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Review

A REVİEW: MORPHOLOGICAL, PHYSIOLOGICAL AND MOLECULAR RESPONSES OF SWEETPOTATO TO DROUGHT

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ABSTRACT

Sweetpotato is a drought resilient crop that is nutritionally important for the economic uplifting of humans. Sweetpotato has high beta-carotene contents and low glycaemic index that are important sources of vision improvement. It regulates blood sugar level and insulin resistance in diabetic patients, serves a homeostatic property, and maintains healthy blood pressure. The storage roots and the leaves have anti-cancer agents, purifies the liver, and reduce the risk of obesity, diabetes, heart disease, prevents constipation and malnourishment in children, and promotes fertility in women due to its high contents of fibre, irons, and phytochemicals. Sweet potato has high yielding capacity per square meter than other root and tuber crops and played a vital role in famine-relief. The agronomic and nutritional versatility of sweet potato makes it very important food security crop. However, abiotic stress such as drought stress mitigate against the biological and potential yield realization of the crop. This article reviews the effect of drought on the yield and yield traits of sweet potato and the morphological, physiological and the molecular response to the crop to drought effect. Drought impedes photosynthetic activities and disturb the metabolic processes of sweet potato plant causing imbalance in photosynthesis, respiration, translocation, stomatal movement, light absorption, and ion uptake. The economic storage root yield is reduced, increase the number of deformed storage root, reduction in the canopy cover, leaf area index, decrease in the stem length among others. These morphological and physiological effects of drought trigger the generation of reactive oxygen species which generate signal transduction as a mechanism to protect the plant. The detrimental effects of the ROS are buffered, minimised, and scavenged by enzymatic and non-enzymatic antioxidants such as superoxide dismutase (SOD), catalase (CAT), peroxidase (POX), glutathione peroxidase (GPX), glutathione reductase (GR), glutathione S-transferases (GST), ascorbate peroxidase (APX), monodehydroascorbate reductase (MDHAR), and dehydro-ascorbate reductase (DHAR) as a defence mechanism to keep the ROS under tight control. In resistant breeding, knowledge of these mechanisms is vital for building resistance and tolerance in sweet potato and other crops to improve yield and yield quality and ensure crop sustainability and food security. The levels of the antioxidants should serve as a guide for agronomists and breeders in selecting and recommending cultivars for drought endemic areas for yield sustainability and food security purposes. In selecting crossing parents for drought tolerance, cultivars with high antioxidants should be used to increase the chance of drought tolerance and eliminate the episode of crop failure due to drought stress.

Keywords: Sweet potato, Drought stress, Yield and yield traits, Antioxidants, ROS

1. INTRODUCTION

Sweet potato (Ipomoea batatas) is a nutritionally important crop belonging to the Convolvulaceae family. Sweet potato occurs cytologically as a diploid 2n=30, tetraploid 2n=60 and hexaploid 2n=90 (cultivated forms) with more than 100 known species. Its origin is in Mexico and Venezuela in the Central or South Americans continent. Sweet potato has become the 3rd largest cultivated root crop after potato and cassava globally based on its nutritional and agronomic resilience and food security properties [1, 2]. The crop was introduced into Europe around 1604, Asia during the Spanish colonial era (1521-1598) and Africa in the early 1600s by the Portuguese traders [3]. Today, sweet potato is cultivated in over 120 countries worldwide [2] with over 133 million tons of annual production. Continentally, Africa and Asia are the leading producers of sweet potato with 95% of production coming from developing countries, bringing sweet potato to the 5th and 6th important food crop respectively in developing countries and in the world.

Sweet potato is nutritionally an important source of beta-carotene, a precursor of vitamin A (vision protection), vitaminsB6, and C [4]. The storage roots are rich starch reservoirs with carbohydrates, dietary fibre, minerals, and vitamins for human consumption and animal feeding [5]. Sweet potato helps in the economic uplifting of humans and serving as a food security crop. It maintains healthy blood pressure, prevents constipation due to its high fibre content, has anti-cancer agents, reduces the risk

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of obesity, diabetes, heart disease, and overall mortality in humans [6] and serve as a liver purification crop. The high fibre, phytochemicals, and iron content in sweet potato promote fertility in women [7, 8]. Sweet potato contains a low glycaemic index scale, regulates blood sugar level and insulin resistance in diabetic patients, and thus serves a homeostatic property in human health. The glycaemic index of sweet potato reduces the risks of stroke by 24% [9]. Only small percentage of the global human population meet the daily dietary potassium requirements, thus the high amounts of potassium in sweet potato help reduce this by 2 %. Potassium also helps mob up excess reactive oxygen species (ROS) in the human system and thus improve their general well-being [10]. Sweet potato is used in the production of alcoholic beverages, flour for biscuits, and noodles [11].

Agronomically, sweet potato has high yield per square meter than any other tuber crop and its yield can be increased by 25% with a slight improvement in agronomic practices [12]. It has high promising dry matter and requires less capital with a short duration of cultivation and is the foremost root vegetable with the highest calories. In its storage roots, carbohydrate and ethanol quantities are triple that of corn though cultivar dependent [2]. Sweet potato is especially earmarked for its higher yield than other root and tuber crops and played a vital role in famine-relief over the years and has been re-evaluated as a health-promoting food [13, 14]. The agronomic and nutritional versatility of sweet potato captured global attention as a resilient food crop to fight food and nutrition insecurities which is expectant of bridging food shortages because it is a high-yielding energy crop per unit area [12].

However, abiotic stress such as drought stress over the years has been a major setback affecting sweet potato production. Drought in agricultural sense is the inadequacy of soil moisture for crop plants utilization for maximum yield output. In the meteorological sense, drought is the shortage of precipitation. In agricultural production, drought, or water deficiency is a major limiting factor that prevents crops from achieving their genetically determined potential maximum yield. Drought adversely affects crop growth and yield, and it has been identified as the primary constraint on rainfed crop production especially in rice, potato, soybean, wheat, maize, groundnut, and sweet potato [15, 16].

Globally, drought is a major abiotic stress factor prevalent in sweet potato production areas. About two-thirds of the world including Southern Africa, West- and North-Africa, central America, west and mid-west of North America, southern and eastern parts of South America, the Near East, and Central Asia; and three-quarters of Western Europe, India, Western Australia, and Northern China are prone to drought and desertification due to drought affecting 52 million humans annually [17, 18]. Drought negatively affects plant growth through various biochemical, morphological, and physiological processes. It inhibits photosynthetic activities and disturb the metabolic processes of the crops, resulting in imbalance in photosynthesis, respiration, translocation, stomatal movement, light absorption, and ion uptake [19, 20, 21]. It also causes reduction in mineral nutrient uptake and disorder in many various metabolic processes.

Although sweet potato is generally said to be a drought tolerant crop, selection of appropriate genotypes for drought conditions is still essential. Almost all plants have drought tolerance however the degree of the tolerance varies from one species to another and even within the same species due to i) different severity of drought, ii) duration of drought, iii) the organizational level of the plant, and iv) the developmental stage of the plant species [22, 23, 24]. Hence, understanding the morphological and physiological responses of sweet potato to drought can help to determine the traits to be used as selection criteria in breeding programs for yield improvement under drought conditions [25].

In the context of climate change, the frequency and the severity of drought is expected to rise most especially in Africa in the coming decades [18, 26]. Different drought response mechanisms in sweet potatoes such as drought escape (earliness), drought avoidance (root depth), and drought tolerance (maintaining assimilation under drought conditions) need to be identified and new sweet potato varieties should be improved using this information to combat the negative effect of drought to the sweet potato cultivation [18]. Therefore, this review assessed the effect of drought on the morphological, physiological, and molecular traits in sweet potato.

2. EFFECT OF DROUGHT ON STORAGE ROOT OF SWEET POTATO

Plants respond to soil moisture deficit conditions through the root zone by sensing the soil drought at the cellular level and through the whole root system architecture [27, 28]. The sensed changes in the soil moisture caused morphological and physiological changes aimed to absorb water and nutrients in the soil [29, 30]. The growth and architecture of roots are plastically a complex system. It is associated with several gene interaction and expression with an array of factors such as biological, and physical, features in the soil.

In crop production, root system serves as a good selection criterion for drought tolerant crop varieties under drought and is stated to positively correlate with yield under drought stress. In sweet potato, the drought tolerance ability is affected by root quantity, morphology, distribution, and physiology. Deep and dense rooting system increases the drought tolerance and root yield of sweet potato due to maximum uptake of mineral nutrients and water. In sweet potato, drought causes extensive structural changes in the root by increasing the root branching and density. In sweet potato as in other crops, drought stress causes small root system configuration and reduction in size of root system which is dependent on the magnitude of water shortage. The rate and availability of nitrogen (N) affect sweet potato root architecture development [28] especially at storage root formation stage

[31]. The adventitious roots of sweet potato differentiate into storage roots from the stems during the early growth period. This is affected by environmental factors such as soil moisture and N content [31].

[27, 28] revealed that the application of N fertilizer caused early development and differentiation of sweet potato roots. They further indicated that drought stress reduced root biomass ranging from 47.23% to 75.19%. Sweet potato root growth and differentiation is delayed in the presence of excess soil nitrogen [28]. This apparently causes great reduction in the storage root yield of sweet potato. The negative effect of drought on the root development of sweet potato can be alleviated by appropriate nitrogen (e.g., N75) treatment and thus enhance their tolerance levels while excess soil nitrogen (N150) significantly reduces root biomass and morphological parameters such as root volume, total root length, and root surface area that affect the number of differentiated marketable (diameter greater than 3.0mm) storage roots [32]. In fact, drought stress has greater negative impact on sweet potato root architecture. The difference in root architectural response of different sweet potato varieties to drought or water deficit present the bases for drought tolerance of the varieties.

For improvement of drought resistance of sweet potato varieties, deep root system is the identified target. Sweet potato varieties with deep and thick roots increase the drought stress tolerance of the plants due to positive correlation with xylem vessel area. The xylem vessel area helps the plants to meet the evaporative demand by conducting soil water to all parts of the of the plant [32, 33]. Drought stress in sweet potato crop is characterised by small root development. For optimum growth and development of crops, [34] reviewed that, large and vigorous root system and the continued production of new root hairs are required for maximum response to nutrients supply and increase dry matter accumulation within the shoot in drought conditions.

Drought stress has two main effects on sweet potato root development. It declines the rate of root meristematic activity and decreases the root elongation. This causes negative effect on mineral nutrient and water uptake and suberization of the root system as whole. Drought stress also adversely reduces the fresh and dry biomass production of sweet potato due to the inhibited uptake of nutrients and water by the small root architecture [35]. This situation disrupts the dry matter portioning and temporal biomass distribution, resulting in a decrease in crop yield. Thus, for drought resistance breeding, high dry weight under water stress conditions is a desirable characteristic for survivability of the plant under water stress conditions.

3. EFFECT OF DROUGHT ON THE STORAGE ROOT YIELD OF SWEET POTATO

Sweet potato breeders aim to obtain varieties with high yielding and nutritious storage root in their breeding program, especially under drought conditions. Under favourable condition, the yield potential of a variety is important to determine the variety's yielding ability under water stress. Drought stress critically affect sweet potato yield at three critical stages, seedling, vegetative development, and anthesis. These stages are highly affected by water stress, resulting in reduction in the yield. Sweet potato is susceptible to water stress at plant establishment stage. At this stage water limitation causes lignification of developing roots which impair the potential lateral thickening, a sign of proper photosynthate sink [36]. Different sweet potato cultivars have varied drought response and sensitivity.

In a field study to assess the response of sweet potato to prolonged drought, [37] stated that drought stress caused cracking on storage roots. This increased the accessibility of insects to the storage roots and caused a greater degree of root damage depending on the cultivar. Drought stress caused three different underground effect on the sweet potato, decreased storage root yield, unmodified storage root yield, and increased storage root yield. Indicating that, the effect of drought stress on the yield and yield components such as the number of storage root and single storage root weight of sweet potato is cultivar dependent. Based on this, sweet potato genotypes have been classified as susceptible, neutral, and resistant sweet potato genotypes based on their drought stress response strategy. [37] demonstrated that under water stress conditions, the marketable yield of sweet potato cultivar "Toka Toka Gold" decreased by 77% under drought conditions. They concluded that drought stress had a severe negative significant impact on storage root yield of all sweet potato cultivars. It has been noticed that severe water deficit caused drastic reduction in storage root yield and biomass of sweet potato. Thus, in paramount to say that sweet potato experiences huge yield losses under severe water stress conditions. This effect can be attributed to the decrease in assimilates translocation towards storage roots.

To select suitable cultivar under water deficient environment, drought tolerance index has been used as an important criterion to screen for drought tolerant varieties [38, 39]. Drought tolerance index is calculated based on the loss of yield under drought condition in correlation to water sufficient condition [40]. Drought stress aside causing drastic storage root yield reduction, it results in a decrease in stem length and leaf size of genotypes.

It is further stated that drought stress has severe effect on the yield of sweet potato, though sweet potato is said to be a drought tolerant crop. This was confirmed by [41] assessing the effect of drought on sweet potato using seven cultivars and six elite lines under three water treatments (100% water availability as control treatment; 60% water availability as mild stress; and 30% water as severe stress). Very great effect and significant genotypic differences were found for the storage root yield, leaf area index (LAI), stem length and stomatal conductance (gs). In terms of stem length, Purple Sunset and Blesbok sweet potato cultivars were found to strive much better under drought stress conditions. For total biomass and root yield, Bophelo, Resisto, and 199062 produced the highest yield at the mild stress with positive significant correlation with LAI, stem length, and stomatal conductance. With this, [41] suggested that the underground traits of sweet potato are directly influenced by the above-ground growth with LAI and stomatal conductance playing a very important role in achieving storage root yield. This can be useful indicators for screening drought tolerant sweet potatoes.

During the storage root development and establishment stage of sweet potato, drought stress is the most limiting factor. It causes reduction in the cells swelling pressure and inhibit cell growth, which reduce the plant root growth and differentiation [28, 34], sweet potato root biomass, total root length, root surface area, and root volume [27, 28]. The intensity and duration of the drought determine the extent of decrease in storage root yield which also depend on the number of lateral roots formed. It also reduces the plant establishment, inhibits the growth, causes damage to the photosynthetic apparatus which leads to decrease in net photosynthesis, and reduction in the mineral nutrient uptake [18]. Many scientific reports stated that sweet potato is particularly susceptible to drought at the root initiation and bulking stages which cause great effect to loss of yield, hence require adequate available soil moisture from planting till harvest [17, 18].

4. EFFECT OF DROUGHT STRESS ON THE LEAF CANOPY AND DENSITY

Drought stress is a very important factor for plant growth and affects both elongation and expansion growth. It is a major abiotic stress affecting agricultural production and productivity around the world, resulting in yield loss. Drought reduces soil water availability and increased evaporation. The cell size, intercellular volume and leaf area of the plants are reduced in drought stress conditions [42], especially in sweet potato. This amongst others is mostly due to the reduction in the soil moisture content which lead to lowering the leaves water content. The water loss in the leaves leads to a decrease in turgor pressure of guard cells, resulting in reduction in the stomatal pore sizes and the stomatal closure.

Drought stress causes reductions in the height and branching of cereals, legumes, root, and tuber crops including sweet potato. It also reduces LAI and biological yield of the plants. In soybean crops, water stress has been noticed to cause reduction of seed weight, total biomass, pods per plant, seeds per plant, seeds per pod, 100-grain weight, and ultimately caused a decline in soybean yield due to reduction in the LAI [39]. The reduced LAI reduced the absorption of photosynthetically active radiation by the plant which results in the yield reduction and the other parameters being affected. In sweet potato, drought stress at the early stages causes early profuse flowering and delayed canopy growth and development. Research by [37] on the effect of drought on sweet potato revealed that, the canopy growth and expansion of some varieties of sweet potato such as Beauregard, S1819, S1818, S1816 and S1787 were highly affected (reduced) throughout the growth period.

5. MORPHOLOGICAL, PHYSIOLOGICAL, AND MOLECULAR EFFECTS OF DROUGHT ON SWEET POTATO

Plants encounters water stress or drought due to no rainfall, high salt concentrations that cause reverse osmosis, low temperature, and transient loss of turgor at midday. At the molecular levels, plants during long-term water stress produce stress proteins, chaperones, up-regulate antioxidants, and accumulates compatible solutes [43, 44]. These mechanisms aim to increase root growth, and reduce leaf and stem growth, as they are the effects of drought stress on the growth of plants. Physiologically, plants during water stress produce and accumulate abscisic acid (ABA) and solutes in the leaves and roots that are transported to the guard cells. Also, reactive oxygen species (ROS) () are produced which inhibits the membrane protein pumps and increase influx of Ca^{2+}, causing efflux of anions and K^+ of the cells [42, 45, 46]. These actions cause conversion of malate to starch leading to reduction in the osmotic potential and turgor pressure, and reduction in the cell volume. The resultant effect of this physiological effect is closure of the stomata. In sweet potato, drought causes stomatal closure that affect the photochemical efficiency in the photosystem I and II and quantum generation this cause metabolic breakdown and reduction of photosynthesis [47, 48]. Drought also disrupts the cyclic and non-cyclic types of electron transport in the light phase. It is stated that, photosynthetic reduction in plants is due to qualitative and quantitative reduction or total stoppage of the photosynthetic pigment chlorophyll and carotenoids pigmentation production, photo-oxidation, and degradation of the pigments [49].

[49] found a significant reduction in the stomatal conductance of the sweet potato under drought conditions. Other researchers also reported that, stomatal conductance of plants is an indication of their drought tolerance ability. In breeding and agronomical evaluations for drought tolerant cultivars, stomatal conductance, relative water content, and photochemical efficiency of photosystem II are the important physiological traits to be considered.

It was stated that drought stress caused a reduction in photosynthesis rate ranging from 71.2% to 98.7%, in stomatal conductance between 34.4% and 47.1% and in chlorophyll content of *Triticum aestivum* [50, 51, 52]. In sweet potato and potato, the closure of the stomata prevents the capture of CO_2 by the carboxylation centre and increases the internal leaf temperature. This causes the degradation of the chlorophyl apparatus and lead to photosynthetic rate depletion and the inhibition of the plant growth. [53] revealed that drought is the main inhibitory factor for growth of plants. This effect among others is caused by the low absorption of CO_2 by the carboxylation centre due to stomata closure. The low or prevention of CO_2 capturing by the plant leaves results in reduced photosynthetic rate leading to the low growth and yield of the crops.

In a study to assess the effect of drought stress on chlorophyll content, proline content, stomatal conductance, photosynthesis and transpiration, and yield characteristics in chickpea cultivars (drought tolerant Bivaniej and ILC482 and drought sensitive Pirouz), [54] concluded that all physiological and biochemical parameters of the varieties were significantly affected. The concentration of chlorophyll, transpiration, stomatal conductance was drastically decreased while the proline content was substantially increased. Among other researchers, [54] stated that plants were especially affected at physiologlal level by drought occurred at vegetative or anthesis stages. Plants physiologically respond to drought by accumulation of proline. The rise in proline indicates impairment of photosynthesis due to stomatal (stomatal closure) and nonstomatal (impairments of metabolic processes) factors [55, 56, 57]. The photosynthetic sensitivity of a plant to drought is determined by its mesophyll resistance under drought conditions.

Morphologically, [58] stated that the above ground biomass and morphology of potato cultivars were greatly affected by drought stress treatments and the impact escalated when combined drought and heat stress. This resulted in less abundant leaves, foliage and shorter stems of the stress treatment as compared to their controls varieties. [58] noticed a drastic reduction in the photosynthetic rates and leaf relative water content of all cultivars (Agria, Desiree, Russet Burbank, and Unica) being either drought resistant or sensitive under drought conditions. Furthermore, a significant increase in proline and MDA was observed in the drought treated cultivars in potato [58, 59]. The results of various research on the effect of drought on crops shows that drought affects at morphological, physiological, and biochemical level.

Plants have evolved several survival mechanisms at morphological, physiological, and molecular levels to withstand the various environmental impediments they encounter due to their sessile nature (Figure 1). Some plants adapt to the effect of drought by developing deep and defused root systems to absorb soil moisture as much as possible, small leaves to prevent much water loss through transpiration. Crops including sweet potato are photosynthetically sensitive to drought and thus exhibit diffusive stomatal resistance, leaf water retention, osmotic adjustment, rolling of the leaves, regulation of the closing and opening of stomata and its position, and leaf senescence. The limited water in the soil serves as a prime signal perception to detect the available water and act as a signal for ABA (abscisic acid) biosynthesis that regulates stomata closure to prevent the water loss [60, 61]. The consequential impact of this response is reduction in the CO_2 absorption and assimilation in the dark reaction of the photosynthesis. It also affects the photons' energy levels in the light reaction in the photosystem II. This restricts the H_2O oxidation in the chloroplastic organelle and lowers the quantum yield (Fv/Fm).

