# Log- Mean Divisia Index Method

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## Abstract

A new method of energy decomposition called Log- Divisia Index Method I (LMDI I) is presented. It has the desirable characteristics of perfect decomposition and aggregation consistency. Perfect decomposition guarantees that the results of the decomposition do not include a residual period. Consistency in aggregation allows sub-group estimates to be aggregated in a consistent manner [1]. To analyze and understand historical changes in economic, environmental, employment or other socio-economic indicators, it is useful to assess the driving forces or determinants that underlie these changes. Index decomposition analysis has been used to analyze changes in indicators such as energy use, CO<sub>2</sub>-emissions, labor demand and value added. The changes in these variables are decomposed into determinants such as technological, demand, and structural effects. LMDI uses aggregate data at the sector-level. The IDA method has developed quite independently, which has resulted in method being characterized by specific, unique techniques and approaches [2].

Keywords: LMDI Method, Decomposition analysis, Energy, Emissions.

## 1. Introduction

The structural decomposition analysis (SDA) and index decomposition analysis (IDA) are two main analysis methods that determine the factors related to energy consumption and carbon dioxide emissions. SDA is based on the input-output model. The Laspeyres exponential decomposition method and the logarithmic mean division index (LMDI) decomposition method are the two most commonly used IDA methods. The traditional Laspeyres method has the problem that high residuals cannot be explained in the decomposition of the carbon emissions history, especially in long-term multivariate analyses [3]. On the basis of previous research, Ang[4] provided practical guidance for the LMDI decomposition method. However, LMDI decomposition still has the problem of how to deal with negative values in the data set. Ang [5] provided a strategy and criteria to deal with negative values that eliminates the deficiency of the only LMDI decomposition method in practical applications. The improved LMDI method has been widely used in existing decomposition systems because of its practicability and accuracy.

## 2. Literature review

In recent years, domestic and foreign scholars have used the LMDI decomposition model in many empirical studies of the influencing factors of energy consumption and carbon emissions. LMDI I is a more recommendable method due to both its theoretical base and its set of properties, which are satisfactory in the case of index decomposition. LMDI I is a "refined," non-parametric approach based on the IDA method, with a weighted logarithmic mean. An additional argument in favor of LMDI I is that it allows perfect decomposition (that is, without residuals) and provides a simple and direct association between the additive and

the multiplicative decomposition form [5]. Many researches has a common idea that decomposition analysis, in addition to being a powerful explanatory tool, offers valuable assistance in assessing drivers of CO2 emissions changes.

Sheinbaum et al.[6] conducted an LMDI decomposition analysis of energy use and carbon dioxide emission changes in the Mexican steel industry from 1970 to 2006 and found that industrial activities contributed to a significant increase in primary energy consumption; energy structure and energy efficiency played important roles in reducing energy consumption and carbon dioxide emissions

Olanrewaju [7], has showed that the activity effect dominated the energy dynamics in South Africa, resulting in intensity and structural effect which exert pressure on the country's energy consumption. It should be noted that as much as it seemed like policies were not efficiently implemented, activity effect played a huge role in the increase of the country's industrial energy consumption. For the country's continuous growth, energy will continue to play a huge role; however, there is need for an energy-saving and environment-conscious mode to solve the energy crisis. This study clearly showed that activity is very informative in the South African industry with respect to the amount of energy consumed. Energy-conservation policy can be improved by considering the information provided by the three factors represented in this study. This study is for the South African policy makers to reconsider industrial energy policy according to this finding. The industry's energy challenges are rooted in two facts: it consumes a lot of energy and it continues to consume more. Unless there are new energy conservation policies or behavior changes, the high industrial energy consumption rate will continue. Finally, he concluded that the application of LMDI made it possible to disentangle and identify the various factors to explain the total change in energy consumed in the South African industrial sectors for the period 1970-1971 to 2015-2016. This will help to point energy policy in the right direction. The significance of this study is to provide policy makers with information to enable the revision and formulation of the country's industrial energy policy.

Román et al.[8] has used an Index Decomposition Analysis-Logarithmic Mean Divisia Index (IDA-LMDI) model was developed to find the drivers behind the changes in CO2 emissions between 1990 and 2012 in Colombia. The results facilitate the assessment of the impact in Colombia of the main measures regarding the mitigation of CO2 emissions. To carry out the decomposition analysis, six effects were taken into consideration: carbonization, the substitution of fossil fuels, the penetration of renewable energy, energy intensity, wealth and population. The effects of income and population appear as drivers of emissions for the period analyzed. A stylized analysis allows richer conclusions to be extracted regarding a battery of recommendations for emission mitigation policies that are compatible with economic growth in Colombia. The results obtained from LMDI analysis provide useful policy guidance for the Colombian authorities. The change in the value of Cemc suggests that an environmental law that prohibits the use of heavy crude oil with elevated sulfur contents has been successful, to a certain degree, in reducing CO2 emissions.

