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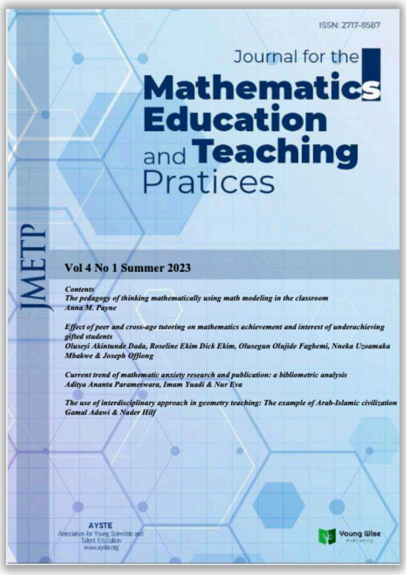


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**Review Article**

# The pedagogy of thinking mathematically using math modeling in the classroom

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**Abstract**

Mathematical modeling is the process of addressing a real-life task using mathematical concepts. Used in applied mathematics, engineering, the natural and social sciences, educators are recognizing that the use of mathematical modeling as a teaching tool gives students the opportunity to work conceptually on topics that are inherently motivating and engaging and are connected to real-life situations. This can support learning in the mathematics classroom in multiple ways, by using learning for transfer, problem-solving, and conceptual understanding. Students are able to incorporate cross-curricular skills and use real-life examples, which can make mathematics significantly more engaging. Modeling also provides students of varying ability levels to access content knowledge and construct new learning with mathematics that will extend beyond the classroom which is important especially in heterogenous classrooms and increasingly an increasingly diverse student population. Students who understand modeling have the potential to become stronger mathematicians and also use mathematics effectively later in life-whether working in a mathematics-related field or not. Providing professional development for teachers is crucial to making sure that modeling will be enacted with fidelity in the classroom. For pre-service teachers to effectively implement modeling in the classroom, they should both experience modeling as students themselves, learn how to create modeling scenarios in methods coursework, and then have multiple opportunities to enact modeling lessons with students. In-service teachers also need to have professional learning experiences with modeling, including creation of modeling activities, ways to implement modeling scenarios effectively, options for grouping students during modeling (including ways to work with language learners and students with special education needs), ways to grade modeling work, and how to connect modeling to curricular demands. Mathematical modeling is a highly-effective, engaging method to connect students to real-life mathematics, but teachers need professional learning supports if modeling is to be implemented effectively.

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## Introduction

In mathematics curriculum, teachers often strive to give students experience with problems that are more than just routine math exercises. There are word problems, standard application problems, and true modeling problems (Niss et al., 2007). Traditional word problems are those often found in textbooks, allowing the student to simply extract the numbers from text and solve without reference to the context of the problem. When looking at the modeling cycle in the context of a word problem, a student has little to solve, and the answer requires little, if any, interpretation. Standard application problems, unlike word problems, do require some level of translation from the real world to the level of

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mathematizing, but the amount of translation required is not very complex. True modeling problems will make use of the full modeling cycle. These start with a question or situation, followed by student development of a model of the scenario, and then the student solves and interprets the results in both the mathematical and contextual situation (Tran & Dougherty, 2014). This paper contains a discussion about the potential of mathematical modeling to support learning for transfer, problem-solving, and conceptual understanding.

**What is mathematical modeling?**

To solve modeling problems, individuals are required to consider a real-world situation and formulate a mathematical model. In short, problem solvers consider what appears to be non-mathematical information and make it mathematical. This process is referred to as mathematizing (Bonotto, 2010). Then, the problem solvers apply their model to the specific scenario (e.g., data) by testing it and revising it, perhaps multiple times, as needed. This revision process is called creating multiple iterations (Lesh et al., 2000). Doerr and Lesh (2003) define modeling as "conceptual systems…that are expressed…and are used to construct, describe, or explain behaviors of other system(s) – perhaps so that the other system can be predicted intelligently" (p. 10).

To effectively model, authentic tasks are required. These are tasks grounded in real-world scenarios, so that modelers are able to work with them mathematically. Tran and Dougherty (2014) list six criteria for tasks to be considered authentic: event, question, purpose, information/data, language use, and tool demands. The simulation that the task represents needs to be an *event* that either has or could take place, and the *question* students are asked to answer is one that could legitimately be posed in such an event. The *purpose* of the task needs to be as clear as it would be in that authentic situation. It should be either explicitly provided or be implicitly assumed from the context of the problem, though some degree of interpretation may be required. The *information/data* provided should describe and include the specific subjects/objects or data as in the simulated situation. While problem statements should be explicit, students may be expected to realize the need and specify the model accordingly in some cases. *Language use* in the situation should not be unreasonable provided that any unfamiliar terms are not part of the simulated situation, so some new vocabulary specific to the situation may be necessary. However, enabling problem solvers the opportunity to access and interpret problem demands is incumbent upon problem writers, as the context and structure of the problem should be familiar to problem solvers. Lastly, Tran and Dougherty discussed the *tool demands* of a task. These are the tools used for task, which should mimic those used in the realistic situation, such as, "What are the actual dimensions and aluminum requirements to create the packaging for a cola can?" (Tran & Dougherty, 2014, p. 767)).

There is a distinction between in-school and out-of-school mathematics (Bonotto, 2010). Realistic, or out-of-school mathematics, is not the mathematics often discussed in schools. Instead, out-of-school mathematics gives people an integrated, graphic, and dynamic way to think, talk about, and work with mathematics. The mathematics understanding and ability needed for modeling is quite different from what has been traditionally taught because the cognitive challenges are much greater in mathematical modeling than they are in entry-level word problems. Engaging learners in mathematical modeling scenarios, therefore, is likely requisite to prepare young students to succeed in today's age of information. Skills that may be associated with success in mathematical modeling may include constructing, describing, communicating, interpreting, mathematizing, sense-making, and explaining their reasoning and understanding, rather than just computing. Mathematical modeling is as much "about quantities as it is about naked numbers" (p. 16) and about making sense of patterns and reasoning about regularity in a system rather than just data.