At the molecular or cellular levels, plants such as sweet potato adjust their osmotic pressure by synthesizing and accumulating carbohydrates, proline, sugar alcohols, polyamine and glycine betaine that serve as osmoprotectants [63, 64, 65]. For instance, proline, a nontoxic and non-enzymatic small neutral aromatic amino acid molecule accumulated during the episode of drought stress, is synthesized by sweet potato in the cells through the glutamate and/or ornithine pathway to play a key role in osmotic adjustment. More so, [10, 66, 67, 68, 69] among other researchers stated that proline among other non-enzymatic compounds such as ascorbic acid, carotenoids, glutathione, and tocopherol are synthesized to reduce the negative effect of reactive oxygen species (ROS) and keep them in balance during drought stress. The activities of enzymatic antioxidants including ascorbate peroxidase, glutathione transferase, catalase, and superoxide dismutase are also increased to scavenage the accumulated ROS under drought [70, 71]. This detoxification of the ROS by the enzymatic and nonenzymatic antioxidant defence systems decreases the harmful effect of ROS. The ROS are mainly generated by the chloroplasts and mitochondria. Plants have developed a versatile and cooperative antioxidant system and defence mechanism that modulates intracellular ROS content and sets the redox status of the cell in balance and under tight control [72, 73, 74, 75]. A slight increase in ROS production under drought conditions initiates a signal transduction to trigger plant defence mechanism which is linked to abscisic acid (ABA) and Ca^{2+} fluxes in the cell resulting in acclimatization to the environment [60, 66, 72, 74]. The higher accumulation of ROS under drought conditions damages the cells, even causes cell death due to protein denaturation, lipid peroxidation, and DNA degradation. Sweet potato and other plants also produce aquaporins (AQP), drought-responsive genes (DRG), late embryogenesis abundant proteins (LEAP), transcription factors (TF), heat shock proteins (HSP), dehydrins, proline (pro), glycine betaine (GB), and cyclic adenosine 50-diphosphate ribose (cADPR), inositol-1, 4, 5-triphosphate (IP3), NO and soluble sugar (SS) [39, 59] in response to drought stress aiming for the crop to survive. These results in morphological and physiological changes through signal transduction directly or indirectly [76] by the action of calcium-dependent protein kinases (CDPKs), mitogen-activated protein kinases (MAPKs), HD-zip/bZIP, AP2/ERF, NAC, MYB, and WRKY [20] known as regulatory gene products that enables the successful survival of the plants in the environment.

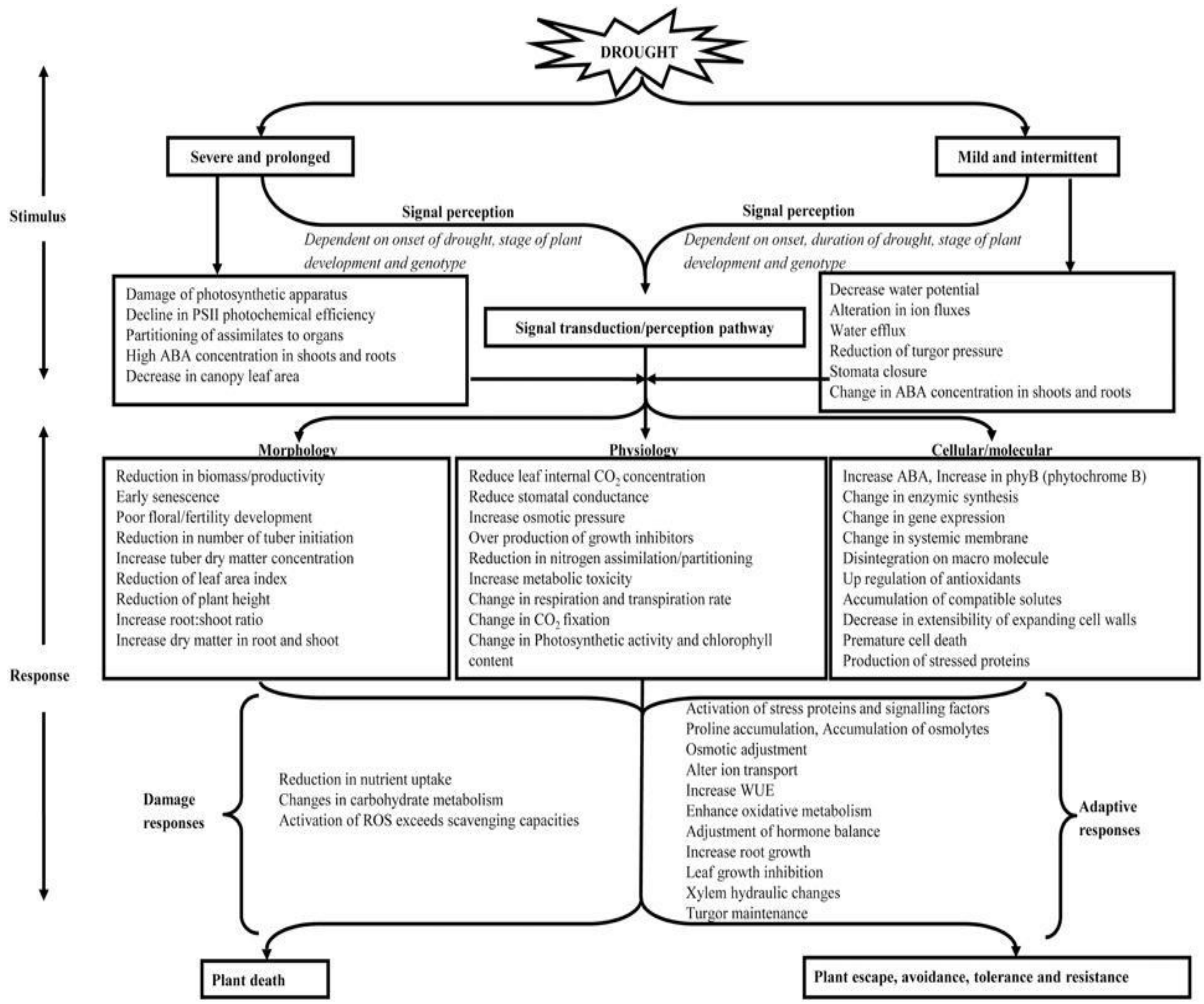

Figure 1: Morphological, physiological, and molecular response of plants to drought stress conditions; obtained from [62]

The enzymatic antioxidants defence system includes superoxide dismutase (SOD, EC 1.15.1.1), catalase (CAT, EC 1.11.1.6), guaiacol peroxidase (POX, EC 1.11.1.7), glutathione peroxidase (GPX, EC 1.11.1.9), glutathione reductase (GR, EC 1.8.1.7), glutathione S-transferases (GST, EC 2.5.1.18), ascorbate peroxidase (APX, EC 1.11.1.11), monodehydroascorbate reductase (MDHAR, EC 1.6.5.4), and dehydroascorbate reductase (DHAR, EC 1.8.5.1) (Rajput et al., 2021). During stress episodes, the antioxidant enzymes effectively buffer, minimize and scavenge the ROS. The superoxide (O_2^-) radical is catalysed by SOD to form molecular oxygen (O_2) and hydrogen peroxide (H_2O_2), a less reactive ROS (Figure 2). The each of CAT, APX, POX, and GPX detoxifies the H_2O_2 to water [70, 77, 78]. The levels of these antioxidants in sweet potato are highly enhanced under drought conditions that makes the sweet potato moderate or highly resistant to drought.

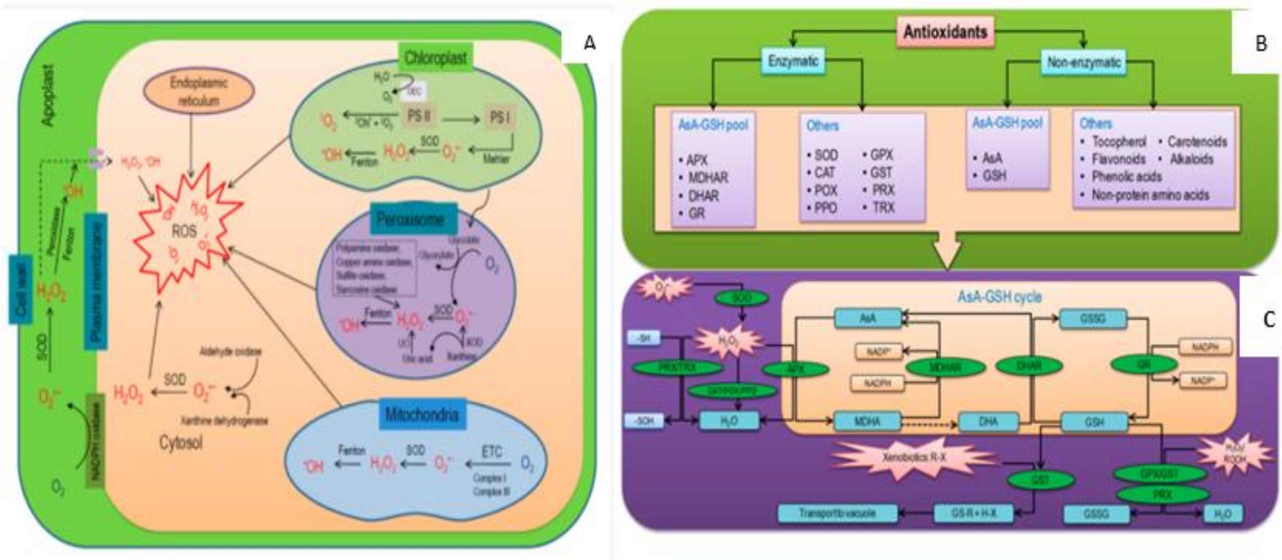

Figure 2: Localization and processes for the generation (A), Enzymatic (B) and non-enzymatic (C) antioxidants and their scavenging activity of ROS; (modified from [70]). Ascorbate peroxidase (APX), Ascorbate (AsA), Catalase (CAT), Dehydro-ascorbate (DHA), , Dehydro-ascorbate reductase (DHAR), Glutathione peroxidase (GPX), glutathione reductase (GR), reduced glutathione (GSH), oxidized glutathione (GSSG), glutathione S-transferase (GST), hydrogen peroxide (H_2O_2) monodehydroascorbate (MDHA), monodehydroascorbate reductase (MDHAR), nicotinamide adenine dinucleotide phosphate (NADPH), superoxide anion ($O_2{\bullet}^-$), peroxidases (POX), peroxiredoxins (PRX), R, aliphatic, aromatic, or heterocyclic group; ROOH, hydroperoxides;, thiolate (–SH), superoxide dismutase (SOD), sulfenic acid (–SOH), thioredoxin (TRX), sulfate (X), nitrite, or halide group. hydroxyl radical (•OH), urate oxidase (UO), xanthine oxidase (XOD), electron transport chain (ETC), PS photosystem I (PSI), photosystem II (PSII), nicotinamide adenine dinucleotide phosphate (NADPH)

Sweet potato also responds to drought by alterations in gene expressions. These genes are classified into functional and regulatory genes [76]. Functional genes directly resist environmental stress, whiles the regulatory genes indirectly respond to stress through the actions of protein kinase genes, protein phosphatase genes, phospholipid metabolism-related genes, and stress-related transcription factor genes assisting in signal transduction and regulation of gene expression [27, 28, 79]. The functional genes include aquaporin genes, osmoregulatory factors synthase genes, and protective proteins genes. These proteins act by participating in plant stress signal transduction pathways or by regulating the expression and activity of other effector molecules.

6. CONCLUSION AND FUTURE PERSPECTİVES

The sessile nature of plants causes them to be highly exposed to environmental effects such as drought stress. Drought stress is the major abiotic stress affecting about two-third of global cultivation land area. In sweet potato cultivation, drought negatively affects the agronomic and economic output due to several morphological, physiological, and biochemical changes. It causes reduction in root yield, branching, canopy cover, leaf area index, stem height and length, stomatal closure, leaf sizes, and photosynthesis in sweet potato. Drought triggers oxidative stress that generate reactive oxygen species (ROS) being harmful to the plants. To survive the effect of these ROS and other negative pressures of drought, sweet potatoes synthesize and accumulate some molecules such as carbohydrates, proline, sugar alcohols, polyamine, and glycine betaine to adjust their osmotic pressure and serve as osmoprotectants. The activities and synthesis other enzymatic and non-enzymatic antioxidant compounds such as ascorbate peroxidase, glutathione transferase, catalase, superoxide dismutase, ascorbic acid, carotenoids, glutathione, and tocopherolare increased during drought stress to scavenge and keep the ROS under tight control. The drought resistant and tolerant nature of sweet potato is mainly due to the high antioxidant levels which effectively keep the generation and effects of the ROS under check. In sweet potato breeding, the levels of the antioxidants in a genotype may serve as a marker for breeders to select drought resistant/tolerant individuals for drought prone areas for yield sustainability and food security purposes. For plant breeders, to increase the drought tolerant levels of cultivar, the genotypes with high antioxidants levels might be used as crossing parents. Such genotypes may have the possibility to increase drought resistance and tolerance levels of sweet potatoes to eliminate the high yield reductions under severe drought conditions.

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Review

A REVIEW ON THE EFFECTS OF IRRIGATION AND NITROGEN FERTILIZATION REGIMES ON POTATO YIELD

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ABSTRACT

Water and nitrogen are the most important factors affecting yield and quality in potato management. Proper irrigation and nitrogen fertilizer management ensures water conservation and reduces nutrient loss. Nitrogen (N) is the most commonly deficient mineral nutrient in agricultural soils to increase potato production. Therefore, the split application of nitrogen fertilizers is important to prevent losses through leaching, evaporation, denitrification, weeding, erosion by running water, and precipitation. Determining the appropriate amounts of nitrogen and irrigation water to improve the efficiency of water and nitrogen use can minimize N losses, minimize costs and increase production profits. Potato is very sensitive to water stress due to its shallow root system and requirement to consume a plenty of water in all growing season. Therefore, regular irrigation of potato is important for obtaining the best tuber yield. This review has been made to identify and analyze in current research on N management practices and irrigation regimes to improve and maintain potato tuber yield. Variability of results from research studies show that determined precise results are not transferred from one place to another because potato varieties and cultivation conditions are not the same. In addition, it was determined that the responses of potato varieties to different nitrogen amounts varied from region to region. According to different irrigation regimes, it has been seen that the best result increasing yield in potato is obtained from applications when the plant is irrigated the most and field capacity is fully saturated. In addition, although the most appropriate nitrogen and irrigation interaction differ from region to region and depending on the variety, it has been determined that the most irrigation is applied and the nitrogen is obtained from application roughly 200 kg/ha.

Keywords: Nitrogen rate, Irrigation regimes, Tuber yield, Solanum tuberosum

1. INTRODUCTION

Potato (Solanum tuberosum L.) is one of the most important plants serving as a food source. It is a good strategy to prevent food insecurity, particularly in disaster conditions [1]. Potato (Solanum tuberosum L.) is the fourth most important food crop and is believed to contribute significantly to maintaining future global food security. The crop has high water needs and is particularly susceptible to drought stress during the tuber growth stage. The low drought tolerance is mainly attributed to the shallow and sparse root system and irrigation is demanded to reach acceptable tuber yield and quality [2, 3]. There are critical growth periods when irrigation is necessary for optimum yield and quality in potato. Nitrogen (N) is the most effective nutrient for growth, development, productivity and tuber quality. Although there is often conflicting information in the sources regarding irrigation and N management of this crop, it is generally confirmed that production and quality are greatly affected by N dose and irrigation amount and that these requirements are related to crop technique [4]. In modern production, the aim is to maximize crop production and optimization of both N and water use to minimize the risk of N leakage into groundwater. Water deficit can result in smaller tuber size and lower yields. Potato yield is greatly affected by the timing and process of water stress at different growth periods. Potato crops are influenced by lack of humidity at all periods of growth, but during tuber initiation and bulking this has a serious impact on yield [5]. Potatoes are susceptible to water deficiency and low water stress leads to decrease in leaf number and size, photosynthesis, which affects tuber number/plant, size and yield. Nitrogen is an important element influencing crop yield and also considerably affects crop yield in water deficient conditions. Hence, researching the source-sink interrelation and its impact on yield under different water and nitrogen conditions aid to maximize crop yield and optimize water and nitrogen use efficiency [6]. Nitrogen deficiency in potato is indicated by a decrease in growth and tuber yield in point of tuber number and size [7]. Soil water is known to affect nutrient transport to the root surface in water flow generated by transpiration and high water uptake by plant roots significantly increases root N uptake by mass flow [8]. Additionally, the water and nitrogen combined effect on the potato yield was also examined in some studies, but these studies have explored to achieve the highest efficiency levels [4, 5, 9].

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2. EFFECT OF NITROGEN FERTILIZATION ON POTATO

Although potatoes are widely grown and adapted to very different climatic conditions, it has need for a stable fertilization, without which the yield and quality of tubers is clearly affected. Potato is intensely fertilized because they have a high nutritional demands. Fertilizer practice relate to soil type, soil fertility, irrigation facilities. Likewise, nutrient uptake by the potato crops depends on the climatic condition, soil type and fertility status and crop management practice [10]. N is one of the most vital macronutrients for growth and biomass development. Plants can benefit N in several forms. Their significant sources are ammonium ($NH4+$) and nitrate ($NO3-$) [11]. N shows an important role in plant growth as it is a component of chlorophyll, amino acids, proteins, nucleic acids, coenzymes and membrane components [12]. Therefore, N supply means bigger tubers for plant. Evaporation, leaching, denitrification, erosion and sedimentation can be prevented by applying nitrogen in a split form. Thus, nitrogen efficiency is increased.

By applying the right source of N fertilizer at the right rate, time and place, N can be managed effectively. The period when the potato plant needs nitrogen the most is the tuber bulking period. About 58-70% of the total N is taken at this stage of development [7]. Nitrogen fertilizers are the most commonly used mineral fertilizers on potato farming lands. Nitrogen fertilizers have positive effects on both growth and plant development when they are in the most appropriate form and amount. However, when nitrogen fertilizer is applied excessively, resistance to diseases and pests decreases, and storage resistance also decreases. In case of insufficient application of nitrogen during tuber formation period, it causes drying of tuber and old leaves and tuber development decreases significantly. Nitrogen fertilization is necessary due to the limited root system of the potato and the low nitrogen utilization efficiency. However, irrigation management should be done appropriate together with nitrogen. Otherwise, nitrogen can be leached away, especially with over-irrigated surface irrigation methods [13]. Potato plant generally requirements more nitrogen fertilizer than other plants to reach high yields. The save of nitrogen fertilizer applied in the developing plant is less than 50% on average [14].

The management of nitrogen fertilization is extremely important both economically and environmentally. Nitrogen deficiency decreases plant growth and also reduces tuber size and yield [15]. However, excess nitrogen can lead to reduced tuber quality, delayed maturation and nitrate leaching. [16]. In addition, the effectiveness of nitrogen fertilization and optimum nitrogen utilization can alter among farming and over the years [15]. This diversity is based on variation in both crop nitrogen demand and soil nitrogen content. As a result, the development of tools to more accurately predict individual field nitrogen needs in potato production will improve tuber yield and quality. In this way, it can be used as a strategy to minimize nitrogen loss and damage to the environment. Plant nutrition is a biological process essential for plant health and efficiency. Fertilizers are the primary method of nutrient management. However, taking into consideration that soil is the main source of nutrients for plants, fertilizers are method to make up for certain deficiencies in the soil. In this case, it is necessary to obtain information about the lack of elements in the soil [17]. Nitrogen, one of these fertilizers, has a direct effect on potato growth and yield [18].

Potato, like other cultivated plants, need adequate and balanced nutrients for good growth and yield. Compared to other plants, the potato is most demanding on nitrogen and potassium. In terms of nutrient consumption, approximately one ton of potatoes tuber removes 4 kg of nitrogen from the soil [19]. Deficient nitrogen content incurs a negative effect on vegetative growth and tuber formation. As a result, a vital decrease in tuber yield occurs. For this reason, nitrogen should be given as much as the plant needs for optimum efficiency. Therefore, the important point is that the nitrogen requirement of the plant changes according to the growth periods. For example, in potato, it changes with the varieties, the period when the plant needs the most macronutrients is the tuber bulking stage, and this period alter between 42 and 70 days after planting [20]. On the other hand, soil, climate and biotic factors are also important factors affecting the nutrient requirement. Nitrogen optimization is very important to increase yield and reduce nutrient loss in potatoes. Ensuring this depends on the correct selection of the nitrogen source, dose and application time, as well as the variety, soil moisture level and soil structure [21, 22, 23, 24].

2.1. Effect of Different Nitrogen Sources On Tuber Yield

In plant production, nitrogen is taken by the plant in the form of nitrate ($NO3$) and ammonium ($NH4$). Different studies have been conducted to determine whether different nitrogen sources have an effect on tuber yield in potato production. Muthoni and Kabira (2011) conducted a two-year study on different nitrogen sources. Nine different nitrogen forms (DAP, DAP+ Farmyard manure, NPK, NPK+ Farmyard manure, Farmyard manure, Farmyard manure + CAN, TSP+CAN, NPK+TSP, DAP) and two different potato varieties (Tigoni and Asante) were used. As a result, they reported that different nitrogen sources have a significant effect on yield and the highest yield was obtained from DAP+ Farmyard Manure application in the first year and NPK application in the second year [25]. Likewise, Gathungu et al. (2000) reported that different nitrogen sources significantly affect tuber yield and that the highest yield was obtained from CAN (Calcium ammonium nitrate) and ASN (Ammonium sulphate nitrate) applications [26]. On the other hand, contrary to these studies, Cambouris et al. (2016) found that the amount of nitrogen application significantly affects the yield. However, they found that the nitrogen source did not affect it [23]. Zebart et al., (2012), similarly, showed that nitrogen source did not affect tuber yield in their study [27].

