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From the Editorial: Teaching Practices Article: New Trend in Math Education

Dear Authors, Readers, Reviewers, Editors

It is not possible to consider mathematics education separately from teaching practices. Perhaps it is now necessary to concentrate only on teaching practices. Because, apart from the life-oriented nature of mathematics, most of the teaching approaches continue to fail. Except for the elite and gifted students, normal students ask, "What are we going to use mathematics for?" they ask the question. How are we going to teach this math? What teaching practices will we use? prompts us (academicians and practitioners) to answer questions. JMETP started its broadcasting life with this philosophical background.

We plan to include teaching practices and designs in each issue. In this respect, we invite practitioners who conduct academic research and teaching to contribute to our journal.

In this issue; Nur Noor and Fiki Alghadari with the article titled “Conceptual technique for comparison figures by geometric thinking in analysis level”, Yousef M. Abd Algani, Younis Abu Al-Haija, and Wafiq Hibi with the article titled “The impact of the use of mathematical problem solving on the development of creative thinking skills for prep school students in Arab schools in Israel”, Hassan Elhouari, Alioune Khoule, And Nana Osei Bonsu with the article titled “Improving students’ mathematics achievements using classroom interventions”, Mehmet Kirmizi with the article titled “What I learned from Budapest Semester in Mathematics Education”, Adnan Aydin with the article titled “Activity for teaching mathematics for students with learning disabilities with analogy method : division with and without a remainder topic” have contributed. In addition, we would like to thank the referee, editor and all my friends who took part in the layout.

Editorial of JMETP

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Research Article

Conceptual technique for comparison figures by geometric thinking in analysis level

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Geometric thinking

Abstract

In underdeveloped countries, research in mathematics education has been mostly focused on students' geometry abilities based on levels, learning approaches, and textbooks. But, thinking process and level are a problem relevant to the low quality of student achievement. The process and level of students' thinking are due to the conceptual system in operating. In this study, a geometry question at the analysis level was designed to investigate conceptual systems. Students represent and compare two figures by their techniques. Data obtained from the survey and narrative study. Data were analyzed based on three components of activity: input, internal processing, and output. Students represent by copying, revising symbols, rummaging objects, and reconstructing properties. They analyze property geometry on the building block or spatial representation. Students compare through one of the two process models of think, namely: object extraction techniques to structure-property connection and inter-object connection to property extraction. The systematic paths of the two models are different. One produces a creative conceptual formulation before extracting geometry properties. Its creativity is involved in comparisons so there is a leap to a more objective point of view. Therefore, conceptual systems and construction for the conceptual formulation are two ideas for learning situations or solving problems.

To cite this article

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Introduction

According to the international assessment report, the geometry achievements of Indonesian students in primary and secondary schools are still in the low international benchmark category (Alghadari, Herman, & Prabawanto, 2020; Sandy, Inganah, & Jamil, 2019), because most students are only able to work up to level 2 of 4 (Hidayah & Forgasz, 2020). The level of student achievement has to do with the status of geometry education in the curriculum and the learning environment related to what is provided to them (Silfverberg, 2019). In the Indonesian educational curriculum, geometry is introduced from elementary school students (Purnomo et al., 2019), to advanced levels (Alghadari & Noor, 2020; Alghadari et al., 2020; Jelatu, Sariyasa, & Ardana, 2018). There are different geometry concepts studied at each school level where these differences are based on considerations of cognitive development theories, such as Piaget and van Hiele's theory (Lesh & Harel, 2003), which are related to students' thinking level and ability. At the basic level, students learn geometry shapes (class of shapes) such as triangles or rectangles (Morales et al., 2018). At the intermediate level, students explore the geometry properties in a shape so that learning leads to conceptual matters (Mahendra et al., 2017). Whereas at an advanced level, students learn geometry concepts based on axioms and definitions. The difference between the three is the thinking stage of students who operate through the conceptual system based on the level of cognitive development required. Based on consideration of the international assessment reports, we can assume how students' geometry thinking ability at an advanced level is due to problems in the conceptual system and their development at the previous level. On the other hand, the modern style learning

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environment has emphasized the use of dynamic geometry with exploratory and experimental activities, but its effectiveness is still not widely known (Silfverberg, 2019).

The focus of our review has been on the question of why students' learning geometry was problematic so that the quality of their achievement was low. We summarize several causes and among them are because: students are less skilled for non-routine problems involving high order thinking (Hidayah & Forgasz, 2020), problems of synchronization between concepts in problem solving (Alghadari et al., 2020; Jupri, 2017; Sumule, Amin, & Fuad, 2018), the conception of the problem is due to limited conceptual knowledge (Alghadari & Herman, 2018; Alghadari & Noor, 2020), the problem of the association between contextual features on the problem and reality context of students understand (Purnomo et al., 2019), problems from students' understanding of the abstract things (Jelatu et al., 2018; Mahendra et al., 2017), as well as problems of the relevance between the level of geometry abstraction and the visualization ability to an abstract object (Riastuti et al., 2017; Sumule et al., 2018). The summary of these problems is relevant to the thinking process and level (Noor & Alghadari, 2021; Silfverberg, 2019). We quote an illustration related to students' thinking processes and levels by Byers (2020), about the conceptual system between integers and rational numbers, where there are children who say that no numbers between 2 and 3, but the other say that there are many numbers between that. Here, differences in processes and levels of students' abilities are due to the operating conceptual system while they think (Noor & Alghadari, 2021; Palmiero, 2020). This difference underlies our study because research has currently more focused on the students' achievement of geometry abilities based on level (e.g. Fitriyani, Widodo, & Hendroanto, 2018; Lutfi & Jupri, 2020; Prayito, Suryadi, & Mulyana, 2019), learning approach (e.g. Alghadari et al., 2020; Jelatu et al., 2018; Mahendra et al., 2017), and textbooks (Hidayah & Forgasz, 2020; Purnomo et al., 2019), so there have not been many studies on operating conceptual systems student do. The conceptual system is one of the causes of differences in students' geometry thinking levels and abilities (Battista et al., 2018; Heyd-Metzuyanim & Schwarz, 2017; Noor & Alghadari, 2021; Olkun et al., 2005).

A coherent conceptual system is the basis of planning (Sierpinska, 2005), and the resulting model in the problem-solving process (Lesh & Harel, 2003). By definition, a conceptual system consists of concepts and relationships at different levels, or a mixed conceptual space if it has physical or mental elements (Burgin & Díaz-Nafría, 2019). The conceptual system is a pathway based on the thought process. When the pathway contains a wide variety of concepts and relationships, as many concepts as considered involved show a structure with a higher degree of abstraction (Burgin & Díaz-Nafría, 2019; Lesh & Harel, 2003), so the conceptual system describes the level of thinking ability. We analyze it in the context when students have been thinking about answering the geometry question posed. The adopted thinking level on the geometry question is based on the relevance between the problems summarized and the geometry achievement category of Indonesian students according to the International assessment report. Therefore, we set this study at the analysis level of geometry thinking. In practice, the transitioning process of conceptual knowledge when students answer geometry questions at the analysis level begins with the class of shape as an object of thought to property of shape as a product they produce (Van de Walle et al., 2017).

Purpose of Study

Class and property of shape are concepts that will be analyzed to transparency the conceptual systems in students' thinking processes from the visual objects to abstract properties. The importance of the thinking process is as a first step towards abstraction thinking to the development of mathematical thinking at the axiomatic level. For this reason, this study aims to investigate the conceptual system when students think geometrically at the analysis level.

Method

Research Model

In this study, we investigate the coherence between the three components of human activity according to Tall (2013), namely input, internal processing, and output. We analyze how the response appears, what plans are made, and what is followed. We break down the systematic stages of the thinking process by students based on how they compare. A coherent and systematic thought process creates a schema, with the relationship between input and output, and is based on the appropriate concept. When the design thinking is coherent, the model is a structured schema that forms a conceptual system (Burgin & Díaz-Nafría, 2019; Lesh & Harel, 2003). Furthermore, Lesh & Harel (2003) stated that van Hiele's theory of conceptual development can be used to help, understand, and explain the many behaviors that students exhibit during the development process in selecting the necessary conceptual tools in modeling activities. Some of these references underlie this research model which we designed following Figure 1.

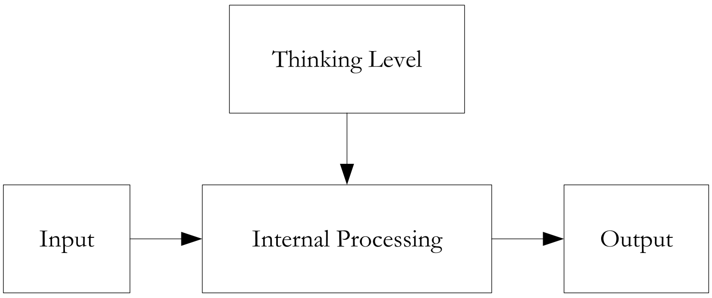


Figure 1.
Model for Synthesizing a Conceptual System in Thinking

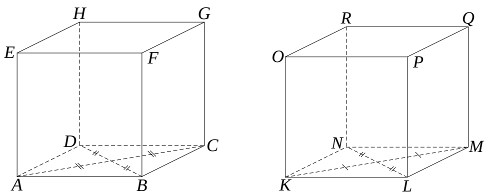
Participants

This study was conducted on 61 people from 12th-grade students at Indonesian Public High Schools in the 2019-2020 academic year. Students who participate are those who have studied the geometry of three-dimensional figures. Some students come from two study groups at a school located in a rural area in Belitung Regency. They were selected based on the major program and the cross-interest subject (minor program) by stratified random sampling.

Data Collection Tools

Various concepts have been studied by students. For example, differences in the properties and shapes of a plane that involve geometry elements such as points, lines, and planes. In the three dimensional-figure, such as cubes and prisms, many concepts are contained abstractly by their representation, including differences in their properties, shape, and relevancy based on the shaped planes of the two figures. Based on these different properties, we designed it to be a geometry question to use for investigating the conceptual system of students' thinking. The question refers to the analysis level according to van Hiele's theory with the classification of objects and product of thought following Van de Walle et al. (2017) namely class and property of shape respectively.

Question: Figure (a) is a cube *ABCD.EFGH*. Figure (b) is a rectangular prism *KLMN.OPQR* with two parallel sides that are rhombus. The lengths of all edges in both figures are the same. Write differences of properties in the model of the pictures, also state your reason why?



(a) (b)

Process

Based on the question above, our first step in obtaining data is through a survey. The survey data were the students' responses for the differences of properties in the model of the pictures. Then, the survey results were followed up with an explanatory narrative study. The data from the results of the narrative study were the students' reasons for answering the survey questions. Both data were analyzed and synchronized. The domain that we identified from the response was conceptual knowledge so that we obtained class and property of shape. Its domain becomes the basic element for students to compare the geometry concepts in two figures. When students compare geometry concepts, there is an internal process in operating and the students' reasons clarify the existence of the comparison process. The underlying technique of conceptual comparison for the two figures, we call that is the conceptual technique for comparison.

Result and Discussion

Our findings are based on an analysis of the internalization of students' thinking contains several processes that occurred before they compare the two figures, namely the copying stage of Figure (a) to produce Figure (b), revising symbols, rummaging objects, which are then accompanied by reconstructing properties. Figure (a) is a familiar solid figure with students' learning experiences. Then, in their learning experiences at school, they often and mostly symbolize the vertices of the cube with *ABCD. EFGH*. However, Figure (b) is not a model they usually encounter when studying geometry or shapes. On the other hand, the two drawing models have been presented in front of them on the question sheet so that the model can be a visual mediator or reference for the construction process. These references support students thinking to represent (Battista et al., 2018; Silfverberg, 2019). Representations have invited students to recall or create experiences and continue with revising, rummaging, and reconstructing. Here, the process in the revision stage is simpler than rummage and can occur in parallel. The rummage stages are from a representation cube *ABCD. EFGH* to prism *KLMN. OPQR*, and at the same time it includes that there is a rummage process from square *ABCD* in Figure (a) to rhombus *KLMN* in Figure (b). Following these findings, Rowlands (2019) states that the process of constructing a new conceptual system is the process of dismantling or complementing the old. In this case, the reconstruction occurs because there is a change in shape in figure (b) and dismantling symbols at the vertices of Figure, from *ABCD* to *KLMN*.

The changes in the object are due to changes in shape and length on the two parallel diagonals in the rhombus as the base and roof of Figure (b). The reconstruction from Figure (a) to Figure (b) is by "rummaging object form", that is from square to rhombus. Relevant to the results of the study by Chotimah & Jannah (2020) that the aspect of internalization when students represent objects abstractly is constructing the appearance of building blocks from spatial shapes. In this internal process, the property figures other than the diagonals of the sides of the base and the roof along with the angles in these shapes are still in the capsulation. We view the process as a process-to-concept encapsulation, which should represent the Figure from the beginning of the construction process, but the processes are encapsulated as a concept from the unchanged Figure representation. Because demolition of property in the representation is not carried out in its entirety but is adjusted to the needs of the change, and the construction of the representation is not carried out from an early stage, so these activities become relevant when we say it with reconstruction. The reconstruction of the Figure representation, in this case, is an internal process for modeling the figure. This case is a model or student's way of representing figures of the same size and shape. Lesh & Harel (2003) stated that a model is a conceptual system that is generally expressed using interacting representations, which may involve writing symbols or experience-based metaphors. Therefore, this finding is a clue about one of the identities of the conceptual system in the geometry thinking process of high school students at the analysis level for the case of representing internally.