> *The cycle from data to deduction to application recurs everywhere math is used, from everyday household tasks such as planning a long automobile trip to major management problems such as scheduling airline traffic or managing investment portfolios. The process of "doing" math is far more than just calculation or deduction; it involves observation of patterns, testing of conjectures, and estimation of results (National Research Council, 1989, p. 31).*

With respect to problem solving, traditional approaches (as math has been taught) have ostensibly specific heuristics that one can use or follow in order to solve a given problem (Polya, 1957). This process removes the inherent quality of novelty (Chamberlin & Chamberlin, 2010) that is requisite in legitimate problem solving. While such information may be helpful, there are multiple stages to the modeling process, and the rigidity of heuristics can stifle creativity in modeling. In fact, it is strongly suggested that modeling problems be utilized when needed mathematical knowledge exists, but problem solvers are asked to assemble it to create a mathematical model. Moreover, the modeling process is not merely a *watch my solution and regurgitate it* process. Modeling connects to the real world in an authentic manner, and leaves students working with messy, ill-defined problems that have no one unique correct answer. This process requires students to investigate and define the situation to predict or explain what is happening. Modeling *starts* in the real world, moves to the classroom or laboratory, and then figuratively moves back to the real world. In mathematical modeling, the problem that needs to be solved encourages the student to move from the real world to the mathematics (Hansen & Hana, 2015), and use the mathematics creatively to explain the phenomenon.

When modeling, a key piece of the cycle is to analyze the results and make sense of the solution. While analyzing the mathematics, choices need to be made. For instance, solvers need to consider which variables are needed, what information is most important, and what assumptions can be made, among other considerations. Students make decisions and move back to the real world and interpret that solution. Does the solution make sense in the context of the initial problem? If it does not, the modeling cycle repeats (Cirillo et al., 2016). There is no *one right answer* and depending on how the variables are interpreted can alter the outcome that a student/group may decide on.

**Why should we use mathematical models?**

Mathematical modeling supports learning in the mathematics classroom in multiple ways. Students are able to connect with the mathematics through realistic, authentic problems, which can be motivating and engaging (Blum & Borromeo Ferri, 2009). Modeling helps students move through developmental stages (Doerr & Lesh, 2003), provides students with multiple entry points into the problem (Bostic, 2015), and allows students to use tacit knowledge (Doerr & Lesh, 2003) to solve real-world problems. These authentic modeling tasks also allow students to work with multiple content and practice standards of the Common Core State Standards-Mathematics (CCSS-M) at one time, allowing students to gain footing with others in the developed world (Schmidt & Houang, 2012).

**Goals of learning with modeling**

While previous work in mathematics problem solving has drawn largely on Polya's 1957 work on heuristics (Lesh & Zawojewski, 2007), the 21st Century goal is to produce mathematical learning through the modeling process (Zawojewski, 2010). Students are able to learn through modeling by seeing the significance of the mathematics while solving realistic problems (Bostic, 2015), which can be motivating in ways traditional text-book driven mathematics may not be. With specific modeling activities known as Model-Eliciting Activities (MEAs) (Lesh et al., 2010), the process of mathematizing is required (Doerr & Lesh, 2003; Lesh & Yoon, 2007). This leads students to construct, explain, justify, conjecture, and represent with mathematics. Modeling requires students to quantify, coordinate, and organize data (English et al., 2005). They insert, extend, refine, and revise constructs that are more powerful than mathematical lessons been taught directly (Doerr & Lesh, 2003) especially when applied to varied authentic situations. Students actually construct mathematical knowledge through the process of mathematical modeling (Zawojewski, 2010), which typically engenders sense-making in mathematics (Hiebert et al., 1997; Skemp, 1976). Modeling can help with significant forms of concept development and is achievable by all students (Lesh & Yoon, 2007). Mathematical modeling problems provide an engaging context which can foster student perseverance. Students develop greater motivation to determine the best value for the model while solving the realistic problem (Blum & Borromeo Ferri, 2009; Bostic, 2015). Authentic uses of modeling can create productive dispositions towards mathematics in general, as well as support the development of mathematical literacy (Carlson et al., 2016; Chamberlin & Parks, 2020). Mathematical modeling can support the learning of mathematics in terms of motivation, comprehension, and increased retention. Students see how math is used in real-world contexts and are able to demonstrate their understanding of mathematical content and practices (Blum & Borromeo Ferri, 2009; Carlson et al., 2016).

Seeing the value of mathematics in these contexts is crucial; without the opportunity to create models in mathematics, students may think that the field is just a series of formulae and equations to memorize and solve for, rather than an opportunity to make sense of mathematics through engaging in modeling situations. Textbooks often present problem solving and real-life situations appear to be simplistic, with specific solutions leading to specific answers (Meyer, 2015), and teachers and schools may propagate this perception by over-relying on textbooks *as* a mathematics curriculum (Remillard, 2005). Textbook treatments of problem-solving and mathematical modeling have critical limitations, explicitly relying on the school use of mathematics rather than the professional use of mathematics (Lesh & Zawojewski, 2007). Pragmatically speaking, this is problematic because companies are looking to hire people who can generate and work with mathematical models (Meyer, 2005). Computers can perform the computational portion of the modeling cycle in a far more efficient manner than humans can, but ironically, computation is the focus of most textbooks' approach to modeling. For instance, computing a derivative in calculus could be done without error and far faster with software or an advanced calculator than it could by hand. However, the human mind may be far better at specifying a preferred mathematical model than a computer might. Mathematics teachers, therefore, need to be teaching identifying, formulating, and validating with models (Meyer, 2005), so that industry needs can be met. Individuals with modeling capabilities are knowledge about problem solving and have the ability to create and modify mathematical models themselves (Lesh & Zawojewski, 2007).

**Multiple options**

In contrast to traditional word problems, modeling allows students to work with authentic problems. Problem solvers need to interpret and describe the problematic situation in mathematical ways (English & Watters, 2005). Modeling often ill-defined problems which can engage student interest (Chamberlin & Parks, 2020) leading to discussion and justification for their reasoning (Cirillo et al., 2016). Students become engaged as they are able to work with multiple representations; specifying the model they have selected can allow for multiple solutions and products depending on the approach adopted. What truly matters is how the solution is justified (Bostic, 2015). Students are able to use multiple approaches, interpretations, and even apply various models to one situation to see which approach fits best. It can be motivating to work in a multidisciplinary situation, with group work and collaboration being essential to a comprehensive modeling approach. When students work in teams on modeling situations, they are able to leverage their real-world knowledge in such problem solving situations (Cirillo et al., 2016), rather than just using book smarts.

