsymbol{y}_M^T(t)]^T \qquad (9)$$

The incomplete data consists of M independent Gaussian vectors having mean $d(\boldsymbol{\Theta}_m)s_m(t)$ and each with identical covariance $v_m \boldsymbol{I}/M$, thus the augmented data is given by

$$\boldsymbol{y}_m(t) = d(\boldsymbol{\Theta}_m)s_m(t) + u_m(t), \quad 1 \le m \le M \qquad (10)$$

Motivation behind this choice is that if one could somehow observe each of the incident waves separately, the estimation of its near-field parameters would be straightforward by performing M parallel maximization. The logarithmic likelihood function of the augmented data is given as below

$$\log f(\boldsymbol{y}(\boldsymbol{\theta}); \boldsymbol{\vartheta}) = -\sum_{m=1}^{M} \Bigg[N \log \pi + N \log; \left(\frac{v}{M} \right) + \frac{M}{v} (\boldsymbol{y}_m(t) - d(\boldsymbol{\theta}_m)s_m(t))^H \times (\boldsymbol{y}_m(t) d(\boldsymbol{\theta}_m)s_m(t)) \Bigg] \qquad (11)$$

$(.)^H$ denotes the Hermitian transpose of a vector. The EM algorithm is an iterative procedure that makes use of the log-likelihood function of the augmented data (11) to obtain ML estimates of the source parameters.

Moreover, the proposed algorithm assumes that sources are moving with constant velocity by a non-linear movement. Thus, the near-field parameters related to the sources are both changes with time which are described by the linear polynomial model as,

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + t\boldsymbol{\theta}_1 \qquad (12)$$

$$\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{r}_1 \qquad (13)$$

where $\boldsymbol{\theta}_0 = [\theta_{01}, \ldots, \theta_{0M}]^T$, $\boldsymbol{\theta}_1 = [\theta_{11}, \ldots, \theta_{1M}]^T$, $\boldsymbol{r}_0 = [r_{01}, \ldots, r_{0M}]^T$, $r_1 = [r_{11}, \ldots, r_{1M}]^T$. The direction of arrivals and the ranges are shown together in $\boldsymbol{\Theta}_m = [\theta_{0m}, \theta_{1m}, r_{0m}, r_{1m}]^T$ and here $\boldsymbol{\Theta} = [\boldsymbol{\Theta}_1^T, \ldots, \boldsymbol{\Theta}_m^T, \ldots, \boldsymbol{\Theta}_M^T]^T$.

Since the recursive expectation maximization algorithm is only the used for estimation of angle and range parameters, we therefore consider only unknown parameter vector $\boldsymbol{\Theta}$ in the development of REM procedures rather than complete unknown set $\boldsymbol{\vartheta} = [\boldsymbol{\theta}(t)^T, \boldsymbol{r}(t)^T, \boldsymbol{s}(t), v]$.

4. PARAMETER ESTIMATION WITH INCOMPLETE DATA

The problem we address in this section is the recursive estimation of the time varying near field parameters. Two different REM approaches lead to corresponding solutions.

4.1. REM Algorithm I

The first approach we propose here uses the stochastic approximation approach which can be thought as a stochastic generalization of an optimization procedure namely the Newton descent method. In this approach, inverse of the true Hessian matrix (gain matrix) and gradient vector of the logarithmic likelihood function of the augmented data provides an adaptive step in a recursion to lead to an asymptotically optimal search direction.

This first REM algorithm maximizes the augmented log likelihood function using a stochastic approximation recursion at iteration i, given by

$$\boldsymbol{\vartheta}^{i+1} = \boldsymbol{\vartheta}^i + \varepsilon_i \boldsymbol{\ell}_{EM}(\boldsymbol{\vartheta}^i)^{-1} \boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^i) \qquad (14)$$

where ε_i is denotes the step size and

$$\boldsymbol{\ell}_{EM}(\boldsymbol{\vartheta}^i) = E[-\nabla_{\vartheta} \nabla_{\vartheta}^T \log f(\boldsymbol{y}; \boldsymbol{\vartheta}) | \boldsymbol{x}(t), \boldsymbol{\vartheta}] \Big|_{\boldsymbol{\vartheta} = \boldsymbol{\vartheta}^i} \qquad (15)$$

$$\boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^i) = \boldsymbol{\nabla}_{\boldsymbol{\vartheta}} \log f(\boldsymbol{x}(t); \boldsymbol{\vartheta}) \Big|_{\boldsymbol{\vartheta} = \boldsymbol{\vartheta}^i} \qquad (16)$$

represent the augmented information matrix (gain matrix) and gradient vector, respectively, both evaluated at point $\boldsymbol{\vartheta}^i$ for the algorithm, and the gain matrix is calculated from the complete data of the EM algorithm . Moreover, $\nabla_{\boldsymbol{\vartheta}}$ is a column gradient operator with respect to $\boldsymbol{\vartheta}$.

