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Carbon Nanotube Beam Model and Free Vibration Analysis

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Abstract

Thanks to their superior properties, the interest on nanostructures has increased. Among the nanostructures, carbon nanotubes have an important place. In this study, the free vibration of Carbon Nanotubes is investigated. CNT is modeled as a beam. Four different cross-sections are selected such as circular, rectangular, triangular and quadratic for use in the solution. The frequency values of the first five modes of these 4 different cross-sections with the same area have been gained by using Euler-Bernoulli Beam Theory for simply supported boundary condition. The results are compared. In this study, it is aimed to understand how the mode number and cross-section change the frequency values.

Keywords: Carbon nanotube, free vibration, Euler-Bernoulli, frequency.

1. Introduction

As a result of rapid developments in technology, nanotechnology emerged and has become one of the most important issues of today. Nanotechnology is mainly concerned with particles smaller than 100 nm. The nanostructures are defined as structures having at least one dimension between 1 and 100 nm [1-2]. One of the most important subjects of nanotechnology is carbon nanotubes. Carbon nanotube was discovered by Iijima in 1991 [3]. The CNT is the cylindrical shape of the graphene and can be single-layered or multi-layered. There are also 3 types of nanotubes depending on the direction of rolling of the graphene layer. These are armchair, zigzag and chiral. The properties of CNT vary depending on the geometry. The very interesting properties such as mechanical, electrical, optical exhibited by CNT attract a lot of attention. It has high Young's modulus and low mass density [4-8]. Analyzes such as buckling, vibration, bending of structures are important. Analysis results help to understand the behavior of structures. Analyzes of macro and nano-sized structures are studied in various methods [9-14]. In this paper, the vibration of CNT is studied by using Euler-Bernoulli Beam Theory. Frequency values of CNT are carried out for 4 different cross-sections and simply supported boundary condition.



2. Vibration of Euler-Bernoulli Beam

The vibration equation of a beam whose material and cross-sectional properties are unchanged is expressed as [15,16]:

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t) \quad (1)$$

Where E is Young Modulus, I is moment of inertia, ρ is mass density, A is cross-sectional area, $w(x,t)$ is transverse deflection of beam and $f(x,t)$ is external force. When the equation is solved for free vibration and the simple supported boundary condition by using separating of variables equations, we obtain the frequency equation

$$\omega_n = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{EI}{\rho A}} \quad (2)$$

Where n is mode number, L is length of beam and ω shows the frequency.

3. Numerical Examples

In this study, the free vibration of CNT with simple supported boundary condition is investigated by using Euler-Bernoulli Beam Theory. Four different cross-sections with same area are used. For a better comparison, the width/height (b/h) ratio is taken as the same for triangle and rectangle. The Young modulus used in calculations is 1054 GPa and mass density is 1.4 g/cm³. The results are shown in table 1 and figure 1.

Table 1. Frequency values for 20 nm length (10⁹ rad/sn)

$A=12.6 \text{ nm}^2, L=20 \text{ nm}$				
<i>mode numbers</i>	<i>circular</i> $r=2 \text{ nm}$	<i>rectangular</i> $b=3 \text{ nm}$ $h=4.2 \text{ nm}$	<i>triangular</i> $b=4.24 \text{ nm}$ $h=5.93 \text{ nm}$	<i>quadratic</i> $b=3.55 \text{ nm}$ $h=3.55 \text{ nm}$
1	677.0115	820.8328	946.2678	693.8005
2	2708.046	3283.3313	3785.0712	2775.2018
3	6093.1034	7387.4953	8516.4102	6244.2041
4	10832.184	13133.325	15140.285	11100.807
5	16925.287	20520.820	23656.695	17345.012

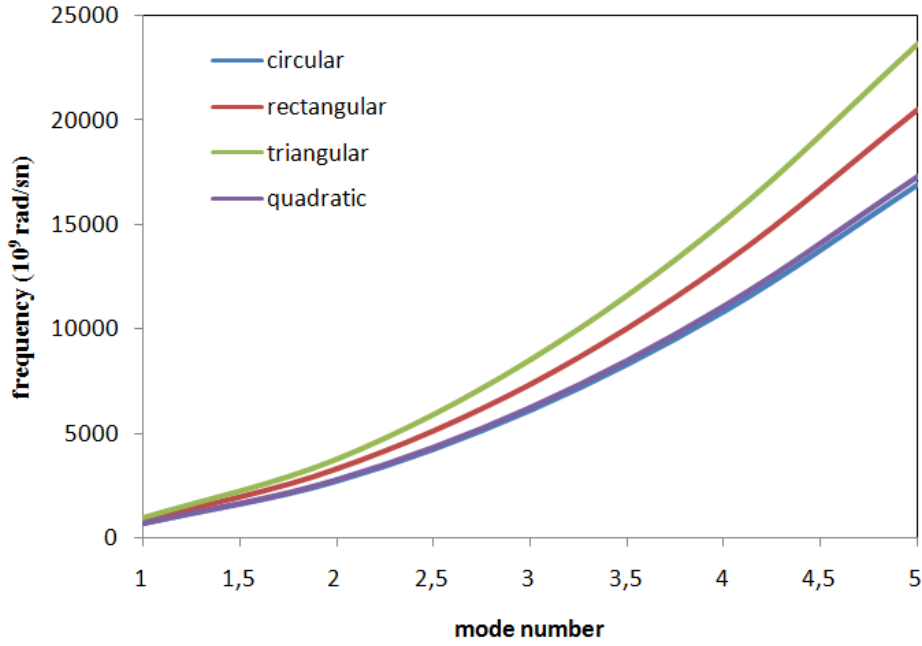


Fig. 1. Frequency values for different cross-sections

4. Conclusions

In this paper, free vibration analysis of CNT is presented. The frequency values of the first five modes of four different cross-sections have been gained by using Euler-Bernoulli Beam Theory for simply supported boundary condition. As can be seen from the results, the frequency values increase with the mode numbers. Using same cross section area, the highest frequency value is seen on the triangular cross-section. Circular cross section has the lowest frequency value.

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Numerical Methods for FGM Composites Shells and Plates

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Abstract

Main formulations for free vibration analysis of functionally graded composite shells have been given in numerical concept. Equations of motions for conical shells are listed in differential form. First-order shear deformation (FSDT) shell theory is used for obtaining the equations. Then two methods have been applied for solution. These methods are differential quadrature (DQ) and discrete singular convolution (DSC). The discrete forms of these equations have been given.

Keywords: Functionally graded composites, frequency, conical shells, annular plates, sector plates, DSC, HDQ.

1. Introduction

Functionally graded materials (FGM) are greatly used in different applications in engineering. Thus, many papers have been published for beams, plate and shell problems in order to obtain reasonable accurate results for design via different numerical methods [1-44]. By using the FSDT, the related governing equation for free vibration of conical shell can be written as

$$L_{11} + L_{12} + L_{13} + L_{14} + L_{15} - \rho h \cdot \omega^2 = 0 \quad (1)$$

$$L_{21} + L_{22} + L_{23} + L_{24} + L_{25} - \rho h \cdot \omega^2 = 0 \quad (2)$$

$$L_{31} \cdot U + L_{32} \cdot V + L_{33} \cdot W + L_{34} \cdot \Phi_x + L_{35} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \quad (3)$$

$$L_{41} \cdot U + L_{42} \cdot V + L_{43} \cdot W + L_{44} \cdot \Phi_x + L_{45} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \quad (4)$$

$$L_{51} \cdot U + L_{52} \cdot V + L_{53} \cdot W + L_{54} \cdot \Phi_x + L_{55} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \quad (5)$$



2. Solution by DSC method

By DSC method, governing differential equation of motion of truncated conical panel, Eqs. (1-5), can be discrete

$${}^{DSC}L_{11} \cdot U + {}^{DSC}L_{12} \cdot V + {}^{DSC}L_{13} \cdot W + {}^{DSC}L_{14} \cdot \Phi_x + {}^{DSC}L_{15} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \quad (6)$$

$${}^{DSC}L_{21} \cdot U + {}^{DSC}L_{22} \cdot V + {}^{DSC}L_{23} \cdot W + {}^{DSC}L_{24} \cdot \Phi_x + {}^{DSC}L_{25} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \quad (7)$$

$${}^{DSC}L_{31} \cdot U + {}^{DSC}L_{32} \cdot V + {}^{DSC}L_{33} \cdot W + {}^{DSC}L_{34} \cdot \Phi_x + {}^{DSC}L_{35} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \quad (8)$$

$${}^{DSC}L_{41} \cdot U + {}^{DSC}L_{42} \cdot V + {}^{DSC}L_{43} \cdot W + {}^{DSC}L_{44} \cdot \Phi_x + {}^{DSC}L_{45} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \quad (9)$$

$${}^{DSC}L_{51} \cdot U + {}^{DSC}L_{52} \cdot V + {}^{DSC}L_{53} \cdot W + {}^{DSC}L_{54} \cdot \Phi_x + {}^{DSC}L_{55} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \quad (10)$$