**Non-traditional knowledge**

Students who are able to memorize formulae, quickly compute answers, and follow algorithms are those who are generally considered to be "good" at mathematics. However, with modeling, those are not the only skills that are important. Students who may not succeed in traditional math classrooms are able to use different abilities (Doerr & Lesh, 2003) effectively in classroom modeling. Students who have been considered underachievers or with less math ability have been shown in numerous studies to be capable of inventing or adapting and modifying constructs to be able to model effectively. This performance has been demonstrated in numerous studies (Doerr & Lesh, 2003). This is the case because when engaging in mathematical modeling activities, such as MEAs, the solution is not revealed or discussed prior to the modeling process.

**Zone of proximal development**

Mathematical modeling is often done in a group setting in schools. This interaction can be beneficial; the group setting can be a social experience that can leads to higher psychological processes. It allows for thinking as well as learning to be internalized and externalized. This interplay between an individual's use of informal, personal knowledge and key knowledge of the problem can strengthen understandings of mathematics (English & Watters, 2005; Zawojewski, 2010). Doerr and Lesh (2003) explain how the natural development of relevant constructs in modeling follows similar stages of development observed by psychologists and educators. When teachers are aware of these stages of development, they can engage their students to relate and extend their students ideas. By helping their learners construct and reconstruct knowledge and ideas in a social setting, students will feel that learning, making mistakes, relearning, and exploring knowledge and understandings are all safe (Hattie, 2012).

**Connected to standards**

The CCSS-M promote mathematical concepts that place the United States in a comparable position to most other developed countries (Schmidt & Houang, 2012). Teaching traditional mathematics may allow teachers to cover the CCSS-M content standards in a more efficient manner than they can through the medium of mathematical modeling, but some of the practice standards can feel artificial or forced. The same occurs with traditional word problems; students may not fully address mathematical knowledge as they follow delineated steps and plugging the numbers into an algorithm. When engaging in authentic modeling situations, however, students use mathematical knowledge, processes, representational fluency, and social skills that are more suited for actual 21st century learning as well as requisite for the content standards. Students working with a mathematical model not only are automatically reaching practice standard 4 (model with mathematics), but also likely meet other practice standards, such as standard 5 (use appropriate tools strategically) and standard 6 (attend to precision) when having to make decisions such as whether to use a calculator or computer program or whether to round an answer to a certain decimal point or to use a fraction or exact answer.

Mathematical modeling is a practice standard found in the Common Core State Standards-Mathematics but is not solely confined to mathematics alone. Focus on the CCSS-M practice standards also allows students to connect with the Next Generation Science Standards (NGSS) Science and Engineering Practices, specifically numbers two (develop and use models) and five (use mathematics and computational thinking). Helping students see the connections between mathematics and science (or mathematics and other subjects) will increase student facility with STEM fields. When presenting the modeling solution (whether to a teacher or some other professional audience), students make use of a variety of English/Language arts standards, including integration of knowledge and ideas, researching to build and present knowledge, and presenting of knowledge and ideas.

**How do we teach modeling?**

It is not sufficient to say that mathematical modeling must be utilized in schools. We need to support our teachers so they are well prepared to utilize modeling through instruction in which models are used, creating their own scenarios, and effectively assessing modeling. If these processes are not engaged, the idea of implementing mathematical modeling in the classroom may be poorly delivered, as has happened with other classroom reforms (Cobb & Jackson, 2011). A majority of in-service teachers may be ill-prepared to infuse mathematical modeling problems and content into the classroom because these have not been a regular staple in teacher preparation programs at either the elementary or secondary level (personal communication with S. Chamberlin, 3 October 2020.)

**Pre-service education**

Many researchers have pointed out the need for pre-service (and in-service) teacher training in mathematical modeling competencies (Blum, 2015; Cetinkaya et al., 2016; Niss et al., 2007), ideally including as much as a course focusing on modeling and teaching competencies in teacher preparation programs. Through specific coursework, pre-service teachers are able to develop an understanding of how to bridge out-of-school mathematics with in-school teaching, and deepen their mathematical conceptual knowledge and their pedagogical content knowledge. (Sevis, 2016).

The Association of Mathematics Teacher Educators (AMTE) (2017) states that "well prepared beginning teachers have solid and flexible knowledge of mathematical processes and practices" (p. 9). A course on modeling would address this standard, as an effective teacher preparation program should provide candidates with "opportunities to learn and work with mathematical process and processes that are appropriate to the content being studied" (p. 31). Borromeo Ferri and Blum (2009) suggest four key themes for a modeling course: 1) theoretical knowledge of modeling; 2) the ability to solve/create modeling tasks; 3) the ability to plan modeling lessons; and 4) the ability to diagnose student challenges and thought processes as they work through modeling tasks. Additional domains of knowledge can be developed during a modeling course as well, including: 1) knowledge of modeling tasks; 2) recognition of students' mathematical thinking; 3) use of information and communication technologies; and 4) classroom management during the modeling process (Cetinkaya et al., 2016). Providing pre-service educators the time and ability to develop this knowledge and put modeling skills into practice before they enter the classroom will improve teacher and student modeling self-efficacy (Jacobs & Durandt, 2016) and likely increase the odds that modeling will be effectively implemented in the classroom. A course on mathematical modeling during undergraduate studies would provide pre-service teachers the opportunity to develop their pedagogical content knowledge in mathematics, specifically related to modeling (Jacobs & Durandt, 2016). This approach would prompt pre-service teachers to spend time thinking about student competencies such as critical thinking and interpreting results (Karali & Durmus, 2015), as well as about how modeling can impact student affective states, which can help students develop a "modeling persona" (Chamberlin et al., 2020).