Since the augmented data associated with the EM algorithm is characterized by the hypothetical data (complete data) assumed to arrive to the sensors separately instead of observed data (incomplete data), the augmented data $\boldsymbol{y}_m$ therefore have a more simple form than the observed data $\boldsymbol{x}$. Therefore, the calculation

of the augmented data information matrix $\boldsymbol{\ell}_{EM}(\boldsymbol{\vartheta}^i)$ can be performed in a more simple way [11].
The convergence rate is achieved by the step size. Therefore, the step size ε_i is chosen as a small positive constant for the stable operation in this work assuming the sources are moving slowly.
Furthermore, the gradient vector $\boldsymbol{\gamma}(\boldsymbol{x}(t);\nu^i)$ corresponding to the mth source DOAs $\boldsymbol{\theta}_m$ is given as below;

$$\frac{\partial}{\partial\theta_{0m}}\log f(\boldsymbol{x}(t);\boldsymbol{\vartheta})\bigg|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^i}=\frac{2}{\nu^i}\mathrm{Re}[(\boldsymbol{x}(t)-\boldsymbol{H}(\boldsymbol{\Theta}^i(t))\boldsymbol{s}^i)^H(d'(\boldsymbol{\Theta}^i_{0m}(t))s^i_{0m})] \quad (17)$$

$$\frac{\partial}{\partial\theta_{1m}}\log f(\boldsymbol{x}(t);\boldsymbol{\vartheta})\bigg|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^i}=\frac{2}{\nu^i}\mathrm{Re}[(\boldsymbol{x}(t)-\boldsymbol{H}(\boldsymbol{\Theta}^i(t))\boldsymbol{s}^i)^H(d'(\boldsymbol{\Theta}^i_{1m}(t))s^i_{1m})] \quad (18)$$

Here the indices $0m$ and $1m$ refer to $\boldsymbol{\theta}_0$ and $\boldsymbol{\theta}_1$ in equations (17)-(18). Similarly, the components of the gradient vector $\boldsymbol{\gamma}(\boldsymbol{x}(t);\nu^i)$ corresponding to the mth source range parameter $\boldsymbol{r}_m$ th are

$$\frac{\partial}{\partial r_{0m}}\log f(\boldsymbol{x}(t);\boldsymbol{\vartheta})\bigg|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^i}=\frac{2}{\nu^i}\mathrm{Re}[(\boldsymbol{x}(t)-\boldsymbol{H}(\boldsymbol{\Theta}^i(t))\boldsymbol{s}^i)^H(d'(\boldsymbol{\Theta}^i_{0m}(t))s^i_{0m})] \quad (19)$$

$$\frac{\partial}{\partial\theta_{1m}}\log f(\boldsymbol{x}(t);\boldsymbol{\vartheta})\bigg|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^i}=\frac{2}{\nu^i}\mathrm{Re}[(\boldsymbol{x}(t)-\boldsymbol{H}(\boldsymbol{\Theta}^i(t))\boldsymbol{s}^i)^H(d'(\boldsymbol{\Theta}^i_{1m}(t))s^i_{1m})] \quad (20)$$

Here the indices $0m$ and $1m$ refer to $\boldsymbol{r}_o$ and $\boldsymbol{r}_1$ in equations (19)-(20). Furthermore, the derivatives of the steering matrix with respect to DOA parameter and the range parameter are given as follows, respectively

$$\boldsymbol{d}'(\boldsymbol{\Theta}^i_m(t))_\theta = \partial\boldsymbol{d}(\boldsymbol{\Theta}^i_m(t))/\partial\theta_m\Big|_{\theta_m=\theta^i_{0m}+t\theta^i_{1m}} = \frac{\partial e^{j(-k\frac{2\pi\Delta}{\lambda}\sin\theta_m+k^2\frac{\pi\Delta^2}{\lambda r_m}\cos^2\theta_m)}}{\partial\theta_m}\Bigg|_{\theta_m}$$
$$= j(-k\frac{2\pi\Delta}{\lambda}\cos\theta_m - 2k^2\frac{\pi\Delta^2}{\lambda r_m}\cos\theta_m\cdot\sin\theta_m)e^{j(-k\frac{2\pi\Delta}{\lambda}\sin\theta_m+k^2\frac{\pi\Delta^2}{\lambda r_m}\cos^2\theta_m)} \quad (21)$$

$$\boldsymbol{d}'(\boldsymbol{\Theta}^i_m(t))_r = \partial\boldsymbol{d}(\boldsymbol{\Theta}^i_m(t))/\partial r_m\Big|_{r_m=r^i_{0m}+tr^i_{1m}} = \frac{\partial e^{j(-k\frac{2\pi\Delta}{\lambda}\sin\theta_m+k^2\frac{\pi\Delta^2}{\lambda r_m}\cos^2\theta_m)}}{\partial r_m}\Bigg|_{r_m} \quad (22)$$
$$= j(-k\frac{2\pi\Delta}{\lambda}\cos\theta_m - 2k^2\frac{\pi\Delta^2}{\lambda r_m}\cos\theta_m\cdot\sin\theta_m)e^{j(-k\frac{2\pi\Delta}{\lambda}\sin\theta_m+k^2\frac{\pi\Delta^2}{\lambda r_m}\cos^2\theta_m)}$$