2.2. Effect of Nitrogen Application Time On Tuber Yield

Even if the main purpose in production is to maximize yield, many factors such as application time, input cost and availability of resources come into play in fertilization. There are many factors that can affect the rate at which photosynthesis products are created and consumed at different growth stages. Nitrogen, in particular, is one of the most important nutrients affecting the production and distribution of assimilates. The application time of nitrogen fertilizer can significantly affect the yield and quality of potato tubers [28, 29]. Since nitrogen is a fertilizer that is constantly needed for plant growth and development, it is not appropriate to give nitrogen at once. For this, field trials for specific conditions and potato varieties can solve the best nitrogen management [30, 31]. Sun et al., (2012) examined the effect of four different application times on yield in potato in their study. At the end of the study, the highest tuber yield was obtained from T1 (150 kg/ha at planting) and T3 (150 kg/ha at planting + 50 kg/ha one week before tuber growth period) [32]. Öztürk et al., (2007), on the other hand, determined that the application time significantly affects the large (more than 5.0 cm in diameter), small (less than 3.5 cm in diameter) and total tuber yield, except for the middle tuber (diameter 3.5-5.0 cm) [33].

2.3. Effect of Different Nitrogen Rates On Tuber Yield

Nitrogen fertilization is the most costly and significantly affecting tuber yield in potato production [34]. Thus, due to the environmental impact and high fertilizer prices, it is necessary to keep nitrogen application in certain varieties between limited values [35, 36, 37]. In some countries, the recommended amount of nitrogen fertilization for potato production varies between 70 and 330 kg/ha. However, it has been reported that the optimum nitrogen rate is between 147 and 207 kg/ha, depending on the variety and nitrogen cost [35]. In the study by Akpınar et al., (2019), in which different nitrogen doses were applied, it was shown that the 20 kg/da nitrogen dose was the most economically and application higher than 20 kg/da did not have any effect on the yield [24]. Ahmed et al., (2017), on the other hand, reported that the highest tuber yield was obtained from nitrogen doses given in four different periods (5 days, 20 days, 35 days and 50 days after planting) [38]. In addition, according to the results of their research, Ghiyal and Bhatia (2018) determined that the highest total tuber yield was 120 kg/ha nitrogen dose [39]. However, Workineh et al., (2017) suggested that 69 kg/ha nitrogen fertilization rate provides the most economical and best tuber yield and that this dose should be used for production in ecologies under the same conditions [40]. When other studies are examined, it is seen that the most appropriate nitrogen dose for potatoes is between about 200 kg/ha (Figure 1). However, these studies were carried out at the regional. Therefore, it should be noted that the amount of nitrogen fertilization in potato production varies from region to region. In addition, the most appropriate dose should be determined by the researchers and recommendations should be given to the farmers in the region according to dose.

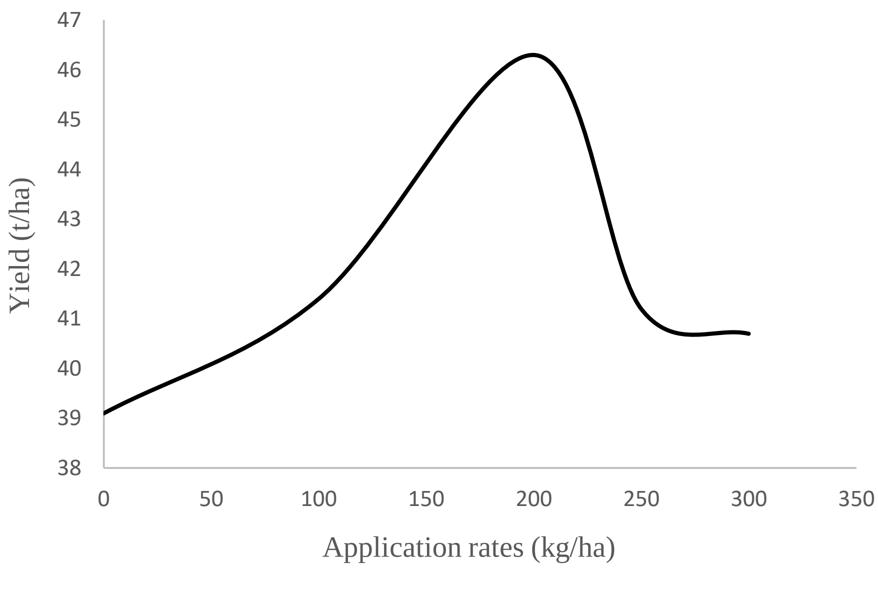

Figure 1. Effect of application amount on potato yield [4, 24, 35, 34, 41, 42, 43, 44, 45, 46]

3. EFFECT OF IRRIGATION REGIMES ON POTATO

During the potato production season, the plant growing period differs according to varieties. Moreover, the plant water consumption (Evapotranspiration, ET) of potato, which is 100-150 days on average, is between 500-700 mm depending on climate conditions [47]. Since potato are great sensitive to moisture deficit in the soil, the available water in the soil should not decrease below 65% in order to obtain the highest yield [48]. For an efficient potato production, irrigation should be done before 35% of the total available moisture is consumed and effective root depth should be taken as 0.4 m in irrigation. [49]. The duration when potatoes need water the most is the period from the beginning of tuber formation to 15 days before harvest. At this stage, if irrigation is not applied regularly, it can cause secondary growth. Irrigation increases the average tuber weight but does not always increase the number of tubers per plant. [50]. Therefore, it is necessary to correctly determine the time and number of irrigations of potato. Three base irrigation methods are used in potato, namely surface (furrow) irrigation, sprinkler irrigation and drip irrigation. Although each method has advantages and disadvantages, which method will be used differs according to the region, purpose and condition of the water source. Despite this difference, the commonly used method in the regions where potato cultivation is done is sprinkler irrigation. Recently, drip irrigation method has started to be used and become widespread. [51].

Different irrigation strategies have been developed for crops due to the decrease in agricultural water resources with the increasing water demand and the effects of ongoing climate change. Two of these are water-saving irrigation techniques such as partial root zone drying (PRD) and deficit irrigation (DI). Gültekin and Ertek (2018) applied five different irrigation levels (I100, I85, I70, I55 and I40) in a two-year study in which they investigated the effect of water deficit practices on tuber development and quality. In the first year, the amount of irrigation water was between 243 mm and 311.9 mm, evapotranspiration was between 337. 1 and 385.9, in the second year the amount of irrigation water was between 166.7 and 223.2, and evapotranspiration was between 204 and 255.7. Yields ranged from 30.85 to 47.13 t/ha in the first year and between 28.77 and 44.45 t/ha in the second year. At the end of the study, they reported that as the irrigation levels decreased, the number of tubers per plant, tuber weight, tuber diameter and length and marketable tuber ratio decreased, and the highest yield values and water use efficiency were obtained from I100 and I85 irrigation levels. [52].

Irrigation can have a direct impact on yield and yield components. Dry matter content in tuber increases with 60% and 100% full irrigation but decreases at 120% of full irrigation. [53]. According to Dervish et al. (2006) reported that restricted irrigation reduced tuber dry matter production and average weight of commercial tuber [54]. Likewise, Nagaz et al. (2007) stated that there is a decrease in tuber number and weight as a result of water scarcity during the formation and development of tubers [55]. In addition, Carli et al. reported that both dry matter and starch content increased with water restriction [56]. According to Sahebi et al. (2012) Starch content greatly enhanced with water limitation compared to control. Contrary to these, Ballmer et al. (2012) reported that water restriction reduced the starch content of tubers [57]. Sahebi et al. (2012) stated that there was no significant change in starch content with irrigation [58]. These different results may have been caused by soil and climatic conditions, stress intensity and timing.

Potato is a very sensitive plant to water shortage in the soil. The highest efficiency is obtained when the usable water in the soil is in the range of 30-50%. If the moisture level in the soil drops below 50%, the yield may decrease. Potatoes are significantly affected by water deficiency during germination, tuber formation and tuber growth periods, while they are less sensitive to water during maturation and early vegetative periods. Potatoes need frequent irrigation for good growth and yield. Yield is significantly affected by storage quality, disease resistance and duration, rate, frequency of irrigation. Variable irrigation level has a significant impact on yield and yield components in potatoes. Hence, applying less water than necessary may cause a decrease in the yield value. If irrigation is applied fully in potatoes, the yield will be at the highest value. However, if the water applied in underground drip irrigation is 80% ETc, yield values similar to the full irrigation conditions applied in above-ground drip irrigation can be obtained. Therefore, it is seen that 20% of irrigation water can be saved in potatoes [59].

Research has shown that the yield and quality of potato tubers will be affected even by short-term water stress. The size of harm to tuber yield and quality will rely on the rigor, timing and lenght of water stress during the growing season. Water stress throughout the growing season has less effect on tuber yield and quality than similar reduction in crop water use over a shorter period of time. To determine the most profitable use of limited water resources, it is critical to understand how water stress affects tuber yield and quality at each growth stage. Special irrigation management according to the growth stages of the potato is given below.

3.1. Sprout Development (Planting to Emergence)

The soil must be moist for the seed tuber to emerge. However, the humidity level should not be excessive. The excess moisture in the soil during this period may result in the plant not being able to take in oxygen and therefore cause the tubers to decay. Irrigation should be done regularly and adequately. In conditions where the temperature is high, the soil temperature can be reduced by watering at short. Excessive irrigation or precipitation may cause irregular or no emergence during this period. Excess moisture will also reduce tuber respiration by placing the seed piece under metabolic stress [60, 61].

3.2. Vegetative Growth (Emergence to Tuber Initiation)

The vegetative growth phase begins with the sprouting of the seed piece and extends to stolon formation. Plant against water stress in this period, generally reduces the effect of water stress in later growth stages and improves tuber quality. Water stress during the this stage decrease leaf area, root expansion, and plant height, and delays canopy development. During this period, the plant does not grow much, but maintain its development rapidly. Therefore, the water consumption of the plant is almost half of a mature plant. Excessive amount of water may cause the formation of exposed roots during this period. Thus, the amount of water to be given should not be more than the plant demand. In case of water is limited, tuber formation occurs earlier, but the number of tuber decreases.

3.3. Tuber Initiation

It is described as the process in which the stolon leaves growing and the tip swells to twice the stolon diameter. It is particularly sensitive to water stress during tuber formation. In addition, water stress can reduce the specific gravity during this period [62]. Water stress can reduce the number of tubers per plant during this period. However, this is not the same for all cultivars. Regular watering has a positive effect on tuber formation. Therefore, the plant needs the most water during this period. If not enough water is given, plants can accelerate maturation. Tubers cannot reach the demand size and yield may decrease. In the drought condition, deformations may occur in the tubers. Hence, in case of insufficient water intake, tuber quality is adversely affected as well as yield.

3.3. Tuber Bulking

The tuber bulking stage continues with a steady increase in tuber size and weight unless restrictive condition. Root growth improves, but the rise in total plant dry matter is considerably depend on tuber growth. Water stress at this stage often affects the total tuber yield more than the quality. Low humidity reduces or stops tuber growth during and after the stress period. So it shortens tuber bulking period and can also led to internal and external tuber defects. Excessive irrigation can inhibit physiological activity and nutrient uptake and may also reduce tuber growth by increasing disease susceptibility[62].

4. COMBINED EFFECT OF NITROGEN APPLICATION AND IRRIGATION REGIMES ON POTATO

Irrigation and fertilization are very important for increasing production in agricultural lands. Irrigation and fertilization are major inputs that increase the efficiency of each other and ensure the requested quality and amount of product. Water rescue by shortage irrigation shows different climatic and crop limitation. A sensitive crop such as potato is especially difficult to control. Because it demonstrated negative reaction to deficit irrigation [63]. Developing nitrogen management just will not be effective in reducing nitrogen leaching in sandy soils. Nitrogen can move below the root zone through irrigation and rainfall. Chemicals that cannot be taken up by the plant or held by the soil move downwards with the water. Rainfall and irrigation are critical in determining the rate at which the chemical moves down the soil profile. Therefore, measured irrigation management is important to minimize leaching of chemicals and nutrients. Hence, wetting depth should be done according to the root depth in each irrigation. In this way, it will also facilitate nitrogen uptake by roots and thereby minimize potential leakage loses below the root zone [64]. Excessive application of mobile nutrients such as nitrogen can cause leaching below the root zone with overwatering. However, it can reduce the yield in case of limited irrigation. So, N fertilizer needs to be optimized according to water availability. Although the information on irrigation and nitrogen management of potatoes is often contradictory in the literature, it is generally accepted that yield and quality are highly affected by N dose and irrigation, and they are related to cultivation technique. In potato production, nitrogen and water management is required to ensure steady growth, high dry matter content and marketable tuber [4]. Water stress has many undesirable effects on nutrient uptake, growth and yield of plants. Nutrients such as nitrogen are effective against water stress, and proper use of nitrogen can prevent a significant reduction in yield under water stress conditions. Da Silva et al., (2018) investigated the effects of irrigation method and application time on tuber yield and nitrogen use efficiency in potato. In the study, a total of 168 kg/ha of ammonium nitrate was divided equally into three applications as 56 kg/ha. According to the results obtained, it was determined that the irrigation method did not have a significant effect on tuber yield and nitrogen use efficiency. Average tuber yield was 32.1 t/ha and average nitrogen utilization efficiency was 41%. It was determined that the nitrogen utilization efficiency was highest at the beginning of the tuber (62%), followed by the emergence (44%) and the lowest (18%) at planting. Researchers stated that the nitrogen applied in the emergence and tuber formation increases the nitrogen use efficiency and tuber yield, but even if there is more loss than nitrogen, some nitrogen is required during planting [65]. In addition, in the study by Elmetwalli et al., the effects of water deficit and nitrogen deficiency on tuber yield were investigated. According to the research results the highest yield was obtained with 1.25 ETc irrigation regime and 200 kg

N/ha application. In addition, potato productivity can be maximized if the appropriate irrigation system, optimum water regime and nitrogen fertilization rate are determined. [43].

5. CONCLUSION

In this review, the effects of irrigation and nitrogen management on potato yield were explored by evaluating previous studies. In general, potato yields are highest at all stages of its development when available soil moisture does not fall below 60% between irrigations. Water stress during tuber initiation and bulking reduces yield by increasing tuber malformations. In this case, it is seen that the most sensitive period to stress is the tuber bulking period. In case of low or excess nitrogen in the vegetative and tuber initiation stages of development can reduce the overall yield of potato. However, high nitrogen availability throughout the entire growing season often delays tuber maturity. This effect on yield and development varies with time and amount of both nitrogen and water availability, region and variety. Different experiments around the world show that the application of the nutrient is positively related to obtaining higher potato yields. Experiments confirmed that adequate nitrogen and irrigation application increase potato production. Therefore, applying adequate N and irrigation is an option to maximize potato production and yield. The final nitrogen ratio, irrigation regime, and nitrogen and water utilization efficiency are interrelated. For sustainable production, collecting these parameters with the yield and yield component of the potato provides sufficient benefit.

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Research

EFFECT OF DEFICIT IRRIGATION AND POTASSIUM ON LEAF AREA, CHLOROPHYLL AND PHOTOSYNTHESIS IN POTATOES

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ABSTRACT

The research was conducted to determine the leaf area index, chlorophyll content (SPAD), and photosynthesis rate of potatoes grown under different irrigation levels and different potassium doses. The study was established in Niğde Ömer Halisdemir University, Ayhan Sahenk Faculty of Agricultural Sciences and Technologies, Department of Plant Production and Technologies Research and Application Area. Three different irrigation levels (I_{100}, I_{66}, and I_{33}) and six different potassium doses (0, 40, 80, 120, 160, and 200 kg/ha) were applied in this study. The experiment was established using a completely randomized design in split plots with four replications. Agria potato variety was used in this experiment. This variety is heavily preferred by the producers in the Niğde region. As a result of the study, leaf area index values changed ranged from 4.50 – 1.01, leaf chlorophyll content (SPAD) values 49.62 – 29.37, and photosynthesis rate values 23.92 – 3.55 µmol m^{-2} s^{-1}. As the amount of irrigation water decreased, it was negatively affected on the investigated properties. While I_{33} application was the irrigation application with the lowest results, the best results were obtained from the I_{100} application. Increasing doses of potassium had a positive effect but the best results were obtained from doses of 120 and 160 kg/ha. Because of these results, it is recommended to apply 120 to 160 kg/ha doses of potassium to the farmers.

Keywords: Irrigation, Potassium, Photosynthesis, Chlorophyll, Potato

1. INTRODUCTION

Globally potato production is approximately 380 million tons and ranked fourth after rice, wheat, and corn [1]. In Turkey, potato production is about 52 million tons and it is one of the most valuable dietary sources for human consumption due to its high nutritional value [2]. It can be consumed in different forms. Mainly, potato is preferred as table purpose potato, mashed potato, and potato flour. Recently, intensification of potato production was increased depending upon high-value food, easy digestion, having many uses, and adaptability to different environmental conditions [3].

Water is one of the most crucial resources for life and agriculture as well. Water resources have been gradually limited due to several reasons in the World and Turkey as well. One of the main reasons is the increase in population, which increases demand for water. Additionally, impact of climate change is also triggering water deficiency globally. Therefore, the reduction of available water causes problems for the growth and development of potatoes [4] due to drought [5]. Potato is sensitive to drought due to shallow root system and it causes devastating yield losses and deteriorates the quality of tubers [6, 7, 8]. Previous studies pointed that available water in soil should not be less than 50% for optimum yield because of sensitivity of potato to soil water deficit conditions [5, 9]. Additionally, potato plants will face drought stress conditions. One of the major responses of plants to drought stress is decreased growth and development of cell. Chlorophyll synthesis, photosynthesis, and respiration are negatively affected by drought and drought induces stomatal closure [10]. In addition, drought promotes leaf senescence, inhibits leaf growth and development, and decreases leaf area that directly decreases the photosynthetic rate of the plant [11]. All phenomenon influences drastically potato yields and quality. Therefore, the determination of efficient irrigation methods, irrigation schedule and water amount to be applied is the most important subject to cope with drought stress and its negative effects on potatoes. Therefore, different irrigation methods with high water use efficiency should be preferred such as drip irrigation [12].

Besides irrigation, other factors that are crucial for robust potato growth including fertilization. The length of the growing period is varied depending on the potato cultivar. The growing period of potatoes is changed between 100 to 150 days. Accordingly, it plays a role in the rate of nutrient uptake and type, time, and amount of fertilizer application [13]. For the production of 1 ton of tubers, the abundant amount of N, P_2O_5, K_2O, and CaO and less amount of MgO, S, Fe, Mn and Zn are used from the soil during the growing period of potato [14, 15, 16]. It is reported that K_2O is uptaken in huge amounts from soil and potato requires a large amount of K_2O compared to other elements. Potassium (K) is the primary and essential element for

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plant growth and physiology among the plant nutritions [3, 17]. Abiotic stress hinders plant productivity due to detrimental effects on plant growth, development, and physiological responses and it causes yield losses [18, 19]. Previous studies reported that potassium played important role in tolerance against abiotic stress and the biological function of plants [17]. In addition, potassium is responsible for biochemical and physiological processes during plant growth and development [20]. One of the functions of potassium is stomatal regulation to carry out the photosynthetic process. It also helps to transport water and nutrition [17]. Potassium assists to sustain water balance in the plant cell [21]. Therefore, potassium also contributes to the maintenance of the turgor pressure of the cell and the regulation of osmotic pressure under drought conditions [20, 22]. Earlier studies demonstrate that sufficient amount of potassium in plants or the supply of potassium to plants positively affected cell membrane stability [23], leaf area [24], chlorophyll contents, and photosynthetic rate [25].

It is obvious that potassium has a positive effect on regulation of biochemical and physiological processes in plants under drought conditions. The use of potassium with different irrigation methods can be beneficial to tolerate drought stress in plants. The current study aimed to determine the effect of potassium fertilization on morpho-physiological characteristics by using drip irrigation under drought conditions.

2. MATERIAL AND METHOD

This study was conducted in the field of research and application areas of the Department of Plant Production and Technologies in Niğde Ömer Halisdemir University in 2019. The field experiment consisted of a drip irrigation method with three different irrigation levels and six different potassium doses. The irrigation levels were full irrigation (I_{100}), 66% of full irrigation (I_{66}; 33% deficit), 33% of full irrigation (I_{33}; 66% deficit). The doses of potassium used as fertilizers were 0, 40, 80, 120, 160 ve 200 kg/ha. The experiment was laid out the randomized split plot experimental design with four replications. Three different irrigation levels in the main plot, different doses of potassium fertilizers were used in sub-plot in the experimental design. Each subplot had 4 rows with 0.30 m width and 5.10 m length. The rows were 0.70 m apart from each other. The six different doses of potassium sulphate consisting of 50% K_2O were applied to plots before planting. Planting was performed by using a semi-automatic potato planter machine in May 2019. The medium-early potato cultivar 'Agria' which is suitable for processing, was used in this study due to characteristic of yellow tuber and flesh colour, medium dry matter and starch content, high yielding, appropriate for french-fried, chips and baked potato, highly preferred producer product. Irrigation applications were carried out every five days. Drip irrigation lines were installed after planting. Soil moisture was determined at 0-30 cm and 30-60 cm soil depth before each irrigation. After 20 days of emergence, plants were irrigated until soil moisture was reached the field capacity. Later, three different irrigation levels (I_{33}, I_{66}, and I_{100}) were applied to all plots. Just after starting deficit irrigation treatment, the measurements of leaf area index, leaf chlorophyll content, and photosynthesis rate have been done at three different dates (28.07.2019, 16.08.2019, and 05.09.2019). The measurements were performed before irrigation. Chlorophyll content, leaf area index and photosynthesis rate were measured by using Minolta SPAD 502, LAI-2200 instrument, and LICOR-6400 portable photosynthesis device, respectively. The analysis of variance (ANOVA) according to the split plot design was used to determine the significance of the difference between the means. Then, the LSD test was used for the determination of different groups.