The geometry properties that construct two figures in each representation are compared by students. Students involve competence from cognitive factors, namely figural property construction (Chotimah & Jannah, 2020; Rivera, 2018), and figural conceptual constructs (Patsiomitou, 2018), which influence the appearance of mathematical structures. The base plane includes given information in the question which is represented by the rectangle in Figure (a) and the rhombus in Figure (b). A shape as a base of two Figures is a comparison object by students has been a basic design to elicit their response. Based on the students' responses, some of the products of thought they produced were about the base of solid as the class of shape, namely the edges or angles of their intersection, the diagonal of the sides or the length, the angle of intersection between the diagonals of the sides or the magnitude, and the area of the base. All of them are property of the building block in the figure. Then, another response from students is their point of view on other areas of the figure, such as the diagonal of space which is also one of the properties of the two figures. The diagonal of space is a spatial property. The results of our investigation show that the response to space diagonal is the focus of the next analysis after switching from the properties in the base plane. Based on these findings, the categories for the two classifications of property of figure as the product of thought are based on students' abilities in spatial dimensions, namely property on the building block or spatial representation (Jirout & Newcombe, 2015).

Conceptual Technique for Comparison

The results of the identification of objects and product of thought provided the basis for us to continue to investigate how students compared the two figures. This investigation is on the relationship between input, internal processing, and output as a component of human activity, namely the systematic operation of geometry concepts in figures. In the results of this investigation, we found two categories of students' techniques comparing Figures. First, the properties between the figures are then encapsulated in a geometry relationship. An example of student response from the first category is that the side diagonal of one figure is longer than the other. We categorize this based on the process of transparency of the properties of the shape of the two structures that precede the comparison process, so we address this fact with the product of thought in the analysis level of geometry thinking as the basis for comparison. In other words, thinking about geometry at the level of analysis precedes the process of mathematical connections. Therefore, an image can be considered alone, as a graphic object, or as one of the possible representations of a geometry object. In the case of this first category, the product appears through empirical abstraction because the student focus is on objects and their properties (Gray & Tall, 2007). Furthermore, Mithalal & Balacheff (2019) explained that figures are associations of theoretical references from the two determinants, namely the relationship between geometry properties and geometry objects. The comparison technique of the first category is based on theoretical references to geometry properties, and we categorize them as object extraction to structure-property connection. Meanwhile, the comparison technique determined by geometry objects, or inter-object connection to property extraction, is categorized as the second conceptual technique as follows.

The second, the properties of each object in capsulation and then compared. Examples of student responses from this second category are the diagonal lengths of the sides and spaces of the two different figures. The results of our confirmation of the students' responses that the response model from the second category is not based on the results of one-by-one analysis properties of shape in each figure but it is the consequences of analysis on the shape of the basic plane of the two figures. The property of the figure encapsulated in the object is reframed by the students because the question is the differences of properties that should direct the focus of their analysis. Byers (2020) calls such framing efforts a conceptual world of mathematics in which logical processes live exclusively and control that effort. Piaget calls cases from this second category an empirical pseudo-abstraction that focuses on actions on objects and the properties of the actions (Fitriani, Suryadi, & Darhim, 2018a,b; Scheiner & Pinto, 2014). The basis for comparison of this second category is the object so that students' geometry thinking, in this case, occurs after they compare the class of shape. Students compare the object of thought representations then think geometrically in the analysis level occurs after the mathematical connection process, and they formulate concepts to construct a concept that can be thought of. In this second case, the geometry properties are encapsulated in a class of shape and then compared between the two figures. Students see the entire properties of shape as individual objects which then recursively extract the properties. Students compare the shape of the base planes in two figures and not their properties directly. Because there are different classes of shape, of course, there are abstract properties that are also different. Silfverberg (2019) explains that conceptualization does require synthesis through association, abstraction, and differentiation between properties before analysis. Internal processes as in this second category are not easy enough for most people and virtually impossible for the many who simply learn the rules by rote, but it is a process that seems to be performed implicitly by those who make sense of the hierarchical structure (Tall, 2013; Tall & Witzke, 2020).

Conceptual System in Representing to Comparing

The next investigation is focused on the base of the figure, namely the process of representing the figure and two conceptual techniques for comparison. These two concepts form a model linked by systematic operation. Lesh & Harel (2003) stated that the model includes accompanying procedures to produce constructs, manipulations, or predictions that are useful for achieving recognized goals. Referring to these references and based on the findings of this study, the model of the representation process is copying, revising symbols, rummaging objects, and reconstructing properties. Models are also a way of treating concepts through thinking about objects, properties, and their relationships (Burgin & Díaz-Nafría, 2019) so that the model for comparison is object extraction to structure-property connection and inter-object connection to property extraction. From this case, there are two process models of think until students respond to questions and both are different because of the consideration of the concept formulation that students think that if there are differences in the class of shape, then there are abstract properties that are also different. The formulation of concepts from the results of students' thoughts becomes an additional concept for consideration in comparing figures. The conceptual formulation has become the first point of different parts in students' conceptual systems. By involving this formula, the comparison stage becomes more objective because there is a leap to a higher point of view, and the process is said by Byers (2020) as creativity. Schoevers et al. (2019) stated that mathematical creativity can be in the form of creating new and meaningful mathematical ideas or concepts through cognitive action combining known concepts in an adequate way. Furthermore, at the conceptualization stage where the situation is more complex, problems that were difficult at first but when brought to the original space with the right logical leaps can become more transparent (Morales et al., 2018). Therefore, conceptual considerations show structures with a higher degree of abstraction (Burgin & Díaz-Nafría, 2019; Lesh & Harel, 2003; Palmiero, 2020) so that the conceptual technique from the second category with the empirical pseudo-abstraction type is superior because it is considered more sophisticated than sensory experiences (Gray & Tall, 2007; Scheiner & Pinto, 2014; Tall, 2013; Tall & Witzke, 2020).

The concept formulation is in the operational phase of the geometry thinking system at the analysis level, occurs before extracting geometry properties, and is an additional concept to the inter-object connection to the property extraction model. Van Hiele's theory has detailed objects and product of thought at the level of analysis of geometry thinking, respectively, shape and property of shape (Van de Walle et al., 2017) where these details are the pathways of the geometry thinking system that underlies the two model process students' thinking as a study finding is subdivision part of the conceptual system for comparison. The operational phase of geometry thinking in the comparison stage is when the object was extracted or the product of thought has been connected. The extraction or connection process sequence in the comparison stage confirms the division path of the geometry thinking system. This case is an example of the progressive sophistication of students' intellectual means to control the representation of the phenomenon of geometry knowledge development, that requires a specific focus on relevant aspects of a situation to name and compress into a thinkable concept (Gray & Tall, 2007; Tall & Witzke, 2020). If the model includes a conceptual system to describe or explain relevant mathematical objects, relationships, actions, patterns, and regularities associated with problem solving situations Lesh & Harel (2003), then the models of the process of think in the findings of this study are two types of conceptual systems, because Burgin & Díaz-Nafría (2019) definitively explain that a conceptual system consists of concepts and relationships at different levels, as well as concepts with their construction based on certain concept models as their basic elements.

Furthermore, Burgin & Díaz-Nafría (2019) state that models of conceptual systems exist in conceptual space such as conceptual representation theory in mind. Therefore, based on the findings of this study, our idea for learning geometry so that students can achieve a higher degree of abstraction is to formulate concepts to take logical leaps into the right conceptual space when in problem solving situations. Lesh & Harel (2003) detail that the resulting model in the problem-solving process includes the conceptual constructs and systems needed to understand complex types of systems. However, in students' thinking, the construction community and conceptual systems often compete to dominate the interpretation to be emphasized. Here, it is explained that solving problems requires both conceptual systems and construction (Schoevers et al., 2019). The first, the conceptual system is a line of systematic thinking that is operated both when students learn and when solving problems. However, when learning only emphasizes systematic thinking, the result will give birth to procedural tendencies. Based on the notes from several research results, it shows that procedural knowledge controls students' conceptual knowledge and not the other way around (Alghadari & Noor, 2020; Morales et al., 2018). The second, construction that produces a conceptual formulation is a phase to enter another different conceptual system in the world of mathematics and an attempt to leap into a more transparent space. Creating new and meaningful ideas in mathematics is by break away from established mindsets (Morales et al., 2018; Palmiero, 2020; Schoevers et al., 2019). All this time, especially in studying geometry, the effective relevance of the learning approach and creative conceptual construction has not been seriously considered (Hidayat et al., 2017; Nugraheni et al., 2018). Thus, one of the pieces of information for researching special thinking skills in students' geometry learning for better achievement can be started by integrating the conceptual system and the construction of creative concept formulations. Whether efforts to improve students' thinking skills have involved the model of the two elements suggested by Lesh & Harel (2003) in learning geometry is a question that needs to be investigated at another stage.

Conclusion

In the geometry thinking process, especially at the analysis level, when students represent and compare figures, there are two types of models of conceptual systems in operating. The model for representing is no different, namely copying, revising symbols, rummaging objects, and reconstructing properties. But the model is different when students start comparing figures. The different models are based on the following conceptual techniques, object extraction to structure-property connection or inter-object connection to property extraction. There is another consideration with a conceptual formulation that appears in the conceptual technique from the type of inter-object connection to property extraction so that it becomes the first point of different parts in students' conceptual system. Concept formulation is an additional concept for figural comparing technique and has resulted in a leap of thinking to the right space and a more objective point of view. The problems are difficult but with a leap of logic so that can become more transparent. For learning or solving geometry problems so that students can achieve a higher degree of abstraction is by formulating concepts to take logical leaps into the right conceptual space. Therefore, it takes both the conceptual formulations constructed and the conceptual system for understanding the types of complex systems when in problem solving situations.

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Research Article

The impact of the use of mathematical problem solving on the development of creative thinking skills for prep school students in Arab schools in Israel

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Abstract

In underdeveloped countries, research in mathematics education has been mostly focused on students' geometry abilities based on levels, learning approaches, and textbooks. But, thinking process and level are a problem relevant to the low quality of student achievement. The process and level of students' thinking are due to the conceptual system in operating. In this study, a geometry question at the analysis level was designed to investigate conceptual systems. Students represent and compare two figures by their techniques. Data obtained from the survey and narrative study. Data were analyzed based on three components of activity: input, internal processing, and output. Students represent by copying, revising symbols, rummaging objects, and reconstructing properties. They analyze property geometry on the building block or spatial representation. Students compare through one of the two process models of think, namely: object extraction techniques to structure-property connection and inter-object connection to property extraction. The systematic paths of the two models are different. One produces a creative conceptual formulation before extracting geometry properties. Its creativity is involved in comparisons so there is a leap to a more objective point of view. Therefore, conceptual systems and construction for the conceptual formulation are two ideas for learning situations or solving problems.

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Introduction

The goals of teaching mathematics have gone through many different stages. In the past the goal of teaching mathematics was to focus on the accuracy and speed in performing calculations. But the technological developments have reduced the importance of this goal since a small calculator can perform all these operations more accurately and quickly. Therefore, the goals of teaching mathematics have changed to focus on understanding and perceiving the meaning, and this requires a focus on understanding mathematics as an independent, interconnected subject with its own fundamentals, problems and self-pleasures. While this goal may be sufficient to create a plenty of theoretical mathematicians, it may not be an excuse to overburden pupils with many subjects of mathematics, the primary goal of education as a whole is to prepare the individual to become a useful member for himself/ herself and his/ her community. The important question here is: How does mathematics contribute to this goal? The continuing and growing problems facing humanity require rapid and growing development in the methods of solving them. Therefore, mathematics helps to prepare a useful individual by developing his/her ability to solve the problems of the renewed life with all their types and times. The importance of solving mathematical problems in school comes from the fact that it is the primary objective of teaching and learning process.

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Knowledge, skills, concepts, mathematical generalizations and even all other school subjects are not only goals by themselves, but also they are means and tools that help the individual to solve his/her real problems. In addition, problem solving is the natural way to practice thinking in general, there is no mathematics without thinking and there is no thinking without problems. So the objectives of teaching mathematics focus on developing understanding, meaning and skills alongside with the basic processes, which contribute to rapid scientific development which result endless problems in one's life. It is considered the main goal of problem solving in mathematics is to train students on some ways and methods that help them to solve problems in general.

The school mathematics Study Group in the USA (SMSG) has developed the following set of problem solving goals:

- Provide the student with different types of strategies that help him/her to solve problems.
- Develop the flexibility of student in the way of processing and initiating problem solving.
- Develop some methods to apply geometric representations in the production of new information about the problem.
- Develop some skills in scheduling and organizing the given information and derivatives to take advantage in the solution.
- Deepen the student's understanding of the problem by accustoming him/her to the work of numerical estimates based as in the light of the problem posed

From the above, the importance of problem-solving methods is marked in the following points:

- Help student to discover new concepts.
- Teach student how to develop the concept to use it in solving a new problem.
- Accustom student to critical scientific thinking.
- Help to join and match mathematical concepts.
- Develop some of the student's mental abilities such as visualization, abstraction, analysis, and synthesis.
- Stimulate student's curiosity and discovery.
- Develop student's ability to analyze situations and make decisions.