Zhang et al. [9] used the LMDI decomposition method to decompose related energy and  $CO_2$  emissions in China for the period 1991– 2006 divided into three equal time intervals. The complete decomposition method developed by Sun is used to analyze the nature of the four factors:  $CO_2$  intensity, energy intensity, structural changes and economic activity. The results show that economic activity has the largest positive effect in  $CO_2$  emission changes in all the major economic sectors and China has achieved a considerable decrease in  $CO_2$  emission mainly due to the improved energy intensity. However, the impact of  $CO_2$  intensity and structural changes is relatively small. Structural changes only exhibit positive effect to the  $CO_2$  mitigation in agricultural sector, and  $CO_2$  intensity also contributes to the decrease of

 $CO_2$  emission in transportation sector. Moreover, a formula about CO2 mitigation is presented in this paper, which shows that China has made a significant contribution to reducing global  $CO_2$  emission.

Ma et al.[10] put forward an LMDI decomposition method based on the Sankey diagram of energy and carbon dioxide distribution and analyzed the influencing factors of China's energy  $CO_2$  emissions on a national level. It was found that the growth of per capita GDP was the main factor that promoted the growth of  $CO_2$  emissions while the reduction of energy intensity, the improvement of energy supply efficiency, and the introduction of non-fossil fuels in heat and electricity generation slowed the growth of  $CO_2$  emissions.

Xu et al.[11] obtained the same conclusion by analyzing the decomposition of the factors that affect energy consumption at different stages and industries in China.

Some scholars also used the LMDI decomposition method to decompose energy-related carbon emissions in different regions of China, most through dividing the influencing factors into economic activities, energy intensity, energy efficiency, industrial structure and so on [12–15].

In manufacturing carbon-related research, Akbostancı [16] used the LMDI decomposition method to decompose the changes in CO<sub>2</sub> emissions in the Turkish manufacturing industry and found that the changes in total industrial activity and energy intensity were the main factors for CO<sub>2</sub> changes during the study period. Kim [17] and Jeong [18] used the LMDI decomposition method to decompose the influential factors of energy consumption and greenhouse gas emissions in the Korean manufacturing industry. It was found that structural effects and intensity effects play major roles in reducing energy consumption and greenhouse gas emissions and that the structural effect is greater than the intensity effect. Hammond et al. [19] divided the UK manufacturing industry into the energy-intensive (EI) subsector and the nonenergy-intensive (NEI) subsector and used the LMDI decomposition methods to classify influencing factors into output scale, industrial structure, energy intensity, fuel mix and electricity emission factor; they found that the decline in energy intensity was the main factor in the reduction of carbon emissions. In addition, on the basis of the Disia index method, Ang and Pandiyan [20] used two common methods to decompose the factors that affect CO2 emission changes into energy intensity effects, energy structure effects, CO2 emission factor effects and industrial structure effects. Schipper [21] used Adaptive-Weighting-Divisia decomposition to analyze the CO2 emissions of the manufacturing sector in 13 International Energy Agency countries in 1994 and decomposed the factors that affect CO2 emissions into energy intensity, industrial structure, energy structure and economic output. The results showed that the energy intensity and output scale effect are the main factors that lead to different CO<sub>2</sub> emission changes in manufacturing industries.

#### 3. The LMDI Methodology

In this study, the LMDI method developed by Ang [4] is used to decompose the driving factors of on Turkish main four combustion sectors  $CO_2$  emissions from four fuel type combustion as follows:

$$C = \sum_{ij} C_{ij} = \sum_{ij} Q \frac{Q_i}{Q} \frac{E_i}{Q_i} \frac{E_{ij}}{E_i} \frac{C_{ij}}{E_{ij}} = \sum_{ij} Q S_i I_i M_{ij} U_{ij}$$
(1)

Where C is the total CO<sub>2</sub> emissions and C<sub>ij</sub> is the CO<sub>2</sub> emissions arising from fuel j in the sector i,  $Q(= \sum Q_i)$  is the total economic activity level,  $Q_i$  is activity level of sector i, E<sub>i</sub> (=

 $\sum E_{ij}$ ) is the use of energy of sector i, and the unit of this variable is TJ ,  $E_{ij}$  is the consumption of fuel j in sector i. Where  $S_i$  (=Q\_i/Q) is the share of sector i,  $I_i$  (=E\_i/Q\_i) represents the energy intensity of sector i; the energy-mix variable is given by  $M_{ij}$  (=E\_{ij}/E\_i) and  $U_{ij}$  (=C\_{ij}/E\_{ij}) represent the CO<sub>2</sub> emissions factor of fuel j consumed by sector i.