**Teacher professional development**

Most teachers have not received preservice training in modeling, so professional development in modeling is crucial as mathematical modeling requires distinct habits of mind from traditional mathematics. Because of the lack of prior practice, teachers are left with few adequate tools when implementing modeling problems (Blomhøj & Kjeldsen, 2006). Resulting misconceptions and blunders in modeling can result in student and teacher frustration, which can lead to the end of modeling being used in the classroom altogether.

Instead of leaving teachers with minimal or no strategies to support themselves and their students, on-the-job professional development is a necessity. Professional development needs to focus on the development of teachers' practice (Blomhøj & Kjeldsen, 2006). Teachers, in-service and pre-service, need to be able to reflect on and challenge their beliefs about what constitutes true mathematical learning. They need to practice anticipating misconceptions that may occur during the modeling process and evaluating the ways students think. It is important that teachers discuss pedagogy and have time to reflect on the approaches and meanings of modeling as mathematicians might (Clark & Lesh, 2003). Teachers need to have time to create modeling scenarios for use in their classroom.

Doerr (2016) suggests designing MEAs, where the activity is intended to help students create a model. She gives the following principles for designing a MEA: begin with a real-life context; construct a model; document the model; self-evaluate; generalize from the model; and create a simple prototype. This leads to significant forms of concept development and can be achievable by all students. A goal of MEAs is that they should be applicable (Lesh & Yoon, 2007). When developing MEAs, teachers should look to the sub-processes of modeling, as it is not necessary that students go through the entire modeling process each time. If teachers choose to pull modeling scenarios from texts, they can use these sub-processes to make the scenario stronger (Gould, 2016; Meyer, 2015), although teachers may find that modeling problems are not readily available in texts.

**Teaching students using modeling**

Using modeling activities to facilitate learning has become important, given an ever-increasing technological-based society. Thirty-five years ago, for example, advertising may not have been as sophisticated as it is now. Advertisements for credit cards or car sales now show increasingly complex data related to loans, leases, and rates. Students need to learn practical uses of mathematics to be knowledgeable consumers in a modern economy (Doerr & Lesh, 2003).

The preparatory work teachers do, for instance, setting the scene and deciding how much information about the modeling scenario to share, is important (Blomhøj & Kjeldsen, 2006). This is also a chance to differentiate the modeling process for different groups of students. Considerable scaffolding and information can be provided to some students, while others may be capable of realizing the full challenge of the problem. It is important to keep in mind that the most successful modeling projects are those where teachers have given clear and explicit intentions for student learning, without detailing aspects of a solution. This does not necessarily mean that they have outlined what students are to do or where they are to end up, but rather what the intended outcomes of the model are and what types of products might be expected from the process.

It is important that teachers use authentic modeling scenarios and do not rely too heavily on textbook problem-solving exercises. Textbook curricula prompt students to simply look back to the previous section for an example of the solution. Then students simply copy the problem, step-by-step, change the numbers, and perhaps make a few alterations. Rather than thinking creatively and using a design process, students follow problem-solving heuristics that may have been carefully explicated as examples. This is not what real-life modeling provides. When faced with a modeling scenario, it is ideal for students to ask questions that help guide their thinking. For instance, questions such as, Where do I begin? What do I need? How can I apply this? What assumptions have I made? Is my conclusion valid? (Pollak, 2003) are germane to success in mathematical modeling. Such questions are generally not common with textbook problem-solving tasks. One positive resource that is in the process of making their rather expansive database available to all educators is the CPALMS website (CPALMS.org). This website contains hundreds of modeling activities, specifically designed for K-12 educators and is believed to be the largest and most comprehensive of its kind in the world.

Teachers can look to the sub-processes of modeling to support students. These processes can include, but are not limited to identifying variables; mathematizing the scenario; analyzing the situation and using mathematics to draw conclusions; interpreting the results; validating the results (and determining if the process needs to be repeated for a better fit model); and reporting the results (Gould, 2016). Not all of these are necessary for every modeling scenario; variables could be provided in a situation and decrease the time spent modeling.

**Modeling processes**

The extent to which modeling is used with students is a decision that individual teachers should make; however, it is crucial to provide students with ample modeling opportunities so that they are able to become adept at the process and prepared for college or the workforce, where they will encounter real-life examples of modeling. Meyer (2015) studied two textbooks that claimed to include modeling and found that while there were many word problems in the texts, most of these examples were asking students to "perform an operation" or "interpret results". These are tasks that can be done by computers because they involve low-level computations, not high-level processing. Tasks in which problems solvers should engage are the tasks most critical to the modeling process, such as identifying essential variables, formulating a model, and validating conclusions. While there are times when problem solvers may require some assistance, they should learn to work independently to develop true modeling competencies on their own. They need to be able to ask questions during the modeling process, questions such as: "What information is needed here? What do I do to answer my question? Is my prediction correct?" (Meyer, 2015, p. 582) As Einstein repeatedly stated, "The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill" (Einstein & Infeld, 1938, p. 95).

Blomhøj and Kjeldsen (2006) agree that students do need to experience a balance between a holistic process, doing the complete modeling cycle every time, or a reductionist/atomistic approach (Geiger et al., 2016), where students will focus on certain parts of the modeling cycle. To develop to full competency in modeling, students need to work with the full cycle often enough that they are able to develop a metacognitive approach (Blomhøj & Kjeldsen, 2006), and for some students this will require more access to the full modeling cycle than required by other students, more advanced in mathematical skills. Providing explicit instruction that allows students to focus on the metacognitive aspects of mathematics and coaching with active interventions will help students develop this awareness (Schoenfeld, 1992). Learning to successfully use metacognition does not happen automatically for most students, so the more access and experience students are able to have, the more they will be able to consciously regulate and willingly work hard to apply the metacognitive ideas (Schukajlow et al., 2015). Being purposeful in training students to successfully implement metacognitive approaches may be fruitful for enhancing the likelihood of success with it.

Blum and Borromeo Ferri (2016) offer some teaching/learning principles for working with modeling units. Firstly, they suggest a change among working units, with variations between groups, partners, individual, and whole-class work, requiring classroom management during modeling activities. Next, they suggest that it is important that the modeling activities are authentic and rich in context, so that students can continue practicing their modeling skills, gaining the required mathematical competencies. It is important that all students are actively engaging with the modeling task; this may mean providing scaffolding for some students, while encouraging others to work independently. Additionally, it is important that students working in groups can take ownership of modeling by developing independent solutions. Having multiple solutions in a classroom "takes into account individual preferences or thinking styles…[and] enables a natural differentiation" (p.71). Finally, Blum and Borromeo Ferri (2016) write that encouraging metacognitive reflection and exploring into solution strategies is helpful for modeling and the general learning of mathematics.