Mathematics also is one of the study materials that aims to develop creativity and creative thinking. Creativity is not achieved out of the blue; it must be preceded by a problem that challenges the mind, so mathematics can be taken as a field for the development of creativity and creative thinking. Its structural nature allows more than one logical conclusion to the same given introductions, and its connotation structure gives some flexibility in the organization of content. Moreover, mathematics is rich in problem situations for which students can find for each situation multiple and diverse solutions, and its study teaches the student to employ objective criticism of the situation. This will provide the student with some of the basic capabilities for the creative process. Creative thinking in mathematics can be learned as a skill, and then developed with more training, since each learner possesses some degree of thinking (Mufti, 1995).

Mathematics represents an important field of education since it reveals the capabilities of creative thinking and its development for learners in all grades. Mathematics is not just a collection of facts and information, but mainly a way and style of thinking to face mental problems; therefore, successful teaching of mathematics affords learners the capabilities and methods of creative thinking (Aladdin & Abdel, 2003).

Mathematical creative thinking as a science is different from creative thinking as a study subject. Creative thinking as a science seems obvious when the learner solves the mathematical problem in an independent way, not previously known to him, but creative thinking in mathematics as a study subject appears if the learner knows that many mathematical questions can be solved in more than one way, and this itself is the essence of creative thinking (Roshka, 1989).

Creative Thinking Skills

A review of most common tests of creative thinking, that Torrance (1966) and Guilford (1967) tests indicate the most important skills and abilities of creative thinking that researchers have tried to measure, that are:

Fluency

Fluency refers to the ability to produce as many ideas and solutions to a problem as possible. Fluency in mathematics means the learners' ability to give several different solutions to a particular topic or issue. It means accustoming pupils to give several different solutions to a particular topic, issue or obstacle, so that they have the ability to recall the largest number of ideas when exposed to a particular mathematical or geometric problem, and then choose the solution or idea that the pupil finds most convincing. Fluency is divided into sub-aspects: verbal fluency, fluency of thoughts and fluency of expression.

Verbal Fluency

The verbal fluency of mathematics may not be as important as in languages, for instance. We mean the speed of the individual to think and to provide words, or mathematical synonyms, or their imperfections, and generate them in a certain format, or the ability of an individual to produce as many mathematical vocabularies as possible within a given specification in a certain period of time (Austin, 1988).

Write as many attempts as possible to solve the following question:

$$y = -x^2 + 4x - 7$$

A man who is now (71) years old and his son is (33) years, how many years ago was the father three times the age of his son?

The answer……………….... years.

1) 14 2) 11 3) 15 4) 21

List the largest number of objects around us whose size can be calculated?

Fluency of Ideas

Fluency of ideas is the individual's ability to give as many mathematical ideas as possible associated with a certain perceived situation, for example (Davis, 1981):

- Mention all the consequences of doubling the population of Israel.
- Write down as many results as possible for doubling the length of the day to 48 hours.
- Every line has to be the same sum, find the value of y (see figure 1),

17			y
13	**15**		
	10	**11**	**17**
	y		**14**

Figure 1.
Find the Value of y

c. Fluency of Shapes (expression):

Fluency of shapes refers to the ability to change shapes with simple additions, and the ability to quickly draw a number of examples and preferences or adjustments in response to a particular visual stimulus. Examples (Brousseau, 1991):

Example: What can be shaped from the following forms (see figure 2)?

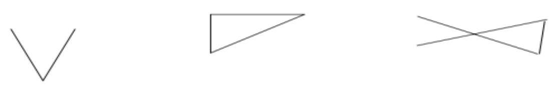

Figure 2.
What can be shaped from the following forms? Question Figure

In the previous example, the student should draw whatever s/he wants to draw, geometric or non-geometric shapes, and the more the answer or the shape s/he draws is meaningful and unique, the more it indicates his/her innovative abilities.

We want to turn the triangle into various drawings that express objects or things. What additions can you add for that (see figure 3)?

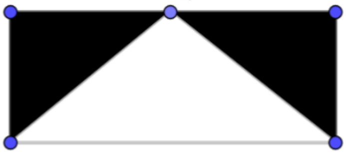

Figure 3.
The Triangle

This is an example of one of the students' solutions (see figure 4):

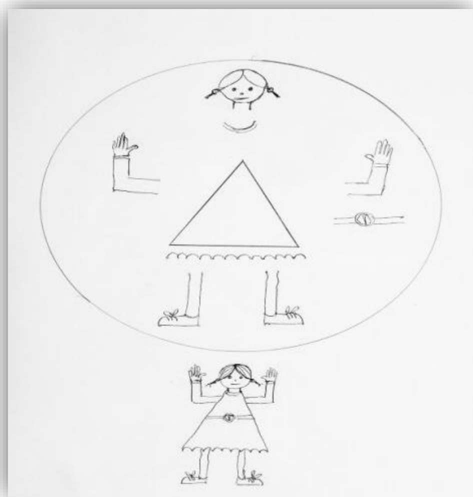

Figure 4.
This is an Example of One of the Student Answer

Flexibility
Flexibility is the ability to vary the mathematical answers and solutions. Flexibility in mathematics means asking learners to mention as many as possible properties of a drawn geometric figure. Here we can notice the development of the student's thinking, and the flexibility that s/he shows in the production of the greatest number of ideas to achieve the presented mathematical situation. One kind of flexibility is that the student can change the way of research, and stop searching in a particular way or narrate preconceived ideas. Another a kind of flexibility is that the student must be aware of the things s/he is looking for, maybe more important than the things s/he needs. Forms of Flexibility:

Automatic Flexibility: It is the speed of the individual to produce as many diverse ideas as possible associated with a problem, and according to this ability the individual tends to automatic initiative in these situations, and does not just respond (Al-Khalili, 2005).

Adaptive Flexibility: It means finding a solution to a problem or facing any situation in the light of the feedback that comes from that situation (Al-Taati, 2004).

The researcher believes that flexibility is the individual's ability to give multiple and different inputs and ideas to solve a problem. Examples of Flexibility (RAMA, 2017):

- Mention the uses of the caliper ruler for (student, tailor, carpenter, blacksmith …….)
- Think of all the ways you can design to weigh very light objects.
- If there were six people at a party and everyone wanted to shake hands with the others only once, how many times did they shake hands at this party?

    a) 36 b) 18 c) 15 d) 12

Originality

Originality is the ability to produce mathematical ideas unfamiliar to one's colleagues. Originality in mathematics means the ability to produce authentic responses, i.e., responses not commonly repeated among the colleagues of the person who comes up with these responses. It can be measured in mathematics by asking the learner to give several different solutions to the same mathematical situation, such as giving more than one method to solve a given geometric exercise, or solving an algebraic question by more than one method, for example, (Crrouse, 1987):

In this figure (5), AB, CD are two diameters of the circle perpendicular, M, N two points, and MX, NK, NG, MV, are columns on AB, CD as shown.

To prove that $GK = XV$, the traditional solutions are either:

- Application of triangles $\Delta XEG \cong \Delta EGK$
- Application of Pythagorean theory.

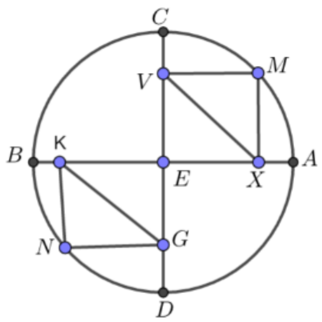

Figure 5.
The circle

Elaboration:

Elaboration means the ability to give many details or make new additions to an idea, problem or mathematical question to develop or enrich it. With this skill, more precise details of the case are discovered or identified and highlighted. For examples (Davis, 1981):

Ahmed bought 10 pens …………….

Add what you want to the question so that it can be solved using a process of:

a) addition b) subtraction c) multiplication d) division

In each item there is a set of triads and there is an adjective to achieve, except one triad has not achieved it. Draw a circle around the irregular triad, and write down what is the adjective: (4, 2, 4), (5, 1, 4), (2, 5, 4), (5, 3, 2), (2, 3, 5), (2, 2, 6).

Sensitivity to Problems

Sensitivity to problems means vigilance to what is in the mind when solving or researching a particular mathematical situation and being attentive to anything new or every change in the path for researching the problem or solving it. Problem sensitivity means awareness of problems, needs, or weaknesses in the educational situation. Sensitivity to problems in mathematics means that some pupils are faster than others in noticing the problem, checking its presence in the situation, and linking the data to their previous experiences. For examples (Brousseau, 1991):

Khalid bought 6 books, 9 notebooks and 7 pens for 15 NIS. If the price of one pen is one NIS. Can you help Khalid to know the price of one book? How?

Notice the outputs of the examples and write down the output of example four?

$$11 \cdot 11 = 4$$
$$22 \cdot 22 = 16$$
$$33 \cdot 33 = 36$$
$$44 \cdot 44 = ?$$

a) 0 b) 16 c) 48 d) 64

The correct answer is b.

Multiple-Choice

Example: Which number, when it is squared, decreases? The answer is ……

a) negative integer
b) positive integer
c) number 1
d) any fraction in which the numerator is less than the denominator

The correct answer is d because:

For example, if we take the number $\frac{1}{2}$ and square it, it becomes $\frac{1}{4}$, which is less. while negative numbers and positive numbers increase when squared, and number 1 remains the same.

Factors Affecting Creative Thinking

We cane summarize the factors that affect the creative thinking in the following chart (figure 6):

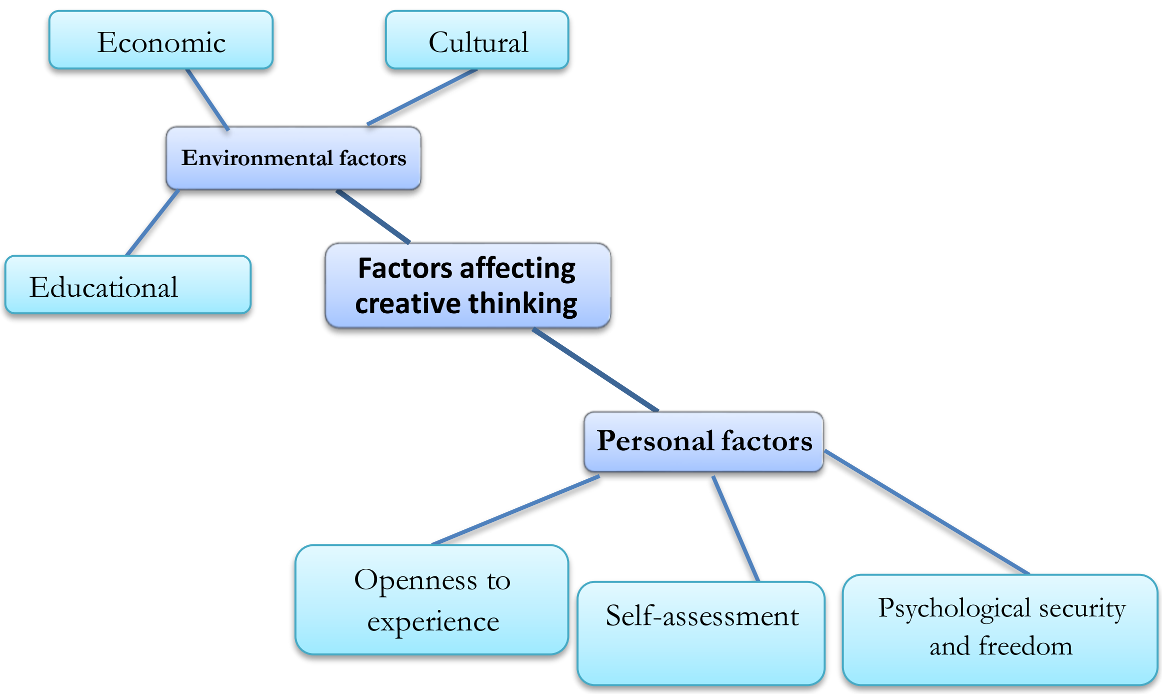

Figure 6.
Factors Affecting Creative Thinking

Personal Factors

There are some factors:

Self-Assessment: Self-assessment means that the individual has a sense of self-confidence, knows well how to evaluate it (high, medium, or low rating), and how an individual sees himself from his personal perspective.

Psychological Security and Freedom: Psychological security is a feeling of reassurance that means lack of fear, absence of anxiety and having self-confidence, while the opposite is called psychological insecurity, which means constant anxiety, lack of confidence and severe phobia from others.

Experience Acquisition (Openness to experience): Experience acquisition relates to many interests and is influenced by imagination, insightful vision, previous knowledge and the acquisition of various experiences.

Environmental Factors

Environmental factors refer to lack of proper place, overcrowding, lack of support from colleagues, lack of material support and socialization in a bossy family.

Educational Patterns: Educational factors refer to the diverse teaching methods and the ways used by an individual in learning, or the teacher in education, as well as the place where the learning process takes place.

Economic Level: A person may not pass through the process of learning, going to university, completing high school, or experience society rejection and criticism for creative ideas, with the lack of alternatives and the lack of appropriate reinforcement for the creators (Al-Ayasr, 2013).