The coefficients of L_{ij} are:

$${}^{DSC}L_{11} = A_{11} \cdot \Xi_x^{(2)} + \frac{A_{11}}{R(x)} \sin \alpha \cdot \Xi_x^{(1)} - \frac{A_{22}}{R^2(x)} \cdot U(i) \cdot \sin^2 \alpha + \frac{A_{33}}{R^2(x)} \cdot \Xi_s^{(2)} \quad (11)$$

$${}^{DSC}L_{12} = \frac{(A_{12} + A_{33})}{R(x)} \sin \alpha \cdot \Xi_{xs}^{(2)} \frac{\partial^2 V}{\partial x \partial s} - \frac{(A_{22} + A_{33})}{R^2(x)} \sin \alpha \cdot \Xi_s^{(1)} \quad (12)$$

$${}^{DSC}L_{13} = \frac{A_{12}}{R(x)} \cos \alpha \cdot \Xi_x^{(1)} - \frac{A_{22}}{R^2(x)} \cdot W(i) \cdot \sin \alpha \cdot \cos \alpha \quad (13)$$

$${}^{DSC}L_{14} = B_{11} \cdot \Xi_x^{(2)} + \frac{B_{11}}{R(x)} \sin \alpha \cdot \Xi_x^{(1)} - \frac{B_{22}}{R^2(x)} \cdot \Psi_x(i) \sin^2 \alpha + \frac{B_{33}}{R^2(x)} \cdot \Xi_s^{(2)} \quad (14)$$

$${}^{DSC}L_{15} = \frac{(B_{12} + B_{33})}{R(x)} \cdot \Xi_x^{(1)} \cdot \Xi_s^{(1)} - \frac{(B_{22} + B_{33})}{R^2(x)} \cdot \Xi_s^{(1)} \cdot \sin \alpha \quad (15)$$

$${}^{DSC}L_{21} = \frac{(A_{12} + A_{33})}{R(x)} \cdot \Xi_x^{(1)} \cdot \Xi_s^{(1)} + \frac{(A_{22} + A_{33})}{R^2(x)} \sin \alpha \cdot \Xi_s^{(1)} \quad (16)$$

$$\begin{aligned} {}^{DSC}L_{22} &= A_{33} \Xi_x^{(2)} + A_{33} \frac{\sin \alpha}{R(x)} \Xi_s^{(1)} \\ &- \frac{A_{33}}{R^2(x)} \cdot V(i) \cdot \sin^2 \alpha + \frac{A_{22}}{R^2(x)} \Xi_s^{(2)} - \frac{A_{44}}{R^2(x)} \cdot V(i) \cdot \cos^2 \alpha \end{aligned} \quad (17)$$

$${}^{DSC}L_{23} = \frac{(A_{22} + A_{44})}{R^2(x)} \cdot \cos \alpha \cdot \Xi_s^{(1)} \quad (18)$$

$${}^{DSC}L_{24} = \frac{(B_{12} + B_{33})}{R(x)} \cdot \Xi_x^{(1)} \cdot \Xi_s^{(1)} + \frac{(B_{22} + B_{33})}{R^2(x)} \sin \alpha \cdot \Xi_s^{(1)} \quad (19)$$

$$\begin{aligned} {}^{DSC}L_{25} &= B_{33} \cdot \Xi_x^{(2)} + B_{33} \frac{\sin \alpha}{R(x)} \cdot \Xi_x^{(1)} \\ &- \frac{B_{33}}{R^2(x)} \cdot \Psi_s(i) \cdot \sin^2 \alpha + \frac{B_{22}}{R^2(x)} \cdot \Xi_s^{(2)} + A_{44} \cdot \frac{\cos \alpha}{R(x)} \cdot \Psi_s(i) \end{aligned} \quad (20)$$

$${}^{DSC}L_{31} = -\frac{A_{12}}{R(x)} \cos \alpha \cdot \Xi_x^{(1)} - \frac{A_{22}}{R^2(x)} \cdot U(i) \cdot \sin \alpha \cdot \cos \alpha \quad (21)$$

$${}^{DSC}L_{32} = -\frac{(A_{22} + A_{44})}{R^2(x)} \cos \alpha \cdot \Xi_s^{(1)} \quad (22)$$

$${}^{DSC}L_{33} = A_{55} \cdot \Xi_x^{(2)} + \frac{A_{55}}{R(x)} \sin \alpha \cdot \Xi_s^{(1)} + \frac{A_{44}}{R^2(x)} \cdot \Xi_s^{(2)} - \frac{A_{22}}{R^2(x)} \cdot W(i) \cdot \cos^2 \alpha \quad (23)$$

$$\begin{aligned} {}^{DSC}L_{34} &= A_{55} \cdot \Xi_x^{(1)} - \frac{B_{12}}{R(x)} \cos \alpha \cdot \Xi_x^{(1)} \\ &+ \frac{A_{55}}{R(x)} \cdot \Psi_x(i) \cdot \sin \alpha - \frac{B_{22}}{R^2(x)} \cdot \Psi_x(i) \cdot \sin \alpha \cdot \cos \alpha \end{aligned} \quad (24)$$

$${}^{DSC}L_{35} = \frac{A_{44}}{R(x)} \cdot \Xi_s^{(1)} - \frac{B_{22}}{R^2(x)} \cdot \cos \alpha \cdot \Xi_s^{(1)} \quad (25)$$

$${}^{DSC}L_{41} = B_{11} \cdot \Xi_x^{(2)} + \frac{B_{11}}{R(x)} \sin \alpha \cdot \Xi_x^{(1)} - \frac{B_{22}}{R^2(x)} \cdot U(i) \cdot \sin^2 \alpha + \frac{B_{33}}{R^2(x)} \cdot \Xi_s^{(2)} \quad (26)$$

$${}^{DSC}L_{42} = \frac{(B_{12} + B_{33})}{R(x)} \cdot \Xi_x^{(1)} \cdot \Xi_s^{(1)} - \frac{(B_{22} + B_{33})}{R^2(x)} \sin \alpha \cdot \Xi_s^{(1)} \quad (27)$$

$${}^{DSC}L_{43} = -A_{55} \cdot \Xi_x^{(1)} + B_{12} \frac{\cos \alpha}{R(x)} \cdot \Xi_x^{(1)} - \frac{B_{22}}{R^2(x)} \cdot W(i) \cdot \sin \alpha \cos \alpha \quad (28)$$

$${}^{DSC}L_{44} = D_{11} \cdot \Xi_x^{(2)} + D_{11} \frac{\sin \alpha}{R(x)} \cdot \Xi_x^{(1)} - \frac{D_{22}}{R^2(x)} \sin^2 \alpha + \frac{D_{33}}{R^2(x)} \cdot \Xi_s^{(2)} - A_{55} \cdot \Psi_x(i) \quad (29)$$

$${}^{DSC}L_{45} = \frac{(D_{12} + D_{33})}{R(x)} \cdot \Xi_x^{(1)} \cdot \Xi_s^{(1)} - \frac{(D_{22} + D_{33})}{R^2(x)} \cdot \Xi_s^{(1)} \sin \alpha \quad (30)$$

$${}^{DSC}L_{51} = \frac{(B_{12} + B_{33})}{R(x)} \cdot \Xi_x^{(1)} \cdot \Xi_s^{(1)} + \frac{(B_{22} + B_{33})}{R^2(x)} \cdot \Xi_s^{(1)} \sin \alpha \quad (31)$$

$$\begin{aligned}
 {}^{DSC}L_{52} &= B_{33} \cdot \Xi_x^{(2)} + B_{33} \frac{\sin \alpha}{R(x)} \cdot \Xi_x^{(1)} \\
 &- B_{33} \cdot \frac{\sin^2 \alpha}{R^2(x)} \cdot V(i) + \frac{B_{22}}{R^2(x)} \cdot \Xi_s^{(2)} + \frac{A_{44}}{R(x)} \cdot V(i) \cdot \cos \alpha
 \end{aligned} \tag{32}$$

$${}^{DSC}L_{53} = -\frac{A_{44}}{R(x)} \cdot \Xi_s^{(1)} + \frac{B_{22}}{R^2(x)} \cos \alpha \cdot \Xi_s^{(1)} \tag{33}$$

$${}^{DSC}L_{54} = \frac{(D_{12} + D_{33})}{R(x)} \cdot \Xi_x^{(1)} \cdot \Xi_s^{(1)} + \frac{(D_{22} + D_{33})}{R^2(x)} \sin \alpha \cdot \Xi_s^{(1)} \tag{34}$$

$$\begin{aligned}
 {}^{DSC}L_{55} &= D_{33} \cdot \Xi_x^{(1)} + D_{33} \frac{\sin \alpha}{R(x)} \cdot \Xi_x^{(1)} \\
 &- \frac{D_{33}}{R^2(x)} \cdot \Psi_s(i) \cdot \sin^2 \alpha + \frac{D_{22}}{R^2(x)} \cdot \Xi_s^{(2)} - A_{44} \cdot \Psi_s(i)
 \end{aligned} \tag{35}$$