### Assessment

It is important to assess students in a manner similar to how they have been taught. It would not make sense to give individual, standardized questions to students after they have been working on authentic modeling problems in a group setting. Eames et al. (2016) states that there is an obvious assessment imbedded in the act of modeling itself. By verifying that the model is viable and then justifying how and why the model works in a given situation, students have already provided one form of (self)assessment.

Self-reflection tools can also be used throughout the entire process of modeling, and can incorporate aspects of metacognitive thinking (Blomhøj & Kjeldsen, 2006; Eames et al., 2016). Students can write status updates on their modeling process as an exit slip on a daily/weekly basis and give justification or an interpretation of their current understanding of how they/their groups are approaching the modeling scenario. Eames et al. (2016) also suggests that students could create a "client letter," where students would write to the client associated with their model and describe/defend their ways of thinking about the solution to explain why they have employed the best or correct approach for the scenario.

## Conclusion

Mathematical modeling is the formulation of a mathematical problem based on a real-world scenario. The problem is then solved, and that mathematical solution is then returned to the real world, where it is tested for accuracy and usability. Teaching the use of modeling is an important 21st century strategy in contemporary classrooms because this is exemplifies how mathematics is experienced in real life. Additionally, modeling engages students more fully than traditional textbook approaches do, by incorporating cross-curricular skills, by using real life examples, by giving students at multiple skill levels the opportunity to engage with mathematics, and by providing a rich background for connecting with the content.

For teachers to be able to effectively utilize modeling in the classroom, quality professional learning needs to occur. Teachers need to learn how to engage with modeling themselves and how to teach using modeling. They need to either learn where to find quality modeling problems or how to create such problems themselves. Teachers also need to learn how to provide appropriate assessment for students when using modeling, so students can grow in their modeling skills, becoming stronger mathematicians and stronger modelers. Students who understand the utility of the mathematics in class will become stronger mathematicians and problem solvers.

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Research Article

# Effect of peer and cross-age tutoring on mathematics achievement and interest of underachieving gifted students

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**Abstract**

Underachievement in the mathematics of gifted students is a disturbing issue that calls for urgent intervention. This study, therefore, investigates the effects of peer and cross-age tutoring as intervention strategies on the mathematics achievement and interest of gifted students. The pretest-posttest control group experimental research design was adopted and three null hypotheses were tested. Fifty underachieving gifted students participated in the study. The study lasted for 12 weeks. Analysis of Covariance (ANCOVA) was used to analyze the data collected. Findings show that peer tutoring and cross-age tutoring are both effective in enhancing the achievement and interest in mathematics of underachieving gifted students. There is also a significant differential effect between peer and cross-age tutoring. Based on the findings, it was recommended that Mathematics teachers should adopt peer and cross-age tutoring to enhance interest and achievement in mathematics of gifted students.

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## Introduction

Gifted students can achieve the peak of their potential in Mathematics if well exposed to appropriately differentiated instruction. Their interest in Mathematics could be skyrocketed come adequately challenged, developed critical solution providers and inventors of improved technology if given the opportunity to develop through appropriate instructional experiences. Gifted and talented students have special abilities and aptitudes, which when harnessed and developed make them great instruments of positive change in society (Dada & Ani, 2019). But if the gifted and talented students are not well nurtured they become underachievers, less productive, and waste their potential. This perhaps is the reason for global concern in evolving ways of nurturing gifted and talented individuals in various subjects and disciplines (Dada & Fagbemi, 2014).

One key subject that cannot be overemphasized in the education of gifted and talented students is Mathematics because it is the basis for all disciplines in the fields of science, technology, engineering, commerce, and economics. This

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is mainly because studying mathematics helps students develop their capacity for problem-solving and analytical thinking while also fostering their logical, practical, and aesthetic thinking. Generally, people use mathematics every day, and this encourages the brain to express ideas, problems, and solutions that are necessary for the survival of the human race. (Awofala & Lawani, 2020; Dada & Akpan, 2019). Many countries in Africa remain underdeveloped because they do not nurture their gifted and talented individuals for strong mathematical and scientific literacy that is capable of turning around their natural endowment. Students' achievement in Mathematics is at low ebb in Nigeria despite the fact that Mathematics is a compulsory subject for all students at both primary and secondary levels of education (Obi & Dada, 2015; Dada & Meremikwu, 2021) and a compulsory requirement in technology, engineering, science and Mathematics careers at the university level.

Scientific and technological advancement is one of the major focuses of Nigeria's National goal and Policy on gifted education (Federal Republic of Nigeria [FRN] 2015) so efforts are channeled towards sustainable Mathematics in the gifted education program. Differential learning in Mathematics for underachieving gifted and talented students should be targeted at increasing interest, providing appropriate and adequate challenges, and developing critical thinking in Mathematics. For the best Mathematics achievement to be recorded for these special students, there a is need for alternative pedagogy and a diversified instructional approach. Unfortunately, many Mathematics teachers in Nigeria are not aware of learning strategies for the gifted and talented for effective teaching and learning of Mathematics but rely absolutely on the conventional methods (Dada & Dada, 2014). This tends to affect the interest and performance of students, especially the underachieving gifted, and talented ones in Mathematics. The inability of Mathematics teachers to differentiate learning for the gifted has resulted in poor learning, underachievement, and threat to interest in the Mathematics of gifted and talented students in (Dada & Ogundare, 2016). The classroom environment is a critical consideration in teaching and learning Mathematics, particularly for underachieving gifted, and talented who require specialized learning intervention (Orim, Dada & Igwe, 2017). Therefore, it is expedient for teachers to provide appropriate intervention by ensuring effective classroom administration and learning optimization.