Cultural Level: This refers to the environment surrounding the person, when all people around him are not educated, or do not have the cultural background that appreciates the importance of education. This cultural poverty creates a kind of frustration for the learner. Family is considered the first cell and the basic building block in the building of society, and the place where the individual receives his/her life lessons. It has an impact on the formation of personality and behavior, so the family plays a big role in the development or suppression of the child's creative abilities. For example, a domineering father who imposes his opinions and does not allow to his children to express their opinions, negatively affects the personality of the children and damages their self-confidence, and serves to suppress the creative abilities of the individual (Abu Latifa, 2009).

Personal Stimuli: Personal stimuli are within the person, giving him/her the motivation to reach the goal s/he wants.

Obstacles of Creative Thinking

Several references have indicated that there are many and varied obstacles that stand in the way of the development of creative thinking skills and effective thinking, and perhaps the first step that teachers, coaches and parents should pay their attention to is identifying these obstacles; so that they can be effectively overcome them when applying an educational or training program aimed to develop creative thinking skills (Treffinger & Isaksen 1985). Obstacles of creative thinking were classified into two main groups – personal obstacles and situational obstacles, which we summarize below:

Personal Obstacles

Poor self-esteem: Poor self-esteem is an important factor in creative thinking, because poor self-esteem leads to fear of failure, risk avoidance, and unsafe consequences situations.

Tendency to Conformity: The tendency to comply with the prevailing norms hinders the use of all sensory inputs, and limits the possibilities of imagination and expectation, and thus sets limits on creative thinking.

Excessive Enthusiasm: A strong desire for success and an overzealous enthusiasm for achievements lead to a rush of results before the situation matures, perhaps jumping to a late stage in the creative process without exhausting the prerequisites that may take more time. For example, (Austin, 1988):

Use numbers from 1 to 9; write one number in each circle so that you get a total of 23 in each direction (see figure 7).

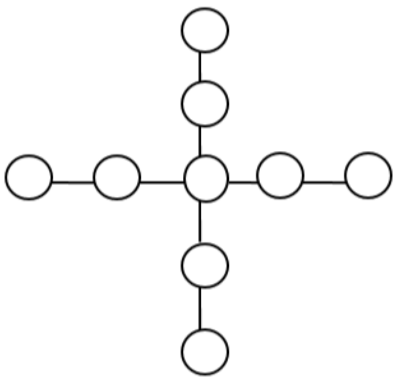

Figure 7.
Example

Saturation

Saturation means reaching a state of overexposure that may result in a loss of awareness of the merits of the status quo, or inaccuracy of views. Saturation is an anti-incubation state or a phased storage of an idea or problem, for example (Austin, 1988):

Use the numbers $\{0, 1, 3, 5, 6\}$ to solve the following code:

$$\begin{array}{ccccc} & & A & A & C \\ & \times & & A & C \\ \hline & N & M & 7 & C \\ N & W & W & C & 0 \\ \hline N & N & 7 & 2 & C \end{array}$$

Figure 8.
"Use numbers from 1 to 9; write one number in each circle so that you get a total of 23 in each direction" Question Figure

Stereotypical Thinking:
Stereotypical thinking is a form of traditional thinking, restricted to habits; Isaksen and Treffinger (1985) considered it the most prominent obstacle to creative thinking. To illustrate the impact of this obstacle, De Bono gives a symbolic example:

A dog used to walk a long way to get the bone his owner put it in the same location behind a fence (see the figure 8). Since the first successful attempt to reach the bone was achieved by taking this long road, the dog held on to it, and it became a habit he did it automatically. If the dog could be guided to this obstacle, he would be able to abandon his habit and find the shortest way to reach his goal.

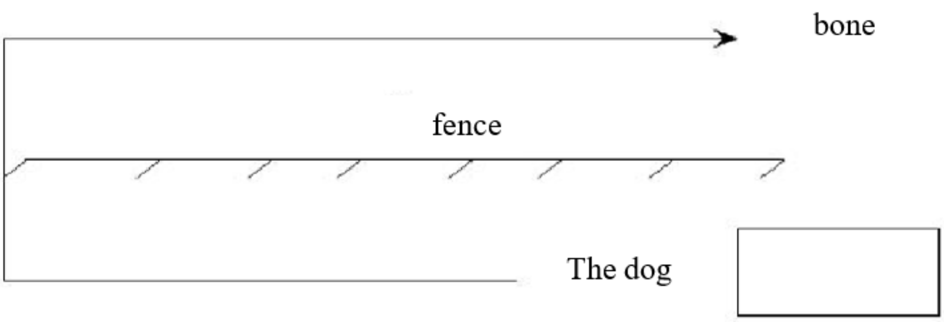

Figure 9.
Example

Another example:
You enter a room with two ropes hanging from the ceiling, and you are asked to tie the two ropes together. There are some tools on a table in the corner of the room, including a hammer, pliers and scissors. You hold the end of one of the ropes and walk towards the other, but you quickly realize that you cannot reach the end of the other rope. You try to extend the range you can reach with the hammer, but it doesn't work. What can you do to solve the problem?

The solution: This problem can be solved using the available tools in an unconventional way. If you attach the hammer to the end of one of the ropes and wave it like a pendulum, you will be able to hold it in the Isaken & Treffinger, (1985) explained this failure by the tendency of individuals to stick to the familiar uses of things, and described this tendency by the terms "inertia" or "functional constancy".

Insensitivity or a Feeling of Helplessness
One of the necessary characteristics of the process of creative thinking is vigilance and a delicate sensitivity to problems. When the sensitivity is weakened by lack of excitement or lack of challenge, the person becomes more inclined to stay in the circle of reactions to what is happening around him, giving up the initiative to explore the dimensions of the problem and to find solutions (Isaken & Treffinger, 1985).

Haste and Improbability of Ambiguity
This trait is related to the desire to find an answer to the problem by taking the first opportunity, without understanding all aspects of the problem, and without working to develop several alternatives or solutions to it, and

then choosing the best one. One of the problems associated with this trait is the unpredictability and evasion of complex or ambiguous situations, and escaping from facing it. Postponement of judgment is also an important characteristic of creative thinking; for example, when brainstorming is practiced, judgment is allowed only after every possible opportunity to generate ideas has been exhausted.

Transfer Habit

When certain mental patterns and structures that have been effective in dealing with new and diverse situations are entrenched, other, more effective strategies are often ignored, and some killer phrases that summarize this obstacle include: "We've always been doing this successfully," or: "We've always been solving the problem this way".

Situational Obstacles

Situational obstacles to creative thinking mean those related to the situation itself, or to prevailing social or cultural aspects. The most important of these obstacles are:

***Resistance to Change*:** There is a general tendency to resist new ideas, and to maintain the status quo by many means for fear of their repercussions on the security and stability of the individual, there are those who believe that modern experience poses a threat to their gains and conditions, and therefore, they respond by using 'deadly' phrases to any new idea (Brousseau, 1991), such as:

- ❖ It's not going to work in statement questions.
- ❖ This idea of solving the question is very long.

For example, three consecutive natural numbers totaling 300. What are they?
Since the quotient of 300 by 3 is 100. There are three possibilities:

100,101,102

99,100,101

98,99,100

The second possibility is the correct answer.

Imbalance between Competition and Cooperation

There is a need to combine the spirit of competition and the spirit of cooperation for both the individual and the group to achieve new achievements, and excessive consideration of either of them may cause loss of contact with the real problem or progress in solving it; therefore, balance between them is a condition of productive or creative thinking (Coxeter, 1986).

Example for the fifth grade students:
The lengths of the sides of this rectangle are 2 cm and 6 cm (see figure 9):

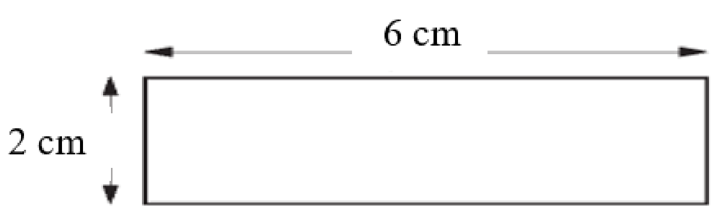

Figure 10.
Example

Indicate the polygon whose perimeter is equal to the perimeter of the rectangle

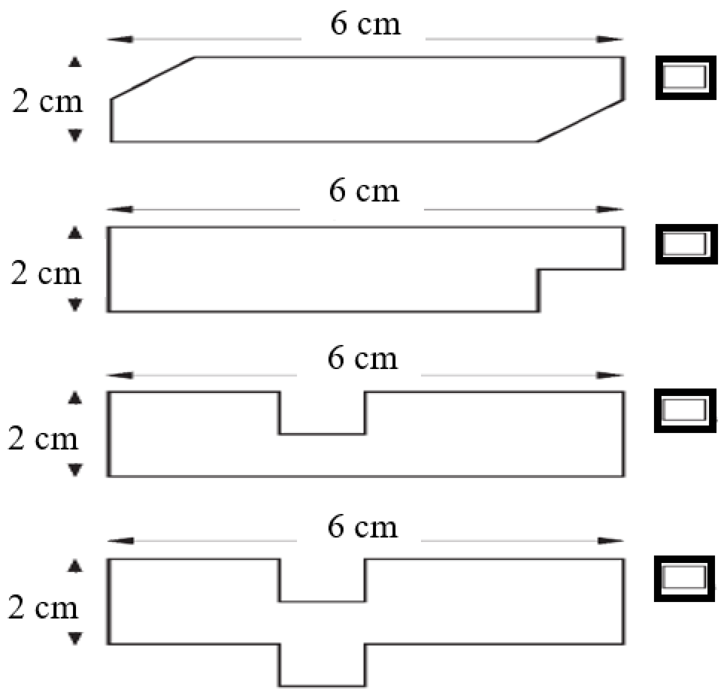

Figure 11.
Example

Comparison between Creative Thinking and Critical Thinking

Perhaps it is not possible to distinguish between creative thinking and critical thinking for the simple reason that any good thinking involves an assessment of quality and the production of what can be described as novelty. It is difficult for the brain to be preoccupied with a complex thinking process without the support of another complex thinking process.

But the outcomes of thinking vary depending on the type of task, and whether it requires creative or critical thinking. Creative thinking must include internal critical thinking.

How critical thinking is involved in creative thinking:

- Self-criticism (for a person): For example, a student criticizes himself for a mistake he made when answering the question.
- External criticism: It comes from other people for a mistake in one's answer/solution.

For example (Austin, 1998): A student solved the following equation: $x^2 - x + 1 = 0$

And from the equation $x^2 - x = -1$ and by analysis a common divisor

$$x^2 = x - 1 \quad (2)$$

$$x \cdot (x - 1) = -1$$

we compensate for the $(x - 1)$ of the equation (2) so

$$x \cdot x^2 = -1$$

$$x^3 = -1$$

$$x = -1$$

When verifying by compensation by the original equation, the solution does not verify, where is the error in which the student falls in?

The correct solution: The mistake lies in the step $x^2 = x - 1$ where this step becomes a requirement to be $(x - 1)$ positive quantity which means x is more than 1.

Trends of Creative Thinking

There are many trends and theories that explain creativity, and the most prominent are (Al-Ayasrah, 2013):

Behavioral Trend: adopted by Skinner, who defined creative thinking as the kind of thinking that is reinforced or substantiated, leading to the possibility of continuity. However, if it does not receive the required reinforcement, this thinking will diminish.

Cognitive Trend: focuses on creative thinking results through the interaction and organization of past and new experiences, while providing a certain maturity, an exciting environment for the individual, and effective training in connectivity and logic.

For example (Edwards, 1979): If a student told to you, I can prove that $5 = 7$ and gave you the following proof:

$1^5 = 1$ as well as $1^7 = 1$ no matter how high the forces = 1
That is $1^5 = 1^7$ because the two results are equal.

And there's a rule in mathematics: if the bases are equal, the exponents are equal, and vice versa. So $5 = 7$

What's wrong with this proof? (The error is that the rule has a complement.)

a) the two quantities are not equal   b) 5, 7 are odd numbers

c) the rectangle is less in perimeter   d) if the basis is greater than 1

Clairvoyance Trend: This trend in interpretation of creativity was adopted by the German scientist Fertimer, who assumed that creative thinking is clairvoyant and intuitive thinking, so the creative idea is the one in which the problem is formulated and the individual suddenly arrives at the solution by active mental processes, where the situation is treated with a new treatment (Saada, 2015).

Intuition is the formation and verification of mathematical guesses.

For example, (Edwards, 1979):

Find the result of dividing (1122) by (11) without performing the division operation:

a)120   b) 12   c)102   d) 1002

The correct answer is (c).

The Levels of Creative Thinking

Researchers' views on the topic of creative thinking differed. Turns attempted to resolve differences between those views, and proposed five levels of thinking, which are (Tashman, 2010; Al-Ayasrah, 2013; Austin, 1988):

Creative Expressions: This level refers to the development of unique ideas, regardless of their type.

For example, (Edwards, 1979):

Find uncompleted square roots: It is known that the square root of the number 9 is 3, but what about the roots of uncompleted squares? To find their value, it is necessary to estimate their approximate value, for example: Find the root of the number 85?

Productive Creativity: In this level there are strong indications of the availability of certain restrictions that regulate the free performance of individuals.

For example (Edwards, 1979):

How do you prove that the sum of the angles of a triangle equals 180^0? Mention 4 different ways to prove that? What is the simplest of these ways?