DSC derivation is given as

$$\Xi_x^n() = \frac{\partial^{(n)}()}{\partial x^{(n)}} = \sum_{k=-M}^M \delta_{\Delta, \sigma}^{(n)}(k \cdot \Delta x)()_{i+k, j} \tag{36}$$

$$\Xi_s^n() = \frac{\partial^{(n)}()}{\partial s^{(n)}} = \sum_{k=-M}^M \delta_{\Delta, \sigma}^{(n)}(k \cdot \Delta s)()_{i, j+k} \tag{37}$$

$$\Xi_x^1 \Xi_s^{(n-1)}() = \frac{\partial^{(n)}()}{\partial x \cdot \partial s^{(n-1)}} = \sum_{k=-M}^M \delta_{\Delta, \sigma}^{(1)}(k \cdot \Delta x)()_{i+k, j} \sum_{k=-M}^M \delta_{\Delta, \sigma}^{(n-1)}(k \cdot \Delta s)()_{i, k+j} \tag{38}$$

$$\Xi_x^{(n-1)} \Xi_s^1() = \frac{\partial^{(n)}()}{\partial x^{(n-1)} \partial s} = \sum_{k=-M}^M \delta_{\Delta, \sigma}^{(n-1)}(k \cdot \Delta x)()_{i+k, j} \sum_{k=-M}^M \delta_{\Delta, \sigma}^{(1)}(k \cdot \Delta s)()_{i, k+j} \tag{39}$$

3. Solution by DQ method

If DQ used above derivation can be define as

$$\Xi_x^n(*) = \frac{\partial^{(n)}(*)}{\partial x^{(n)}} = \sum_{k=1}^N C_{i+k, j}^{(n)}(i)(*)_{i+k, j} \tag{40}$$

$$\Xi_s^n(*) = \frac{\partial^{(n)}(*)}{\partial s^{(n)}} = \sum_{k=1}^N C_{i, j+k}^{(n)}(j)(*)_{i, j+k} \tag{41}$$

$$\Xi_x^1 \Xi_s^{(n-1)}(*) = \frac{\partial^{(n)}(*)}{\partial x \cdot \partial s^{(n-1)}} = \sum_{k=1}^N C_{i+k,j}(i) \sum_{k=1}^N C_{i,k+j}^{(n-1)}(j)(*)_{i,k+j} \quad (42)$$

$$\Xi_x^{(n-1)} \Xi_s^1(*) = \frac{\partial^{(n)}(*)}{\partial x^{(n-1)} \partial s} = \sum_{k=1}^N C_{i,k+j}^{(n-1)}(j) \sum_{k=1}^N C_{i+k,j}^{(1)}(i)(*)_{i,k+j} \quad (43)$$

C_{ijk} are weighting coefficients. The equations of motion are:

$${}^{DQ}L_{11} \cdot U + {}^{DQ}L_{12} \cdot V + {}^{DQ}L_{13} \cdot W + {}^{DQ}L_{14} \cdot \Phi_x + {}^{DQ}L_{15} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \quad (44)$$

$${}^{DQ}L_{21} \cdot U + {}^{DQ}L_{22} \cdot V + {}^{DQ}L_{23} \cdot W + {}^{DQ}L_{24} \cdot \Phi_x + {}^{DQ}L_{25} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \quad (45)$$

$${}^{DQ}L_{31} \cdot U + {}^{DQ}L_{32} \cdot V + {}^{DQ}L_{33} \cdot W + {}^{DQ}L_{34} \cdot \Phi_x + {}^{DQ}L_{35} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \quad (46)$$

$${}^{DQ}L_{41} \cdot U + {}^{DQ}L_{42} \cdot V + {}^{DQ}L_{43} \cdot W + {}^{DQ}L_{44} \cdot \Phi_x + {}^{DQ}L_{45} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \quad (47)$$

$${}^{DQ}L_{51} \cdot U + {}^{DQ}L_{52} \cdot V + {}^{DQ}L_{53} \cdot W + {}^{DQ}L_{54} \cdot \Phi_x + {}^{DQ}L_{55} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \quad (48)$$

In FGM material some properties are not constant:

$$E(z) = (E_c - E_m)V_c + E_m \quad (49)$$

$$\nu(z) = (\nu_c - \nu_m)V_c + \nu_m \quad (50)$$

For example if four-parameter power law is used then volume fractions are given for two cases.

$$\text{Case-1 } V_f = \left[1 - a \left(\frac{z}{h} + \frac{1}{2} \right) + b \left(\frac{z}{h} + \frac{1}{2} \right)^c \right]^p \quad (51)$$

$$\text{Case-2: } V_f = \left[1 - a \left(-\frac{z}{h} + \frac{1}{2} \right) + b \left(-\frac{z}{h} + \frac{1}{2} \right)^c \right]^p \quad (52)$$

4. Conclusion

These equations can also be used for circular cylindrical shell and panel, annular, circular plates, sector and annular sector plates. Each methods have own advantages. But for higher modes, the method of DSC is more effective.

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Elastic Beam Model and Bending Analysis of Silver Nanowires

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Abstract

In this study, bending analysis of silver (Ag) modeled nanowires has been carried out for six-various boundary conditions. Silver nanowires have great importance for Nano-electro-mechanical systems (NEMS) technology. The displacement, rotation of cross-section and bending moment values of elastic beam models of silver nanowires under uniform load have been calculated. Numerical results have been presented as graphics and tables. The influence of boundary conditions on deformation and bending moment has been discussed. As the boundary conditions become rigid, the values of displacement and cross-sectional rotation under uniform load reduce.

Keywords: Silver nanowires, bending, NEMS, elastic beam.

1. Introduction

Nanotechnology is scientific disciplines that have applications that aim all the tools and equipment that we use have superior properties. Finding superior properties by processing the materials at the nano-scale is the basic concept of this discipline. The general investigation area of this discipline is structures with 1 – 100 nm in length. In the Nanotechnology, the physical, electrical, optical, elastic and thermal properties of one-dimensional structures such as carbon nanotubes, boron nitride nanotubes, silica carbide nanotubes, zinc oxide nanowire, gold nanorod as well as two-dimensional nanostructures such as graphene and silicene have been intensively investigated. The discovery of the superior properties of these materials enables the production and using of materials with new properties. Carbon nanotubes (CNT) material has a very popular research field in nanotechnology. Carbon (C) atoms are arranged to form a two-dimensional structure called “Graphene”, one or more of graphene are wrapped like cylinder in space. These tubes form the carbon nanotubes structure by intertwining. Japanese scientist Iijima discovered these materials in 1991 and from this date forward, carbon nanotubes have been an intense research topic [1]. Chemical sensors, medical and industrial applications, quality control, detection of war and security threats exemplify to its potential uses [2]. Another material that is intensively studied in nanotechnologies researches is boron nitride nanotubes (BNNT). Similar to structure of carbon nanotubes, they consist of equal numbers of boron (B) and nitrogen (N) atoms. They are synthetically produced because they are not found naturally. Their production can be grouped under two headings: Synthesis



at high temperature such as arc-discharge, and synthesis at medium or low temperatures such as carbo-metallic methods and chemical vapor synthesis [3]. Because of strong insulating properties, it is an important material for nanocables. These cables are used in complex electronic circuits [4]. Optical devices operating with UV-light are another using area [5].

The element of silver (Ag) is a transition metal in group-1B and period-5 of the periodic table. It reflects the light, very well. It shows ductile behavior and high resistance to oxidation. It is used intensively in electrical wires [6]. The nanowire structure can be described as a one-dimensional structure with a diameter of less than 1 nm and its ratio of length-width is roughly equal to 1000. Silver nanowires can be used in optical industry, conductive materials, anti-bacterial applications [7]. There are several methods about its synthesis: Hard template methods and soft template methods. Soft template methods are divided into two. One of the methods is typical soft templates, another of methods is the polyol method [8]. It is very important to know the behavior of nano-scale material based devices such as micro-processor, transistor, sensor, conductor wire structures under external influences. Researchers have worked intensively with the mechanical analyses of beam and rod models of one-dimensional nanostructures and plate models of two-dimensional nanostructures [25-36]. Because of the nanowires are one-dimensional structures, they can be modeled as beams and rods. In this study, the analysis of the beams modeled with silver nanowires under the uniform load has been carried out and obtained numerical results have been discussed.