To prevent the singularity and simplify the iterations instead of the whole block diagonal matrix, $\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^i)$ with only diagonal components of $\boldsymbol{\ell}_{EM}(\boldsymbol{\vartheta}^i)$ used in equation (14). The components of diagonal matrix $\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^i)$ corresponding to mth source are derived with respect to the DOA and range parameters and related equation is given in (23):

$$diag\left[\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^i)\right]\Big|_{\boldsymbol{\theta}_0,\boldsymbol{\theta}_1,\boldsymbol{r}_0,\boldsymbol{r}_1} = \frac{2}{\nu^i}\mathrm{Re}\Big[(-d''(\boldsymbol{\Theta}^i_m(t))s^i_m)^H(\boldsymbol{x}(t)-\boldsymbol{H}(\boldsymbol{\Theta}^i(t))\boldsymbol{s}^i)^H + M\left\|d'(\boldsymbol{\Theta}^i_m(t)s^i_m)\right\|^2\Big] \quad (23)$$

where the second derivatives of the DOA parameters and the range parameters are given as follows, respectively

$$\boldsymbol{d}''(\boldsymbol{\Theta}^i_m(t)) = \partial^2\boldsymbol{d}(\boldsymbol{\Theta}^i_m(t))/\partial\theta^2_m\Big|_{\theta_m} = \frac{\partial^2 e^{j(-k\frac{2\pi\Delta}{\lambda}\sin\theta_m+k^2\frac{\pi\Delta^2}{\lambda r_m}\cos^2\theta_m)}}{\partial\theta^2_m}\Bigg|_{\theta_m}$$
$$= j(k\frac{2\pi\Delta}{\lambda}\sin\theta_m - 2k^2\frac{\pi\Delta^2}{\lambda r_m}(\cos^2\theta_m-\sin^2\theta_m))e^{j(-k\frac{2\pi\Delta}{\lambda}\sin\theta_m+k^2\frac{\pi\Delta^2}{\lambda r_m}\cos^2\theta_m)} \quad (24)$$

$$\boldsymbol{d}''(\boldsymbol{\Theta}_m^i(t)) = \partial^2 \boldsymbol{d}(\boldsymbol{\Theta}_m^i(t)) / \partial r_m^2 \Big|_{r_m} = \frac{\partial^2 e^{j(-k\frac{2\pi\Delta}{\lambda}\sin\theta_m + k^2\frac{\pi\Delta^2}{\lambda r_m}\cos^2\theta_m)}}{\partial r_m^2}\Bigg|_{r_m}$$

$$= (j(2k^2 \frac{\pi\Delta^2}{\lambda r_m^3}\cos^2\theta_m) -$$

$$(k^2 \frac{\pi\Delta^2}{\lambda r_m^2}\cos^2\theta_m)^2) e^{j(-k\frac{2\pi\Delta}{\lambda}\sin\theta_m + k^2\frac{\pi\Delta^2}{\lambda r_m}\cos^2\theta_m)} \quad (25)$$

When the $\boldsymbol{\Theta}^{i+1}$ parameter is estimated, the signal and noise parameters are calculated by means of ML estimation with respect to $\boldsymbol{\Theta}^{i+1}$ and $\boldsymbol{x}(t)$ given as follows;

$$\boldsymbol{s}^{i+1} = \boldsymbol{H}(\boldsymbol{\theta}^{i+1})^{\#}\boldsymbol{x}(t) \quad (26)$$

$$v^{i+1} = \frac{1}{N} tr\left[\boldsymbol{P}(\boldsymbol{\theta}^{i+1})^{\perp}\widehat{\boldsymbol{C}}_x(t)\right] \quad (27)$$

where $\boldsymbol{H}(\boldsymbol{\theta}^{i+1})^{\#}$ is the pseudo inverse of $\boldsymbol{H}(\boldsymbol{\theta}^{i+1})$, and $\boldsymbol{P}(\boldsymbol{\theta}^{i+1})^{\perp} = \boldsymbol{I} - \boldsymbol{P}(\boldsymbol{\theta}^{i+1})$ denotes the orthogonal complement of the following projection matrix: $\boldsymbol{P}(\boldsymbol{\theta}^{i+1}) = \boldsymbol{H}(\boldsymbol{\theta}^{i+1})\boldsymbol{H}(\boldsymbol{\theta}^{i+1})^{\#}$ and $\widehat{\boldsymbol{C}}_x(t) = \boldsymbol{x}(t)\boldsymbol{x}(t)^H$. The update equation then has the following form,

$$\boldsymbol{\Theta}^{i+1} = \boldsymbol{\Theta}^i + \varepsilon_t \ell_{EM}(\boldsymbol{\Theta}^i)^{-1}\gamma(\boldsymbol{x}(t), \boldsymbol{\Theta}^i) \quad (28)$$

The steps of the proposed algorithm are summarized as follows;

Step1: Choose the initial values of DOA, $\boldsymbol{\theta}^0$ and range $\boldsymbol{r}^0$ parameters.