Regenerative Creativity: This level represents the individuals' ability to penetrate intellectual principles, and includes substantial improvements through modifications incorporated in new starting point's skills and concepts.

For example (Hoffman, 1988):

Is $(x+y)^2 = x^2 + y^2$ true for every real number x, y?

Innovative Creativity: It refers to the proficiency in using materials.

For example (Hoffman, 1988): Prove that $\frac{\boldsymbol{n(n+1)}}{\boldsymbol{2}}$ is a natural number for every natural number $\boldsymbol{n}$?

This level includes principles and assumptions, and it is considered the highest degree of creativity

Two numbers x, y square sum of 41 expressed this by symbols
The answer: $x^2 + y^2 = 41$ (Hoffman, 1988).

Modern methods that teachers apply to develop creative thinking in mathematics:

- Encourage the learner to think in a collective way in order to get as many ideas as possible from the discussion with the group.
- Accept the suggested ideas and help the learner to modify and develop them.
- Help the learner assume answers and test them to reach the right solution
- Do not provide ready-made solutions for the problems or ready-made proofs for theorems.
- Give questions that require deep thinking and are open ended.
- Encourage the pupil to produce something new from his/her imagination and invention.

Problem of Research

From all of the above, we can deduce the research question as follows:
What is the impact of the use of mathematical problem-solving on the development of creative thinking skills among prep school students in Arab schools in northern Israel?
The following research hypotheses can be derived:

- There are statistically significant differences at the level of the $\alpha \leq 0.05$ function between the average scores of students in the experimental group and the control group in the post application of the creative thinking test on the fluency skill in favor of the experimental group.
- There are statistically significant differences at the level of the $\alpha \leq 0.05$ function between the average scores of students in the experimental group and the control group in the post application of the creative thinking test on the flexibility skill in favor of the experimental group.
- There are statistically significant differences at the level of the $\alpha \leq 0.05$ function between the average scores of students in the experimental group and the control group in the post application of the creative thinking test on the originality skill in favor of the experimental group.
- There are statistically significant differences at the level of the $\alpha \leq 0.05$ function between the average scores of students in the experimental group and the control group in the post application of the creative thinking test as a whole in favor of the experimental group.

Methods

This chapter discusses the procedures followed in the implementation of the study, the study community and sample, the preparation of the study tool, the statistical methods used, and ensuring its reliability and stability.

Study Procedure

The researcher used the experimental method, which is defined as a method that studies a current phenomenon while inserting changes in one of the factors and monitoring the results of the change. (Al-Aga, Al-Ostath, 2020). The study design was a control group and an experimental group with pre- and post-tests. The independent variable in the study was "solving mathematical problems" and its effect on the dependent variable "creative thinking skills".

Study Community

The study community consists of all preparatory school students in northern Israel, numbering 26,600 students for the 2019-2020 academic year.

Study Sample

The sample was selected from different Arab preparatory schools in northern Israel during the first semester of the 2019-2020 school year. The researcher divided the study sample into two groups: the experimental group (40 students) and the control group (40 students), as presented in Table 1.

Table 1.
The Sample

School	Class	Experimental / Control Group	Number of the students
Arab prep schools from northern Israel	E	Experimental	40
	C	Control	40
Total			80

Teacher's Guide

The teacher's guide is considered a monitor and assistant in the implementation of lessons without problems and flops. It provides directions and guidance that help the teacher to facilitate the educational process and its progress in the right direction. It has been prepared according to the following steps:

Guide Aim: To provide a comprehensive presentation of the role of the teacher in applying the steps of solving mathematical problems in order to achieve the educational goals of the unit. It also helps the teacher to develop creative thinking skills in mathematics in general and in the unit of algebraic fractions in particular for prep school students according to the curriculum of Israeli Ministry of Education.

Guide Content: It consists of the unit of algebraic fractions and solving equations for the preparatory stage according to the Israeli Ministry of Education curriculum in mathematics, as shown in Table 2:

Table 2.
Guide Content

No.	Subject
1	Algebraic fractions
2	Simplifying algebraic fractions
3	Addition of algebraic fractions
4	Subtracting algebraic fractions
5	Multiplying algebraic fractions
6	Dividing algebraic fractions
7	Solving algebraic equations

The guide was written according to the following:

- The objectives of each subject are formulated in a behavioral way, so that the teacher can measure the achievement of class goals and student performance.
- Tools and educational means: the researcher prepared the means that suit the nature of the educational situation according to student's needs.
- Assessment and evaluation: To assess students' understanding of the educational materials.

Data Collection Tool

Creative Thinking Test

The test consisted of 12 questions, to identify and evaluate thinking skills. The objective of the test was to measure the extent to which the prep school students possess creative thinking skills. The researcher prepared the test items according to previous studies, and placed emphasis on examining the following:

Fluency: Refers to the ability to give as many answers as possible to the mathematical problem in a specified period of time.

Flexibility: Means the ability to generate varied thoughts in solving problems.

Originality: Refers to the students' ability to find unique solutions for the group s/he belongs to.

The test was prepared in its initial form with written instructions, and then presented to a competent committee in the field of mathematics including teachers in the field and experts in evaluation and language. The final form of the test was amended according to the recommendations of the expert committee. The test was given to an exploratory sample to determine the level and time required to solve it and the difficulties that it may include. The time required for the test was 120 minutes. The researcher checked the tests, and determined the stability of the test using the alpha Cronbach equation. The result was alpha Cronbach's $\alpha=0.63$, which is an indicator of the test's validity.

Equivalence of Two Study Groups
The researcher confirms the equivalence of the experimental group and the control group using the following variables:

Mathematical achievement: Through students' achievements in previous exams, as presented in Table 3:

Table 3.
Mathematical Achievement

Group	*Number*	*Mean*	*SD*	*T(79)*
Experimental	40	66.93	16.9	**0.43**
Control	40	65.5	18.48	

Cultural, economic and social level: The experimental and control groups were from the same schools and from a close socio-economic and cultural environment.

Results

The current study aims to reveal the impact of the use of mathematical problem-solving on the development of creative thinking skills for prep school students in Arab schools in northern Israel. To achieve this goal, the researcher's applied his creative thinking test to the study students. And after the application was completed, data was collected to examine the validity of study hypotheses.

First Hypothesis: The first hypothesis states that there are statistically significant differences at $\alpha \leq 0.05$ between the average scores of the students in the experimental group and the control group in the post application of the creative thinking test in the skill of fluency in favor of the experimental group.

In order to test this hypothesis, a t-test was used to calculate the significance of differences between two independent groups to identify the impact of the use of problem-solving in the development of creative thinking skills for prep school students in Arab schools in northern Israel.
Table 4 shows the results:

Table 4.
First Hypothesis Results

Group	Number of Students	Arithmetic Mean	Standard Deviation	T-test	η^2
Experimental	40	37.86	13.55	11.52	**0.62**
Control	40	12.98	1.8		

This confirms the obvious impact of the use of mathematical problem-solving on the development of fluency; and also confirms the first hypothesis.

Second Hypothesis: The second hypothesis states that there are statistically significant differences at $\alpha \leq 0.05$ between the average scores of the students in the experimental group and the control group in the post application of the creative thinking test in the flexibility skill in favor of the experimental group.
In order to test this hypothesis, a t–test was used to calculate the significance of differences between two independent groups to identify the impact of the use of problem-solving in the development of creative thinking skills (flexibility) among prep school students in Arab schools in northern Israel.
Table 5 shows the results:

Table 5.
Second Hypothesis' Results

Group	Number of Students	Arithmetic Mean	Standard Deviation	T-test	η^2
Experimental	40	27.16	8.33	11.21	**0.6**
Control	40	12.38	0.54		

This confirms the obvious impact of the use of mathematical problem-solving on the development of flexibility, and thus corroborates the second hypothesis too.

Third Hypothesis: the third hypothesis states that there are statistically significant differences at $\alpha \leq 0.05$ between the average scores of the students in the experimental group and the control group in the post application of the creative thinking test in originality skill in favor of the experimental group.

In order to test this hypothesis, a t-test was used to calculate the significance of differences between two independent groups to identify the impact of the use of problem-solving in the development of creative thinking skills (originality) among prep school students in Arab schools in northern Israel. Table 6 shows the results:

Table 6.
Third Hypothesis Results

Group	Number of Students	Arithmetic Mean	Standard Deviation	T-test	η^2
Experimental	40	65	12.88	8.53	**0.47**
Control	40	29.5	2.89		

This confirms the obvious impact of the use of mathematical problem-solving on the development of originality, and consequently confirms the third hypothesis.

Fourth Hypothesis' Results: The fourth hypothesis states that there are statistically significant differences at $\alpha \leq 0.05$ between the average scores of students in the experimental group and the control group in the post application of the creative thinking test as a whole in favor of the experimental group.

To test this hypothesis, a t-test was used to calculate the significance of the differences between two independent groups to identify the impact of the use of problem-solving in the development of creative thinking skills in general among prep school students in Arab schools in northern Israel. Table 7 depicts the results:

Table 7.
Fourth Hypothesis Results

Group	Number of Students	Arithmetic Mean	Standard Deviation	T-test	η^2
Experimental	40	78	10.18	9.43	**0.52**
Control	40	56.6	6.47		

This confirms the obvious impact of the use of mathematical problem-solving on the creative thinking test as a whole, and substantiates the fourth hypothesis.

Discussion and Conclusion

The results of the first hypothesis, whose results found an effect for the use of mathematical problem-solving between the average results of students in the experimental group and the average of the students of control group in favor of the experimental group, coincide with the results of Abu Athrah (2010) and Mustafa (2009), regarding the use of some strategies that work on the development of creative thinking skills, including fluency in the field of mathematics, and this can be explained by the following reasons:

- The use of mathematical problem-solving gives the students an opportunity to solve life problems related to the reality of their lives, which improve the students' fluency of thinking.
- Mathematical problem-solving helps students to apply knowledge in new life situations and increase their connection to this knowledge, leading them to improve their performance.
- The use of mathematical problem-solving improves students' handling of life problems compared to the control group students.
- Mathematical problem-solving increases students' awareness of the importance of studying mathematics in solving daily life problems, prompting students to develop their creative thinking.

The results of the second hypothesis, which found an effect for the use of mathematical problem-solving between the average results of the students of the experimental group and the average of the students of the control group in favor of the experimental group, are consistent with the results of Abu thrah (2010), Giordano (2003), Mann (2005) and Rosa (2000), in the use of some strategies that develop creative thinking skills, including flexibility, in the field of mathematics. This can be explained by the following reasons:

- Solving mathematical problems allows students to take organized steps that greatly contribute to the development of their ability to express and participate effectively, resulting in the development of students' flexibility in producing ideas.
- The use of solving mathematical problems gives the issue a vital character by accustoming students to formulate the issue in their own language, to draw an appropriate diagram of it, and to explain it with a model or sensory means. The diversity in the means of presentation of objects and life situations, and the use of symbols, increase students' creativity.
- Solving mathematical problems is concerned with students' understanding of the topics and problems posed, which develops a spirit of creativity, makes them abler and skilled in dealing with life and its problems, and expands their life experiences and links them to what they had learned at school.
- The use of mathematical problem-solving made students able to solve unfamiliar and more complex problems with more confidence and mental flexibility compared to the control group students.

The results of the third hypothesis, which found an effect for the use of mathematical problem-solving between the average results of the students of the experimental group and the average of the students of the control group in favor of the experimental group, are consistent with the results of the studies of Abu thrah (2010), Giordano (2003), Mann (2005) and Rosa (2000), in the use of some strategies that develop creative thinking skills including originality, in the field of mathematics. This can be explained by the following reasons:

- The use of mathematical problem-solving helps to develop the aesthetic sense of mathematics, to appreciate its importance in real life, and develops a positive inclination towards it, which led to an originality in students' thinking when solving life problems.
- Mathematical problem-solving is used to ask meaningful questions, and gives enough time to think about the answer and clarify the meaning of each question, so that students learn how to address any problem they face by themselves.
- The use of mathematical problem-solving allows students to appreciate reasonable answers, and to use them inversely towards the data, which contributed to the formation of their own issues or problems, which increased their authenticity.

The results of the forth hypothesis, which found an effect in the use of mathematical problem-solving between the average results of the students of the experimental group and the average of the students of the control group in favor of the experimental group, are consistent with the results of the studies of Abu thrah (2010), Giordano (2003), Mann (2005) and Rosa (2000), in the use of some strategies that develop creative thinking skills as a whole in the field of mathematics. This can be explained through the following reasons:

- The diversity of activities and life problems leads to a rise in creativity among students.
- Solving mathematical problems with their multiple, progressive and interrelated steps broadens students' perception, and their inclusion in the steps has helped them to develop their creative thinking.
- Solving mathematical problems increases students' understanding of the problems posed, which developed their fluency of solutions, flexibility of entrances, and originality of solving them, which developed their creative thinking.