2. Bending Analysis of Nanobeams

The deformations of one-dimensional bending elements under uniform load constitute a continuous function. In the differential geometry, this function is expressed as below

$$\frac{1}{\rho} = \pm \frac{w''}{[1 + (w')^2]^{3/2}} \quad (1)$$

Here, w is elastic curve of beams, namely geometric locus, ρ is radius of curvature. Eq. (1) which is being seen, can be reduced by two reasons. Firstly, extremely flat curves become $w \cong 0$ because elastic curves are extremely flat in applications. On the other hand, second derivative of elastic curve become $w'' < 0$ under positive bending moment. Hereby, Eq. (2) is written,

$$\frac{1}{\rho} = -w'' \quad (2)$$

The curvature expression of one – directional bending according to Euler – Bernoulli beam theory,

$$\frac{1}{\rho} = \frac{M}{EI} \quad (3)$$

Here, M is bending moment, E is elasticity modulus and I is moment of inertia. If Eq. (3) is replaced in the Eq. (2), Bending moment-displacement relation is obtained as below,

$$M = -EIw'' \quad (4)$$

According to differential equilibrium of elastic body under external forces,

$$V = \frac{dM}{dz}, \quad q = -\frac{dV}{dz} \quad (5)$$

If Eq. (5) is replaced in the Eq. (4), Eq. (6) is obtained and Eq. (6) is elastic curve equation of beams

$$q = EIw^{(4)}(z) \quad (6)$$

where, q is uniform load in this equation. If Eq. (6) is integrated four times one after another, Displacement equation is obtained. First-order derivative of displacement equation is rotation equation. These equations are as below,

$$w' = \frac{1}{EI} \left[\frac{q}{6} z^3 + \frac{C_1}{2} z^2 + C_2 z + C_3 \right] \quad (7)$$

$$w = \frac{1}{EI} \left[\frac{q}{24} z^4 + \frac{C_1}{6} z^3 + \frac{C_2}{2} z^2 + C_3 z + C_4 \right] \quad (8)$$

Eq. (7) and Eq. (8) are expresses cross-sectional rotation and displacement, respectively. C_i ($i = 1,2,3,4$) is constant of indefinite integral and is found with the help of boundary conditions. The boundary conditions are as below

- Free end (F) $w'' = 0$ and $w''' = 0$
- Simply end (S) $w = 0$ and $w'' = 0$
- Cantilever end (C) $w = 0$ and $w' = 0$
- Guided end (G) $w' = 0$ and $w''' = 0$

3. Numerical Examples

The beam models to be analyzed are as shown in Fig.1. For the all numerical examples, elasticity modulus of Ag nanowires is $E = 0.102$ TPa [9]. The diameter of circular cross-section has been chosen as $d = 5$ nm. When we examine the Results in the Table-1, as the length of the beam increases, we understand the maximum displacement value increases. On the other hand, it is seen the most displacement is in the S-G beam, the least displacement is in the C-C beam. The elastic curves of the beams with different boundary conditions are given in Fig.2. The displacements of S-S, C-C and C-S beams are very low alongside other beam types. Rotation curves are plotted in Fig.3. S-G beam have made most cross-sectional rotation. The least rotation values have been shown in C-C beams. When we look to Results of Fig.4, The most bending moment have been shown guided end of S-G beam and cantilever end of C-F beams and theirs values are equal. The maximum bending moment of C-S and S-S are equal. In addition, C-C beams have the least bending moment.

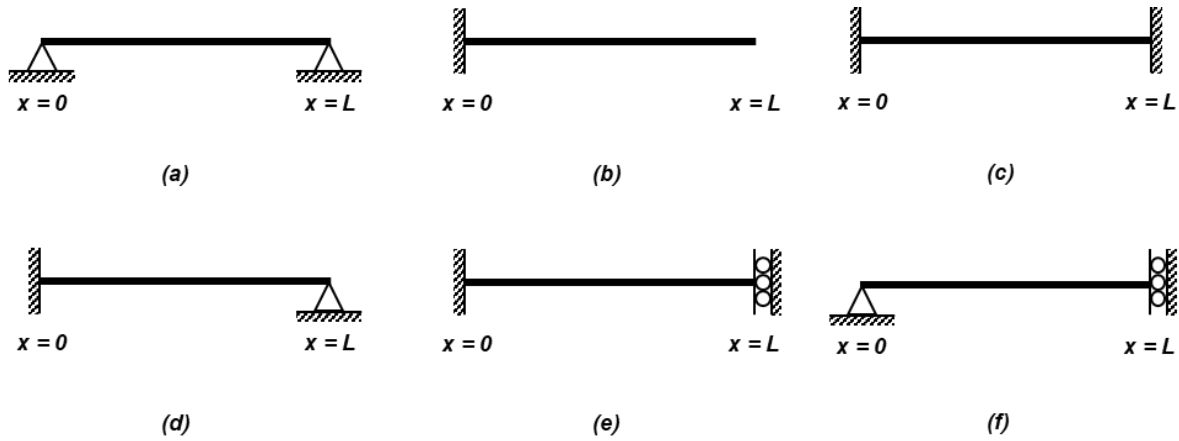


Fig.1. Nanobeams with various boundary conditions

(a) S – S (b) C – F (c) C – C (d) C – S (e) C – G (f) S – G

Table 1. The displacement values of nanobeams with various boundary conditions under $q=0.05$ nN/nm uniform load along span (nm)

Length (nm)	S – S	C – F	C – C	C – S	C – G	S – G
10	0.00208	0.01997	0.00080	0.00042	0.03329	0.00666
20	0.03329	0.31956	0.01278	0.00666	0.53260	0.10652
30	0.16852	1.61776	0.06471	0.03370	2.69627	0.53925
40	0.53260	5.11293	0.20452	0.10652	8.52155	1.70431
50	1.30029	12.48274	0.49931	0.26006	20.80457	4.16091

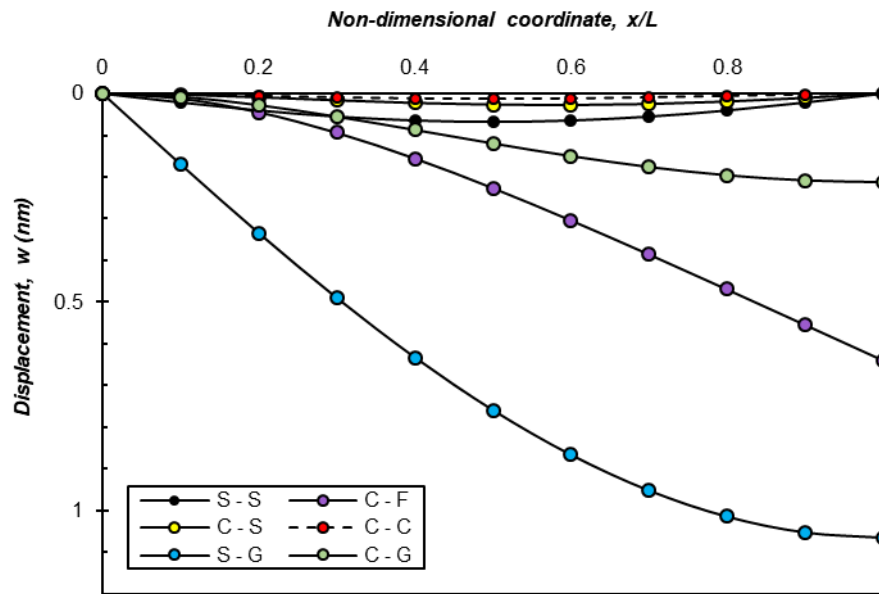


Fig. 2. Plotting the elastic curves of nanobeams with various boundary conditions under $q=0.1$ nN/nm uniform load

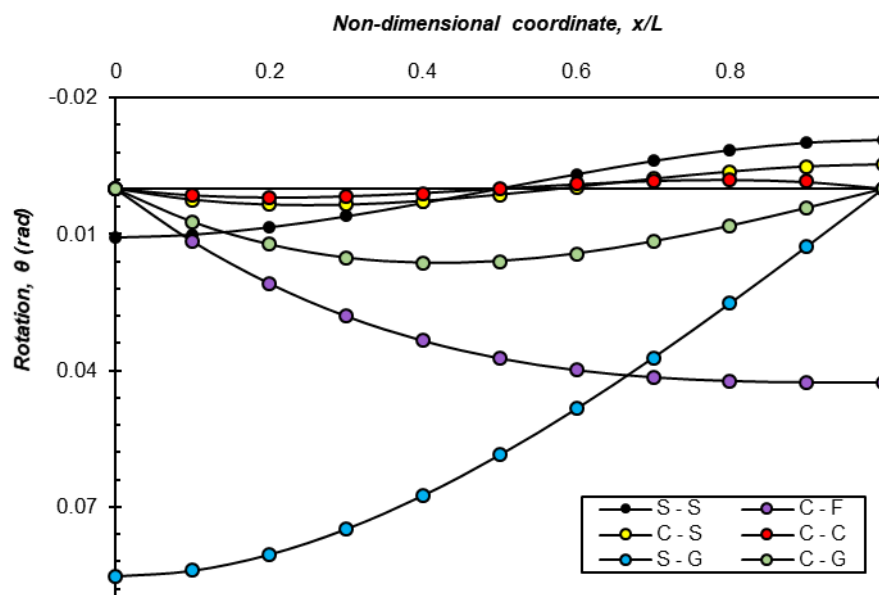


Fig. 3. Plotting the cross-sectional rotations of nanobeams with various boundary conditions under $q=0.1$ nN/nm uniform load