The general index decomposition analysis (IDA) identity is given by

$$V = \sum_{i} V_{i} = \sum_{i} x_{1,i} x_{2,i} \dots \dots x_{n,i}$$
(2)

In additive decomposition the difference is decomposes as:

$$\Delta \mathbf{V}_{\text{tot}} = \mathbf{V}^{\mathrm{T}} - \mathbf{V}^{0} = \Delta \mathbf{V}_{\mathrm{x1}} + \Delta \mathbf{V}_{\mathrm{x2}} + \dots + \Delta \mathbf{V}_{\mathrm{xn}}$$
(3)

where subscript tot represents the total or overall change and the superscript T refers to period T and 0 refers to period 0.

In the LMDI approach, the general formulae for the effect of the kth factor on the right-hand side of Equations (2) and (3) are respectively:

$$D_{x_{k}} = \exp\left(\sum_{i} \frac{L(V_{i}^{T}, V_{i}^{0})}{L(V^{T}, V^{0})} ln\left(\frac{x_{k,i}^{T}}{x_{k,i}^{0}}\right)\right)$$
  
$$= \exp\left(\sum_{i} \frac{(V_{i}^{T} - V_{i}^{0})/(lnV_{i}^{T} - lnV_{i}^{0})}{(V^{T} - V^{0})/(lnV^{T} - lnV^{0})} x ln\left(\frac{x_{k,i}^{T}}{x_{k,i}^{0}}\right)\right)$$
  
$$AV_{k} = \sum_{i} L(V_{i}^{T}, V_{i}^{0}) ln\left(\frac{x_{k,i}^{T}}{i}\right)$$
(4)

$$\Delta V_{x_{k}} = \sum_{i} L(V_{i}^{T}, V_{i}^{0}) ln\left(\frac{x_{k,i}}{x_{k,i}^{0}}\right)$$
$$= \sum_{i} \frac{V_{i}^{T} - V_{i}^{0}}{ln(V_{i}^{T}, V_{i}^{0})} ln\left(\frac{x_{k,i}^{T}}{x_{k,i}^{0}}\right)$$
(5)

Where  $L(a,b)=(a-b)/(\ln a - \ln b)$ ,

Specifically, the additive decomposition for CO2 emissions takes the following form

$$\Delta C_{\text{tot}} = C^T - C^0 = \Delta C_{\text{act}} + \Delta C_{\text{str}} + \Delta C_{\text{int}} + \Delta C_{\text{mix}} + \Delta C_{\text{emf}}$$
(6)

The subscripts on the right hand side of the above equation, act, str, int, mix and emf denote the effects associated with overall activity, activity structure, sectoral energy intensity, sectoral energy-mix and emission factors, respectively

These component can be expressed as:

$$\Delta C_{act} = \sum_{ij} \frac{C_{ij}^{T} - C_{ij}^{0}}{\ln C_{ij}^{T} - \ln C_{ij}^{0}} \ln \left(\frac{Q^{T}}{Q^{0}}\right)$$
(7)

$$\Delta C_{\rm str} = \sum_{ij} \frac{C_{ij}^{\rm T} - C_{ij}^{\rm 0}}{\ln C_{ij}^{\rm T} - \ln C_{ij}^{\rm 0}} \ln \left( \frac{S_i^{\rm T}}{S_i^{\rm 0}} \right)$$
(8)

$$\Delta C_{\rm int} = \sum_{ij} \frac{C_{ij}^{\rm T} - C_{ij}^{\rm 0}}{\ln C_{ij}^{\rm T} - \ln C_{ij}^{\rm 0}} \ln \left( \frac{I_i^{\rm T}}{I_i^{\rm 0}} \right)$$
(9)

$$\Delta C_{\rm mix} = \sum_{ij} \frac{C_{ij}^{\rm T} - C_{ij}^{\rm 0}}{\ln C_{ij}^{\rm T} - \ln C_{ij}^{\rm 0}} \ln \left( \frac{M_{ij}^{\rm T}}{M_{ij}^{\rm 0}} \right)$$
(10)

$$\Delta C_{\rm emf} = \sum_{ij} \frac{C_{ij}^{\rm T} - C_{ij}^{\rm 0}}{\ln C_{ij}^{\rm T} - \ln C_{ij}^{\rm 0}} \ln \left( \frac{U_{ij}^{\rm T}}{U_{ij}^{\rm 0}} \right)$$
(11)

#### 4. Conclusion

Many studies have utilized the LMDI method to decompose the total CO<sub>2</sub> growth of various sectors and regions. For example; China, Brazil [22], Turkey[16], [23–25], Ireland [26], Spain [27], EU [25,26], USA [30,31], Greece[32], Philippine [33], Tunisian [349, India [35], Nigeria [36], Mexican [37].

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