We can conclude that the diversity of activities led better understanding of the material, which in turn led to conceptual learning among students and to more creative thinking among students. All of the above can be summarized in the Abd Algani, Hibi and Abo Al-Haija' Instructional Model Offer (see figure 10):

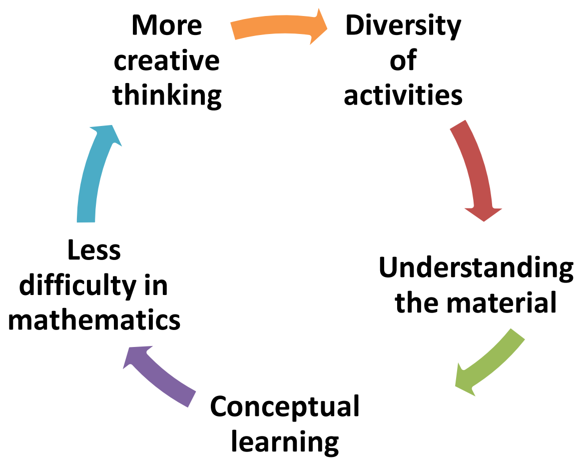

Figure 12.
Abd Algani, Hibi and Abo Al-Haija' Mathematics Instructional Model Offer

Recommendations

In light of the results of the study the researcher recommends:

- To use mathematical modeling in the mathematics curriculum to show the role of mathematical knowledge in solving real-life problems.
- To train students in the faculties of education and teacher training colleges on how to use modeling and mathematical problem-solving in solving life problems.
- Teachers should work to discover students' abilities and inclinations, develop their curiosity, and work to develop these abilities in the right direction.
- The authors of the math curriculum should boost teacher's attention to the importance of mathematical problem-solving, to increase the student's motivation to study mathematics.
- There should be a specialized team to select problems and activities that develop creativity, and to include them in the math curriculum in an appropriate manner that takes into account students' individual differences.
- Apply gradualness in posing problems in the curriculum, so that there are problems solved mentally; some need paper and pen and some need calculators in order to develop creativity in students.
- To focus on organizing the content of the mathematics curriculum in the preparatory stage according to mathematical problem-solving.
- To prepare guides for teachers to teach the mathematics curriculum at the preparatory stage using mathematical problem-solving.

For Further Research

In the light of the objectives of the current study and its results, we can propose the following future studies and research:

- Study the impact of the use of mathematical modeling in teaching other subjects at other educational stages.
- Study the impact of the use of mathematical modeling on the development of visual thinking skills among primary school students.
- Study the effectiveness of training programs for teachers to use mathematical modeling in teaching different school subjects.
- Study and identify the awareness of workers in the educational field concerning the importance of mathematical modeling.

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Research Article

Improving students' mathematics achievements using classroom interventions

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Abstract

Increasing retention and success rates, using innovative techniques, is one of the core objectives in community colleges. Several researches have shown that interventions strategies boost such rates in high and middle schools. The purpose of this study is to examine how classroom interventions impacts student's mathematics achievements in a remedial mathematics course at a community college. This quantitative research used cluster sampling to obtain a sample of 44 students from two fundamental algebra sections in Fall 2018 semester at LaGuardia Community College, New York City, US. The instruments used in this study are the first and second attempts of two departmental exams and the final exam scores. After each departmental exam, students were divided into two groups, a basic and advance group, based on their performances on the exam. Each group had a 4-hour intervention session separately before they were given a second attempt for the departmental exam. The difference between the first and second attempt of departmental exam 1 and departmental exam 2 average scores were calculated to evaluate students' improvements after the interventions. Correlations were performed to assess the strength of the relationship between each departmental exam and the final exam. The result of this quantitative research showed that the classroom interventions helped students improve their departmental exams scores. However, departmental exam 2 intervention had more positive impact on the final exam scores than departmental exam 1 intervention.

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Introduction

Over the last few years, an increasing emphasis on low student success rates in remedial mathematics courses at the community college level has created an urgency to develop new strategies for improvement. Most community colleges have a 30 percent progress rate in these remedial courses, according to the National Council on Academic Transformation (Twigg, 2003). Furthermore, the overall completion rate, in three-years, for community college students nationwide was 24 percent for the 2000 cohort and 20 percent for the 2010 cohort (NCES, 2014). Although data on success rates vary among community colleges, the number of students placed in remedial mathematics courses is increasing, while completion and retention rates remain substandard or decreasing. Students are less likely to complete college mathematics across various stages of developmental courses (Bailey, 2009). Thus, these remedial mathematics courses have become an academic and career unyielding gatekeeper to college students' success. Colleges constantly search for solutions to increase the retention and success rates of students through mathematical curriculum and pedagogies. Nationwide, efforts to increase student success rates have been focused on implementing new strategies. For students experiencing mathematics difficulties, classroom interventions have become an essential aspect to improve student's mathematics ability, hence preventing subsequent failure.

In general terms, classroom intervention is a set of measures an instructor takes to help students improve in their area of weakness by removing educational barriers. This type of intervention which specifically addresses an

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"observed weakness" is called an Intentional Intervention (Lynch, 2019). Research indicates that students who struggle in mathematics can be successful given they receive additional instructional time and support (Burris, Heubert, & Levin, 2006). To be effective, this additional instruction and support must be in addition to and integrated with the regular classroom (Balfanz, Mac Iver, & Byrnes, 2006). A study provides eight concrete suggestions for instructors, principals, and school administrators, to improve their ability to succeed in the classroom (Gersten et al. 2009). This study used "Response to Intervention" to identify students who need assistance in mathematics and addressed the needs of these students through focused interventions. A study investigated whether students who are exposed to interventions programs will show growth in their mathematics achievements (Hines, 2016). Using a qualitative and quantitative method, the results revealed the effectiveness of the two interventions used and the relationships between academic growth and teacher perception. The target population was made up of middle school students.

Problem of the Study

Based on previous studies, whole-class mathematics interventions have shown the longest enduring effects on student performance (Griffin, 2004; Conner et al. 2009). The common goals for these interventions were to increase students' self-efficacy in mathematics and re-teach critical concepts and skills. Yet, most of these studies focused on middle school or high school students. This study will examine the effects of classrooms interventions on community college students' mathematics achievements in remedial courses.

Methodology

Research design

There are three major exams (2 departmental Exams and a CUNY Final Exam) in fundamental algebra that students must successfully complete to have a great chance of passing the class. Each exam includes 25 multiple choice questions. The departmental exams are taken online in the computer lab, during the fourth and the eighth weeks, using Lumen platform and the CUNY Final Exam is taken at the end of the semester in the school testing center. Students have two chances to take the departmental exams and the highest score is counted toward their grades.

The second chance of the exam is usually given a week after students take the first attempt of the exam. The departmental exams and final exam represent 65% of students' final grade and a student needs an average of 70% or higher to pass the course.

The group interventions were designed to re-teach missed concepts and skills based on student's performance on departmental exam 1 and departmental exam 2. The goals were to increase students' grades as well as their procedural flexibility in mathematics. The first intervention ran during the fifth week of class (after departmental exam 1) and the second intervention took place during the ninth week (after departmental exam 2). The step-by-step process of the interventions were as follows:

Departmental Exam 1:

Step 1: Students take the departmental exam 1 during week 4 of classes.

Step 2: Students, in each section, were divided into two groups based on their performance on the exam. Students who scored below 70% were placed in basic group (group 1) and those who scored above 70% were placed in the more advance group (group 2)

Step 3: Investigators designed a practice-problem worksheet for each group based on topics where students under performed

Step 4: Instructors and teacher assistants each ran a 4-hour intervention during week 7. For group 1, instructors conducted the intervention by giving a mini lecture of each of the topics needed, followed by a practice of the designed worksheet. Teacher assistants led the 4-hour intervention for group 2 in a separate room by just working on the designed worksheet.

Step 5: Students were given the second attempt to take the departmental exam 1 at the end of week 7

The same process was repeated for second departmental exam.

Participants

This study was conducted at LaGuardia Community College located in New York City, where the principal investigators worked as full-time faculty. The target population is made up of students who are enrolled in fundamentals algebra course. This course is a remedial mathematics course which consist of 9 sections during the 12-week session of Fall 2018. The average enrolment per section is 30 students.

Sampling and Sampling Procedure: The sampling strategy used in this quantitative research study was a cluster sampling. Out of 9 fundamental algebra sections offered in Fall 2018, two were randomly selected to form the samples of this research. In this study, the two fundamental Algebra sections were made up of 58 students. After withdrawals and drops, 44 students took all exams and participated in all research activities. The two sections were taught by adjuncts who had the same level of experience (7 years on average) teaching the course. A teacher assistant was assigned to each section to help students in and outside of the classroom. The fundamental algebra course meets seven hours per week: 4 hours lecture, 2 hours computer lab, and one tutoring lab hour led by the teacher assistant (TA). Students used a mathematics platform, OHM Lumen, a free Open Educational Resource (OER) software to complete their assignments including homeworks, quizzes and departmental exams.

Data Collection Tools

The instruments used in this study includes students' scores on departmental exam 1(1st and 2nd attempts), departmental exam 2 (1st and 2nd attempts) and final exam. The investigators designed four worksheets for the group interventions. Each group (group 1 and group 2) used two worksheets, one for the first intervention and one for the second intervention.

Data Analysis

The average scores on departmental exams and final exam for the basic group (group 1) and the advance group (group 2) were calculated to compare the groups' performances. The difference between the first and second attempt of departmental exam 1 and departmental exam 2 were calculated to evaluate students' improvements after the interventions. Finally, the correlations between the second attempt of each of the departmental exams (departmental exam 1 and departmental exam 2) and the final exam scores were performed to measure the strength of the relationship between each of the departmental exams and the final exam scores.

Results

Departmental Exam 1 Intervention

The first attempt of departmental exam1 was given during the fourth week of classes. Of the 44 students from both fundamental algebra sections that fully participated in this study, 22 students scored below 70 out of 100 points and were placed in group 1 and the remaining 22 students who scored at least 70 points were in group 2. The two instructors for both sections, each ran a 4-hour intervention for group 1 and their teacher assistants led the 4-hour intervention for group 2. The average scores of the first attempt of departmental exam 1 for both groups are summarized in Table 1. The exam report showed that students in group 1 had an average score of 54 out of 100 points, and struggled on questions related to algebraic sentences, linear inequality, rational equations, and linear equations. During the intervention, instructors focused on these topics by reviewing key formulas or concepts as well as practicing more related problems using the prepared worksheet.

Table 1.

Departmental Exam 1 Scores (First and Second Attempts Each Out of 100 Points)

Departmental Exam 1			
Group	1st attempt average score	2nd attempt average score	Difference = 2nd attempt – 1st attempt
1 (n = 22)	54	69	15
2 (n = 22)	83	93	10

Students in group 2 averaged 83 out of 100 points on the first attempt of departmental exam 1 and generally struggled on questions related to algebraic expressions and linear inequality. The teacher assistants for both classes helped students in group 2 practice related problems using the prepared worksheet.

At the end of the interventions, students took the second chance of departmental exam 1 which had the same format but was a different version. Students in both groups performed better on the second attempt as compared to the first attempt (see table 1). The average score for students in group 1 went up by 15 points on the second attempt as compared to 10 points for students in group 2.

Departmental Exam 2 Intervention

For the first trial of departmental exam 2, given during the 8th week of classes, 16 out of 44 students scored below 70 out of 100 points and were placed in group 1. The remaining 28 students who scored 70 points or more, were placed in group 2. The average score on the first attempt for group 1 was 57/100 compared to 86/100 for group 2. Following the same procedure used in the departmental exam intervention, instructors ran a 4-hour intervention on slope and equation of a line, system of equations, and factoring for group 1. In group 2, the teacher assistants focused their intervention on equation of a line and factoring.

Table 2.
Departmental Exam 2 scores (first and second attempts each out of 100 points)

Departmental Exam 2			
Group	1st attempt average score	2nd attempt average score	Difference = 2nd attempt – 1st attempt
1 (n = 16)	57	78	21
2 (n = 28)	86	94	8

After the interventions, students took the 2nd attempt of departmental exam 2. Students in group 1 averaged 78 out of 100 points, 21 points higher than the first attempt as compared to group 2 average which went up by 8 points (Table 2).

Summary of Exams by Group
For the intervention of departmental exam 1, students in group 1 scored on average 81 out of 100 points on the final exam as compared to 86 points for students placed in group 2 (Table 3).

Table 3.
Departmental Exam 1 vs Final Exam Scores (Each Out of 100 Points)

Departmental Exam 1 vs Final Exam Scores			
Group	1st attempt average score	2nd attempt average score	Final exam score
1 (n = 22)	54	69	81
2 (n = 22)	83	93	86

In the second intervention of departmental exam 2, students in group 1 averaged 79 points on the final exam as compared to 94 points for those placed in group 2 (Table 4).