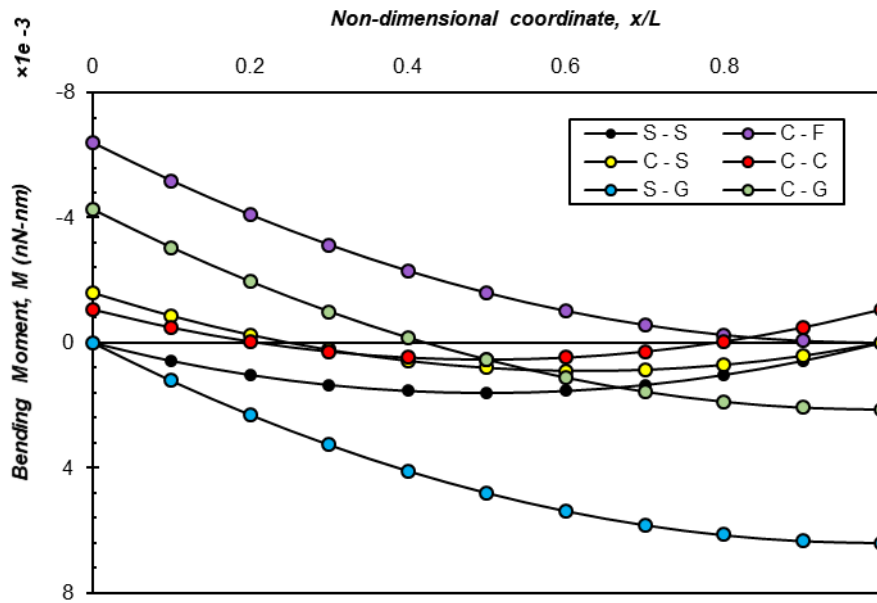


Fig. 4. Plotting the bending moment diagram of nanobeams with various boundary conditions under $q=0.1$ nN/nm uniform load

4. Concluding Remarks

As the ends of the beam become rigid, the displacement rotation and bending moment values have decreased. On the other hand, as the length increases in all beam types, the ratio of displacement to length of nanobeam increases. It is hoped that these results will contribute to the research on the design problem of nano-scaled elements under bending effect.

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Analytical Buckling of FG Nanobeams on The Basis of A New One Variable First-Order Shear Deformation Beam Theory

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Abstract

In this work, buckling analysis of functionally graded (FG) nanobeams based on a new refined beam theory has been analyzed. The beam is modeled as an elastic beam subjected to unidirectional compressive loads. To achieve this aim, the new obtained beam theory has only one variable which lead to one equation similar to Euler beam theory and also is free of any shear correction factor. The equilibrium equation has been formulated by the nonlocal theory of Eringen to predict small-scale effects. The equation has been solved by Navier's approach by which critical buckling loads have been obtained for simple boundaries. Finally, to approve the results of the new beam theory, various beam theories have been compared.

Keywords: Buckling analysis, FG nanobeams, A new refined beam theory, Nonlocal elasticity theory, Navier's approach

1. Introduction

Carbon nanotubes (CNTs) are seamless cylinders included one to multi-graphene layers with open or close ending that they are called single-walled (SWCNT) or multi-walled carbon nanotubes (MWCNT) [1]. Today, the most manufactured CNTs are used in composite materials and thin films [1]. The SWCNT is remarkably strong and hard [2], conducting electric current and directing heat [3-5], which has led to the use of these materials in the electronics industry [6-7]. The carbon nanotube promises a bright future in cellular experiments because they can be used as nano-pipes to distribute very small volumes of fluid or gas into living cells or on surfaces [8-10].

To exploit the industrial amazing properties of nanostructures, it can be highly recommended that the mechanical behavior of them should be analyzed. In last years, this issue has been taken into consideration by researchers around the world in order to identify the behavior of them under various mechanical conditions. Among these researchers, Reddy [11] reformulated beam theories by using nonlocal elasticity theory for vibrations, buckling and bending analyses. Civalek et al. [12] analyzed natural frequencies of a skew symmetric composite plate using discrete convolution method (DSC). Malikan et al. [13] published stability of bi-layer graphene nanoplates subjected to shear and thermal forces on the basis of a medium using numerical solutions. Malikan investigated stability analysis of a micro



sandwich plate with graphene coating using the refined couple stress theory [14] and buckling of graphene sheets subjected to nonuniform compression based on the four-variable plate theory using an analytical approach [15]. Yao and Han [16] presented buckling of double-walled carbon nanotubes with considering thermal influences. They obtained critical buckling loads on the basis of Donnell's equilibrium equation and solved the equation for simply-supported boundary condition. Ansari et al. [17] studied coupled natural frequency analysis of post stability functionally graded micro/nanobeams on the basis of the strain gradient theory. Wang et al. [18] presented exact modes for post stability characteristics of nonlocal nanobeams in a longitudinal magnetic field. Wang et al. [19] utilized both stress and strain gradient continuum theories to consider buckling of nanotube which was embedded in an elastic foundation. Timoshenko beam theory and Navier solution method were employed in their study. They proved that both stress gradient and strain gradient predict the same results if the nonlocal effect is not taken into account. Xiang et al. [20] used nonlocal elasticity theory for studying nonlinear free vibration of double-walled carbon nanotubes based on Timoshenko beam theory. Ansari et al. [21] developed Rayleigh–Ritz method for buckling of carbon nanotubes considering thermal effects. They classical Donnell shell theory was incorporated in conjunction with nonlocal elasticity theory of Eringen. Ansari et al. [22] employed Timoshenko beam model to consider buckling and postbuckling of nanotubes using nonlocal elasticity theory. The equations were solved with generalized differential quadrature method and the pseudo arc-length technique for several boundary conditions. Ansari and Arjangpay [23] presented using the meshless local Petrov–Galerkin method for various boundary conditions to analyze carbon nanotubes under buckling and vibrations. The vibration of thermally post-buckled carbon nanotube-reinforced composite beams resting on elastic foundations has been examined by Shen et al. [24]. Beni et al. [25] studied vibration of shell nanotubes using nonlocal strain gradient theory and molecular dynamics simulation. Wang et al. [26] presented nonlinear vibration of nonlocal carbon nanotubes placed on the visco-Pasternak foundation under excitation frequency. Civalek et al. [27] investigated laminated composites in static conditions on the basis of nonlinear first-order shear deformation theory. The equations were discretized and solved with the singular convolution method (DSC). Reddy [28] developed couple stress theories for functionally graded Euler-Bernoulli and Timoshenko microbeams. Reddy and Arbind [29] derived a couple stress theory for bending analysis of Euler and Timoshenko functionally graded beams. Stability analysis of nanotubes made of boron nitride embedded on the elastic matrix using DSC has been presented by Mercan and Civalek [30]. Akgöz and Civalek [31] studied nonlocal buckling of carbon nanotubes subjected to an axial compressive load surrounded by Pasternak matrix. In their study various beam theories were applied and governing equations were analytically solved by Navier solution method. Civalek et al. [32] developed the modified couple stress, the strain gradient and nonlocal elasticity theories for buckling of silicon carbide nanowires-based Euler beam theory. Akgöz and Civalek [33] considers influences of thermal and shear deformations on the vibrations of a functionally graded thick micro composite beam.

In this theoretical work, we report a new beam theory by reducing the unknown variables from a regenerated shear deformation theory. The functionally graded (FG) nanobeam is modeled as an elastic beam which is subjected to unidirectional compressive load. The influence of stress nonlocality is examined by using nonlocal elasticity theory of Eringen which leads to a size-dependent equation. Furthermore, Navier's technique is exerted to solve the stability equation by assuming simply-supported boundary condition for both edges of the beam. To approve the present formulation, various beam theories have been analyzed resulted from several well-known references.