Among the favourite learning interventions for the gifted and talented are peer tutoring and cross-age tutoring (Dada, 2010) to motivate their interest and increase their achievement. Peer tutoring, according to Topping (2005), is the active process of acquiring knowledge and skills through mutual support and assistance among peers who are equal in status and have similar ages and social groups. The concept originated from the recognition that students can learn better when they teach each other. Peer tutoring involves non-professional teachers, who are learners themselves, helping one another to learn. The peer tutors may be of the same or different ability levels but are of the same learning grade and similar age. Golding, Lisa, and Tennant (2006) further explained that peer tutoring is a technique in which students, with guidance from their teacher, teach one or more peers to acquire new skills or concepts. This approach emphasizes the use of peers to solve problems and can be effective in promoting creativity, experimentation, problem-solving skills, and deep learning of concepts.

One significant advantage of peer tutoring is that it can be easily implemented in an inclusive classroom of diverse abilities (Dada, 2016). Peer tutoring has been recognized by Nathern and Liz (2007) as a useful tool for teachers to address the diverse needs of learners and enhance academic achievement across various subjects and skill levels. Additionally, Miller and Miller (1995) suggest that peer tutoring is an effective and cost-efficient intervention that can benefit both struggling and high-achieving students by boosting both the tutor and the tutee's social and educational development and motivation to learn. When peer tutoring is skilfully supervised by a teacher, it can promote interaction among individuals and groups in the classroom, resulting in a more comprehensive understanding of scientific concepts among the students.

Cross-age tutoring involves pairing students of different grade levels, with the older student serving as a tutor for the younger student(s). This approach is similar to other peer learning programs, which are considered a form of differentiated instruction for both struggling and high-achieving students. In cross-age tutoring, an older student who has already mastered certain mathematics concepts facilitates learning through small group interactions. Specifically, cross-age tutoring employs an older student who has previously mastered specific mathematical concepts to facilitate and promote learning through small group interactions. While some studies have examined the academic outcomes of cross-age tutoring, others have focused on psychological effects such as changes in self-esteem, self-concept, and feelings of academic efficacy among learners. By employing a cooperative learning model, cross-age tutoring fosters communication, cooperation, independence, and responsibility, as both the tutor and tutee engage with the course content and employ appropriate and useful study strategies (Arendale, 2014). There is abundant evidence supporting the benefits of cross-age tutoring, including increased grades and pass rates and decreased withdrawal and failure rates (Dawson, Vander-Meer, Skalicky, & Cowley, 2014). However, its effects on mathematics achievement and interest among gifted and talented students are not yet fully understood.

Early research and meta-analysis on peer and cross-age tutoring programs found that they were effective in improving student outcomes (Britz, Dixon, and McLaughlin, 1989; Cohen, Kulik & Kulik, 1982). A more recent meta-analysis examining the impact of peer-assisted learning on elementary school students also found evidence supporting the efficacy of such learning strategies (Rohrbeck, 2003). Although the aforementioned meta-analysis highlighted significant academic gains, particularly for minority students (Rohrbeck, 2003), the effects of peer and cross-age tutoring programs involving gifted and talented students at the secondary school level in Nigeria have yet to be documented. The study is therefore motivated to investigate the differential effects of peer and cross-age tutoring on Mathematics achievement and interest of underachieving gifted and talented students in Nigeria.

### Hypotheses

- There is no significant effect of peer tutoring on Mathematics achievement and interest of underachieving gifted students.
- There is no significant effect of cross-age tutoring on Mathematics achievement and interest of underachieving gifted students.
- There is no significant differential effect of peer and cross-age tutoring on Mathematics achievement and interest of underachieving gifted students.

## Method

### Research Model

The **p**retest-posttest experimental design was adopted. Participants were from Junior Secondary School classes with the age range of 9-12 years who have been earlier identified by experts to be underachieving gifted but are not doing so well in Mathematics as expected. Their Mathematics underachievement was revealed by their school records.

### Sampling

There were 50 participants in the study. The participants were randomly assigned into two experimental groups according to their age range: those who were within the same age bracket (9-10) in the peer tutoring group and those who are older (above 10 years) in the cross-age tutoring. The participants were divided into three groups in the study: Group A for peer tutoring, Group B for cross-age tutoring, and Group C as the control group. Group A and B were offered the treatment based on peer and cross-age tutoring interventions respectively. The control group is not given any special intervention but a placebo of conventional teaching. The study took 12 weeks including the pre-test, treatment, and post-test.

### Data Collection

The data for the study was collected via two validated instruments namely, Mathematics Achievement Test (MAT: Reliability estimate = .92) and Mathematics Interest Scale (MIS: Reliability estimate = .89). The MAT was developed from the Mathematics curriculum for Junior Secondary School and based on the treatment package by the researchers to measure Mathematics achievement of the participants. The MAT has 25 multiple-choice objective questions and four essay questions with a total scoring point of 50 marks. The MIS is a scale of 10 items with four response options to indicate the Mathematics interest of the participants.

### Data Analysis

Data collected were analyzed using Analysis of Covariance (ANCOVA) and Independent Sample t-test.

## Results

The result of the descriptive data analysis is presented in table 1. Table 1 provides the post-test mean and standard deviation of each group. The findings from the descriptive results indicate.

**Table 1.** Mean and standard deviation of post-test scores in Mathematics achievement and interest

| Group | | Mean | Std. Deviation | N |
|---|---|---|---|---|
| Peer tutoring | Mathematics Achievement | 41.19 | 2.064 | 21 |
| Cross-age tutoring | Mathematics Achievement | 40.63 | 3.451 | 19 |
| Peer tutoring | Mathematics Interest | 35.10 | 3.208 | 21 |
| Cross-age tutoring | Mathematics Interest | 34.32 | 4.110 | 19 |
| Control | Mathematics Achievement | 36.70 | 1.567 | 10 |
| | Mathematics Interest | 30.00 | 2.211 | 10 |
| Total | Mathematics Achievement | 40.08 | 3.089 | 50 |
| | Mathematics Interest | 33.78 | 3.882 | 50 |

Table 2 revealed that there is a statistically significant effect of peer tutoring on Mathematics achievement and interests of underachieving gifted students (p = .00) when adjusted for pre-test and pre-assessment scores respectively. The calculated $F_{(1, 28)} = 46.709$ and 19.734 for Mathematics achievement and Mathematics interest respectively, at p= .00; < .05. The result of the analysis indicates that there was a significant mean difference between control and experimental groups in their post-test scores on both achievement and interest in Mathematics while adjusting for the pre-test. The partial Eta Squared ($\eta^2$) value if compared with Cohen's guidelines (0.2 – small effect, 0.5 – moderate effect, 0.8 – large effect) indicated that the calculated $\eta^2$ on the treatment groups was .625 and .413 on Mathematics achievement and interest respectively. This indicates a moderate and small effect but significant. The $\eta^2$ revealed that 62.5% and 41.3% of the variance in the post-test scores on Mathematics achievement and Mathematics interest are explained by peer tutoring.