3. RESULTS AND DISCUSSION

The effect of different irrigation levels and potassium doses on leaf area index was given in Table 1. The results revealed that the limited irrigation and potassium doses had significant effects on leaf area at each of three different periods. The limited irrigation negatively affected the leaf area and as the severity of limited irrigation increased, the leaf area index decreased more. The lowest leaf area index was observed in I_{33} treatment for all three measurements. In addition, the mean value of leaf area index in I_{66} treatment was negatively affected compared to control (full irrigation). When first measurements were evaluated, increasing doses of potassium did not affect the leaf area index in I_{33} and I_{66} treatments. In I_{100} treatment, increasing doses of potassium, however, had a positive effect on leaf area index, and the highest value was observed at 200 kg/ha potassium. Second measurements showed that potassium doses were not significant for leaf area index in I_{33} treatment, unlike the first measurement. In I_{66} treatment, 80, 120, 160, and 200 kg/ha K doses resulted in a higher leaf area index than 0 and 40 kg/ha. In I_{100} treatment, increasing potassium doses had significant effect on leaf area index and increased the value of this trait however, the highest leaf area index was observed in 120, 160 and 200 kg/ha K. Considering the results of third measurement, there was a decrease in all applications as the extent of water scarcity and drought stress affected plants severely. The highest leaf area index was recorded at 200 kg/ha K for I_{33}, 200 kg/ha K for I_{66} and 200 kg/ha K for I_{100} applications. Drought stress negatively affects cell growth and development, therefore alters overall plant growth [26]. Upon continuous exposure to stress factors, plants close their stomata and photosynthesis is negatively affected. The plant growth and development retards and leaf area decreases as a consequence of a morphological outcome [27, 28, 29]. Potassium application to plants under drought stress help reducing the negative effects of stress and also has a positive effect on leaf area [24, 30]. In our study, although it was observed that limited irrigation decreased leaf area, increasing doses of potassium promoted increase in leaf area index.

Table 1. Mean values with LSD groups and ANOVA results of leaf area index under different irrigation levels, potassium doses and irrigation x potassium interaction

Applications		**Leaf area index**		
Irrigation levels (I) (%)	**Potassium doses (K) (kg/ha)**	**1 (28.07.2019)**	**2 (16.08.2019)**	**3 (05.09.2019)**
I_{33}	0	3.57 e	2.22 e	1.01 l
	40	3.57 e	2.32 e	1.07 l
	80	3.57 e	2.32 e	1.36 k
	120	3.70 e	2.32 e	1.60 j
	160	3.65 e	2.25 e	1.77 ıj
	200	3.67 e	2.20 e	1.90 ıh
	Avg.	**3.62**	**2.27**	**1.45**
I_{66}	0	3.72 de	2.37 e	1.82 ı
	40	3.70 de	2.42 e	2.07 gh
	80	3.82 de	2.75 d	2.20 gf
	120	3.77 de	2.85 d	2.35 f
	160	3.77 de	2.92 d	2.57 e
	200	3.82 de	2.80 d	2.65 de
	Avg.	**3.78**	**2.68**	**2.27**
I_{100}	0	3.80 de	3.47 c	2.67 cde
	40	4.00 dc	3.53 bc	2.80 dc
	80	4.20 bc	3.75 ba	2.87 c
	120	4.45 ba	3.80 a	3.27 b
	160	4.45 ba	3.92 a	3.45 ba
	200	4.50 a	3.88 a	3.50 a
	Avg.	**4.23**	**3.72**	**3.09**
LSD (%5)		0.27	0.25	0.20
ANOVA				
	Degre of Freedom	**Mean Square**	**Mean Square**	**Mean Square**
Replication	3	0.95	2.17	2.05
Irrigation	2	72.79**	404.30**	790.49**
Potassium	5	5.67**	6.52	68.38**
Irrigation x Potassium	10	2.45*	2.09*	0.78*
Error		0.03	0.03	0.02
Coefficient of variation		4.71	6.32	6.28

*P < 0.05, **P < 0.01

The effect of different irrigation regimes and potassium doses on chlorophyll content is given in Table 2. Deficit irrigation regimes and potassium doses had no significant effect on chlorophyll content according to data taken during first measurement day. Interaction between irrigation and potassium doses were found to be significant regarding chlorophyll content. Chlorophyll content was decreased based on reduction of irrigation levels. Potassium doses applied up to 120 kg/ha was increased chlorophyll content. However, the decrease was observed in chlorophyll content after treatment with 160 kg/ha and 200 kg/ha K doses. The highest chlorophyll content was determined during second measurement day under I_{33}-K_{120}, I_{66}-K_{120} and I_{100}-K_{120}. On the other hand, the highest chlorophyll content was recorded during third measurement day under I_{33}-K_{40}, I_{33}-K_{80}, I_{33}-K_{120}, I_{66}-K_{120} and I_{100}-K_{120}. Leaf chlorophyll content is one of the most important parameters affected by drought stress [11, 31]. It is known that chlorophyll content declines due to decreasing relative water content of plants under drought stress. However, it is known that potassium applied in increasing doses affects the chlorophyll content positively and tolerates stress in case of drought [25, 32]. In this study, chlorophyll content decreased as irrigation restriction increased, but it was determined that potassium applied up to 120 kg/ha K dose increased chlorophyll content in all irrigation applications compared to other potassium doses.

Table 2. Mean values with LSD groups and ANOVA results of leaf chlorophyll content (SPAD) under different irrigation levels, potassium doses and irrigation x potassium interaction

Applications		**Leaf chlorophyll content (SPAD)**		
Irrigation levels (I) (%)	**Potassium doses (K) (kg/ha)**	**1 (28.07.2019)**	**2 (16.08.2019)**	**3 (05.09.2019)**
I_{33}	0	48.00	42.15 k	30.72 gf
	40	48.47	42.23 jk	31.65 f
	80	48.87	43.90 ıj	31.95 f
	120	49.22	44.77 ıh	32.02 f
	160	48.07	43.32 ıjk	30.27 gf
	200	47.95	43.22 ıjk	29.37 g
	Avg.	**48.43**	**43.26**	**30.99**
I_{66}	0	48.37	46.40 hg	34.70 e
	40	48.42	46.72 fg	35.50 e
	80	48.70	46.72 fg	35.52 e
	120	49.07	48.30 efd	38.87 d
	160	48.22	47.32 efg	34.79 e
	200	48.15	47.27 efg	34.70 e
	Avg.	**48.48**	**47.12**	**35.68**
I_{100}	0	48.30	49.35 bcd	42.27 bc
	40	48.25	50.00 bc	44.17 ba
	80	48.45	51.00 ab	44.25 ba
	120	49.62	52.25 a	45.05 a
	160	48.48	48.70 ecd	41.42 c
	200	48.52	48.43 ecd	41.85 c
	Avg.	**48.60**	**49.95**	**43.16**
LSD (%5)		1.74	1.67	2.00
		ANOVA		
	Degre of Freedom	**Mean Square**	**Mean Square**	**Mean Square**
Replication	3	2.15	1.03	2.33
Irrigation	2	0.12	201.93**	496.13**
Potassium	5	1.48	7.43**	10.85**
Irrigation x Potassium	10	0.14	1.63*	1.13*
Error		1.48	1.33	1.82
Coefficient of variation		2.51	2.47	3.68

*P < 0.05, **P < 0.01

The effect of different irrigation regimes and potassium doses on photosynthesis rate is presented in Table 3. The differences were observed between different irrigation regimes and potassium doses according to results. The decrease of irrigation levels induced photosynthesis rate. The rate of photosynthesis measured in I_{33} was founded to be lower as compare to other irrigation regimes. On the other hand, the highest rate of photosynthesis was recorded in I_{100}.Drying and harvesting stage started depending upon the of deficit irrigation applied during field experiment. It is observed that potassium doses applied under different irrigation regimes were found to be statistically significant except first application date of I_{33}. The increase of potassium doses was positively affected photosynthesis rate. Additionally, the highest values occurred in 120 kg/ha K with $I_{33.}$ According to results of first measurement date, the highest values were obtained with application of all doses of potassium under I_{66} as compare to control group. Treatment with 120 kg/ha K increase photosynthesis rate for other measurement date as well. The highest photosynthesis rate was noted with different doses of potassium in the second measurement date under I_{100} as compare to control group. Besides, 120 kg/ha and 160 kg/ha K showed better results for other measurements days as well. Plants subjected to drought stress due to deficit irrigation regimes. Therefore, the decline of water in plants are negatively affected photosynthesis [33]. One of the efficient way is that plant could avoid drought by closing stomata. Additionally, photosynthesis rate is negatively affected due to lack of sufficient amount of CO_2 [10, 34, 35]. Potassium have essential role on not only photosynthesis but also CO_2 fixation and transportation of photosynthesis product [22]. The increase of potassium in plants cause to increase of osmotic potential in plant cell. As a result, water enters to cell through opening of stomata and photosynthesis rate is positively affected [25]. Finally, current study suggested that potassium doses had positive effect on photosynthesis rate under deficit irrigation regimes.

Table 3. Mean values with LSD groups and ANOVA results of photosynthesis rate of potato under different irrigation levels, potassium doses and irrigation x potassium interaction

Applications		**Photosynthesis rate (µmol m^{-2} s^{-1})**		
Irrigation levels (I) (%)	**Potassium doses (K) (kg/ha)**	**1 (28.07.2019)**	**2 (16.08.2019)**	**3 (05.09.2019)**
I_{33}	0	21.62 d	13.30 g	3.92 hg
	40	21.60 d	14.00 gf	4.05 hg
	80	21.92 d	14.02 gf	4.22 hg
	120	21.85 d	14.60 f	4.52 g
	160	21.57 d	13.37 g	3.95 hg
	200	21.62 d	13.50 g	3.55 h
	Avg.	**21.69**	**13.79**	**4.03**
I_{66}	0	21.90 d	15.87 f	5.65 f
	40	22.15 dc	16.30 de	5.71 f
	80	22.17 dc	16.52 de	6.15 fe
	120	22.42 dc	17.65 c	7.17 dc
	160	22.27 dc	17.02 dc	5.67 f
	200	22.42 dc	16.82 dc	5.90 fe
	Avg.	**22.22**	**16.69**	**5.04**
I_{100}	0	22.35 dc	20.80 b	6.62 de
	40	23.15 bac	22.17 a	7.57 c
	80	23.25 bac	22.72 a	9.00 b
	120	23.90 a	22.85 a	10.15 a
	160	23.95 a	22.40 a	8.69 b
	200	23.60 ba	22.15 a	8.87 b
	Avg.	**23.36**	**22.18**	**8.48**
LSD (%5)		1.14	0.87	0.87
		ANOVA		
	Degre of Freedom	**Mean Square**	**Mean Square**	**Mean Square**
Replication	3	0.55	1.08	1.05
Irrigation	2	26.16**	1349.16**	310.59**
Potassium	5	1.55**	11.27**	12.98**
Irrigation x Potassium	10	0.58**	1.62**	3.69**
Error		0.65	0.32	0.38
Coefficient of variation		3.61	3.23	10.00

*P < 0.05, **P < 0.01

4. CONCLUSION

The reduction of water supplies induces to less irrigation of plants that needed for their life cycle. It is obvious that plants suffer from drought stress during their growth and development. It is known that potassium provides to maintain water balance and to tolerate against drought stress. The three different irrigation regimes and six different potassium doses were used for current study. The important parameters like leaf area index, chlorophyll content and photosynthesis rate was observed. Those parameters were negatively affected by limiting plant growth and development under deficit irrigation regimes. The increase of potassium dosses had significant both full irrigation and deficit irrigation regimes on those parameters except first measurement day for I_{33}. As a consequence, positive effect of 120 kg/ha and160 kg/ha K applied under full irrigation (I_{100}) and deficit irrigation conditions (I_{66} and I_{33}) were determined and recommended to producer.

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Research

CAMERA SELF-CALIBRATION BY USING SfM BASED DENSE MATCHING FOR CLOSE-RANGE IMAGES

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ABSTRACT

The camera calibration is an important issue that must be overcome to getting metric scene measurement. The imaging parameters are estimated by calibration of the camera. Basically, the camera calibration is performed individually from the photogrammetric evaluation. Today, 3-D point cloud generation and the camera calibration are usually attained simultaneously by using SfM approach photogrammetric evaluation. Stereo images that do not have camera intrinsic parameters can also be evaluated by SfM based photogrammetry. In this study, camera calibration models were investigated for point cloud generation of close-range photogrammetry. The results shown that self-calibration of loop-close images enables the close results to the pre-calibration. Otherwise, the images should be convergent as far as possible or projection-to-sparse point cloud ratio must be raised. The results show that the projection-to-sparse point cloud ratio of 13.22 created high accuracy to self-calibration. Consequently, the pre-calibration requires extra computation and time. However the self-calibration can be implemented for high accuracy measurement subject to convergence imaging or sufficient number of projection.

Keywords: Camera calibration, Dense matching, Image matching, Self-calibration, Point cloud.

1. INTRODUCTION

Three-dimensional (3-D) measurement is prevalent task in many disciplines for the aim of 3-D modelling and visualization. The users are benefited from different instrument and techniques to carry out the operation. The photogrammetry is highly accurate and low-cost method for terrestrial 3-D measurement. Many innovations in computer vision have been adopted to photogrammetry for acquire more dense measurement data in a short time. The photogrammetry uses structure-from-motion (SfM) based on automatic measurement [1, 2]. It extracts measurement data from metric or non-metric camera images. Any source of overlapping images [3] or imageries [4] can be evaluated with SfM based photogrammetry for generating high density 3D measurement data.

The SfM matches unordered overlapping images automatically without camera calibration data [5]. After the image keypoints (feature points) which does not change with respect to perspective, scale and orientation of the image are detected, they are matched with their similarities. The number of matched keypoints affects the measurement accuracy from stereo images of photogrammetry [6]. The images should have perspective projection and, any deviations from the perspective projection must be modelled as mathematically for high accuracy from the photogrammetric evaluation [7]. The deviation from the perspective projection is called as distortion and it can be removed from the images by using the camera calibration parameters. Thus the calibration parameters of the camera must be known for right spatial measurement from the photogrammetry. The images, which have the camera calibration parameters, are called metric, non-metric otherwise. The camera calibration performed with the special test field before the photogrammetric measurement is named pre-calibration. The pre-calibration requires to design a special test field with location known control points. It is time and labour consuming task. The camera calibration which is performed together with object images on the photogrammetric evaluation is called as self-calibration [8]. The images are matched as automatically by using SfM algorithm and the calibration parameters are estimated with photogrammetric bundle adjustment in the self-calibration. This study investigates the camera calibration methods and its relation to photogrammetric measurement accuracy.

The camera convergence and number of projection must be in proper one to performing correct self-calibration and 3-D measurement from SfM based photogrammetry. The pre-calibration enables high accuracy to photogrammetric measurement [9,10]. However, professional or usually non-professional applicants prefer to self-calibration based photogrammetric measurement since its easy, practical and has very fast applications. The SfM gives rise to generation high accuracy self-calibration parameters and measurement data with depends on imaging surface geometry.

In earlier 3-D measurement studies, 0.2 mm accuracy could be achieved from photogrammetry by using self-calibration of non-metric cameras [11]. The high accuracy makes enable to the close-range photogrammetry to use in earth surface deformation [12] and industrial measurement [13]. The smart phone stereo images have also been used for photogrammetric

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measurement. The calibrated smart phone cameras exhibited lower accuracy than the metric cameras, but the results were relatively satisfactory in comparison to studies using other inexpensive cameras [14]. [10] shown that the incorporation of dGPS supported control points (CPs) and a pre-calibrated camera model can result in systematic distortion being reduced to below detection levels. Nevertheless the commercial softwares do not provide a one-size-fits-all solution to getting high accuracy from pre- or self- calibration and workflows should be adjusted so to topography. In this study SfM based point cloud generation workflow for non-, pre- and self- calibrated camera images were investigated. The self-calibration is highly related to image convergence and number of corresponding points between the overlapping images. The self-calibration was performed with different image configurations related to convergence and projection numbers. The distortions and CP residuals were compared for accuracy evaluations.

2. METHODOLOGY

2.1. Stereo-view Measurement

The close-range photogrammetry has interdisciplinary characters with its varied applications [12,13,15,16]. Its industrial applications have 0.1 mm measurement accuracy, and the accuracy varies around 1 cm for large-scale object measurement. The measurement accuracy changes with respect to applied evaluation model, imaging properties (metric or non-metric), image scale and ground sapling distance (GSD).

The main purpose of a photogrammetric measurement is the 3-D reconstruction of an object in digital form with coordinates or wireframe geometric elements. The measurement data are generated from overlapping area of a stereo image that has central projection properties. The 3-D visualization is acquired from stereo images by imitating the human eye interpretation. The photogrammetric evaluation is performed basically in three steps such as inner, relative and absolute orientations.

At the inner orientation, image generation optical properties that were estimated by the camera calibration are implemented to photogrammetric evaluation. These optical values which are called camera intrinsic parameters are focal length *f*, principal point (PP) coordinates (x_o,y_o) and pixel dimensions (Figure 1). The principle point is defined as the image points where the optical axis intersects with the image plane. Its image coordinates are estimated by calibration of the camera.

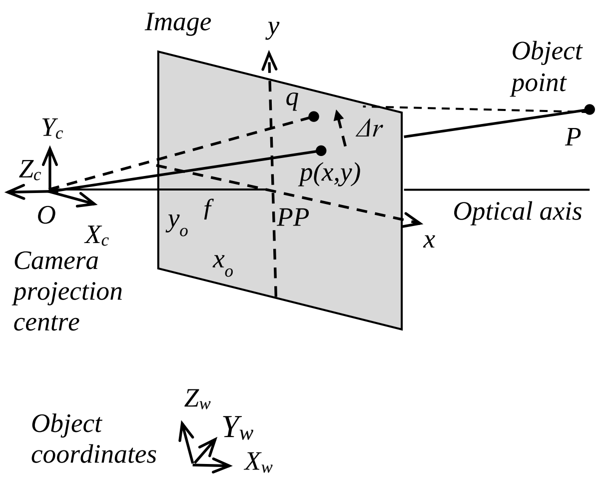

Figure 1. The camera imaging geometry and distortion error

The 3-D visualization from stereo images is getting via relative orientation by applying the co-planarity constraint. The relationship between the camera projection centre 0, image point *p* and object point *P* are expressed well-known colinearity model in perspective transformation (Eq. 1).

$$\begin{pmatrix} x - x_o + \Delta x \\ y - y_o + \Delta y \\ -f \end{pmatrix} = \lambda R \begin{pmatrix} X_w - X_w^c \\ Y_w - Y_w^c \\ Z_w - Z_w^c \end{pmatrix} \quad (1)$$

Where; *x,y* are image coordinates; X_w, Y_w, Z_w are object coordinates; X_w^o, Y_w^o, Z_w^o are the place of the projection centre and *R* is rotation matrix (3x3 dimensions). The additional parameters Δx and Δy are account distortions from colinearity condition. The colinearity condition for all the object points must be realized in every images of the stereo view for 3-D visualization. In this way, the projection centres, object points and its image points are lie on the same plane (Figure 2). This geometrical condition in photogrammetry is named as co-planarity constraint for the stereo-view. It is realized to relative orientation and translation of the images with respect to each other. It has five unknown parameters which are estimated with least five conjugate points. The relative orientation provides 3-D object model at unknown arbitrary scale. The scale has been attained to the model with the absolute orientation.

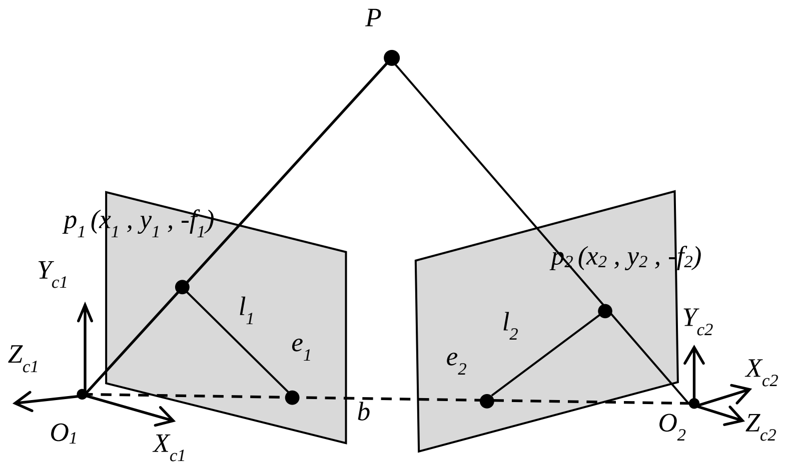

Figure 2. The epipolar geometry

2.2. Camera Calibration

The camera calibration covers the estimation of intrinsic and distortion parameters. It is an active research topic in geomatics and computer science community to get high accuracy from the photogrammetry. The automatic calibration methods had been applied to digital cameras with colour code or special shape targets in the literature [17,18]. Moreover the validity of calibration data over a time should be carefully assessed before next photogrammetric measurement [19].