2. Mathematical Formulation

Fig. 1 displays a realistic model for the nanobeam subjected to unidirectional compressive loads with length L , outer diameter d and thickness h parallel to x and z -axes, respectively. First, according to first-order shear deformation beam (FSDT) theory, the displacement field is presented as below [13, 34-36]:

$$\begin{Bmatrix} U(x, z) \\ V(x, z) \\ W(x, z) \end{Bmatrix} = \begin{Bmatrix} u(x) + z\varphi(x) \\ 0 \\ w(x) \end{Bmatrix} \quad (1a-c)$$

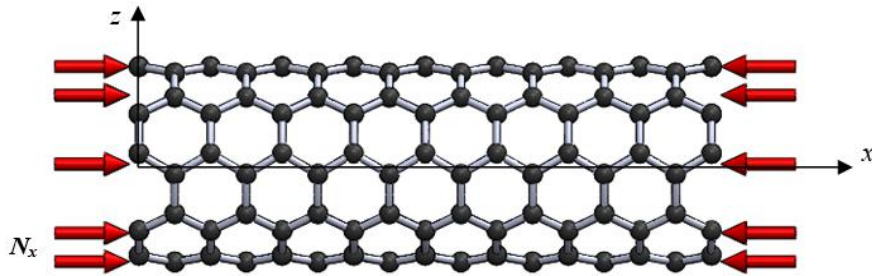


Fig. 1. The SWCNT subjected to the unidirectional compressive load

In Eq. (1), the vector quantities of the neutral axis at directions of x and z are u and w , respectively. Moreover, for defining of the rotation of beam elements around the x axis, φ is used. First off, let us reconsider the simple first-order shear deformation theory (S-FSDT) by which the deflections were re-formulated in the following equation [37-39]:

$$w = w(\text{bending}) + w(\text{shear}) \quad (2)$$

Also, φ parameter was developed as below:

$$\{\varphi\} = \left\{ -\frac{dw_b}{dx} \right\} \quad (3)$$

By replacement Eqs. (2-3) in Eq. (1) the displacement field of the S-FSDT was rewritten as follows [37-39]:

$$\begin{Bmatrix} U(x, z) \\ W(x, z) \end{Bmatrix} = \begin{Bmatrix} u(x) - z \frac{dw_b(x)}{dx} \\ w_b(x) + w_s(x) \end{Bmatrix} \quad (4a-b)$$

Use of $w = w_b + w_s$ might not be conceptual; Therefore, Eq. 4 would be refined in the following:

$$\begin{Bmatrix} U(x, z) \\ W(x, z) \end{Bmatrix} = \begin{Bmatrix} u(x) - z \frac{dw_b(x)}{dx} \\ w_b(x) + W' \end{Bmatrix} \quad (5a-b)$$

So, we could use bending deflection to find the value of w_s :

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{xz} \end{Bmatrix} = \begin{Bmatrix} E(z) \varepsilon_{xx} \\ 2G(z) \gamma_{xz} \end{Bmatrix} \quad (6a-b)$$

After obtaining Eq. (6) from S-FSDT the stresses can be found and then by substituting Eq. (6) in the S-FSDT stress resultants, Eq. 7 will be calculated:

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \int_A \begin{Bmatrix} \sigma_x z \\ \sigma_{xz} \end{Bmatrix} dA \quad (7a-b)$$

Let us use fourth equation of FSDT's governing equations in order to calculate w_s based on w_b :

$$\frac{dM_x}{dx} - Q_x = 0 \quad (8)$$

By imposing Eq. (8) into the stress resultants of Eq. (7):

$$E(z)I_c \frac{d^3 w_b}{dx^3} - AG(z) \frac{dw_s}{dx} = 0 \quad (9)$$

By integrating from Eq. (9) based on x , simplifying and then ignoring the integral constant terms, the shear deflection will now be obtained as follows:

$$w_s = W' = B \frac{d^2 w_b}{dx^2} \quad (10)$$

Term B could be in both positive and negative signs that is explained:

$$B = \frac{E(z)I_c}{AG}, \quad G(z) = \frac{E(z)}{2(1+\nu)} \quad (11)$$

where G represents the shear modulus, E is the Young's modulus, $I_c (\pi d^4/64)$ denotes the moment of area of the cross-section, A is the cross-sectional area and ν is the Poisson's ratio for isotropic nanobeams. Afterwards, the new beam theory will now be achieved as:

$$\text{Now: } w_b = w \quad ; \quad \begin{Bmatrix} U(x, z) \\ W(x, z) \end{Bmatrix} = \begin{Bmatrix} u(x) - z \frac{dw(x)}{dx} \\ w(x) + B \frac{d^2 w(x)}{dx^2} \end{Bmatrix} \quad (12a-b)$$

Regarding Hamilton's principle, the potential energy in the whole domain of the beam (V) is made available and is written in the variational form as below [40]:

$$\delta V = \delta S + \delta \Omega = 0 \quad (13)$$

In which δS is the variation of strain energy and δV is the variation of works, which are done by external forces. The strain energy by variational formulation will be calculated:

$$\delta S = \iiint_v \sigma_{ij} \delta \varepsilon_{ij} dV = 0 \quad (14)$$

The strain tensor in Eq. (14) is expanded as follows:

$$\left\{ \begin{array}{l} \varepsilon_{xx} \\ \gamma_{xz} \end{array} \right\} = \left\{ \begin{array}{l} \frac{du}{dx} - z \frac{d^2 w}{dx^2} + \frac{1}{2} \left(B \frac{d^3 w}{dx^3} + \frac{dw}{dx} \right)^2 \\ B \frac{d^3 w}{dx^3} \end{array} \right\} \quad (15a-b)$$

With applying the variational formulation ($\delta V=0$) the nonlinear governing equation of motion is derived:

$$\delta w = 0; \quad \frac{d^2 M_x}{dx^2} - B \frac{d^3 Q_x}{dx^3} - N_x \left(B^2 \frac{d^6 w}{dx^6} + 2B \frac{d^4 w}{dx^4} + \frac{d^2 w}{dx^2} \right) = q_0 \quad (16)$$

In which M_x , Q_x , and N_x are nonlocal stress resultants, respectively and q_0 is the transverse static load which is ignored in this paper. Here, the quantity N_x is the resultant with respect to the axial applied compressive force. With regard to nonlocal theory of Eringen, the following equation is employed [13, 40]:

$$(1 - \mu \nabla^2) \sigma_{ij} = C_{ijkl} \varepsilon_{kl} ; \quad \mu (nm^2) = (e_0 a)^2, \quad \nabla^2 = \frac{d^2}{dx^2} \quad (17)$$

where μ is the nonlocality factor and a is an interior determined length.

The material property gradation considering power law in the FG nanobeams is expressed as [41-43]:

$$E(z) = E_m + (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h} \right)^k \quad (18)$$

Here E_c and E_m are the Young's modulus corresponding to ceramic and metal, respectively, and k is volume fraction exponent or material grading/power law index. Due to insignificant variation of the Poisson's ratio, this variant is assumed to be constant along the thickness ($\nu(z) = \nu$). From Eq. (18), when $k \rightarrow \infty$, the FG nanobeam reduces to a pure metal one and for case $k=0$, the plate becomes pure ceramic.

The stress resultants in local form are specified by relations below:

$$\left\{ \begin{array}{l} M_x \\ Q_x \end{array} \right\} = \int_A \left\{ \begin{array}{l} \sigma_{xz} \\ \sigma_{xz} \end{array} \right\} dA \quad (19a-b)$$

Now, by substituting Eq. (15) into the Eq. (19) the stress resultants will be given as follows:

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \begin{Bmatrix} -EI_c \frac{d^2 w}{dx^2} \\ AGB \frac{d^3 w}{dx^3} \end{Bmatrix} \quad (20a-b)$$

The compressive force is assumed as follows [40, 44]:

$$N_x = -P_{Cr} \quad (21)$$

Now, incorporating Eq. (17, 20-21) and inserting them into Eq. (16) and also some manipulating, lead to the stability equation of one variable first-order shear deformation theory (OVFSDT) as:

$$\begin{aligned} \delta w = 0: EI_c \frac{d^4 w}{dx^4} + B^2 AG \frac{d^6 w}{dx^6} - P_{Cr} \left(B^2 \frac{d^6 w}{dx^6} + 2B \frac{d^4 w}{dx^4} + \frac{d^2 w}{dx^2} \right) + \\ \mu P_{Cr} \left(B^2 \frac{d^8 w}{dx^8} + 2B \frac{d^6 w}{dx^6} + \frac{d^4 w}{dx^4} \right) = 0 \end{aligned} \quad (22)$$

Also by using Eq. (4) the S-FSDT equations could be obtained as follows:

$$\begin{aligned} \delta w_b = 0: EI_c \frac{d^4 w_b}{dx^4} - P_{Cr} \left(\frac{d^2 w_b}{dx^2} + \frac{d^2 w_s}{dx^2} \right) + \mu P_{Cr} \left(\frac{d^4 w_b}{dx^4} + \frac{d^4 w_s}{dx^4} \right) = 0 \\ \delta w_s = 0: AG \frac{d^2 w_s}{dx^2} - P_{Cr} \left(\frac{d^2 w_b}{dx^2} + \frac{d^2 w_s}{dx^2} \right) + \mu P_{Cr} \left(\frac{d^4 w_b}{dx^4} + \frac{d^4 w_s}{dx^4} \right) = 0 \end{aligned} \quad (23a-b)$$