**Hypothesis 1.** There is no significant effect of peer tutoring on Mathematics achievement and interests of underachieving gifted students.

**Table 2.** ANCOVA result of the effect of peer tutoring on Mathematics achievement and interest

| **Mathematics achievement** | | | | | | |
|---|---|---|---|---|---|---|
| Source | Type III Sum of Squares | Df | Mean Square | F-ratio | p-value | Partial Eta Squared |
| Corrected Model | 155.220[a] | 2 | 77.610 | 24.495 | .000 | .636 |
| Intercept | 117.071 | 1 | 117.071 | 36.950 | .000 | .569 |
| Pre-test | 18.623 | 1 | 18.623 | 5.878 | .022 | .173 |
| Peer Tutoring | 147.994 | 1 | 147.994 | 46.709 | .000 | .625 |
| Error | 88.715 | 28 | 3.168 | | | |
| Total | 49206.000 | 31 | | | | |
| Corrected Total | 243.935 | 30 | | | | |
| **Mathematics interest** | | | | | | |
| Source | Type III Sum of Squares | Df | Mean Square | F-ratio | P-value. | Partial Eta Squared |
| Corrected Model | 180.220[a] | 2 | 90.110 | 10.279 | .000 | .423 |
| Intercept | 1050.867 | 1 | 1050.867 | 119.875 | .000 | .811 |
| Pre-test | 4.352 | 1 | 4.352 | .496 | .487 | .017 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Interest | 172.996 | 1 | 172.996 | 19.734 | .000 | .413 |
| Error | 245.458 | 28 | 8.766 | | | |
| Total | 35115.000 | 31 | | | | |
| Corrected Total | 425.677 | 30 | | | | |

The result of the analysis was significant at .05 because the calculated p-value of .00 on Mathematics achievement and Mathematics interest was less than .05, hence the null hypothesis which states that there is no significant effect of peer tutoring on Mathematics achievement and interest of underachieving gifted students was rejected. This implies that there is a significant effect of peer tutoring on underachieving gifted students' interest and achievement in Mathematics.

Table 3 indicates that the calculated $F_{(1, 28)} = 11.158$ and 6.909 for Mathematics achievement and Mathematics interest respectively at p= .00; < .05. The result revealed that there is a statistically significant effect of cross-age tutoring on Mathematics interests and achievement of underachieving gifted students when adjusted for pre-test scores for Mathematics achievement and Mathematics interest respectively.

**Hypothesis 2.** There is no significant effect of cross-age tutoring on Mathematics achievement and interest of underachieving gifted students.

**Table 3.** ANCOVA result of the effect of cross-age tutoring on Mathematics achievement and interest

| **Mathematics achievement** | | | | | | |
|---|---|---|---|---|---|---|
| Source | Type III Sum of Squares | Df | Mean Square | F | Sig. | Partial Eta Squared |
| Corrected Model | 102.103[a] | 2 | 51.052 | 5.632 | .009 | .302 |
| Intercept | 219.916 | 1 | 219.916 | 24.260 | .000 | .483 |
| MAT pre-test | .831 | 1 | .831 | .092 | .764 | .004 |
| Treatments | 101.149 | 1 | 101.149 | 11.158 | .003 | .300 |
| Error | 235.690 | 26 | 9.065 | | | |
| Total | 45073.000 | 29 | | | | |
| Corrected Total | 337.793 | 28 | | | | |
| **Mathematics interest** | | | | | | |
| Source | Type III Sum of Squares | Df | Mean Square | F | Sig. | Partial Eta Squared |
| Corrected Model | 123.719[a] | 2 | 61.860 | 4.643 | .019 | .263 |
| Intercept | 728.544 | 1 | 728.544 | 54.680 | .000 | .678 |
| MIS Pre-assessment | 1.686 | 1 | 1.686 | .127 | .725 | .005 |
| Treatments | 92.051 | 1 | 92.051 | 6.909 | .014 | .210 |
| Error | 346.419 | 26 | 13.324 | | | |
| Total | 31722.000 | 29 | | | | |
| Corrected Total | 470.138 | 28 | | | | |

This reveals that there is a significant mean difference between the control and experimental groups in their post-test scores on both Mathematics achievement and Mathematics interest while adjusting for the pre-test. The partial Eta Squared ($\eta^2$) were .300 and .210 for MAT and MIS respectively which indicate a small effect size but significant. The $\eta^2$ values show that 30% and 21% of the variance in the post-test scores on MAT and MIS are explained by cross-age tutoring. The null hypothesis which states that there is no significant effect of cross-age tutoring on Mathematics achievement and interests of underachieving gifted students was rejected. This implies that there is a significant effect of cross-age tutoring on underachieving gifted students' interest and achievement in Mathematics.