Step2: Calculate the gradient vector for DOAs and range parameters by the equations (12) and (13).

Step3: Calculate the $\tilde{\ell}_{EM}(\boldsymbol{\vartheta}^i)$ augmented data information matrix for DOAs and range parameters by the equation (23).

Step4: Update the parameters to be estimated by using the equation (28).

Step5: Update the signal and noise parameters $\boldsymbol{s}^i$, v^i by the equations (26)-(27) using $\boldsymbol{\Theta}^i(t)$.

The REM algorithm I is more convenient for slowly moving sources; however it suffers from the value of the step size. The step size must be chosen properly in order to get more accurate parameter estimates. Moreover, the signal and noise variance are also estimated together with azimuth and range parameters due to structure of the first algorithm.

4.2. REM Algorithm II

The second REM approach is related with the sufficient statistics of the complete data. The main idea in this recursive parameter estimation approach is to use the complete data set generated empirically. For the second REM algorithm, M independent sources $m = 1,\ldots,M$ with powers $\boldsymbol{\alpha} = [\alpha_1,\ldots,\alpha_M]$, time varying directions $\boldsymbol{\theta} = [\theta_1(1),\ldots,\theta_M(t)]^T$ and distances from $\boldsymbol{r}(t) = [r_1(t),\ldots,r_M(t)]^T$ arrive at an array of N sensors. The movement of the sources is described by the linear polynomial model and given by equations (12) and (13) as before. The parameter vector is given by $\boldsymbol{\vartheta} = [\boldsymbol{\theta}(t), \boldsymbol{r}(t), \boldsymbol{\alpha}(t), v(t)]$. We assume the likelihood function of the observations is complex multivariate Gaussian distribution and given as follows $\boldsymbol{g}(\boldsymbol{x};\boldsymbol{\vartheta}) = N(0, \boldsymbol{\Gamma}(\boldsymbol{\vartheta}))$ and covariance matrix is given as

$$\boldsymbol{\Gamma}(\boldsymbol{\vartheta}) = \boldsymbol{H}(\boldsymbol{\Theta}(t))\boldsymbol{P}(\boldsymbol{\alpha})\boldsymbol{H}^H(\boldsymbol{\Theta}(t)) + v\boldsymbol{I}_N \quad (29)$$

where $\boldsymbol{H}(\boldsymbol{\Theta}(t))$ is the steering matrix consisting of M steering vectors and given by (2) as before. Steering matrix is a function of unknown parameter vector and steering vector written for mth source and an array of N sensors, $k = 1,\ldots,N$ is given by (5). Similarly, the steering vector parameters are denoted by (6). The power matrix of the source $\boldsymbol{P}(\boldsymbol{\alpha}) = diag[\alpha_1,\ldots,\alpha_M]$ and $\boldsymbol{I}_N$ is the N dimensional identity matrix. By means of the complete data concept the array response can be written as the summation of M independent complex Gaussian source signal with covariance,

$$\boldsymbol{\Gamma}_m(\boldsymbol{\vartheta}) = \alpha_m \boldsymbol{d}(\boldsymbol{\Theta}_m(t))\boldsymbol{d}^H(\boldsymbol{\Theta}_m(t)) + v_m \boldsymbol{I}_N \quad (30)$$

where $\sum_{m=1}^{M} v_m(t) = v$ assuming $v_m(t) = v/M$. The complete data is defined by the (8), (9) and (10). The joint pdf (probability density function) of the likelihood function is given as follows

$$\log(f(\boldsymbol{y};\boldsymbol{\vartheta})) = -N\log\pi - \psi(\boldsymbol{\vartheta}) + \sum_{m=1}^{M} trace[\boldsymbol{S}(\boldsymbol{y}_{t,m})\phi_m(\boldsymbol{\vartheta})] \quad (31)$$

where

$$\phi(\boldsymbol{\vartheta}) = N\sum_{m=1}^{M}\log(\nu / M) + \sum_{m=1}^{M}\log(1 + CM\nu^{-1}\alpha_m) \quad (32)$$

$$S(\boldsymbol{y}) = \boldsymbol{y}\boldsymbol{y}^{\boldsymbol{H}} \quad (33)$$

$$\phi_m(\boldsymbol{\vartheta}) = -M\nu^{-1}\boldsymbol{I}_N + \frac{M^2\nu^{-2}\alpha_m}{1 + CM\nu^{-1}\alpha_m}\boldsymbol{d}(\boldsymbol{\Theta}_m(t))\boldsymbol{d}(\boldsymbol{\Theta}_m(t)) \quad (34)$$

and

$$C = \boldsymbol{d}(\boldsymbol{\Theta}(t))^H \boldsymbol{d}(\boldsymbol{\Theta}(t)) \quad (35)$$

$S(y_{t,1}),\ldots,S(y_{t,M})$ are the empirical covariance matrices. The E-step of the algorithm can be obtained by using the following definition of $\overline{s}_m(X_t;\boldsymbol{\vartheta}) = E[Y_{t,m}Y_{t,m}^H \,|\, X_t;\boldsymbol{\vartheta}]$.