The calibration models are classified into two categories such as linear and non-linear [7,20]. The linear models, i.e. Hall and Faugeras–Toscani, use a least-squares technique to get the parameters of the model [21-24]. Non-linear calibrating methods as with Faugeras with distortion, Tsai and Weng, use a two-stages as a linear approximation and then iterative algorithm to optimize the parameters [25-28].

The camera calibration is outperformed by using single or multi-view perspective images. The extensive approach for a single image camera calibration is to adopt vanishing points and vanishing lines [29,30]. It can be applied with and without any special calibration pattern [31]. The multi-view calibration is applied with many images from different perspectives of the same imaging area by applying linear or non-linear methods.

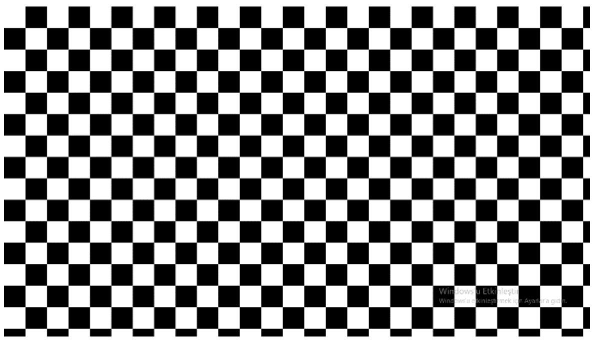

Figure 3. Test grid on computer screen for Agisoft Lens

The calibration is performed with the special test area which has target points localized in 2-D (Figure 3) or 3-D space. The line segmentation has also been used to match multi-view images for the calibration [32,33]. However the point based calibration is popular in multi-view photogrammetry [34]. The point based method is, in addition, applied for a single image camera calibration [35]. The point based calibration method usually use special target shape for automatic detection, and is exploited in pre-calibration of the camera. The pre-calibration can be performed with manually selected target points also.

The SfM based automatic image matching have created an opportunity for the camera calibration by using images of a survey object [10,36,37]. The colinearity condition enables that the camera calibration can be performed together with the object measurement using the bundle adjustment evaluation model. This calibration model of the camera is called as self-calibration. Theoretically known and unknown intrinsic parameters of the camera show the necessary camera numbers for the calibration. The most popular approach is only focal lengths to be unknown and varying, and all the other parameters to be known [38]. Number of counting argument gives a necessary condition for self-calibration. If it is assumed as m the number of cameras, n_k the number of known internal parameters, and n_f the number of constant (but unknown) internal parameters, a necessary condition is given [9] by

$$mn_k + (m \quad 1)n_f \geq 8 \tag{2}$$

In Eq. (2), when n_k=3 (focal length and principal point coordinates) and n_f=0, m must be three or more. The number of unknown parameters to inner orientation and distortion are eight for non-metric images, and require least eight images to estimation all of these parameters. Accordingly, full calibration could not be procured with the self-calibration from two-view stereo images.

2.3. Distortion Error

The image distortions must be corrected as mathematically to getting high accuracy from the photogrammetric measurement. Δx and Δy include colinearity condition's physical departures such as symmetric radial distortion (Δr), decentring (tangential) distortion (Δd), image plane unflatness (Δu) and in-plane image distortion (Δf) [39].

$$\Delta x = \Delta x_r + \Delta x_d + \Delta x_u + \Delta x_f \tag{3}$$

$$\Delta y = \Delta y_r + \Delta y_d + \Delta y_u + \Delta y_f \tag{4}$$

The net image displacement at any point will amount to the cumulative influence of each perturbations in Eq (3) and Eq (4). The effect of tangential, unflatness and in-plane distortions are so small and insignificant magnitude in these perturbations and are not taken into account in CCD or CMOS relevant digital images. The radial lens distortion has usually large effect in digital images.

Radial lens distortion has symmetric effect according to the principal point. It is represented as polynomial Eq. (5);

$$\Delta r = K_1 r^3 + K_2 r^5 + K_3 r^7 \tag{5}$$

Where; K_i terms are the coefficients of radial distortion and r is the radial distance from the principal point. The perturbation related to radial distortion is increasing according to radial distance r and it reaches largest at the border of the image. K_1 term suffice for medium level accuracy in photogrammetric measurement. K_2 and K_3 coefficients improve the accuracy.

The related correction to the x,y image coordinates are proportional to their magnitude (Eq. 6)

$$\Delta x_r = \bar{x}\frac{\Delta r}{r} \qquad \textit{and} \qquad \Delta y_r = \bar{y}\frac{\Delta r}{r} \tag{6}$$

$$r = \sqrt{\bar{x}^2 + \bar{y}^2} \tag{7}$$

Then distortion corrected image coordinates are obtained by Eq (8)

$$x = \bar{x} + \Delta x_r \qquad \text{and} \qquad y = \bar{y} + \Delta y_r \tag{8}$$

Tangential distortion are given by Eq (9)

$$\Delta d = r^2\sqrt{P_1^2 + P_2^2} \tag{9}$$

Where; P_1 and P_2 are the coefficients of tangential distortion. Tangential distortion rarely exceeds 10 micron as determined in a self-calibration. The resulting image coordinate perturbations are very small and the distortion variation is generally ignored in photogrammetry. Out-of-plane distortion occurs due to unflatness of sensor surface and reaches highest one or two micron height differences in sensor surface. In-plane image distortion expresses orthogonality (shear) and scale differences between the sensor pixels on rows and columns [39].

2.4. Essential and Fundamental Matrix

The Essential (*E*) and Fundamental (*F*) matrices have 3x3 dimensions that represent the epipolar geometry between stereo images. They indicate epipolar line to search along in the second image for given a point in the first image (Figure 2). The *E* matrix had been introduced first and then *F* matrix [40]. The *F* matrix is generalization of the *E* matrix in which the calibrated camera is removed. The *E* matrix has fewer degrees of freedom compared to *F* matrix.

If the camera is pre-calibrated, the SfM uses *E* matrix to search possible match point in the other image, otherwise *F* matrix is exploited to SfM. The *E* matrix uses extrinsic camera parameters while the *F* matrix uses both intrinsic and extrinsic camera parameters in this task. In other words, *E* matrix uses camera coordinates while *F* matrix is using the image coordinates for matching the image points.

After the principal point offset and camera-specific distortions are corrected, the relationship between corresponding points p_1 (x_1, y_1,-f_1) and p_2 (x_2,y_2,-f_2) is described with *E* matrix given by Eq (10).

$$p_2^T E p_1 = 0 \tag{10}$$

$$\det E = 0 \tag{11}$$

The *E* matrix has five degrees of freedom and incorporated within a RANSAC procedure for deriving exterior orientation parameters of the stereo images. It is estimated with eight point correspondences with constraint $e^T e = 1$ and its singular value decomposition (SVD). *E* matrix uses normalized image coordinates.

The *F* matrix, that comprises both intrinsic and extrinsic camera parameters, connects two corresponding points from the stereo images as Eq (12)

$$p_2^T F p_1 = 0 \tag{12}$$

$$\det F = 0 \tag{13}$$

F matrix has nine degrees of freedom and estimated with eight point correspondences [9]. It also is incorporated within RANSAC procedure for solving the correspondence problem between the stereo images.

2.5. Point Cloud Generation

The self-calibration cannot be performed with a photogrammetric evaluation of two-view stereo images. Two-view photogrammetric measurement is executed as pre-calibration or without calibration parameters (as non-metric) of the images (Figure 4).

The keypoints are detected by feature point operators such as SIFT [41]. The SfM approach generates tie points between overlapping images by matching the possible candidate keypoints as automatically. The sparse point cloud is generated by estimating the object coordinates of these tie points. The SfM estimates order and orientation of the cameras with respect to unknown local coordinates. The self-calibration is performed with bundle adjustment of multi-view (least 3 views) stereo images. The known intrinsic and distortion parameters can be excluded from the self-calibration. Thereby, a multi-view photogrammetric evaluation is executed as metric images with pre-calibration. In addition, multi-view photogrammetric evaluation can also be executed to non-metric (without calibration) images (Figure 5).

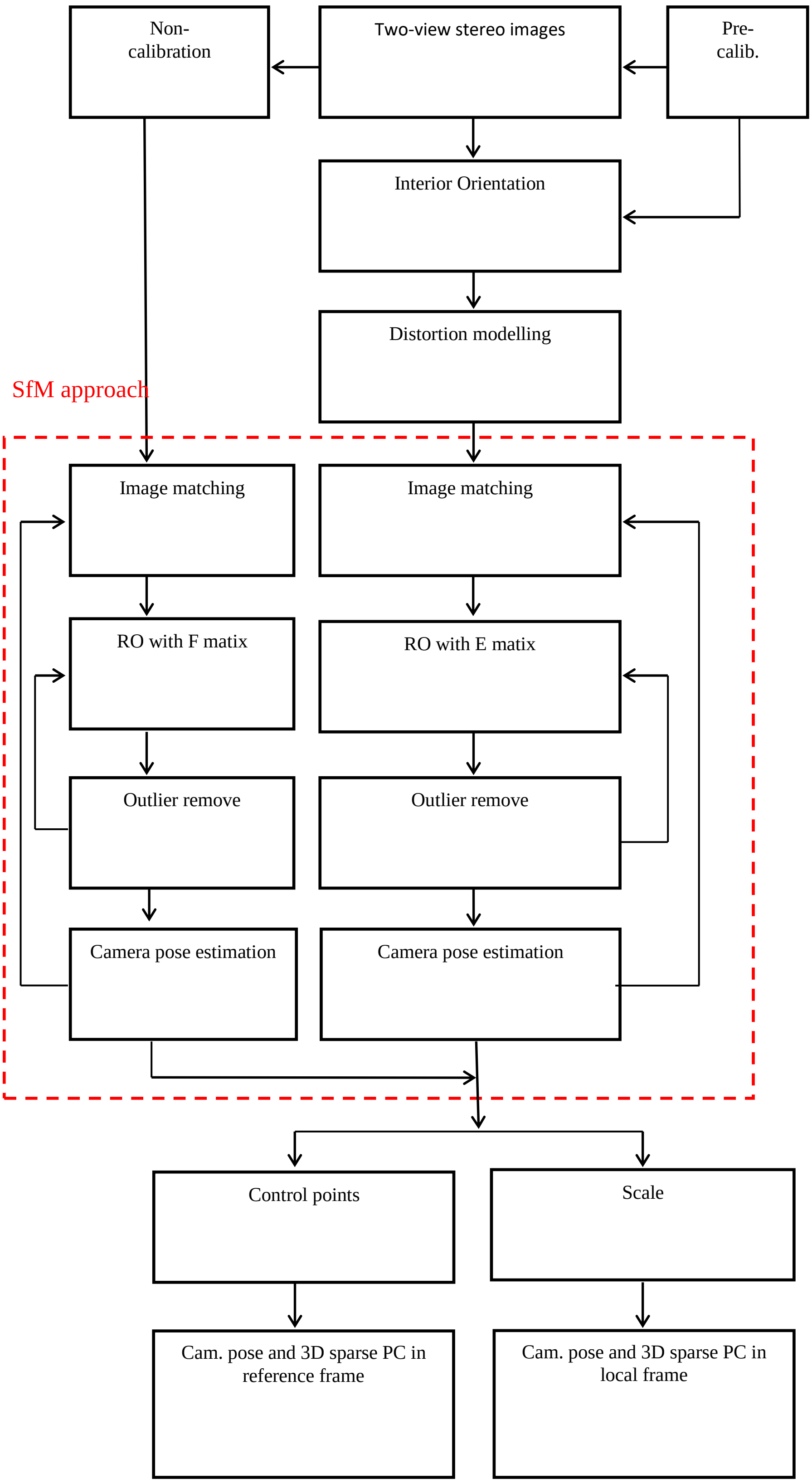

Figure 4. The workflow of SfM based photogrammetric evaluation of two view stereo images. The camera self-calibration is not possible for two view images.

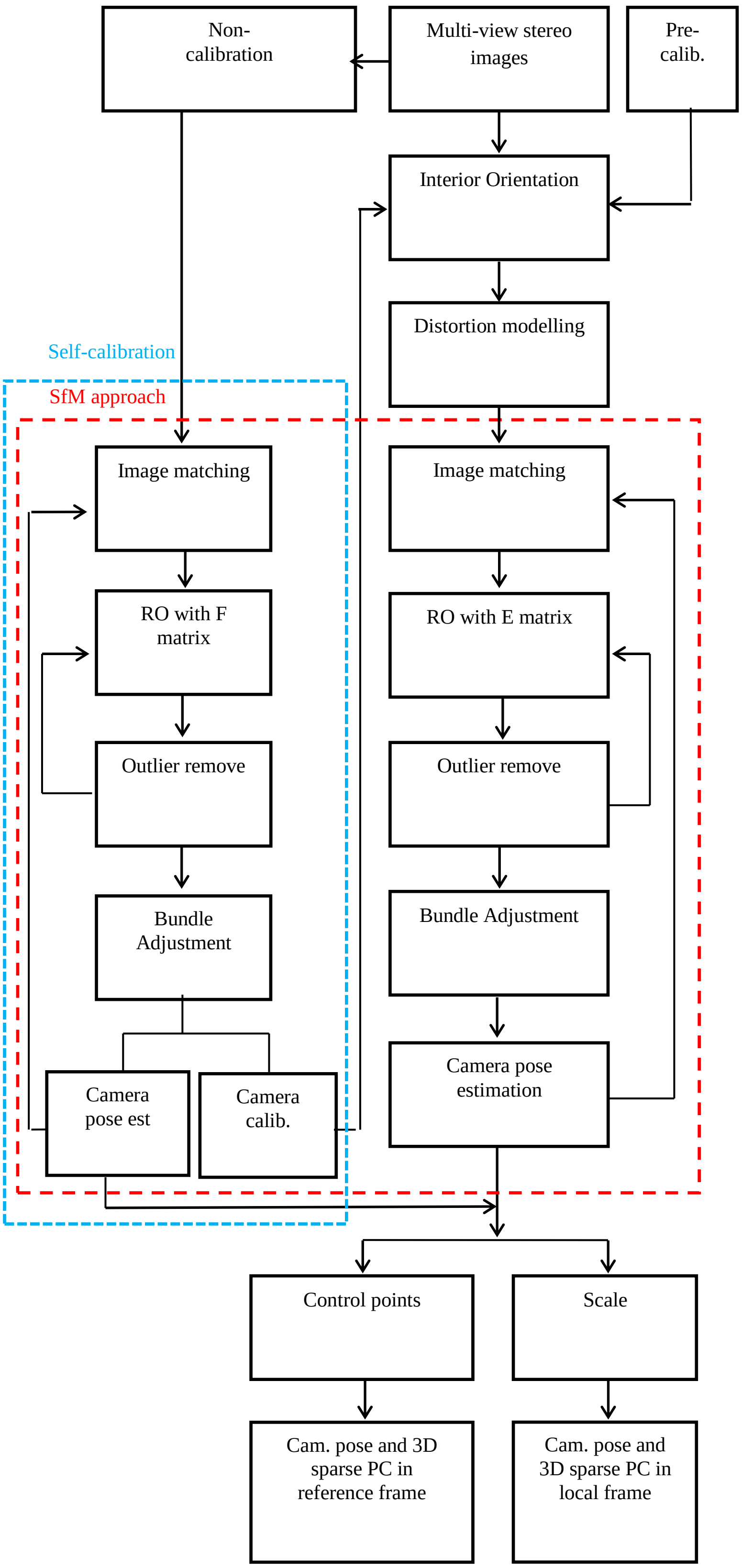

Figure 5. The workflow of SfM based photogrammetric evaluation of multi-view stereo images. The camera self-calibration parameters can also be estimated in multi-view evaluation. The estimated camera calibration parameters can be applied to again evaluation.

3. RESULTS

3.1. Pre-Calibration

The pre-calibration is usually performed with special design test field which has control points with known target shape. The Agisoft lens software uses chessboard plane to do pre-calibration (Figure 3). The chessboard is shown on computer screen and nine convergent images are recorded from different point of view. The images are matched automatically and the calibration parameters are estimated with bundle adjustment. The estimated camera intrinsic parameters and distortion graphics are given on Table 1 and Figure 6.

Table 1. Agisoft Lens pre-calibration results of Nikon P50 camera (3264x2448 pixel array)

Parameters	Quantitate
f (pixel)	2700.220
x_o (pixel)	1604.060
y_o (pixel)	1229.040
Radial K_1	-0.111631000
Radial K_2	-0.002199310
Radial K_3	0.088830800
Tangential P_1	-0.000520235
Tangential P_2	0.000246494
b_1	-0.556039
b_2	0.337776

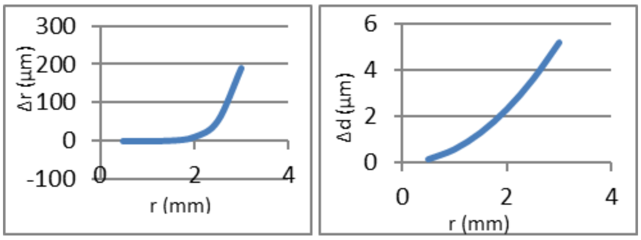

Figure 6. The radial and tangential distortion for pre-calibration of Nikon P50 camera

3.2. Convergent Related Self-Calibration

The convergent angle and related base-to-height ratio effect the accuracy of photogrammetric measurement. The large convergent angle, that base-to-height ratio is around 1.5, increases the measurement accuracy. The multi-image camera geometry is, as a rule, guaranty measurement accuracy that comprised base-to-height ratio. However the convergent degrees of the images also affect the accuracy, especially in structure measurement with loop close or loop open image configurations. The camera self-calibration and measurement were tested on a historical mosque. The structure has roughly rectangular shape in 37x47 m^2 dimensions. The images were collected by Nikon P50 camera as partially overlapping from 14.7 m distance away in ground resolution of 5.42 mm/pixel. The image based point cloud measurement data were generated as loop close and various convergence degree images as loop open configurations. The loop open convergence images were evaluated with four configurations as four, three, two and one (line) façades (Figure 7). The self-calibration that includes camera intrinsic and distortion parameters were carried out together with the estimation of tie point object coordinates by applying multi-image bundle adjustment (Table 2, Figure 8). The point cloud based on bundle adjustment was, also, created using the camera pre-calibration and without calibration as non-metric images. As mentioned above the epipolar constraint was applied to stereo images using E and F matrix for metric and non-metric images respectively. The measurement accuracy was evaluated with the residuals between measured and estimated coordinates of the CPs (Table 3).