Table 4.
Departmental Exam 2 vs Final Exam Scores (Each Out of 100 Points)

Departmental Exam 2 vs Final Exam Scores			
Group	1st attempt average score	2nd attempt average score	Final exam score
1 (n = 16)	57	78	79
2 (n = 28)	86	94	94

Correlations: The correlation between the 2nd attempt of departmental exam 1 and the final exam score was investigated

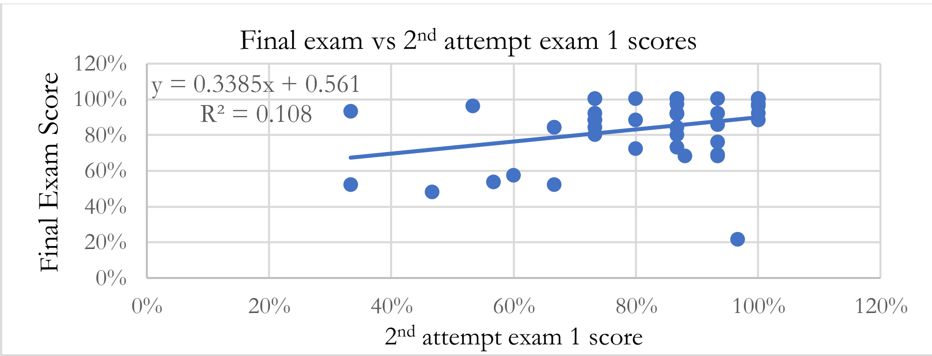

Figure 1.
Final Exam vs 2nd Attempt of Exam 1 Scores

The coefficient of determination, R^2 is 0.1080 indicating about 10.8% of the variability in the final exam scores is explained by the second attempt of departmental Exam 1 scores (figure 1). The corresponding correlation coefficient is equivalent to 0.3286 which indicates a weak positive relationship between the 2nd attempt of departmental exam 1 and the final exam scores

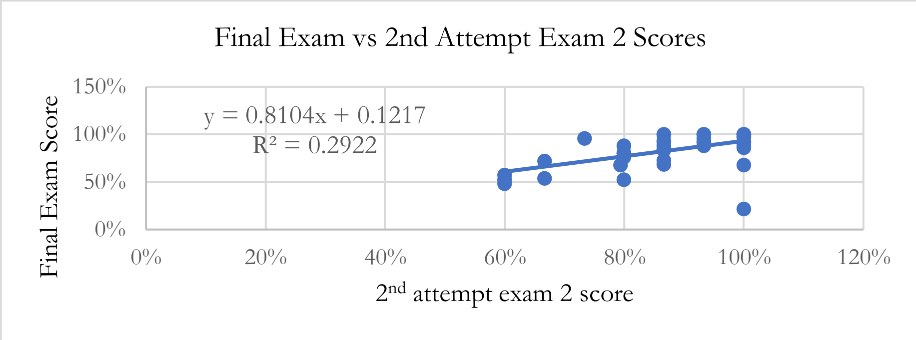

Figure 2.
Final Exam vs 2nd Attempt Exam 2 Scores

The correlation between the 2nd attempt of departmental exam 2 and the final exam score revealed a coefficient of determination, $R^2 = 0.2922$ (figure 2). Thus, about 29.2% of the variability in the final exam scores is explained by the scores of the second attempt of departmental exam 2. The correlation coefficient between these two variables is equivalent to 0.5406. This implies the two variables have a moderate positive relationship.

Discussion and Conclusion

The main goal for this study was to find a new way to help students improve their exam scores during the second chance as well as nurture their ability to apply procedures accurately, efficiently, and flexibly in mathematics. Based on the results previously shown (Table 1 and Table 2), the interventions helped students improve their exams scores. After 4 hours of intervention, the average scores for students in group 1 increased by 15 points and 21 points respectively for departmental exam 1 and departmental exam 2, while the average for group 2 increased by 10 points and 8 points, respectively. These findings align with the results of a previous study that found students who are exposed to interventions programs will show growth in their mathematics achievements (Hines, 2016). As a result, the classroom interventions had a positive effect on students' pass rates in these two fundamental algebra sections. The three major exams (the two departmental exams and the final exam) constitute 65% of students' grade: 15% for each departmental exam and 35% for the final exam. The final exam was given three weeks after the second departmental exam and contains nearly 80% of questions related to topics covered in departmental exam 2. This explains the moderate relationship between departmental exam 2 and the final exam scores has compared to the relationship between departmental exam 1 and the final scores (figure 1 and figure 2). Thus, the second intervention helped students on the 2nd attempt of the departmental exam 2 and the final exam. In addition, students' average scores for both groups, were nearly the same for the 2nd attempt of the second departmental exam and the final exam (Table 4). Of the 44 participants, 34 (77%) passed the class. This pass rate is higher than the average pass rate for fundamental mathematics (61%) course according to the college Institutional research.

Departmental exam 2 was cumulative, therefore contains questions related to topics covered in the first exam. However, students did better on the second departmental exam compared to the first departmental exam. This shows that the first intervention improved students' procedural flexibility which might explain their high performance on the first attempt of departmental exam 2.

The interventions were integrated during the classrooms hours and showed huge impact on students' mathematics achievements. As revealed in a previous research, intervention programs are effective when integrated with the regular classroom (Balfanz, Mac Iver, & Byrnes, 2006). How about individual intervention (online intervention) outside of the classroom? Future research must explore the effect of individual intervention outside of the classroom as compared to in-class group intervention.

The intervention strategy used in this study, identifying students' weaknesses through exams results, contributed to the positive outcomes. As recommended in previous research, instructors must first identify students' weaknesses then address them through classroom interventions that focused just on those issues (Gersten et al. 2009). A key component to the success of any intervention is matching the student with the appropriate supports (Danielson, 2009).

Recommendations

The outcomes of this study revealed the positive effects of classroom interventions on students' procedural flexibility but does not justify students gain of conceptual understanding which is the mental connections among mathematical facts, procedures, and ideas (Hiebert & Carpenter, 1992). Research have shown that conceptual knowledge has had a stronger influence on procedural knowledge than vice versa (Hecht & Vagi, 2010; Matthews & Rittle-Johnson, 2009). In addition, with a strong conceptual understanding, students can better generalize skills and understand algorithms (Gersten et al., 2009; Jones, Inglis, Gilmore, & Evans, 2013; Miller & Hudson, 2007). Thus, a further research is needed that focused on the effects of classroom interventions on students 'conceptual knowledge. Furthermore, this may require monitoring students' progress in learning and comprehension of the subject by assessing each intervention effectiveness throughout the semester.

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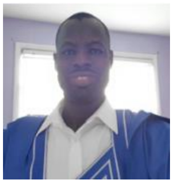

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Opinion Article

What I learned from Budapest Semester in Mathematics Education

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Abstract

In this paper, I wanted to share my personal experience in Budapest Semester in Mathematics Education as an mathematics educator.

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Introduction

Hungary is such a powerhouse when it comes to mathematics and mathematics education. Hungarian mathematicians, including Polya and Paul Erdős, have contributed significantly to mathematics and its teaching. They have a long tradition of teaching mathematics, and they are proud of their traditions. The essence of Hungarian tradition is problem-solving, mathematical creativity, and communication. Nurturing mathematical talent is a part of the Hungarian tradition. Studying mathematics is very respectful, and many opportunities are given to mathematicians.

To learn more about Hungarian mathematics education and their mathematical tradition, I attended Budapest Semester in Mathematics Education (BSME) this summer. BSME is a study abroad program in Budapest, Hungary. The goal of the BSME is to make participants experience the general mathematical climate in Hungary and the Hungarian pedagogy. The course that I was attended was called The Discovery Learning: The Posa Method. Lajos Posa is a Hungarian mathematician, and he is well-known for his studies in Combinatorics and mathematics education. Even this course was only two weeks, it was a good learning experience for me. I wanted to share the two lessons that I learned from BSME.

Experiencing the Struggle

My first lesson was, after a while, experiencing the struggle in mathematics. The traditional view of struggles in mathematics is somehow harmful. The current understanding is that struggles are necessary for deep understanding and sense-making in learning mathematics (Hiebert & Grouws, 2007). Learning mathematics is not an easy journey. Instead, it is a challenging experience. The benefit of struggles is that they make you aware of the intellectual need for a solution or an idea. This intellectual need might be aware of a lack of knowledge or creative ways to solve the problem (Kapur, 2008; Warshauer, 2014).

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The Posa method is a form of Inquiry Oriented Learning, and the difference is that the IBL is more oriented to general education, where Posa Method is more oriented for gifted Students. According to Posa, learning mathematics is not just about the acquisition of some procedures and rules. It is about sense-making and rediscovering the essential mathematical ideas and problem-solving and the joy of mathematics. However, learning mathematics in this way is a challenging task. In the course, we worked on few cognitively challenging problems for each class. Each problem is unique, and the instructor did not expect you to solve problems at that moment. As a learner, you need to think of some creative ways to solve these problems. If we had a correct solution, we could confer the solution. However, if you did not have a correct solution, the instructor did not help us. May be some question to redirect our thinking, that is it. This forces you to think; I struggled a lot. That is the joy of mathematics. The joy is that you know there is a challenging problem, and there are a group of smart people you wanted to be the first one to solve. It is like solving a puzzle. At the end of each lesson, whenever somebody explained the solution, I made sense of it.

This experience made me think about my teaching. As a teacher, I used to believe that a teacher's job is to make students comfortable with the material, and I was helping my students a lot when they had struggles. However, this BSME made me experience the benefits of challenge from the firsthand. I believe BSME will positively influence my teaching.

Society and Tradition

My second lesson is the importance of society and tradition. Since the beginning of history, learning has usually been described as acquiring knowledge. In this acquisition-based learning, teachers are the source of the knowledge, and students are usually the passive receiver of the knowledge. The acquisition is an umbrella term, and many learning theories are fallen under this category (Sfard, 1998). However, learning is not just an accumulation of knowledge. Learning is a social event, and knowledge is inseparable from the context being developed. Learning is a participation of a particular community of practice (Lave & Wenger, 1991).

George Polya (1887-1987) is one of the most famous mathematicians, and his famous book is How to Solve It?

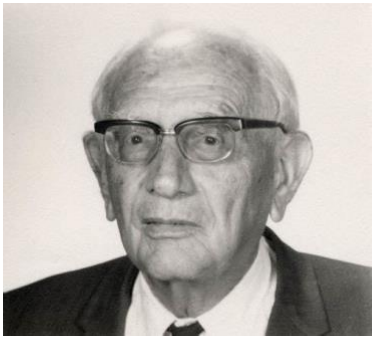

Figure 1.
George Polya (1887-1987) was One of the Most Famous Mathematicians

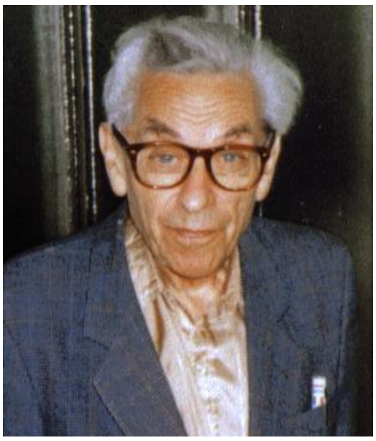

Figure 2.
Paul Erdos (1913-1996) was One of the Most Productive Mathematicians at

In Hungarian tradition, mathematics is considered a dynamic body of continuously evolving and growing knowledge. The teaching of mathematics should reflect the constant reconstruction side of science. They emphasized the importance of intuition and gaining experience in mathematics. In Hungarian pedagogy, teachers make students experience various problems to help them to rediscover the essential mathematical ideas. In this rediscovery process, problem-solving and creativity play an essential role. Hungarian mathematicians believe that playfulness is inseparable from learning mathematics. Mathematics can be considered a form of art. Since Hungarian understanding of mathematics is formed around intuition, creativity, and playfulness, they tend to use less formal language and new notations (Gosztonyi, 2016). Being a mathematician is a highly respected career in Hungary. The instructor, Peter Juhasz, also took some classes from Posa himself when he was a teenager, and he told many times how his participation in Posa's lessons influenced him to be a mathematician.

The lesson is that individuals do not grow in isolation; regardless of talent, individuals need tradition and a supportive environment to grow. Recently, some countries, like Turkey, invest heavily in gifted education, hoping that this "gifted" induvial will become a world class scientist and mathematicians and then contribute to society. However, this naïve belief is missing in one essential piece in the equation: society and opportunities. Many people study mathematics, not just its internal beauty; it gives people opportunities (Apple, 1992). There is a reason why it is called the queen of science. Once one has a certain level of mathematical understanding, one can seek a career in almost every field. Unfortunately, in many countries, studying mathematics at college or higher is neither respectful nor gives opportunities. Mathematics is merely a tool to go medicine schools. If those countries, including Turkey, want the recent efforts and investments in gifted education is to be successful, they should discuss the importance of society and opportunities.

In this paper, I wanted to share what I have learned from the BSME. The first lesson was to experience the cognitive benefits of struggles firsthand. The second lesson is the importance of society and tradition in learning.

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Teaching Practices Article

Activity for teaching mathematics for students with learning disabilities with analogy method : division with and without a remainder topic

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Abstract

One of the subjects that students with mathematics learning difficulties have difficulty with is division with and without remainders. In teaching this subject, concretization, use of analogy and gamification can be effective in understanding. Concretization, semi-concrete, abstract teaching strategy is applied. In this article, an activity for teaching division in students with mathematics learning difficulties is presented.

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Introduction

Special Learning Disability (SLD) is an umbrella term that includes many sub-categories. One of these subcategories is learning disability in mathematics. Students who have problems in this category; It has been observed that they have difficulties in acquiring skills in calculating and counting, they have difficulties in performing arithmetic operations and they encounter problems in remembering, and because of all these issues, they are in the background compared to their peers in mathematics lessons (Olkun & Akkurt-Denizli, 2015).