Thus the conditional expectation is calculated as,

$$\overline{s}_m(X_t;\boldsymbol{\vartheta}) = E[Y_{t,m}Y_{t,m}^H \,|\, X_t;\boldsymbol{\vartheta}] \quad (36)$$

The following equation is written by using equation (36) as following,

$$\overline{s}_m(\boldsymbol{X};\boldsymbol{\vartheta}) = \boldsymbol{\Gamma}_m(\boldsymbol{\vartheta}) - \boldsymbol{\Gamma}_m(\boldsymbol{\vartheta})\boldsymbol{\Gamma}^{-1}(\boldsymbol{\vartheta})\boldsymbol{\Gamma}_m^H(\boldsymbol{\vartheta}) + \boldsymbol{\Gamma}_m(\boldsymbol{\vartheta})\boldsymbol{\Gamma}^{-1}(\boldsymbol{\vartheta})(\boldsymbol{Y}\boldsymbol{Y}^{\boldsymbol{H}})\boldsymbol{\Gamma}^{-1}(\boldsymbol{\vartheta})\boldsymbol{\Gamma}_m^H(\boldsymbol{\vartheta}) \quad (37)$$

$\boldsymbol{\Gamma}(\boldsymbol{\vartheta})$ and $\boldsymbol{\Gamma}_m(\boldsymbol{\vartheta})$ are given as in (29) and (30). In the M-step of the algorithm

$$\widehat{s}_t = \widehat{s}_{t-1} + \gamma_t(\overline{s}_m(\boldsymbol{Y}_t;\widehat{\boldsymbol{\vartheta}}_{t-1}) - \widehat{s}_{t-1}), \quad \widehat{\boldsymbol{\vartheta}}_t = \overline{\boldsymbol{\theta}}(\widehat{s}_t) \quad (38)$$

is maximized. Here, γ_t illustrates the step size [12]. In order to maximizing conditional likelihood function of the complete data, the following functions are maximized with respect to the parameters to be estimated, $\theta_m(t)$ and $r_m(t)$

$$m_m = max\, \boldsymbol{d}^{\boldsymbol{H}}(\boldsymbol{\Theta}_m(t))s_m\boldsymbol{d}(\boldsymbol{\Theta}_m(t)), \quad (39)$$

$$\overline{\theta}_m(s_m) = \underset{\boldsymbol{\theta}}{arg\, max}\, \boldsymbol{d}^{\boldsymbol{H}}(\boldsymbol{\Theta}_m(t))s_m\boldsymbol{d}(\boldsymbol{\Theta}_m(t)), \quad (40)$$

$$\overline{r}_m(s_m) = \underset{\boldsymbol{r}}{arg\, max}\, \boldsymbol{d}^{\boldsymbol{H}}(\boldsymbol{\Theta}_m(t))s_m\boldsymbol{d}(\boldsymbol{\Theta}_m(t)) \quad (41)$$

The maximization algorithm is based on Golden Section search and parabolic interpolation [13].Then the noise variance and the powers are computed

$$\overline{\nu}(s_1,\ldots,s_M) = \frac{1}{(N-1)}\sum_{m=1}^{M}(trace(s_m) - m_m / C) \quad (42)$$

and

$$\overline{\alpha}(s_1,\ldots,s_M) = \frac{m_m - C\overline{\nu}(s_1,\ldots,s_M)/M}{C^2} \quad (43)$$

Finally, by virtue of the sufficient statistics $\widehat{s}_t = \widehat{s}_{t-1} + \gamma_t(\overline{s}_m(\boldsymbol{Y}_t;\widehat{\boldsymbol{\vartheta}}_{t-1}) - \widehat{s}_{t-1}),\ \widehat{\boldsymbol{\vartheta}}_t = \overline{\boldsymbol{\theta}}(\widehat{s}_t)$ is evaluated by using (37) and, $\widehat{\boldsymbol{\vartheta}}_t$ is computed by (39)-(43) [12].

The steps of the second proposed algorithm can be summarized as follows;

Step1: Choose the initial values of DOA, $\boldsymbol{\theta}^0$ and range $\boldsymbol{r}^0$ parameters.

Step2: Calculate the covariance matrices by the equations (29) and (30).

Step3: Calculate the conditional expectation by the equation (37).

Step4: Maximize the equations (39)-(41).

Step5: Calculate the noise variance and power values by using the equation (42) and (43).

Step6: Put the calculated parameter values at the necessary equations and update the $\boldsymbol{\vartheta}$.

This REM approach requires the sufficient statistics of the complete data and is a stochastic parameter estimation procedure. The power and noise variance estimations are evaluated at each iteration step to obtain the covariance matrices of the sources. Thus, the covariance matrix provides useful information about the source signals. Moreover, in the M-step of the algorithm the optimization equations (39)-(41) is maximized and the parameters to be estimated are calculated.