On the other hand, by using Eq. (1) the FSDT equations could be obtained as follows:

$$\delta w = 0: k_s AG \left(\frac{d^2 w}{dx^2} - \frac{d\varphi}{dx} \right) - P_{Cr} \frac{d^2 w}{dx^2} + \mu P_{Cr} \frac{d^4 w}{dx^4} = 0 \quad (24a)$$

$$\delta\varphi = 0: EI_c \frac{d^2 \varphi}{dx^2} + k_s AG \left(\frac{dw}{dx} - \varphi \right) = 0 \quad (24b)$$

Furthermore, for CPT the stability equation is obtained in the following form:

$$\delta w = 0: EI_c \frac{d^4 w}{dx^4} - P_{Cr} \frac{d^2 w}{dx^2} + \mu P_{Cr} \frac{d^4 w}{dx^4} = 0 \quad (25)$$

3. Navier's technique

The Navier solution method has been applied to present simply-supported boundary condition according to Eq. (26) [44].

$$w(x, t) = \sum_{m=1}^{\infty} W_m \sin\left(\frac{m\pi}{L}x\right) e^{i\omega t} \quad (26a)$$

$$\varphi(x, t) = \sum_{m=1}^{\infty} \Phi_m \cos\left(\frac{m\pi}{L}x\right) e^{i\omega t} \quad (26a)$$

where m is the half-wave number as a integer one, W_m and Φ_m are the unknown terms which should be determined and also ω is the natural frequency in vibrational analysis. Substituting Eq. (26) into Eqs. (22-25), the algebraic equation is obtained from which the critical buckling load equation is calculated as follows:

- OVFSDT:

$$P_{Cr} = \frac{EI_c \left(\frac{m\pi}{L}\right)^4 - \frac{(EI_c)^2}{AG} \left(\frac{m\pi}{L}\right)^6}{\left(\frac{EI_c}{AG}\right)^2 \left(\frac{m\pi}{L}\right)^6 - \frac{2EI_c}{AG} \left(\frac{m\pi}{L}\right)^4 + \left(\frac{m\pi}{L}\right)^2 + \mu \left(\left(\frac{EI_c}{AG}\right)^2 \left(\frac{m\pi}{L}\right)^8 - \frac{2EI_c}{AG} \left(\frac{m\pi}{L}\right)^6 + \left(\frac{m\pi}{L}\right)^4 \right)} \quad (27)$$

- CPT:

$$P_{Cr} = \frac{EI_c \left(\frac{m\pi}{L}\right)^4}{\left(\frac{m\pi}{L}\right)^2 + \mu \left(\frac{m\pi}{L}\right)^4} \quad (28)$$

- S-FSDT:

$$\begin{bmatrix} EI_c \left(\frac{m\pi}{L}\right)^4 + P_{Cr} \left(\frac{m\pi}{L}\right)^2 + \mu P_{Cr} \left(\frac{m\pi}{L}\right)^4 & P_{Cr} \left(\frac{m\pi}{L}\right)^2 + \mu P_{Cr} \left(\frac{m\pi}{L}\right)^4 \\ P_{Cr} \left(\frac{m\pi}{L}\right)^2 + \mu P_{Cr} \left(\frac{m\pi}{L}\right)^4 & -AG \left(\frac{m\pi}{L}\right)^2 + P_{Cr} \left(\frac{m\pi}{L}\right)^2 + \mu P_{Cr} \left(\frac{m\pi}{L}\right)^4 \end{bmatrix} \begin{Bmatrix} w_b \\ w_s \end{Bmatrix} = 0 \quad (29)$$

- FSDT:

$$\begin{bmatrix} -k_s AG \left(\frac{m\pi}{L}\right)^2 + P_{Cr} \left(\frac{m\pi}{L}\right)^2 + \mu P_{Cr} \left(\frac{m\pi}{L}\right)^4 & k_s AG \left(\frac{m\pi}{L}\right) \\ k_s AG \left(\frac{m\pi}{L}\right) & -EI_c \left(\frac{m\pi}{L}\right)^2 - k_s AG \end{bmatrix} \begin{Bmatrix} w \\ \varphi \end{Bmatrix} = 0 \quad (30)$$

If determinant of coefficients of Eqs. (29) and (30) is set to zero, the critical buckling load of S-FSDT and FSDT can be calculated.

4. Numerical results

In the first glance it is required to consider the precision of the numerical results obtained from the proposed beam theory with other theories. Hence, as can be seen in Tables 1 and 2, references [45-47] are employed. In [45] a nano rod was based on the both Euler and Timoshenko (Table 1) beam theories and the equations were solved by using an explicit analytical method and differential transform method. On the other hand, in ref. [47] Euler and Timoshenko nano rods were modeled and Navier solution method was utilized in order to obtain numerical results. In fact, both thin and moderately thick beams are compared and carried out with both ends simple boundaries. It is worth noting that with increasing length to diameter ratio of the nano rod the results in the Tables are becoming closer to one another. This means that for thin beams the proposed theory makes same predictions with Euler beam theory which is an acceptable conclusion. Because thin beam theories like Euler can predict appropriate results only for thin beams due to lack of considering transverse strain influences in such a theory. This strain is fundamentally required for response of moderately thick beams which is embedded in proposed theory. It can be seen that for lower values of length to diameter ratio which the rod goes into moderately thick and thick cases the results of Euler beam theory are in a major difference with present formulation. Furthermore, increasing small-scale parameter decreases the gap between the results of current beam theory and others. It is interesting to note that the results of S-FSDT and OVFSDT are corresponded to each other completely. Note that the shear correction factor used in Timoshenko theory can be a serious defect in light of the approximate quantity of it ($k_s=5/6$). Although this value has been applied for moderately thick models, it cannot be an exact value to analyze several cases, in particular nanostructures. But in the proposed beam theory this extra factor is vanished from the governing equation leads to further accurate results.

To have further comparison, Table 3 is presented in which the proposed theory is compared with ref. [48] within which a functionally graded nanobeam was analyzed with both Euler and Timoshenko beam theories and the equations were solved by Navier solution method. This Table approved the results of previous Tables for thin beams in light of the proximity of all of the beam theories to one another. Moreover, it can be seen that by an increase in the material grading index the difference of the present theory with others will be increased; however, this difference for moderately thick beams is further than thin ones. Generally, Tables 1 to 3 show the close numerical results between the present theory and others from which the theory can be confirmed. Although the new theory of beam which is used could not be a complete theory, by carrying out the errors and refining them the more appropriate numerical results will be obtained.

Table 1. Results of critical buckling load (nN) developed from several theories for a rod ($E=1TPa$, $\nu=0.19$, $d=1nm$)

		$P_{Cr} (nN)$									
		$e_0a=0 nm$		$e_0a=0.5 nm$		$e_0a=1 nm$		$e_0a=1.5 nm$		$e_0a=2 nm$	
L (nm)		EB^*		EB^*		EB^*		EB^*		EB^*	
		[45-	OVFSDT	[45-	OVFSDT	[45-	OVFSDT	[45-	OVFSDT	[45-	OVFSDT
		46],	FSDT***	46],	FSDT***	46],	FSDT***	46],	FSDT***	46],	FSDT***
		TB**	S-FSDT	TB**	S-FSDT	TB**	S-FSDT	TB**	S-FSDT	TB**	S-FSDT
		[45]	[45]	[45]	[45]	[45]	[45]	[45]	[45]	[45]	
10		4.8447	4.7609	4.7281	4.7985	4.4095	4.4752	3.9644	4.0234	3.4735	3.5252
		4.7670	4.7609	4.654	4.7985	4.3450	4.4752	3.9121	4.0234	3.4333	3.5252
12		3.3644	3.3991	3.3077	3.3418	3.1486	3.181	2.9149	2.9449	2.6405	2.6677
		3.3267	3.3237	3.2713	3.2677	3.1156	3.1105	2.8865	2.8797	2.6172	2.6086
14		2.4718	2.4905	2.4411	2.4595	2.3533	2.3711	2.2202	2.237	2.0574	2.0729
		2.4514	2.4498	2.4212	2.4193	2.3348	2.3323	2.2038	2.2005	2.0432	2.0391
16		1.8925	1.9034	1.8744	1.8852	1.8222	1.8327	1.7414	1.7515	1.6396	1.6491
		1.8805	1.8795	1.8626	1.8616	1.8111	1.8098	1.7313	1.7295	1.6306	1.6284
18		1.4953	1.5021	1.484	1.4907	1.4511	1.4577	1.3994	1.4057	1.3329	1.3389
		1.4878	1.4872	1.4766	1.476	1.4440	1.4432	1.3928	1.3918	1.3269	1.3257
20		1.2112	1.2156	1.2038	1.2082	1.182	1.1864	1.1475	1.1517	1.1024	1.1064
		1.2063	1.2059	1.1989	1.1985	1.1773	1.1768	1.1431	1.1424	1.0983	1.0975
		1.2156	1.2082	1.2082	1.2082	1.1864	1.1864	1.1517	1.1517	1.1064	1.1064

* Euler beam (EB).