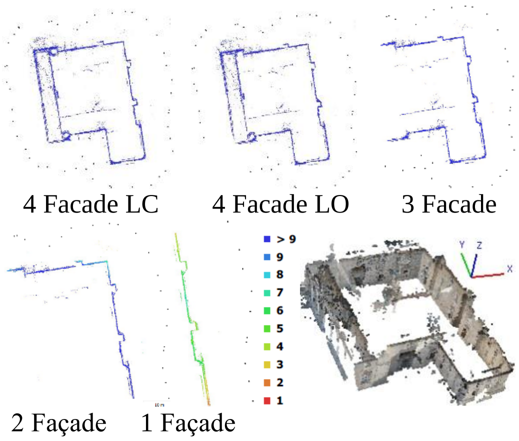

Figure 7. The different level convergence imaging for multi-image photogrammetric measurement. (Colour legend shows number of overlapping images)

Table 2. The estimated interior parameters relation to convergence images

Calibration	***f*** **(pixel)**	**x_o (pixel)**	**y_o (pixel)**
Pre-	2700.220	-27.935	5.042
Self- 4Facade-LoopClose	2700.370	-22.157	5.826
Self- 4Facade-LoopOpen	2703.950	-23.281	6.788
Self- 3Facade	2705.320	-18.102	3.358
Self- 2Facade	2704.410	-20.378	8.107
Self- 1Facade	2702.730	-18.465	10.141

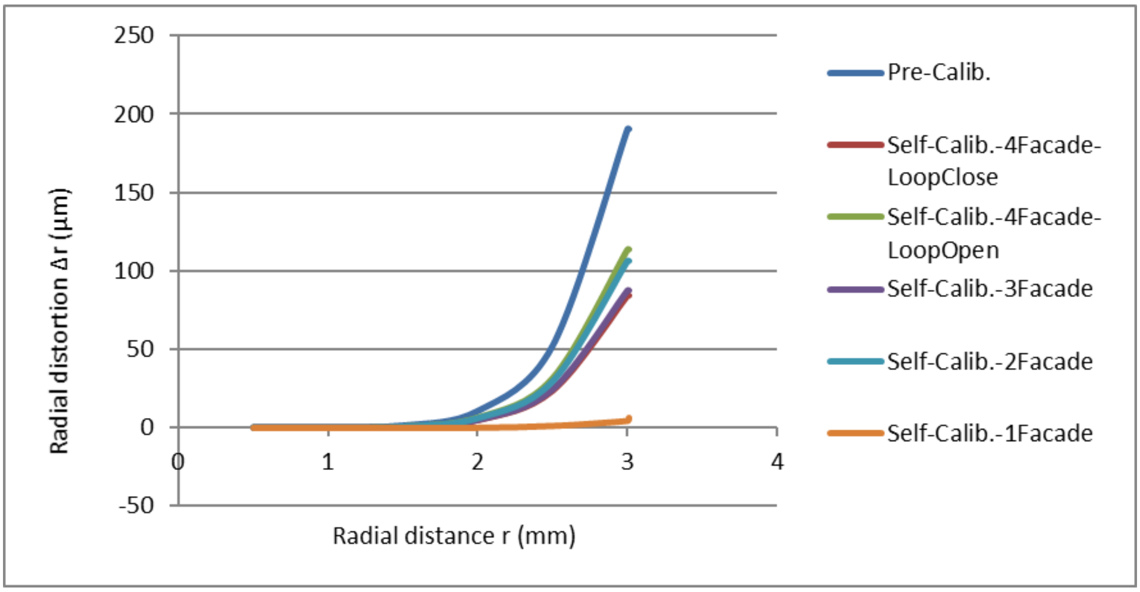

Figure 8. Image convergences and related radial distortions for pre- and self- calibrations

Table 3. The building convergence images and point cloud generation from pre- self- and non- calibrated images

Façade#	Calibration	CP#	RMSE_XYZ (cm)	Image#	Sparse Point (SP)#	Projection#	Projection# /SP#	Projection# /Image#
4 Façade-Loop close-Convergent	Pre-	8	0.86	50	67223	161607	2.40	3232
	Self-		0.68					
	Non-		14.30					
4 Façade-Loop open-Convergent	Pre-	8	0.68	46	61543	148256	2.41	3223
	Self-		0.32					
	Non-		15.38					
3 Façade-Convergent	Pre-	6	0.72	36	47722	116708	2.44	3242
	Self-		0.35					
	Non-		14.44					
2 Façade-Convergent	Pre-	4	0.66	22	30546	75692	2.48	3441
	Self-		0.23					
	Non-		19.71					
1 Façade-Plane	Pre-	3	0.53	11	15418	35534	2.30	3230
	Self-		0.08					
	Non-		25.49					

3.3. Projection Related Self-Calibration

The experiments were made on façade (22x12 m2 dimensions) of a historical structure. Thirty images were recorded from the structure façade as regular space projection centres in overlapping positions. The images were taken 13.7 m distance away from the object surface (Figure 9). The scale is around 3028 and, ground resolution is 5.06 mm/pixel for all the images.

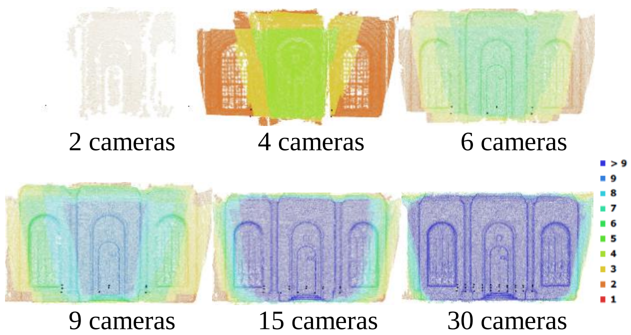

Figure 9. Camera stations and point projections (Colour legend shows number of overlapping images and projections as well)

Table 4. The pre- and self- calibration interior parameters for different number multi-view images

Calibration	*f* (pixel)	x_o (pixel)	y_o (pixel)
Pre-	2700.220	-27.935	5.042
Self-30 cameras	2702.770	-23.229	1.793
Self-15 cameras	2705.660	-23.432	0.385
Self-9 cameras	2700.370	-21.481	-0.975
Self-6 cameras	2703.980	-22.548	-3.661
Self-4 cameras	2710.590	-31.625	-5.614

The image based point cloud data can be created from two-view or multi-view stereo images using SfM approach. However, self-calibration data can be estimated only for multi-view photogrammetric evaluation of partially overlapping three or more images. Every extra image to the evaluated image block increases the accuracy of point projection and measurement. A projection number of any connected point indicates that how many images are covering and connected to this point. The self-calibration and point cloud creation were performed with the varying number of images. The self-calibration parameters were estimated for image sets of 4, 6, 9, 15 and 30 cameras (Table 4, Figure 10). The image blocks are, also, evaluated with the pre-calibration as metric images and without calibration as non-metric images. The measurement accuracy was evaluated with the residuals on the CPs (Table 5).

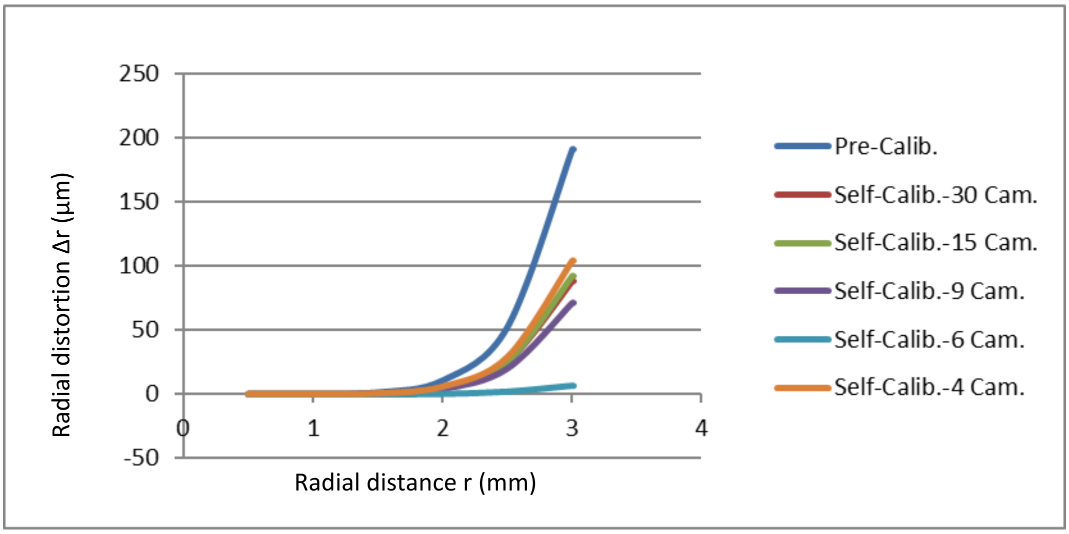

Figure 10. The pre- and self- calibration radial distortions for different number multi-view images

Table 5. The plane surface images and point cloud generation from pre-, self-, and non- calibrated images

Camera (Image)#	Calibration	CP#	RMSE_ XYZ (cm)	Sparse point (SP)#	Projection#	Projection#. /SP#	Projection# /Image#
30	Pre-	7	0.76	7054	93235	13.22	3108
	Self-		0.70				
	Non-		24.63				
15	Pre-	7	0.88	7621	54596	7.16	3640
	Self-		0.58				
	Non-		25.60				
9	Pre-	7	1.04	7893	33347	4.22	3705
	Self-		0.59				
	Non-		26.93				
6	Pre-	7	1.02	7375	22876	3.10	3813
	Self-		0.55				
	Non-		27.39				
4	Pre-	7	1.30	6986	15283	2.19	3821
	Self-		0.79				
	Non-		28.69				
2	Pre-	7	1.11	306	612	2.00	306
	Self-		na				
	Non-		7.18				

4. DISCUSSION

The SfM approach photogrammetric evaluation is an easy and simple way to getting image based point cloud data in the absence of pre-calibration parameters. However the measurements are in low accuracy without the camera calibration. The camera's intrinsic and distortion parameters must be known for high accuracy measurement. The focal length has big effects on the accuracy individually. It is achieved from header of the image file for non-metric images or arbitrary value to an imagery evaluation. The camera calibration parameters are attained from pre- or self- calibration. The pre-calibration raises labour and time requirement in photogrammetric measurement. At this time, the self-calibration can be performed for fast and low cost measurement. But the convergent imaging or enough number of projections for plane surface should be considered for high level accuracy of self-camera calibration and executed photogrammetric measurement of multi-view images. The correct pre-calibration is also performed with these image configurations. Nowadays, all the software packages use SfM algorithm, and can perform self-calibration together with estimation the object coordinates of conjugate points.

The self-calibration needs huge computation that forces to computer capacity. Its execution is very hard by the manually selected conjugated object points. However, the image based measurement and camera calibration can be automatically performed by using SfM approach. The object feature points are detected from overlapping images and then they are matched by their similarities in SfM flowchart. The E matrix is used to finding the possible match points using the co-planarity constraint of the overlapping images when the calibration parameters are known, and F matrix is used otherwise.

The self-calibration requires multi-view images to estimate the camera's intrinsic and distortion parameters. The convergent image geometry ensures high accuracy for self-calibration. If the multiple image geometry is retired from convergence imaging, the calibration accuracy will be reduced (Table 2, Figure 8). On the other hand, depending to the measured object geometry such as building, loop close imaging should be performed for high accuracy of the measurement and self-calibration (Table 3). Another issue on the self-calibration is the number of projections and related images. The projection depicts that how many images are connected to a point which a member of the sparse point cloud, and therefore related to number of images. More projections mean high accuracy for image based point cloud and also self-calibration (Table 4, Figure 10). The results of the study indicate that the multi-view evaluation with self-calibration has better or similar level accuracy with respect to pre-calibration when the projection-to-sparse point cloud ratio is more than ten (Table 5).

The previous studies in the literature are presented the potential of consumer-grade digital cameras to maintain their internal geometry in terms of temporal stability and manufacturing consistency [19]. Nevertheless, the camera can be subject of forces that cause to deformation of internal geometry eventually. Thus the self-calibration photogrammetry is guaranty a high accuracy in the measurement under the condition that the imaging geometry is not far away from the proper one.

The computation time for point cloud generation and self-calibration is usually very short. It is depending to image number and computer configurations. In this study, maximum image number (50 images) were processed at 3 minutes 57 seconds by a computer which has 3.10GHz CPU and 8 GB Ram.

5. CONCLUSION

The wide spectrum of applications in close range photogrammetry entails the image acquisition of more complex network geometry with a much lower standardization. The pre-calibration ensures acceptable level accuracy in measurement. It necessitates proper test field, imaging and computation before the photogrammetry evaluation. In addition the camera individual calibration should be performed in particular time periods for checking the stabilization of the camera parameters. On the other hand, high accuracy measurement can be performed with the self-calibration of proper imaging which has convergence or high projections in SfM based point cloud generation. The self-calibration was performed with high accuracy by using loop-close images. However loop close imaging may not possible for every measurement conditions. Thus the images should be convergent as far as possible, or projection-to-sparse point cloud ratio must be raised. Here, projection-to-sparse point cloud ratio of 13.22 created high accuracy to self-calibration. The consumer-grade digital cameras can be used with self-calibration for metric measurement of photogrammetry. For this, the camera intrinsic parameters of non-metric images must be estimated together with point cloud generation by using the SfM algorithm. The non-metric images cause decrease to accuracy on measurement of photogrammetric point cloud.

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Research

APPROXIMATION PROPERTIES OF MODIFIED SZASZ-SCHURER BASKAKOV TYPE OPERATORS

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ABSTRACT

In the present paper, we study some approximation properties of modified Szász-Schurer-Baskakov type operators. We estimate the moments for these operators using the Hypergeometric series, which are related to Laguerre polynomials. We give approximation properties of derivatives of these operators. Finally, we obtain the Voronovskaya type theorem for derivatives of these operators.

Keywords: Modified Szász-Schurer-Baskakov operator; Szász operator; point-wise convergence; approximation of derivatives; Modulus of continuity; Voronovskaya type theorem.

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1. INTRODUCTION

In 1962, Schurer defined the Bernstein-Schurer operators for any $n \in \mathbb{N},, f \in C[0,1+p]$ and non-negative integers p using well-known Bernstein operators[7]. He defined Bernstein-Schurer operators for $B_{n,p}: C[0,1+p] \to C[0,1]$as follows

$$B_{n,p}(f)(x) = \sum_{k=0}^{n+p} f\left(\frac{k}{n}\right)\binom{n+p}{k} x^k (1-x)^{n+p-k}, x \in [0,1]$$

and studied the approximation properties of these operators. It should be noted that the special case of $p = 0$ here gives classical Bernstein operators.

Later in 1965. [6], Schurer generalized the well-known Szasz operator and the Baskakov operator for every $p \in \mathbb{N}_0, f \in E_2, x \in [0,\infty), n \geq 1$, the $n-th$ Schurer-Szász-Mirakjan operator $M_{n,p}: E_2 \to C[0,\infty)$ and the $n-th$ Baskakov-Schurer operator $A_{n,p}: E_2 \to C[0,\infty)$ as

$$M_{n,p}(f)(x) := exp[-(n+p)x] \sum_{k=0}^{\infty} \frac{[(n+p)x]^k}{k!} f\left(\frac{k}{n}\right)$$

and

$$A_{n,p}(f)(x) := (1+x)^{-n-p} \sum_{k=0}^{\infty} \binom{n+p+k-1}{k} \left(\frac{x}{1+x}\right)^k f\left(\frac{k}{n}\right)$$

repectively. Where $E_2 := \left\{f \in C[0,\infty): \frac{f(x)}{1+x^2} is\ convergent\ as\ x \to \infty\right\}$

Many studies have been made on various generalizations of the above operators and their convergence properties, and such studies are still ongoing, [1], [2], [10], [11], [13], [16], [17], [20], [22] .

In 1967, J. L. Durmeyer defined one-dimensional Bernstein-Durrmeyer operators as

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$$D_n(f;x) := (n+1)\sum_{k=0}^{\infty}\binom{n}{k}x^k(1-x)^{n-k}\int_0^1\binom{n}{k}u^k(1-u)^{n-k}f(u)du, \qquad x\in[0,1], n=1,2,...$$

and studied the approximation properties of these operators. Where f is any real-valued function on [0,1], which is integrable with respect to the kernel [8].

In 1995 [21],.Gupta and Srivastava studied the convergence properties of the derivatives of the Szász-Mirakyan-Baskakov type operators defined as follows;

$$M_n(f;x) := (n-1)\sum_{k=0}^{\infty}e^{-nx}\frac{(nx)^k}{k!}\frac{(n+k-1)!}{k!(n-1)!}\int_0^{\infty}\frac{t^k}{(1+t)^{n+k}}f(t)dt,\ x\in[0,\infty). \tag{1.1}$$

Later, many researchers defined Durrmeyer-type extensions and many modifications of other well-known operators and studied their convergence properties, [3], [9], [12], [14], [15].

Also, in 2006, for $f\in C_{\gamma}[0,\infty)\equiv\{f\in C[0,\infty): |f(t)|\le Mt^{\gamma}, \gamma>0\}$, Gupta et al. defined the Baskakov-Durmeyerr operators as an integral modification of the Baskakov operators as follows

$$B_n(f;x) := \sum_{k=1}^{\infty}\binom{n+k-1}{k}\frac{x^k}{(1+x)^{n+k}}\frac{1}{B(n+1,k)}\int_0^{\infty}\frac{t^{k-1}}{(1+t)^{n+k+1}}f(t)dt+\frac{f(0)}{(1+x)^n}, \tag{1.2}$$

and studied the convergence properties of the derivatives of these operators, [8].

In 2012, Verma et al. defined a Stancu type extension of operator of (1.1) type, gave the approximation properties of these operators, the error estimation and obtained some recurrence relations [4]. In 2012, Gupta et al. examined the point convergence properties of the Stancu extension of the operators (1.2) and gave Voronovskaya type theorems,[19]. In 2014, Agraval et al. gave a different generalization of the (1.2) operators and examined the properties of uniform convergence and point convergence, [12].

In 2015, Pandey et al. a Szàsz-Baskakov Stancu type generalization of operators is defined as follows:

$$M_n(f;x) := (n-1)\sum_{k=0}^{\infty}e^{-nx}\frac{(nx)^k}{k!}\frac{(n+k-1)!}{k!(n-1)!}\int_0^{\infty}\frac{t^k}{(1+t)^{n+k}}f\left(\frac{nt+\alpha}{n+\beta}\right)dt,\ x\in[0,\infty). \tag{1.3}$$

and examined various approximation properties,[5]. They also benefited from the properties of hypergeometric functions while doing these investigations.

By defining the Schurer generalization of the (1.1) operator, we wanted to achieve similar results.

For $f\in C[0,\infty)$, we define the Schurer type generalization of the Szász-Mirakyan-Baskakov type operator given in (1.1) as follows

$$A_{n,a,\beta_n}(f;x) := (n+a-1)\sum_{k=0}^{\infty}p_{n,\beta_n,k}(x)\int_0^{\infty}q_{n,a,k}(t)f(t)dt \ ;\ a\in N_0, x\in[0,\infty), \tag{1.4}$$

where β_n is a sequence of positive numbers such that

$$\lim_{n\to\infty}\beta_n = 0 \tag{1.5}$$

and

$$p_{n,\beta_n,k}(x) = e^{-(n+\beta_n)x}\frac{[(n+\beta_n)x]^k}{k!},\quad q_{n,a,k}(t) = \frac{(n+a+k-1)!}{k!(n+a-1)!}\frac{t^k}{(1+t)^{n+a+k}} \tag{1.6}$$

It is clear that for $x\in[0,\infty)$ the operators A_{n,a,β_n} are linear and positive.

Now, let's give some important information that we will benefit from when examining the convergence properties of the operator we have defined.

Well known Beta and Gamma functions provides the equality

$$B(x,y)=\int_0^\infty \frac{t^{x-1}}{(1+t)^{x+y}}dt=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}=\frac{(x-1)!(y-1)!}{(x+y-1)!}. \tag{1.7}$$

We can represent the operator defined by (1.4) in a different way by using the hyper-geometric functions defined as

$$ {}_1F_1(\alpha;\beta;t)=\sum_{k=0}^\infty \frac{(\alpha)_k}{(\beta)_k}\frac{x^k}{k!} \tag{1.8}$$

Where $(\alpha)_k$ is called Pochhammer symbol defined as

$$(\alpha)_k=\alpha(\alpha+1)(\alpha+2)\dots(\alpha+k-1) \tag{1.9}$$

and $(1)_k=k!$. Using hyper-geometric functions, we can give an equivalent definition of the operators $A_{n,a,\beta_n}(f;x)$ as follows;

$$A_{n,a,\beta_n}(f;x)=(n+a-1)\int_0^\infty \frac{e^{-(n+\beta_n)x}}{(1+t)^{n+a}}f(t)\ {}_1F_1\left(n+a;1;\frac{(n+\beta_n)xt}{1+t}\right)dt.$$

Our aim in this study is to examine the convergence properties of the operator (1.4), which we define as a generalization of the (1.1) operator, and its derivatives, taking into account the study of Pandey et al., and evaluate the results. Let us give some recurrence relations and lemmas that we need for this.

2. SOME MOMENTS AND RECURRENCE RELATIONS

In this section, we give some lemmas that we will use in this study.

Lemma 1. For all $x\in[0,\infty)$, the operators A_{n,a,β_n} defined by (1.4) satisfy the followings:

$$A_{n,a,\beta_n}(1;x)=1 \tag{2.1}$$

$$A_{n,a,\beta_n}(t;x)=\frac{(n+\beta_n)x+1}{(n+a-2)} \tag{2.2}$$

$$A_{n,a,\beta_n}(t^2;x)=\frac{[(n+\beta_n)x]^2+4[(n+\beta_n)x]+2}{(n+a-2)(n+a-3)} \tag{2.3}$$

The proof can be easily done from the definition of the operator A_{n,a,β_n}.

Lemma 2. Let $r\in N_0$. If the $r-th$ order moment is defined as

$$u_{n+\beta_n,r}(x)=\sum_{k=0}^\infty p_{n,\beta_n,k}(x)\left(\frac{k}{n+\beta_n}-x\right)^r=\sum_{k=0}^\infty e^{-(n+\beta_n)x}\frac{[(n+\beta_n)x]^k}{k!}\left(\frac{k}{n+\beta_n}-x\right)^r \tag{2.4}$$

then there exists a reccurence relation

$$(n+\beta_n)u_{n+\beta_n,r+1}(x)=x\left[u'_{n+\beta_n,r}(x)+ru_{n+\beta_n,r-1}(x)\right]. \tag{2.5}$$

Consequently

1. $u_{n+\beta_n,r}(x)$ is a polynomial in x of degree $\le r$.
2. $u_{n+\beta_n,r}(x)=O\left(\frac{1}{n^{\left[\frac{r+1}{2}\right]}}\right)$; as $n\to\infty$.

The proof can be done using the definition.