These students; They also have problems in understanding basic mathematical operations such as telling time, using money, addition, subtraction, multiplication and division, and more abstract problems (for example +, -, × or ÷) (Pandey & Agarwal, 2014). Although these characteristics and inadequacies of students with mathematics learning difficulties are known, not applying any intervention program to students who have difficulties in learning mathematics may cause these students to both experience failure and feel unsuccessful in some subjects in mathematics in the following years. It is stated that the potential of mathematics to affect an individual's self-confidence, self-esteem and living standards is much more than any other discipline. When the studies are examined, it can be deduced that low performance in mathematics causes high mathematics anxiety, low self-confidence, differentiation in the choice of profession in the child, and that high mathematics performance also creates a basis for the child's self-confidence and self-esteem to lead a life in much better economic conditions in the future (Mutlu, 2020). Although estimates vary widely, it is currently thought that between 5 and 8 percent of populations worldwide have a mathematical learning disability (Butterworth, 2005; Shalev, 2007; Shalev, Manor, and Gross-Tsur, 2005).

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In order to talk about mathematics, it is necessary to talk about procedural skills. Division is a concept that students encounter at an early stage. It is possible to predict the general mathematics and algebra knowledge of these students at the high school level, with the knowledge of the division process gained by the students in the primary school period (Siegler et al., 2012; cited in Oral, 2020). Siegler et al. (2012; as cited in Oral, 2020), as a result of their research, determined that students' knowledge about division is a strong factor predicted for their competence in mathematics later in life. Students who have not learned the subject of "dividing natural numbers" have questions about what to do in their minds when they are faced with problems involving radical numbers, exponential numbers, rational numbers, integers, and division operations with fractions. The division operation is the most difficult one among the four operations, both in terms of meaning and understanding the steps of the operation (Baykul, 2002). Albayrak (2010), on the other hand, stated that the operation with the most rules among the four operations is the division operation and the division operation has a different structure from the others. Teaching the division process, which is in an important position in creating the basis for the mathematical concepts required in terms of the higher mathematical skills that students with a learning disability specific to mathematics will need in their later school life, and at the same time reducing the student's concerns about mathematics, is extremely important.

School programs, which have been under the influence of the structuralist approach since 2005 in Turkey, have made efforts to enrich the lessons with new teaching methods and techniques. One of these methods and techniques is the "analogy" method. The analogy method contributes to the systematic categorization and organization of information (Saka, Ayas, & Enginar, 2002). The development of analogies consists of 5 stages (Glynn et al. 2005);

- The target concept is specified.
- The source concept is arranged according to the target concept.
- Similar features between the source concept and the target concept are determined
- Similar features are compared.
- If there is a place or places where the analogy is broken, it is determined
- The result is drawn (as cited in Şahin, 2016).

Analogy method is mostly used in science lessons. Teaching division with the analogy method is also important in that it is prepared to be used in teaching division skills to all students, whether they need special education or not. In addition, the study includes the search for solutions to the problem rather than reflecting the existing problems in the teaching of division in mathematics learning difficulties. With this research, it is hoped that the teaching process of division in mathematics learning difficulties will become more efficient, the materials used in the research will be a resource for teachers, and the use of apology method in teaching division to students with mathematics learning difficulties will facilitate learning the division operation skill.

Implementation of Teaching Activity

According to Şahin (2000), there are four types of analogies: simple analogies, pictorial analogies, game-style analogies and story-style analogies. The purpose of this activity is to teach the skill of division using the game-style analogy method. In game-style analogies event are gamified. In this wise students will be able to learn the beginnings in division skill in a more enjoyable, meaningful and permanent way.

The materials to be used for this activity are 50 toy soldiers, 1 toy tank and 5 beads, pencils, paper. In order to arouse the interest and curiosity of the students, the activity will be started by asking some questions before the application. It was asked to make some evaluations on the students' thoughts on wars, the rules to be followed in wars, and examples of the use of analogy in lessons. The necessary materials related to this activity are prepared and the activity is started.

First Stage: Teaching Division by Analogy Method: Concretization Phase

Before starting the teaching of the concretization stage, the tools/toys representing the analogy to be used in the teaching of the basic division process were kept ready on the table.

Photo 1.
Activity of Implementation Stages

Teaching begins by saying "Welcome" to the participant. After the information about the session is presented (For example; "Today we will try to divide by playing games with you"), the session starts with the consent of the participant. Teaching begins with the practice of being a model. In the practice of being a model, the practitioner performs sample division. reads the number of divisors and divisors. The number of soldiers divided is placed on the table. It is said that as many tanks as the number of divisors can be fired in one shot. Balls are positioned where necessary. Then the basic division operation is read. It is explained that it is necessary to shoot as many guns as the number of divided enemy soldiers, and how many cannons we need to shoot to destroy all the divided soldiers. It is noted that the first shot will start from the group of soldiers farthest from the tank. Rhythmic counting begins with the number of divisors. It is continued until the appropriate number in the divided number is reached. It is said that we can throw as many balls as the number of soldiers in the divided number, and if we fire more than the number of soldiers, the balls will be wasted. However, it is reminded that it is "correct" to shoot the number closest to the divided number. In other words, it is said that the most correct shot will be to shoot the number closest to the divided number, not less than the divided number but not more than the divided number. We are asked to write the number of shots fired in the section section, and to calculate how many enemy soldiers were destroyed by subtracting the total number of balls fired from the divided number by writing under the divider. It is stated that the number written in the number of chapters is the result. It is read again with the result of the basic division operation. After the practice of being a model, the participant is given a skill instruction by saying "Now it's your turn". The participant is asked to read the basic division operation and repeat the same action that the teacher did with the toys. Correct responses are reinforced by social reinforcement, and the participant's participation in the session is reinforced by primary reinforcement after five trials. When the criteria are met in the sessions, the semi-concrete stage is started.

Second Stage: Teaching Division by Analogy Method: Semi-concretization Stage

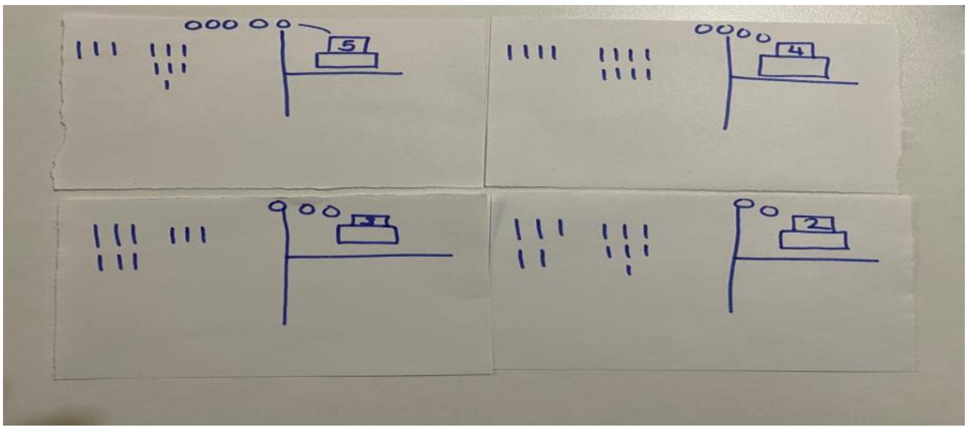

Photo 2.
Activity of Semi-concretization Stages

In the semi-concrete stage, basic division cards with pictures of soldiers, tank and gun used in analogy are used. These materials are presented to the participant one by one. Teaching begins by saying "Welcome" to the participant. Information about the session is presented. Then (For example; "Today we will work on the basic division with the cards you saw"), the session starts with the consent of the participant. Teaching begins with the practice of being a model. In the modeling application, the basic division process is first explained by the practitioner. Divider and divisor numbers are indicated. From the picture cards with the pictures of the soldiers to the divided number, the card is selected. The number of shots to be fired using the tank image is determined by rhythmic counting. When the rhythmic counting process is finished, it is checked how many times it has been counted rhythmically. and this is indicated as the result. is read again along with the result of the basic division operation. After the modeling practice, the participant is told "Now it's your turn" and the skill instruction is presented. The participant is asked to read the basic division process and choose the number of soldier picture cards as the number of divisions. and selects the tank image. Considering the number of divisors, he is asked to find how many shots he needs to make by counting rhythmically. As a result of rhythmic counting, it is said that how many times he counted rhythmically, that number is the answer of the division part of the division operation. It is expected to be read again with the result of the division operation. Correct responses are reinforced by social reinforcement, and the participant's participation in the session is reinforced by primary reinforcement after five trials. When the criteria are met in the sessions, it can be passed to the abstract stage.

Third Stage: Teaching Division by Analogy Method: Abstract Stage

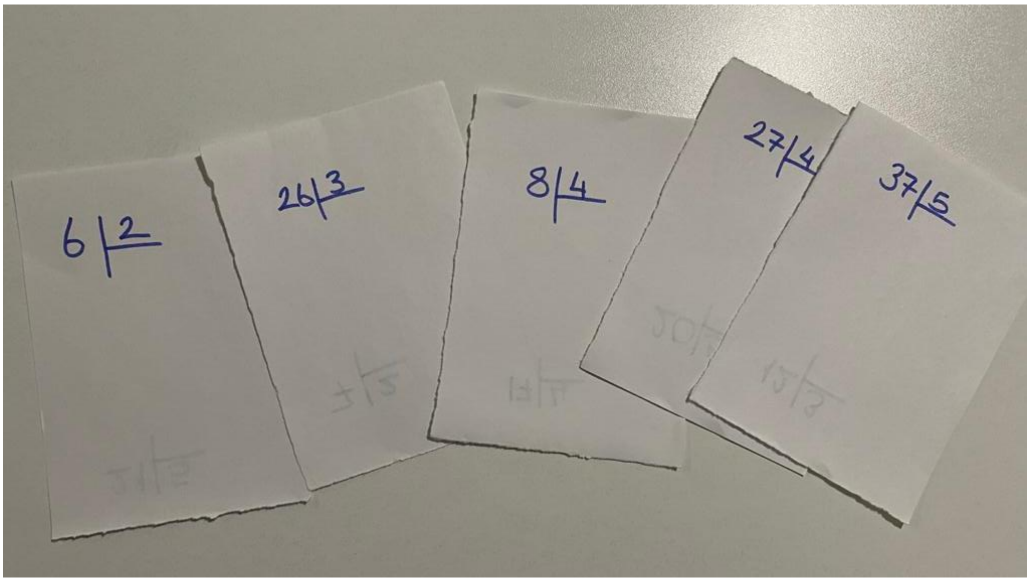

Photo 2.
Activity of Abstract Stages

In the abstract stage, only the basic division operation cards formed from the symbolic values of the numbers are used as material. Teaching begins by saying "Welcome" to the participant. After the information about the session is presented (For example; "Today we will try to divide with you"), the session starts with the consent of the participant. Teaching begins with the practice of being a model. In the modeling application, the practitioner is reminded of the basic division process first. The number of shots fired by the tank is determined by taking into account the number of dividing and dividing. Then, the divided number is imaginatively likened to soldiers, and rhythmic counting is started by taking into account the number of divisors. When it is determined how many times it is counted rhythmically, it is said that the determined number will be written in the section part. The number of how many balls were thrown is written under the number in the dividing part. We are told that we need to perform subtraction. After the practice of being a model, the participant is given a skill instruction by saying "Now it's your turn". The participant is expected to read the basic division operation. The participant is given the opportunity to mentally tell the result after reading the basic division process. If he tells the truth, it is reinforced and a new trial is started. If he does not tell the result in his mind, the participant is asked to make an imaginary likeness to a soldier by looking at the number of divisions, and to start counting rhythmically up to the divided number by considering the number of divisors. It is said that he should determine how many times he counted rhythmically and write the determined number in the section part. It is said that he has to subtract the number of balls he has thrown by writing the number under the number in the dividing part. The correct responses of the participant are reinforced with social reinforcement. the participant's participation in the session is reinforced with primary reinforcement at the end of five trials. When the criteria are met in the abstract stage, the teaching sessions are finished.

Chart to be Used for Measurements at the Event

Data Collection Form																
Teaching Set 1		Sessions														
Question No	Teaching Set Questions	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	6:2=															
2	8:4=															
3	12:3=															
4	20:5=															
5	7:2=															
Session Duration																
Correct Response Percentage																
Percentage of False Responses																
Percentage of No Response																

Figure 1.

Measurement Card

Conclusion

The abstract and sequential nature of mathematical skills and operations emerges as a factor that makes it difficult for students to learn mathematics (Karabulut & Yıkmış, 2010). There are many possible reasons for this situation, such as inadequate use of strategies, less motivation than expected, problems in reading and memory skills (Kumaş, Dada, & Yıkmış, 2019).

Filiz (2021) in her article titled "Examination of the Effects of Instructional Interventions on Students with Mathematics Learning Disability on the Academic Achievement of Students", it is noteworthy that students with mathematical learning disabilities encounter important problems in basic arithmetic skills and concepts, and in this activity article, most of these students are at primary school level. In addition, when the academic studies in this field are examined, it is seen that different teaching methods such as "fraction intervention program" and "schema-based teaching" have a positive effect on overcoming this difficulty (Monei & Pedro, 2017; Marita & Hord, 2016; Chodura et al. 2015; Gersten et al. 2009). The "analogy" method used in this teaching activity article is expected to have positive effects on students.

Similar instructional designs should be planned, taking into account that students with learning disabilities have strong imaginations and weak memories at the same time. Thanks to this proposed activity, the strengths of the students with learning difficulties were utilized while their weaknesses were tried to be strengthened.

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