5. SIMULATONS AND RESULTS

The near-field scenario taken into consideration consists of 5 sensors (identical antennas), and 2 sources which are emitting different signals in the simulation. The moving sources emit signals at different locations, i.e., have different directions of arrival and range parameter values. In this scenario, the targets (sources) are followed by 1000 time steps and the experiments are repeated 100 times. Both algorithms are maintained for the same initial parameter values and the source signals are generated for each step, randomly. The azimuth angle is defined in degree and, the range is defined in Δ/λ unit. The rates of angle increment values are $\boldsymbol{\theta}_{1m} = [0.003°, \ -0.01°]$ and the rates of distance increments are $\boldsymbol{r}_{1m} = [0.005, \ -0.007]$ given for each time step for the both moving objects. Initially, the sources are located at, $\boldsymbol{r}_{0m} = [2, \ 4]$ and the azimuths (angle parameters) are given as $\boldsymbol{\theta}_{0m} = [-10°, \ 45°]$. The range vector denotes the distances from the origin point of the array (the center of the antenna array) to the both sources.

The special issues about both approaches can be summarized as follows,

REM Algoritms I: The augmented data matrix and the gradient vector are calculated at each step of the algorithms. For the step size an appropriate value is chosen to provide a stable operation.

REM Algoritms II: The covariance matrix is calculated to provide some useful information about the source signal. The azimuth, range, noise variance and source power values are computed for each iteration step and parameter vector is updated. Here, the statistical properties related with the source signals are utilized for estimating the angle and range parameter.

The proposed algorithms are tested for a range of SNR values which changes from 0 to 50 dB. The results obtained from the simulations are presented in related figures. In all cases, the following MSEs are used for the $\boldsymbol{\theta}$ and $\boldsymbol{r}$,

$$MSE_{\boldsymbol{\theta}_m} = \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{\theta}_m - \hat{\boldsymbol{\theta}}_{m_n})^2, \ m = 1,2,\ldots,M \qquad (44)$$

$$MSE_{r_m} = \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{r}_m - \hat{\boldsymbol{r}}_{m_n})^2, \ m = 1,2,\ldots,M \qquad (45)$$

The algorithm II yields the better performance when examining the MSE characteristics for the azimuth angle and range parameters, on the other hand the first one needs a little bit less computation time comparing the both algorithm at the same time and the same parameter values. In addition, the fact can be inferred from the some fulfilled computer simulations that the increase of the number of sensors do not provide any performance gain, even may cause some performance degradation. Besides, MSE values can not be reduced rather by increasing the SNR value after about 20 dB. Considering the first approach, true movement trajectories and the estimated trajectories of two non-linear moving sources are shown in figure 1. The tracking of both DOA and range parameter values of the sources are illustrated in this figure. The objects have the both azimuth angle values and defined distance increments so their movements are non-linear, as can be seen in the following figure 1 and figure 4 for the both approach.

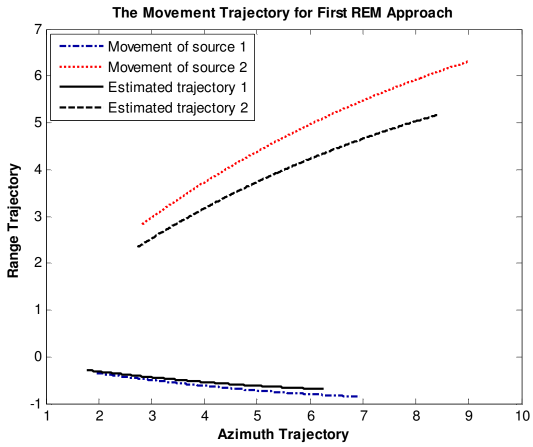

Figure 1. For the REM algorithm I, the true movement trajectory and the estimated trajectory of the non-linear moving sources. The unit of the x axis and y axis are degree and Δ/λ, respectively

The tracking trajectory is calculated by virtue of the real azimuth angles and range parameter values of the both sources and the estimated values for the time instants. The mean square error (MSE) of the estimated direction of the arrival values and range parameter values of the non-linear moving sources for SNR value changing from 0 to 40 dB in the near field are given in figure 2 and figure 3, respectively. True movement trajectories and the estimated trajectories of the sources are shown in the figure 4 for the second approach. And also MSE of the estimated direction of arrival values and range parameter values SNR value are given in the figure 5 and figure 6, successively.

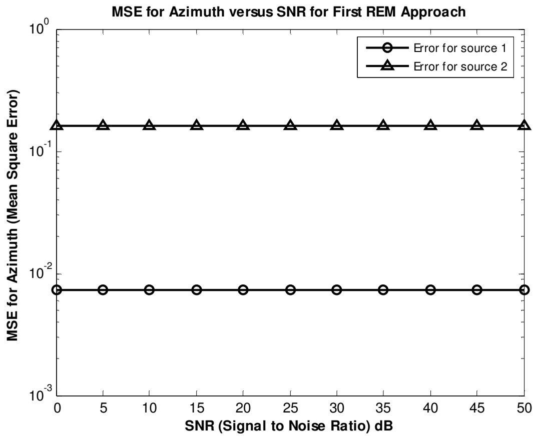

Figure 2. For the REM algorithm I the MSE for the DOA parameters of the non-linear moving sources (the logarithmic axis)