Lemma 3. If wedefine the central moments as

$T_{n,a,\beta_n,r}(x) = A_{n,a,\beta_n}((t-x)^2;x) = (n+a-1)\sum_{k=0}^{\infty} p_{n,\beta_n,k}(x)\int_0^{\infty} q_{n,a,k}(t)(t-x)^r dt,$ (2.6)

then

$$T_{n,a,\beta_n,0}(x) = 1$$

$$T_{n,a,\beta_n,1}(x) = \frac{(\beta_n - a + 2)x + 1}{(n+a-2)}$$

$$T_{n,a,\beta_n,2}(x) = \frac{[(n-a-3)+(\beta_n - a + 3)^2]x^2 + (2n + 4\beta_n - 2a + 6)x + 2}{(n+a-2)(n+a-3)}$$

and for $n > 1$ we have the following recurrence relation;

$(n+a-r-2)T_{n,a,\beta_n,r+1}(x) = xT'_{n,a,\beta_n,r}(x) + [(2r+a-\beta_n+2)x + r + 1]T_{n,a,\beta_n,r}(x) + (x^2+2x)rT_{n,a,\beta_n,r-1}(x).$ (2.7)

From the recurrence relation, it can be easily verified that for all $x \in [0,\infty)$, we have $T_{n,a,\beta_n,r}(x) = O\left(\frac{1}{n^{\left[\frac{r+1}{2}\right]}}\right)$.

Proof. Since the operatör A_{n,a,β_n} is linear, we get $T_{n,a,\beta_n,0}(x) = 1$, $T_{n,a,\beta_n,1}(x) = \frac{(\beta_n - a+2)x+1}{(n+a-2)}$,

$T_{n,a,\beta_n,2}(x) = \frac{[(n-a-3)+(\beta_n-a+3)^2]x^2+(2n+4\beta_n-2a+6)x+2}{(n+a-2)(n+a-3)}$. From (2.6), we can write

$$T'_{n,a,\beta_n,r}(x) = \frac{k}{x}T_{n,a,\beta_n,r}(x) - (n+\beta_n)T_{n,a,\beta_n,r}(x) - rT_{n,a,\beta_n,r-1}(x)$$

$\Longrightarrow$

$$xT'_{n,a,\beta_n,r}(x) = [k-(n+\beta_n)]T_{n,a,\beta_n,r}(x) - xrT_{n,a,\beta_n,r-1}(x).$$

On the other hand, using $t(1+t)q'_{n,a,k}(t) = [k-(n+a)t]q_{n,a,k}(t)$ and $xp'_{n,\beta_n,k}(x) = [k-(n+\beta_n)x]p_{n,\beta_n,k}(x)$, we obtain

$$xT'_{n,a,\beta_n,r}(x) + rxT_{n,a,\beta_n,r-1}(x) = (n+a-1)\sum_{k=0}^{\infty} p_{n,\beta_n,k}(x)\int_0^{\infty} q'_{n,a,k}(t)(t-x)^{r+2}dt$$
$$+(n+a-1)(2x+1)\sum_{k=0}^{\infty} p_{n,\beta_n,k}(x)\int_0^{\infty} q'_{n,a,k}(t)(t-x)^{r+1}dt$$
$$+(n+a-1)(3x^2+x)\sum_{k=0}^{\infty} p_{n,\beta_n,k}(x)\int_0^{\infty} q'_{n,a,k}(t)(t-x)^{r}dt$$
$$+(n+a)T_{n,a,\beta_n,r+1}(x) - (\beta_n - a)xT_{n,a,\beta_n,r}(x).$$

If partial integration is applied to the integrals here, we get

$$xT'_{n,a,\beta_n,r}(x) + rxT_{n,a,\beta_n,r-1}(x)$$
$$= (n+a-r-2)T_{n,a,\beta_n,r+1}(x) - (2xr+2x+r+1-\beta_n x + ax)T_{n,a,\beta_n,r}(x) - (x^2+x)rT_{n,a,\beta_n,r-1}(x).$$

Then we obtain (2.7) and

$$T_{n,a,\beta_n,r}(x) = O\left(n^{-\left[\frac{r+1}{2}\right]}\right).$$

Lemma 4. For all $r \in N_0$,

$$A_{n,a,\beta_n}(t^r;x) = \frac{r!\,(n+a-r-2)!}{(n+a-2)!}\sum_{j=0}^{r}\binom{r}{j}\frac{[(n+\beta_n)x]^j}{j!}$$

is a polynomial in x of degree exactly r.

Proof. By using (1.4), (1.9) and the equations $\Gamma(k+r+1) = \Gamma(r+1)(r+1)_k = r!\,(r+1)_k$, $k! = (1)_k$, we have

$$A_{n,a,\beta_n}(t^r;x) = (n+a-1)\sum_{k=0}^{\infty} e^{-(n+\beta_n)x}\frac{[(n+\beta_n)x]^k}{k!}\frac{(n+a+k-1)!}{k!\,(n+a-1)!}\int_0^{\infty}\frac{t^{k+r}}{(1+t)^{n+a+k}}dt$$

$$= (n+a-1)\sum_{k=0}^{\infty} e^{-(n+\beta_n)x}\frac{[(n+\beta_n)x]^k}{k!}\frac{(n+a+k-1)!}{k!\,(n+a-1)!}\frac{r!\,(r+1)_k(n+a-r-2)!}{(n+a+k-1)!}$$

$$= e^{-(n+\beta_n)x}\frac{r!\,(n+a-r-2)!}{(n+a-2)!}\sum_{k=0}^{\infty}\frac{[(n+\beta_n)x]^k}{k!}\frac{(r+1)_k}{k!}$$

$$= e^{-(n+\beta_n)x}\frac{r!\,(n+a-r-2)!}{(n+a-2)!}\ {}_1F_1(r+a;1;(n+\beta_n)x)$$

Using the Kummer transformation

$${}_1F_1(\alpha;\beta;x) = e^x\,{}_1F_1(\beta-\alpha;\beta;-x)$$

and

$${}_1F_1(1;1;x) = e^x$$

We have

$$e^{-(n+\beta_n)x}\,{}_1F_1(r+a;1;(n+\beta_n)x) = {}_1F_1(-r;1;-(n+\beta_n)x).$$

Then, we can write

$$A_{n,a,\beta_n}(t^r;x) = \frac{r!\,(n+a-r-2)!}{(n+a-2)!}\,{}_1F_1(-r;1;-(n+\beta_n)x).$$

Since the confluent hypergeometric function is related with generalized Laguerre polynomial with the relation

$$L_n^m(x) = \binom{m+n}{n}\,{}_1F_1(-n;m+1;x) = \frac{(m+n)!}{m!\,n!}\,{}_1F_1(-n;m+1;x),$$

and

$$L_r^0(x) = L_r(x) = {}_1F_1(-r;1;x) \Rightarrow L_r(-(n+\beta_n)x) = {}_1F_1(-r;1;-(n+\beta_n)x)$$

we obtain

$$A_{n,a,\beta_n}(t^r;x) = \frac{r!(n+a-r-2)!}{(n+a-2)!}L_r(-(n+\beta_n)x),$$

where

$L_r(-(n+\beta_n)x)=\sum_{j=0}^{r}(-1)^j\binom{r}{r-j}\frac{[-(n+\beta_n)x]^j}{j!}=\sum_{j=0}^{r}\binom{r}{j}\frac{[(n+\beta_n)x]^j}{j!}.$

Then we have

$$A_{n,a,\beta_n}(t^r;x)=\frac{r!(n+a-r-2)!}{(n+a-2)!}\sum_{j=0}^{r}\binom{r}{j}\frac{[(n+\beta_n)x]^j}{j!}=\frac{(n+\beta_n)^r(n+a-r-2)!}{(n+a-2)!}x^r+\frac{r^2(n+\beta_n)^{r-1}(n+a-r-2)!}{(n+a-2)!}x^{r-1}$$
$$+\frac{r(r-1)(n+\beta_n)^{r-2}(n+a-r-2)!}{(n+a-2)!}x^{r-2}+O(n^{-2}).$$

So the proof is completed.

Lemma 5. [21] There exist a polynomial $\phi_{i,j,r}(x)$ independent of n and k such that

$$x^r\frac{d^r}{dx^r}\left[e^{-(n+\beta_n)x}[(n+\beta_n)x]^k\right]=\sum_{\substack{2i+j\le r\\ i,j\ge 0}}(n+\beta_n)^i[k-(n+\beta_n)x]^j\phi_{i,j,r}(x)e^{-(n+\beta_n)x}[(n+\beta_n)x]^k.$$

The proof can be easily done using the induction method.

Lemma 6.Let f be r-times differentiable on $[0,\infty)$ such that $f^{(r-1)}=O(t^\alpha)$, for some $\alpha>0$ as $t\to\infty$. Then for $r=1,2,\ldots$ and $n>\alpha+r$, we have

$$A_{n,a,\beta_n}^{(r)}(f;x)=\frac{(n+\beta_n)^r(n+a-r-1)!}{(n+a-2)!}\sum_{k=0}^{\infty}p_{n,\beta_n,k}(x)\int_0^\infty(-1)^rq_{n-r,a,k+r}(t)f^{(r)}(t)dt.$$

Proof. Using Leibnitz theorem, we can write

$$A_{n,a,\beta_n}^{(r)}(f;x)=$$
$$(n+a-1)\sum_{i=0}^{r}\sum_{k=i}^{\infty}\binom{r}{i}\frac{(-1)^{r-i}(n+\beta_n)^r[(n+\beta_n)x]^{k-i}}{(k-i)!}e^{-(n+\beta_n)x}\frac{(n+a+k-1)!}{k!\,(n+a-1)!}\int_0^\infty\frac{t^k}{(1+t)^{n+a+k}}f(t)dt$$
$$=(n+a-1)\sum_{k=0}^{\infty}\sum_{i=0}^{r}\binom{r}{i}\frac{(-1)^{r-i}(n+\beta_n)^r[(n+\beta_n)x]^k}{k!}e^{-(n+\beta_n)x}\frac{(n+a+k+i-1)!}{(k+i)!\,(n+a-1)!}\int_0^\infty\frac{t^k}{(1+t)^{n+a+k+i}}f(t)dt$$
$$=(n+a-1)\sum_{k=0}^{\infty}p_{n,\beta_n,k}(x)\int_0^\infty(-1)^r\sum_{i=0}^{r}\binom{r}{i}(-1)^i(n+\beta_n)^r(-1)^rq_{n,a,k+i}(t)f(t)dt.$$

By Leibnitz theorem, we have

$$q_{n-r,a,k+r}^{(r)}(t)=\frac{(n+a-1)!}{(n+a-r-1)!}\sum_{i=0}^{r}\binom{r}{i}(-1)^i\,q_{n,a,k+i}(t)$$

If this expression is substituted in $A_{n,a,\beta_n}^{(r)}(f;x)$, we get

$$A_{n,a,\beta_n}^{(r)}(f;x)=\frac{(n+\beta_n)^r(n+a-r-1)!}{(n+a-2)!}\sum_{k=0}^{\infty}p_{n,\beta_n,k}(x)\int_0^\infty(-1)^rq_{n-r,a,k+r}^{(r)}(t)f(t)dt.$$

If partial integration is applied to this expression, we obtain the desired result

$$A_{n,a,\beta_n}^{(r)}(f;x)=\frac{(n+\beta_n)^r(n+a-r-1)!}{(n+a-2)!}\sum_{k=0}^{\infty}p_{n,\beta_n,k}(x)\int_0^\infty(-1)^rq_{n-r,a,k+r}(t)f^{(r)}(t)dt. \quad (2.8)$$

Remark 1. A simple consequence of (2.8) is

$$A_{n,a,\beta_n}(t^r;x) = \frac{r!(n+\beta_n)^r(n+a-r-2)!}{(n+a-2)!} \tag{2.9}$$

Lemma 7. If we define

$$\lambda_r(n) = \frac{\prod_{j=0}^{r}(n+\beta_n)^j(n+a-j-2)}{(n+a-2)!} = \frac{(n+\beta_n)^r(n+a-r-2)!}{(n+a-2)!} \tag{2.10}$$

then we have the following recurrence relations

$$[\lambda_{r+1}(n) - \lambda_r(n)]x = \frac{r+1}{n+\beta_n}\lambda_{r+1}(n) = \lambda_r(n)\left[\frac{(\beta_n-a+r+2)x+(r+1)}{n+a-r-2}\right] \tag{2.11}$$

$$[\lambda_r(n) - 2\lambda_{r+1}(n) - \lambda_{r+2}(n)] = \lambda_r(n)\left[\frac{(r-a)^2+(r+\beta_n)^2-r^2+5(r-a)+6(1+\beta_n)-2a\beta_n+n}{(n+a-r-2)(n+a-r-3)}\right] \tag{2.12}$$

$$\frac{r+2}{n+\beta_n}\lambda_{r+1}(n) - \frac{r+1}{n+\beta_n}\lambda_{r+2}(n) = \lambda_r(n)\left[\frac{r(a-\beta_n)-r^2-5r+2a-6+\beta_n+n}{(n+a-r-2)(n+a-r-3)}\right]. \tag{2.13}$$

The proof can be easily done with simple operations.
Let us now give some definitions that we will use.

Definition 1. [19] The $m-th$ order modulus of continuity $\omega_m(f;\delta:[a,b])$ for a function f which is continuous on $[a,b]$ is defined by

$$\omega_m(f;\delta:[a,b]) = sup\{|\Delta_h^m f(x)|; |h| < \delta; x, x+h \in [a,b]\}. \tag{2.14}$$

For $m = 1, \omega_m(f;\delta)$ is usual modulus of continuity.

Definition 2. [5] Let $C_\gamma[0,\infty) = \{f \in [0,\infty); |f(t)| \le Mt^\gamma,\ \gamma > 0\}$. The norm ($\|.\|$) on $C_\gamma[0,\infty)$ is defined by;

$$\|f\|_\gamma = \sup_{0\le t<\infty} |f(t)|t^{-\gamma}. \tag{2.15}$$

Definition 3. [19] Let $0 < a < a_1 < b_1 < b < \infty$. For sufficiently small $\eta > 0$, the $2-nd$ order Steklov mean $f_{\eta,2}$ corresponding to $f \in C_\gamma[a,b]$ and $t \in I_1$ is defined as;

$$f_{\eta,2}(t) = \eta^{-2}\int_{-\frac{\eta}{2}}^{\frac{\eta}{2}}\int_{-\frac{\eta}{2}}^{\frac{\eta}{2}}[f(t) - \Delta_h^2 f(t)]dt_1dt_2, \tag{2.16}$$

Where $h = \frac{t_1+t_2}{2}$ and Δ_h^2 is the second order forward difference operator with step lenght h. For $f \in C[a,b]$, $f_{\eta,2}$ satisfy the following properties:

1. $f_{\eta,2}$, has continuous derivatives up to order 2 over $[a_1,b_1]$.
2. $\|f_{\eta,2}\|_{C[a_1,b_1]} \le C\omega_r(f;\delta:[a,b]),\ r = 1,2$
3. $\|f - f_{\eta,2}\|_{C[a_1,b_1]} \le C\omega_2(f;\delta:[a,b])$
4. $\|f_{\eta,2}\|_{C[a_1,b_1]} \le C\eta^{-2}\|f\|_{C[a,b]}$
5. $\|f_{\eta,2}\|_{C[a_1,b_1]} \le C\|f\|_\gamma$

where C are certain constants which are different in each occurence and are independent of f and η.

3. DIRECT RESULTS

Theorem 1. (Pointwise convergence) Let $a \in N_0$, $x \in [0,\infty)$ and β_n be a sequence satisfying (1.5). If $r \in N_0, f \in C_\gamma[0,\infty)$ for some $\gamma > 0$ and $f^{(r)}$ exists at a point $x \in (0,\infty)$, then

$$\lim_{n\to\infty}\left|A_{n,a,\beta_n}^{(r)}(f;x)\right| = f^{(r)}(x).$$

Proof. By Taylor expansion of f, we have

$$f(t) = \sum_{i=0}^{r}\frac{f^{(i)}(x)}{i!}(t-x)^i + \varepsilon(t,x)(t-x)^r$$

where $\varepsilon(t,x) \to 0$ as $t \to x$. If the operator $A_{n,a,\beta_n}(f;x)$ is applied to this expression, we get

$$A_{n,a,\beta_n}(f;x) = \sum_{i=0}^{r}\frac{f^{(i)}(x)}{i!}A_{n,a,\beta_n}((t-x)^i;x) + A_{n,a,\beta_n}(\varepsilon(t,x)(t-x)^r;x).$$

By taking the derivative of this expression r times, we obtain

$$\frac{d^r}{dx^r}\left[A_{n,a,\beta_n}(f;x)\right] = \sum_{i=0}^{r}\frac{f^{(i)}(x)}{i!}A_{n,a,\beta_n}^{(r)}((t-x)^i;x) + A_{n,a,\beta_n}^{(r)}(\varepsilon(t,x)(t-x)^r;x) = I_1 + I_2$$

Using Lemma 4, Lemma 5 and (2.9),

$$\begin{aligned} I_1 &= \sum_{i=0}^{r}\frac{f^{(i)}(x)}{i!}A_{n,a,\beta_n}^{(r)}((t-x)^i;x) \\ &= \sum_{i=0}^{r-1}\frac{f^{(i)}(x)}{i!}\sum_{j=0}^{i}\binom{i}{j}(-x)^{i-j}\frac{i!\,(n+\beta_n)^i(n+a-i-2)!}{(n+a-2)!} + \frac{f^{(r)}(x)}{r!}\sum_{j=0}^{r-1}\binom{r}{j}(-x)^{r-j}A_{n,a,\beta_n}^{(r)}(t^j;x) + \frac{f^{(r)}(x)}{r!}A_{n,a,\beta_n}^{(r)}(t^r;x) \\ &= I_1 + I_2 + I_3 \end{aligned}$$

is obtained. Since $I_3 = O((1/n)), I_4 = O((1/n)), I_5 = \frac{f^{(r)}(x)}{r!}A_{n,a,\beta_n}^{(r)}(t^r;x)$, we have

$$I_1 = \frac{f^{(r)}(x)}{r!}A_{n,a,\beta_n}^{(r)}(t^r;x) = \frac{f^{(r)}(x)}{r!}\frac{r!\,(n+\beta_n)^r(n+a-r-2)!}{(n+a-2)!} = \frac{(n+\beta_n)^r(n+a-r-2)!}{(n+a-2)!}f^{(r)}(x).$$

thus $f^{(r)}(x)$ for $n \to \infty$. Now let's set an upper bound for I_2 by using Lemma 6.

$$\begin{aligned} I_2 &= A_{n,a,\beta_n}^{(r)}(\varepsilon(t,x)(t-x)^r;x) \\ &= (n+a-1)\sum_{\substack{2i+j\le r\\ i,j\ge 0}}\frac{(n+\beta_n)^i}{x^r}\left|\phi_{i,j,r}(x)\right|\sum_{k=0}^{\infty}[k-(n+\beta_n)x]^j\,e^{-(n+\beta_n)x}\frac{[(n+\beta_n)x]^k}{k!} \\ &\quad\times\frac{(n+a+k-1)!}{k!(n+a-1)!}\int_0^\infty \varepsilon(t,x)\frac{t^k(t-x)^r}{(1+t)^{n+a+k}}dt \end{aligned}$$

So we can write I_2 as:

$$\begin{aligned} |I_2| &\le (n+a-1)\sum_{\substack{2i+j\le r\\ i,j\ge 0}}\frac{(n+\beta_n)^i}{x^r}\left|\phi_{i,j,r}(x)\right|\sum_{k=0}^{\infty}\left|[k-(n+\beta_n)x]^j\right|e^{-(n+\beta_n)x}\frac{[(n+\beta_n)x]^k}{k!} \\ &\quad\times\frac{(n+a+k-1)!}{k!(n+a-1)!}\int_0^\infty|\varepsilon(t,x)|\frac{t^k|t-x|^r}{(1+t)^{n+a+k}}dt \end{aligned}$$

Since $\varepsilon(t,x) \to 0$ as $t \to x$, for a given $\varepsilon > 0$ there exists a $\delta > 0$ such that $|\varepsilon(t,x)|$ whenever $|t-x| < \delta$, morever if $\lambda \geq max\{\gamma, r\}$ is any integer, then we find a constant $K > 0$ such that $|\varepsilon(t,x)||t-x|^r \leq K|t-x|^\gamma$. Thus we obtain

$$|I_2| \leq (n+a-1)C_1 \sum_{\substack{2i+j\leq r\\ i,j\geq 0}} (n+\beta_n)^i \sum_{k=0}^{\infty} \left|[k-(n+\beta_n)x]^j\right| e^{-(n+\beta_n)x} \frac{[(n+\beta_n)x]^k}{k!}$$
$$\times \left\{\frac{(n+a+k-1)!}{k!(n+a-1)!}\int_{|t-x|<\delta} |\varepsilon(t,x)| \frac{t^k|t-x|^r}{(1+t)^{n+a+k}}dt + \frac{(n+a+k-1)!}{k!(n+a-1)!}\int_{|t-x|\geq\delta} K \frac{t^k|t-x|^r}{(1+t)^{n+a+k}}dt\right\} = I_6 + I_7$$

where

$$C_1 = \sup_{\substack{2i+j\leq r\\ i,j\geq 0}} \frac{|\phi_{i,j,r}(x)|}{x^r} > 0$$

and K is a constant independent of C_1. If we use the Schwarz inequality first for the integration and then for the summation to calculate I_6, we have

$$I_6 \leq \varepsilon C_1 \sum_{\substack{2i+j\leq r\\ i,j\geq 0}} (n+\beta_n)^i \left[\sum_{k=0}^{\infty} [k-(n+\beta_n)x]^{2j} e^{-(n+\beta_n)x} \frac{[(n+\beta_n)x]^k}{k!}\right]^{\frac{1}{2}}$$
$$\times \left[(n+a-1)\sum_{k=0}^{\infty} [k-(n+\beta_n)x]^{2j} e^{-(n+\beta_n)x} \frac{[(n+\beta_n)x]^k}{k!} [(n+\beta_n)x]^k \frac{(n+a+k-1)!}{k!(n+a-1)!}\int_0^\infty \frac{t^k(t-x)^r}{(1+t)^{n+a+k}}dt\right]^{\frac{1}{2}},$$

as

$$\int_0^\infty q_{n,a,k}(t)dt = \frac{1}{(n+a-1)}$$

By using Lemma 2, we get

$$\sum_{k=0}^{\infty} [k-(n+\beta_n)x]^{2j} e^{-(n+\beta_n)x} \frac{[(n+\beta_n)x]^k}{k!} = (n+\beta_n)^{2j} \sum_{k=0}^{\infty} \left[\frac{k}{(n+\beta_n)}\right]^{2j} e^{-(n+\beta_n)x} \frac{[(n+\beta_n)x]^k}{k!}$$
$$= (n+\beta_n)^{2j}[O((n+\beta_n)^{-j})] = O((n+\beta_n)^j).$$