** Timoshenko beam (TB), $k_s=5/6$.

*** Timoshenko beam (FSDT), Navier, $k_s=5/6$.

Note that in [45] an explicit solution and in [46] differential transform method (DTM) were applied, respectively. Also for EB in ref. [45-46] for $e_0a=0, 1$ and $2 nm$ only the validation was existed, but others are appeared by solving CPT in this paper.

Table 2. Results of dimensionless critical buckling load developed from several theories for a rod

$$(E=1TPa, \nu=0.3, d=1nm, \overline{P_{Cr}} = \frac{P_{Cr}L^2}{EI_c})$$

		$\overline{P_{Cr}}$									
		$\mu=0 \text{ nm}^2$		$\mu=1 \text{ nm}^2$		$\mu=2 \text{ nm}^2$		$\mu=3 \text{ nm}^2$		$\mu=4 \text{ nm}^2$	
L/d		EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT	EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT	EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT	EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT	EB[47], TB[47]	OVFSDT FSDT S-FSDT CPT
	10			10.0305		9.1294		8.3769		7.739	
		9.8696	9.6832	8.9830	8.8134	8.2426	8.0869	7.6149	7.4711	7.0761	6.9424
		9.6227	10.0305	8.7583	9.1294	8.0364	8.3769	7.4244	7.739	6.8990	7.1914
20			9.9093		9.6707		9.4433		9.2263		9.0191
		9.8696	9.8223	9.6319	9.5858	9.4055	9.3604	9.1894	9.1453	8.9830	8.94
		9.8067	9.9093	9.5705	9.6707	9.3455	9.4433	9.1308	9.2263	8.9258	9.0191
50			9.8759		9.8371		9.7985		9.7603		9.7224
		9.8696	9.8620	9.8308	9.8232	9.7923	9.7847	9.7541	9.7466	9.7161	9.7087
		9.8595	9.8759	9.8207	9.8371	9.7822	9.7985	9.7440	9.7603	9.7062	9.7224
		9.8696		9.8308		9.7923		9.7541		9.7161	

Table 3. Results of dimensionless critical buckling load developed from several theories for a FG

$$\text{nanobeam } (E_1=1TPa, E_2=0.25TPa, \nu=0.3, \overline{P_{Cr}} = \frac{P_{Cr}L^2}{EI_c})$$

		$\overline{P_{Cr}}$									
		$e_0a=0 \text{ nm}$		$e_0a=0.5 \text{ nm}$		$e_0a=1 \text{ nm}$		$e_0a=1.5 \text{ nm}$		$e_0a=2 \text{ nm}$	
k		EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT	EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT	EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT	EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT	EB[48], TB[48]	OVFSDT FSDT S-FSDT CPT
	$L/h=10$										
0			2.5213		2.4606		2.2948		2.0631		1.8076
		2.4674	2.4056	2.4079	2.3477	2.2457	2.1895	2.0190	1.9685	1.7690	1.7247
		2.4056	2.5213	2.3477	2.4606	2.1895	2.2948	1.9685	2.0631	1.7247	1.8076
0.3			4.1820		4.0813		3.8063		3.4219		2.9982
		4.0925	3.9901	3.9940	3.8941	3.7249	3.6317	3.3488	3.2650	2.9341	2.8607
		3.9921	4.1820	3.8959	4.0813	3.6335	3.8063	3.2667	3.4219	2.8621	2.9982
1			5.5468		5.4133		5.0485		4.5389		3.9769
		5.4282	5.2924	5.2975	5.1650	4.9406	4.8170	4.4418	4.3307	3.8918	3.7944
		5.3084	5.5468	5.1805	5.4133	4.8315	5.0485	4.3437	4.5389	3.805	3.9769
3			6.9666		6.7988		6.3407		5.7006		4.9947
		6.8176	6.6470	6.6534	6.4870	6.2051	6.0498	5.5787	5.4391	4.8879	4.7656
		6.6720	6.9666	6.5113	6.7988	6.0727	6.3407	5.4596	5.7006	4.7835	4.9947
10			8.4993		8.2947		7.7358		6.9549		6.0936
		8.3176	8.1095	8.1173	7.9142	7.5704	7.3810	6.8062	6.6359	5.9633	5.8141
		8.1289	8.4993	7.9332	8.2947	7.3987	7.7358	6.6518	6.9549	5.8281	6.0936
		8.3176		8.1173		7.5704		6.8062		5.9633	

<i>L/h=30</i>										
0		2.4732		2.4665		2.4464		2.4137		2.3693
	2.4674	2.4603	2.4606	2.4536	2.4406	2.4336	2.4079	2.4011	2.3637	2.3569
	2.4603	2.4732	2.4536	2.4665	2.4336	2.4464	2.4011	2.4137	2.3569	2.3693
0.3		4.1022		4.0910		4.0577		4.0035		3.9298
	4.0925	4.0808	4.0813	4.0697	4.0481	4.0366	3.9940	3.9826	3.9205	3.9094
	4.0811	4.1022	4.0699	4.0910	4.0368	4.0577	3.9828	4.0035	3.9096	3.9298
1		5.4412		5.4263		5.3821		5.3101		5.2124
	5.4282	5.4128	5.4134	5.3980	5.3694	5.3541	5.2975	5.2824	5.2001	5.1853
	5.4146	5.4412	5.3998	5.4263	5.3559	5.3821	5.2843	5.3101	5.1871	5.2124
3		6.8338		6.8151		6.7596		6.6693		6.5466
	6.8176	6.7982	6.7989	6.7796	6.7436	6.7244	6.6534	6.6345	6.5311	6.5125
	6.8011	6.8338	6.7825	6.8151	6.7273	6.7596	6.6373	6.6693	6.5153	6.5466
10		8.3374		8.3147		8.2470		8.1366		7.9871
	8.3176	8.2939	8.2949	8.2713	8.2274	8.2040	8.1173	8.0942	7.9681	7.9454
	8.2962	8.3374	8.2735	8.3147	8.2062	8.2470	8.0964	8.1366	7.9476	7.9871
		8.3176		8.2949		8.2274		8.1173		7.9681
<i>L/h=100</i>										
0		2.4679		2.4673		2.4654		2.4624		2.4582
	2.4674	2.4667	2.4667	2.4661	2.4649	2.4643	2.4619	2.4613	2.4576	2.4570
	2.4667	2.4679	2.4661	2.4673	2.4643	2.4654	2.4613	2.4624	2.4570	2.4582
0.3		4.0934		4.0924		4.0894		4.0842		4.0773
	4.0925	4.0914	4.0915	4.0905	4.0885	4.0874	4.0834	4.0823	4.0764	4.0753
	4.0915	4.0934	4.0905	4.0924	4.0874	4.0894	4.0824	4.0842	4.0754	4.0773
1		5.4294		5.4281		5.4241		5.4173		5.4080
	5.4282	5.4268	5.4269	5.4255	5.4229	5.4215	5.4162	5.4148	5.4069	5.4055
	5.4270	5.4294	5.4257	5.4281	5.4217	5.4241	5.4150	5.4173	5.4057	5.4080
3		6.8191		6.8173		6.8123		6.8039		6.7922
	6.8176	6.8159	6.8159	6.8141	6.8108	6.8090	6.8025	6.8007	6.7908	6.7890
	6.8161	6.8191	6.8144	6.8173	6.8094	6.8123	6.8010	6.8039	6.7893	6.7922
10		8.3194		8.3173		8.3111		8.3010		8.2867
	8.3176	8.3155	8.3155	8.3134	8.3094	8.3072	8.2992	8.2971	8.2849	8.2828
	8.3157	8.3194	8.3136	8.3173	8.3075	8.3111	8.2972	8.3010	8.2830	8.2867
		8.3176		8.3155		8.3094		8.2992		8.2849

6. Conclusions

This article investigated stability of functionally graded nanobeams exposed to the axial compressive loads. To obtain this, a novel beam approach was re-formulated to present governing equations. Nanoscale influences were evaluated by use of a non-classical elasticity theory. Moreover, to calculate the numerical results the Navier's approach was used. The greatness outcomes proved that the Euler beam theory has not satisfactory results for moderately thick and thick beams. On the other hand, although the impacts of transverse shear strains has been taken into account by Timoshenko beam, the used shear correction factor

deviates outcomes of this beam approach slightly. The appropriate amount of this factor for nanostructures has not been already calculated and the used value cannot be appropriate at all.

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