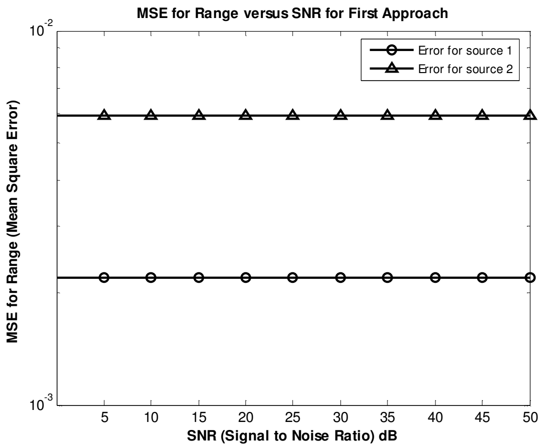

Figure 3. For the REM algorithm I the MSE for the range parameters of the non-linear moving sources (the logarithmic axis)

6. CONCLUSION

In this study, two REM algorithms are proposed to estimate the directions of arrival and the range parameters of the near-field sources at the same time. Simulation results showed that the second proposed method presents excellent results in slowly time varying parameters. On the other hand, simulation results illustrated that if the amount of change in the localization parameters to be estimated (DOA and range) are increased or two source directions across with each other, the both algorithms have performance degradation.

Moreover, we expose from the computer simulations that the calculation load changes with the step size, and the step size is chosen as a constant for the first approach. The estimated trajectories follow the true movement trajectories closely. It can be inferred that the MSEs for the DOAs and also the range parameters of the sources do not decrease too much by the increasing SNR values. Besides, the MSE values of the range parameters of both sources is very close to each other with changing SNR values for the first algorithm. The number of iteration steps affects a little bit the computation time for both algorithms.

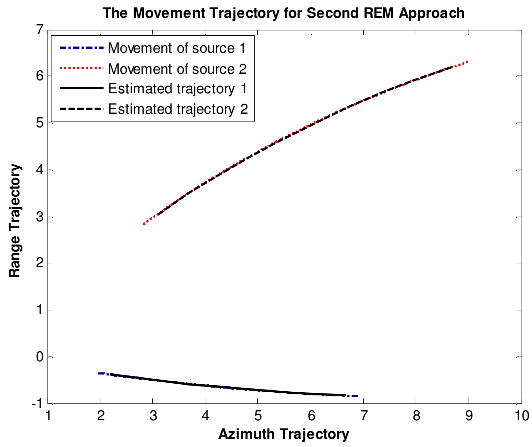

Figure 4. For the REM algorithm II the true movement trajectory and the estimated trajectory of the non-linear moving sources. The unit of the x-axis and y-axis are degree and Δ/λ, respectively

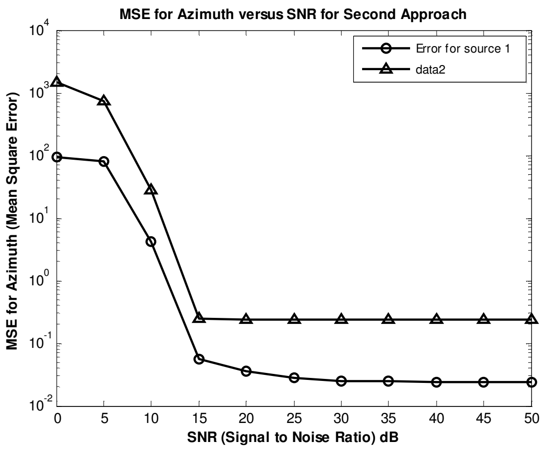

Figure 5. For the REM algorithm II the MSE for the DOA parameters of the non-linear moving sources (the logarithmic axis).

For time varying parameters, the tracking ability of a stochastic approximation procedure depends mainly on the dynamics of the true parameters, the gain matrix, and the step size [14]. Therefore, choosing suitable initial values plays an important role for the performances of the both algorithms.

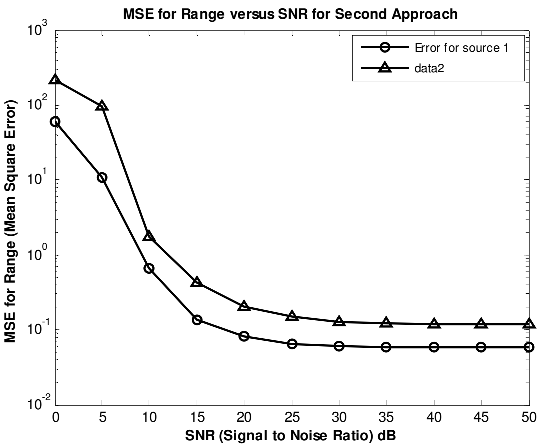

Figure 6. For the 2nd REM algorithm the MSE for the range parameters of the non-linear moving sources (the logarithmic axis).