Since

$$\left[(n+a-1)\sum_{k=0}^{\infty} [k-(n+\beta_n)x]^{2j} e^{-(n+\beta_n)x} \frac{[(n+\beta_n)x]^k}{k!} [(n+\beta_n)x]^k \frac{(n+a+k-1)!}{k!(n+a-1)!}\int_0^\infty \frac{t^k(t-x)^r}{(1+t)^{n+a+k}}dt\right]^{\frac{1}{2}} = O((n+\beta_n)^{-r})$$

can be written as a result of Lemma 2, we have

$$I_6 \leq \varepsilon C_1 \sum_{\substack{2i+j\leq r\\ i,j\geq 0}} (n+\beta_n)^i O\left((n+\beta_n)^{\frac{j}{2}}\right) O\left((n+\beta_n)^{-\frac{r}{2}}\right) = \varepsilon O(1).$$

Next, using Lemma 2 and Schwarz inequality for the integration and summation, in view of of the above results, we have

$$I_7 \leq (n+a-1)C_2 \sum_{\substack{2i+j\leq r\\ i,j\geq 0}} (n+\beta_n)^i \sum_{k=0}^{\infty} \left|[k-(n+\beta_n)x]^j\right| e^{-(n+\beta_n)x} \frac{[(n+\beta_n)x]^k}{k!}$$
$$\times \left[\frac{(n+a+k-1)!}{k!\,(n+a-1)!}\int_{|t-x|\geq\delta} \frac{t^k}{(1+t)^{n+a+k}}dt\right]^{\frac{1}{2}} \left[\frac{(n+a+k-1)!}{k!\,(n+a-1)!}\int_{|t-x|\geq\delta} \frac{t^k(t-x)^{2r}}{(1+t)^{n+a+k}}dt\right]^{\frac{1}{2}}$$
$$\leq C_2 \sum_{\substack{2i+j\leq r\\ i,j\geq 0}} (n+\beta_n)^i \left[\sum_{k=0}^{\infty} [k-(n+\beta_n)x]^{2j} e^{-(n+\beta_n)x} \frac{[(n+\beta_n)x]^k}{k!}\right]^{\frac{1}{2}}$$

$$\times\left[\sum_{k=0}^{\infty}e^{-(n+\beta_n)x}\frac{[(n+\beta_n)x]^k}{k!}\frac{(n+a+k-1)!}{k!\,(n+a-1)!}\int_{|t-x|\geq\delta}\frac{t^k(t-x)^{2r}}{(1+t)^{n+a+k}}dt\right]^{\frac{1}{2}}$$
$$=\sum_{\substack{2i+j\leq r\\ i,j\geq 0}}(n+\beta_n)^iO\left((n+\beta_n)^{\frac{j}{2}}\right)O\left((n+\beta_n)^{-\frac{r}{2}}\right)=O(1).$$

Thus the proof is completed by using the results obtained for I_1 and I_2.

Theorem 2. (Asymptotic expansion) Let $f\in C_\gamma[0,\infty)$ be bounded for every finite sub-interval of $[0,\infty)$ and has the derivative of order $(r+2)$ at a fixed $x\in(0,\infty)$. For some $\gamma>0$, let $f(t)=O(t^\gamma)$ as $t\to\infty$. Then we have,

$$\lim_{n\to\infty}n\left[A_{n,a,\beta_n}^{(r)}(f;x)-f^{(r)}(x)\right]=[(2+r-a)x+r+1]\,f^{(r+1)}(x)+\left(x+\frac{x^2}{2}\right)f^{(r+2)}(x).$$

Proof. By Taylor expansion of f, we have

$$f(t)=\sum_{i=0}^{r+2}\frac{f^{(i)}(x)}{i!}(t-x)^i+\varepsilon(t,x)(t-x)^{r+2}$$

where $\varepsilon(t,x)\to 0$ as $t\to x$. If the operator $A_{n,a,\beta_n}^{(r)}(f;.)$ is applied to this expression, we have

$$A_{n,a,\beta_n}^{(r)}(f;x)=\left\{\sum_{i=0}^{r+2}\frac{f^{(i)}(x)}{i!}A_{n,a,\beta_n}^{(r)}((t-x)^i;x)\right\}+A_{n,a,\beta_n}^{(r)}[\varepsilon(t,x)(t-x)^{r+2};x]=I_1+I_2$$

Using Lemma 7, we get

$$I_1=\left\{\sum_{i=0}^{r+2}\frac{f^{(i)}(x)}{i!}A_{n,a,\beta_n}^{(r)}((t-x)^i;x)\right\}=\sum_{i=0}^{r+2}\frac{f^{(i)}(x)}{i!}\left\{A_{n,a,\beta_n}^{(r)}\left(\sum_{j=1}^{i}\binom{i}{j}(-x)^{i-j}t^i;x\right)\right\}$$
$$=\sum_{i=0}^{r+2}\frac{f^{(i)}(x)}{i!}\sum_{j=1}^{i}\binom{i}{j}(-x)^{i-j}A_{n,a,\beta_n}^{(r)}(t^i;x)$$
$$=\sum_{i=0}^{r-1}\frac{f^{(i)}(x)}{i!}\sum_{j=1}^{i}\binom{i}{j}(-x)^{i-j}A_{n,a,\beta_n}^{(r)}(t^i;x)+\sum_{i=0}^{r}\frac{f^{(i)}(x)}{i!}\sum_{j=1}^{i}\binom{i}{j}(-x)^{i-j}A_{n,a,\beta_n}^{(r)}(t^r;x)$$
$$+\frac{f^{(r+1)}(x)}{(r+1)!}\sum_{j=1}^{r+1}\binom{r+1}{j}(-x)^{r+1-j}A_{n,a,\beta_n}^{(r)}(t^r;x)+\frac{f^{(r+2)}(x)}{(r+2)!}\sum_{j=1}^{r+2}\binom{r+2}{j}(-x)^{r+2-j}A_{n,a,\beta_n}^{(r)}(t^r;x)$$
$$=\sum_{i=0}^{r-1}\frac{f^{(i)}(x)}{i!}O\left(\frac{1}{n}\right)+\frac{f^{(r)}(x)}{r!}A_{n,a,\beta_n}^{(r)}(t^r;x)+\frac{f^{(r+1)}(x)}{(r+1)!}\left[(r+1)(-x)A_{n,a,\beta_n}^{(r)}(t^r;x)+A_{n,a,\beta_n}^{(r)}(t^{r+1};x)\right]$$
$$+\frac{f^{(r+2)}(x)}{(r+2)!}\left[\frac{(r+1)(r+2)}{2}x^2A_{n,a,\beta_n}^{(r)}(t^r;x)+(r+2)(-x)A_{n,a,\beta_n}^{(r)}(t^{r+1};x)+A_{n,a,\beta_n}^{(r)}(t^{r+2};x)\right]$$
$$=\frac{f^{(r)}(x)}{r!}\frac{(n+\beta_n)^r(n+a-r-2)!\,r!}{(n+a-2)!}+O(n^{-1})+\frac{f^{(r+1)}(x)}{(r+1)!}\left[\frac{(r+1)(-x)(n+\beta_n)^r(n+a-r-2)!\,r!}{(n+a-2)!}\right.$$
$$\left.+\frac{(n+\beta_n)^{r+1}(n+a-r-3)!\,(r+1)!}{(n+a-2)!}x+(r+1)^2\frac{(n+\beta_n)^{r+1}(n+a-r-3)!\,r!}{(n+a-2)!}x+O(n^{-1})\right]$$
$$+\frac{f^{(r+2)}(x)}{(r+2)!}\left\{\frac{(r+1)(r+2)}{2}x^2\frac{(n+\beta_n)^r(n+a-r-2)!\,r!}{(n+a-2)!}+(r+2)(-x)\left[\frac{(n+\beta_n)^{r+1}(n+a-r-3)!\,(r+1)!}{(n+a-2)!}x\right.\right.$$
$$\left.+(r+1)^2\frac{(n+\beta_n)^r(n+a-r-3)!\,r!}{(n+a-2)!}\right]+\frac{(n+\beta_n)^{r+2}(n+a-r-4)!\,(r+2)!}{(n+a-2)!}x$$

$$+\frac{(r+1)^2(n+\beta_n)^{r+1}(n+a-r-4)!(r+1)!}{(n+a-2)!}x+(r+1)(r+2)\frac{(n+\beta_n)^r(n+a-r-4)!r!}{(n+a-2)!}+O(n^{-1})\Big\}.$$

and using (2.10), (2.11), (2.12) and (2.13) we obtain

$$I_1=\frac{f^{(r)}(x)}{r!}\left[\lambda_r(n)r!+O\left(\frac{1}{n}\right)\right]+\frac{f^{(r+1)}(x)}{(r+1)!}\left[(r+1)(-x)\lambda_r(n)r!+\lambda_{r+1}(n)(r+1)!\,x+\frac{r+1}{n+\beta_n}\lambda_{r+1}(n)+O\left(\frac{1}{n}\right)\right]$$
$$+\frac{f^{(r+2)}(x)}{(r+2)!}\left[\frac{(r+2)!}{2}x^2\lambda_r(n)-(r+2)!\,x^2\lambda_{r+1}(n)+\frac{(r+2)!}{2}x^2\lambda_{r+2}(n)+\left(\frac{r+2}{n+\beta_n}\lambda_{r+1}(n)-\frac{r+1}{n+\beta_n}\lambda_{r+2}(n)\right)x\right.$$
$$\left.+\frac{1}{(n+\beta_n)^2}\lambda_{r+2}(n)+O\left(\frac{1}{n}\right)\right]$$
$$=f^{(r)}(x)\lambda_r(n)+f^{(r+1)}(x)\left[\left(\lambda_{r+1}(n)-\lambda_r(n)\right)x-\frac{r+1}{n+\beta_n}\lambda_{r+1}(n)\right]+f^{(r+2)}(x)\left\{[\lambda_r(n)-2\lambda_{r+1}(n)+\lambda_{r+2}(n)]\frac{x^2}{2}\right.$$
$$\left.+\left(\frac{r+2}{n+\beta_n}\lambda_{r+1}(n)-\frac{r+1}{n+\beta_n}\lambda_{r+2}(n)\right)x+\frac{1}{(n+\beta_n)^2}\lambda_{r+2}(n)+O\left(\frac{1}{n}\right)\right]\Big\}$$
$$=f^{(r)}(x)\lambda_r(n)+f^{(r+1)}(x)\left[\lambda_r(n)\frac{(\beta_n-a+r+2)x+(r+1)}{n+a-r-2}\right]$$
$$+f^{(r+2)}(x)\left\{\lambda_r(n)\left[\frac{(r-a)^2+(r+\beta_n)^2-r^2+5(r-a)+6(1+\beta_n)-2a\beta_n+n}{(n+a-r-2)(n+a-r-3)}\right]\frac{x^2}{2}+\lambda_r(n)\left[\frac{r(a-\beta_n)-r^2-5r+2a-6+\beta_n+n}{(n+a-r-2)(n+a-r-3)}\right]x\right.$$
$$\left.+\frac{\lambda_r(n)}{(n+a-r-2)(n+a-r-3)}\right\}+O\left(\frac{1}{n}\right)$$

Taking limit for $n\to\infty$, we obtain

$$\lim_{n\to\infty}n\left[A^{(r)}_{n,a,\beta_n}(f;x)-f^{(r)}(x)\right]=[(2+r-a)x+r+1]\,f^{(r+1)}(x)+\left(x+\frac{x^2}{2}\right)f^{(r+2)}(x).$$

Theorem 3. (Eror estimation) Let $f\in C_\gamma[0,\infty)$ for some $\gamma>0$ and $0<a<a_1<b_1<b<\infty$. Then for sufficiently large n, we have

$$\left\|A^{(r)}_{n,a,\beta_n}(f;.)-f^{(r)}\right\|_{C[a_1,b_1]}\le C_1\omega_2\left(f^{(r)},n^{-\frac{1}{2}}[a,b]\right)+C_2n^{-1}\|f\|_\gamma,$$

where $C_1=C_1(r)$ and $C_2=C_2(r,f)$.

Proof. The equation

$$f(t)=f^{(r)}(x)+f(t)-f_{\eta,2}(t)+f_{\eta,2}(t)-f^{(r)}_{\eta,2}(x)+f^{(r)}_{\eta,2}(x)-f^{(r)}(x)$$
$$f(t)-f^{(r)}(x)=f(t)-f_{\eta,2}(t)+f_{\eta,2}(t)-f^{(r)}_{\eta,2}(x)+f^{(r)}_{\eta,2}(x)-f^{(r)}(x)$$

can be written by considering (2.16). If we apply the operator $A^{(r)}_{n,a,\beta_n}$ to each side of this equation and using the linearity of the operator, we can write

$$A^{(r)}_{n,a,\beta_n}(f;.)-f^{(r)}(x)A^{(r)}_{n,a,\beta_n}(1;.)$$
$$=A^{(r)}_{n,a,\beta_n}(f-f_{\eta,2};.)+A^{(r)}_{n,a,\beta_n}(f_{\eta,2};.)-f^{(r)}_{\eta,2}(x)A^{(r)}_{n,a,\beta_n}(1;.)+f^{(r)}_{\eta,2}(x)-f^{(r)}(x)A^{(r)}_{n,a,\beta_n}(1;.)$$

If we first take the absolute value of the equation and then add both sides of the equation on $C[a_1,b_1]$, we obtain

$$\left\|A^{(r)}_{n,a,\beta_n}(f;.)-f^{(r)}\right\|_{C[a_1,b_1]}\le\left\|A^{(r)}_{n,a,\beta_n}(f-f_{\eta,2};.)\right\|_{C[a_1,b_1]}+\left\|A^{(r)}_{n,a,\beta_n}(f_{\eta,2};.)-f^{(r)}_{\eta,2}\right\|_{C[a_1,b_1]}+\left\|f^{(r)}-f^{(r)}_{\eta,2}\right\|_{C[a_1,b_1]}$$
$$:=H_1+H_2+H_3$$

By property (3) of Steklov mean, we get

$H_3 \leq K\omega_2(f,[a,b])$.

Next, using Theorem 2, we obtain

$$H_2 \leq K_1 n^{-1}\sum_{j=r}^{r+2}\left\|f_{\eta,2}^{(j)}\right\|_{C[a,b]} \leq K_1 n^{-1}\left[\left\|f_{\eta,2}\right\|_{C[a,b]} + \left\|f_{\eta,2}^{(r+2)}\right\|_{C[a,b]}\right].$$

Now by properties (2) and (4) of Steklov mean, we have

$$H_2 \leq K_2 n^{-1}\left[\eta^{-2}\|f\|_{C[a,b]} + \eta^{-2}\omega_r\left(f^{(r)};\eta;[a,b]\right)\right] \leq K_3 n^{-1}\left[\|f\|_\gamma + \eta^{-2}\omega_r\left(f^{(r)};\eta;[a,b]\right)\right].$$

Finally, we estimate H_1 choosing a, b satisfiying the condition $0 < a < a^* < a_1 < b_1 < b^* < b < \infty$. Let $\chi(t)$ denotes the characteristic function in the interval $[a^*, b^*]$, then

$$H_1 \leq \left\|A_{n,a,\beta_n}^{(r)}\left[\chi(t)\left(f - f_{\eta,2};.\right)\right]\right\|_{C[a_1,b_1]} + \left\|A_{n,a,\beta_n}^{(r)}\left[\left(1-\chi(t)\right)\left(f - f_{\eta,2};.\right)\right]\right\|_{C[a_1,b_1]} := H_4 + H_5$$

From (2.8), we have

$$A_{n,a,\beta_n}^{(r)}\left[\chi(t)\left(f(t) - f_{\eta,2}(t);x\right)\right] = \frac{(n+\beta_n)^r(n+a-r-1)!}{(n+a-2)!}\sum_{k=0}^{\infty} p_{n,\beta_n,k}(x)\int_0^\infty q_{n-r,a,k+r}(t)\left(\chi(t)\left(f^{(r)}(t) - f_{\eta,2}^{(r)}(t);x\right)\right)dt.$$

Hence

$$H_4 \leq K_4\left\|f^{(r)} - f_{\eta,2}^{(r)}\right\|_{C[a^*,b^*]}.$$

Now for $x \in [a_1, b_1]$ and $t \in [0,\infty)\backslash[a^*, b^*]$, we choose a $\delta_1 > 0$ satisfying $|t-x| > \delta_1$. By Lemma 5 and Schwarz inequality, we have

$$\left|\frac{d^r}{dx^r}A_{n,a,\beta_n}\left[\left(1-\chi(t)\right)\left(f(t) - f_{\eta,2}(t);x\right)\right]\right|$$
$$\leq (n+a-1)\sum_{\substack{2i+j\leq r\\ i,j\geq 0}}\frac{(n+\beta_n)^i}{x^r}\phi_{i,j,r}(x)\sum_{k=0}^{\infty}[k-(n+\beta_n)x]^j e^{-(n+\beta_n)x}[(n+\beta_n)x]^k$$
$$\times\int_0^\infty q_{n,a,k}(t)\left(\left(1-\chi(t)\right)\left|f(t) - f_{\eta,2}(t)\right|\right)dt.$$
$$\leq K_5\|f\|_\gamma(n+a-1)\sum_{\substack{2i+j\leq r\\ i,j\geq 0}}(n+\beta_n)^i\sum_{k=0}^{\infty}[k-(n+\beta_n)x]^j e^{-(n+\beta_n)x}[(n+\beta_n)x]^k\int_{|t-x|\geq\delta_1} q_{n,a,k}(t)dt$$
$$\leq \frac{K_5}{\delta^{2l}}\|f\|_\gamma(n+a-1)\sum_{\substack{2i+j\leq r\\ i,j\geq 0}}(n+\beta_n)^i\sum_{k=0}^{\infty}[k-(n+\beta_n)x]^j e^{-(n+\beta_n)x}[(n+\beta_n)x]^k$$
$$\times\left[\int_0^\infty q_{n,a,k}(t)dt\right]^{\frac{1}{2}}\left[\int_0^\infty q_{n,a,k}(t)(t-x)^{4l}dt\right]^{\frac{1}{2}}$$
$$\leq \frac{K_5}{\delta^{2l}}\|f\|_\gamma(n+a-1)\sum_{\substack{2i+j\leq r\\ i,j\geq 0}}(n+\beta_n)^i\left[\sum_{k=0}^{\infty}[k-(n+\beta_n)x]^j e^{-(n+\beta_n)x}[(n+\beta_n)x]^k\right]^{\frac{1}{2}}$$
$$\times\left[(n+a-1)\sum_{k=0}^{\infty}e^{-(n+\beta_n)x}[(n+\beta_n)x]^k\int_0^\infty q_{n,a,k}(t)(t-x)^{4l}dt\right]^{\frac{1}{2}}.$$

Hence by Lemma 1 and Lemma 2 , we get

$$H_5 \leq K_6 \|f\|_\gamma \sum_{\substack{2i+j\leq r \\ i,j\geq 0}} (n+\beta_n)^{i+\frac{j}{2}-1} \leq K_6 \|f\|_\gamma (n+\beta_n)^{-v} \leq K_6 \|f\|_\gamma (n+\beta_n)^{-1},$$

where $v = \left(l - \frac{r}{2}\right) > 1, \eta > 0.$

Therefore, by property (3) of Steklov mean, we obtain

$$H_1 \leq K_7 \left\| f^{(r)} - f_{\eta,2}^{(r)} \right\|_{C[a^*,b^*]} + K_6 \|f\|_\gamma (n+\beta_n)^{-1} \leq K_8 \omega_2(f;\eta;[a,b]) K_6 \|f\|_\gamma (n+\beta_n)^{-1}.$$

The proof is done by choosing $\eta = (n+\beta_n)^{-\frac{1}{2}}$.

As a result, when the results obtained for the operators defined by (1.4) are compared with the results given for the (1.1) operator, it is seen that the convergence properties are preserved and the error rate decreases by changing compared to β_n.

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