7. REFERENCES

[1] H. Jiang, S.X. Wang, H.J. Lu, "A Novel Higher Order Cyclic Cumulant Based Direction Finding Algorithm In Multipath" *Signal Processing, Procs. ICSP '04. 7th Int. Conf.*, Vol: 1, pp. 411-414, 31 Aug.-4 Sept. 2004.

[2] M.J.D. Rendas, J.M.F. Moura, "Path Resolution By Coherent Averaging: Trading Spatial And Temporal Degrees Of Freedom", *ICASSP-88, Int. Conf. Acou., Speech, Sig.Proc.*, Vol: 5, pp. 2626-2629, 11-14 April 1988.

[3] N. Kabaoğlu, H.A. Çırpan, S. Paker, "Unconditional Maximum Likelihood Approach For Localization Of Near-Field Sources In 3D Space", *Proce. of the Fourth IEEE International Symp. on Sig. Processing and Inf. Tech.*, pp. 233-237, 18-21 Dec., 2004.

[4] A.P. Dempster, N. Laird, D.B. Rubin, "Maximum Likelihood From Incomplete Data Via The EM Algorithm", *J. Roy. Statist. Soc. Ser. B*, Vol: 39, No. 1, pp. 1-38, 1977.

[5] D.M. Titterington, "Recursive Parameter Estimation Using Incomplete Data" *J. Roy. Stat. Soc. Ser. B*, Vol: 46, No. 2, pp. 257-267, 1984.

[6] M.I. Miller, D.R. Fuhrmann, "Maximum-Likelihood Narrow-Band Direction Finding And The EM Algorithm", *IEEE Trans. Acou. Speech, Sig. Proc.*, Vol: 38, pp. 1560-1577, Sept. 1990.

[7] E. Çekli, H.A. Çırpan, "Unconditional Maximum Likelihood Approach for Localization of Near-Field Sources: Algorithm and Performance Analysis", *AEÜ Int. Journal Elect. and Comm.*, Vol. 57, No. 1, pp. 9-15, 2003.

[8] N. Kabaoglu, H.A. Cirpan, S. Paker, "EM Based Stochastic Maximum Likelihood Approach for Localization of Near-Field Sources in 3-D," *Frequenz, Journal Telecommunications*, Vol. 58, pp. 178-184, 7-8/2004.

[9] J.A. Olague, D.C. Rosales, J.L. Rivera, "Efficiency Evaluation Of The Unconditional Maximum Likelihood Estimator For Near-Field DOA Estimation" *ETRI Journal*, Vol: 28, No. 6, pp. 761-769, Dec. 2006.

[10] L. Frenkel, M. Feder, "Recursive Expectation-Maximization (EM) Algorithms For Time Varying Parameters With Applications To Multiple Target Tracking", *IEEE Trans. Signal Proc.*, Vol: 47, No. 2, pp. 306-320, Feb. 1999.

[11] P. J. Chung, J. F. Böhme, A. O. Hero, "Tracking Of Multiple Moving Sources Using Recursive EM Algorithm", *EURASIP Journal on Applied Signal Proces.*, No.1, 50-60., 2005

[12] O. Cappe, M. Charbit, E. Moulines, "Recursive EM Algorithm With Applications To DOA Estimation", *ICASSP Proc. IEEE Int. Conf. Acoustics, Speech Signal Proc.*, 2006.

[13] G.E. Forsythe, M.A. Malcolm, C.B. Moler, *Computer Methods For Mathematical Computations*, Prentice Hall, 1976.

[14] A. Benveniste, M. M´etivier, P. Priouret, "Adaptive Algorithms and Stochastic Approximations", *Source Springer Applications of Mathematics Series, Springer-Verlag*, New York, NY, USA, 1990.

[15] P. Stoica, B. Ottersten, M. Viberg, and R. L., Moses, “Maximum Likelihood Array processing for stochastic Coherent Sources”, *IEEE Trans. Sig. Proc.*, Vol: 44, No. 1, Jan., 1996

[16] M. Harrdt, “Efficient One-, Two, and Multidimensional High-Resolution Array Signal Processing”, *Ph.D. dissertation, Technische Universitat München*, 1997.

[17] H. A. Çırpan, E. Çekli, “Deterministic Maximum Likelihood Approach for Localization of Near-Field Sources”, *AEÜ Int. Journal Electronics and Com.*, Vol: 56, No. 1, pp. 1-10, 2002.

[18] A.L. Swindlehurst and T. Kailath, “Azimuth / Elevation Direction Finding Using Regular Array Geometries”, *IEEE Trans. Acou. Speech, and Sig. Proc.*, Vol: 29, pp. 145-156, Jan 1993.

Serap ÇEKLİ was born in Germany in 1978. She received his B.Sc. degree in Electronics Engineering from Istanbul University in 2000 and M.Sc. degree in Electronics and Communication Engineering from Istanbul Technical University in 2003. She is a Ph.D. student at Istanbul University at Electrical Electronics Engineering Department. Since 2001, she works at Istanbul University Engineering Faculty as a research assistant. Her research interests are estimation theory, sensor networks, ML.