**Hypothesis 3.** There is no significant differential effect between peer and cross-age tutoring on Mathematics achievement and interest of underachieving gifted students

The result of the analysis as presented in Table 4 revealed that there is a statistically significant effect of peer tutoring and cross-age tutoring on Mathematics achievement and interests of underachieving gifted students (p= .00) when adjusted for pre-test and pre-assessment scores respectively. From the result, the calculated $F_{(2, 46)} = 10.882$ and 7.358 for Mathematics achievement and Mathematics interest respectively, p= .000 and .002; < .05. The result of the analysis indicates that there was a significant mean difference between control and experimental groups in their post-test scores on both Mathematics achievement and Mathematics interest while adjusting for the pre-test. The calculated $\eta^2$ on the treatment groups (experimental and control groups) was .321 and .242 for Mathematics achievement and Mathematics interest respectively which indicate a small but significant effect size. Furthermore, $\eta^2$ revealed that 32.1% and 24.2% of the variance in the post-test scores on Mathematics achievement and Mathematics interest are explained by both peer tutoring and cross-age tutoring. The result of the analysis was significant at .05 because the calculated p-value of .000 and .002 on Mathematics achievement and Mathematics interest was less than the p-value of .05, hence both peer tutoring and cross-age tutoring have a significant effect on Mathematics achievement and interests of underachieving gifted students after control for the pre-test. Meanwhile, considering the significant difference between the two groups for Mathematics achievement and Mathematics interest, at .05 and df of 38there is no significant difference in the effect between peer tutoring and cross-age tutoring in Mathematics achievement(t= .0629; p>.05) and Mathematics interest (t= .672; p>.05).

**Table 4.** ANCOVA result of the effect of peer and cross-age tutoring on Mathematics achievement and interest

| Source | Type III Sum of Squares | Df | Mean Square | F-ratio | P-value. | Partial Eta Squared |
|---|---|---|---|---|---|---|
| **Mathematics Achievement** | | | | | | |
| Corrected Model | 153.156[a] | 3 | 51.052 | 7.467 | .000 | .327 |
| Intercept | 295.748 | 1 | 295.748 | 43.254 | .000 | .485 |
| Pre-test | 7.236 | 1 | 7.236 | 1.058 | .309 | .022 |
| Treatments | 148.812 | 2 | 74.406 | 10.882 | .000 | .321 |
| Error | 314.524 | 46 | 6.837 | | | |
| Total | 80788.000 | 50 | | | | |
| Corrected Total | 467.680 | 49 | | | | |
| **Mathematics interest** | | | | | | |
| Source | Type III Sum of Squares | Df | Mean Square | F-ratio | P-value. | Partial Eta Squared |
| Corrected Model | 186.304[a] | 3 | 62.101 | 5.173 | .004 | .252 |
| Intercept | 1467.512 | 1 | 1467.512 | 122.232 | .000 | .727 |
| Pre-test | 1.639 | 1 | 1.639 | .136 | .713 | .003 |
| Treatments | 176.685 | 2 | 88.343 | 7.358 | .002 | .242 |
| Error | 552.276 | 46 | 12.006 | | | |
| Total | 57793.000 | 50 | | | | |
| Corrected Total | 738.580 | 49 | | | | |

**Table 5.** Independent t-test of the effect of peer and cross-age tutoring on Mathematics achievement and interest

| Group | Variable | Mean | Std. Deviation | N | T | Df | p-value |
|---|---|---|---|---|---|---|---|
| Peer tutoring | Mathematics | 41.19 | 2.064 | 21 | | | |
| | Achievement | | | | .629 | 38 | .533 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Cross-age tutoring | Mathematics Achievement | 40.63 | 3.451 | 19 | | | |
| Peer tutoring | Mathematics Interest | 35.10 | 3.208 | 21 | .672 | 38 | .506 |
| Cross-age tutoring | Mathematics Interest | 34.32 | 4.110 | 19 | | | |

This result implies that the two interventions do not differ significantly in their effects. So the null hypothesis that there is no significant differential effect between peer and cross-age tutoring on Mathematics achievement and interest of underachieving gifted students is accepted.

## Discussion and Conclusion

The researchers investigated the effects of peer and cross-age tutoring in Mathematics achievement and interest of underachieving gifted and talented students in the Ibarapa Division of Oyo State. It was found that there is a significant effect of peer tutoring on underachieving gifted student interest in Mathematics and Mathematics achievement. This present finding has implications for Mathematics instruction. The separate findings of Topping (2004) and White (2000) which reported that peer tutoring as a differentiated instruction strategy enhances Mathematics achievement and the interest of students lend credence to this finding. Topping (2004) reported that students are able to ask questions and develop more interest in any concept taught to them by their peers because of the level of rapport and freedom

A significant effect of cross-age tutoring on students' interest in Mathematics and the Mathematics achievement of the participants was also found. This finding supports that of Topping and Whiteley (1993) who found that although tutors academic performance improved regardless of whether they tutored peers, younger or older tutees in reading. However, older tutors experienced greater academic improvements when they tutored younger tutees. Ehly and Bratton (1981) reported significant achievement and interest in older tutees when cross-age tutoring was used. This interest extended to perceptions of competence, with well-liked tutors being perceived as more competent, though it remains unclear whether there were any real differences in competence among the tutors. However, Fogarty and Wang (1982) noted that mixed-age pairs had the potential to produce negative attitudes toward tutoring, particularly among female tutors working with male tutees. Therefore, further research is needed to explore the effects of cross-age tutoring outcomes and the mechanisms underlying them.

There is a significant differential effect of peer tutoring and cross-age tutoring on students' interest in Mathematics. This finding is in alignment with that of Nazzal (2002), Topping (2004), and Heller and Fantuzzo, (1993) who reported significant improvement in the Mathematics achievement of the participants when exposed to peer tutoring and cross-age tutoring. It also tallies with that of Torrado, Manrique and Ayala (2016) who reported a significant effect of both strategies on Mathematics performance and Mathematics interest of underachieving gifted and non-gifted high school students. In a study of mathematics students, Melero and Fernandez (1995) found that students who received peer tutoring performed significantly better than those who did not. Rudland and Rennie (2014) found that cross-age tutoring, in which older students tutor younger students, was also effective in improving academic performance in biology and mathematics. In addition to academic benefits, peer tutoring can also help students develop important social skills, such as communication, cooperation, and empathy. Durán (2009) found that students who participated in peer tutoring programs were more likely to report feeling motivated to study, having higher average grades, and valuing solidarity and communication. They were also more likely to report feeling responsible and having high self-esteem. Additionally, peer tutoring has been shown to promote responsibility and self-esteem, especially among students who take on the role of tutors (Rudland & Rennie, 2014; Durán, 2009).

It is concluded that both peer tutoring and cross-age tutoring are effective interventions for enhancing Mathematics achievement and Mathematics interest of underachieving gifted and talented students. The effect of the two interventions is relatively the same.

## Recommendations

Based on the findings, it is imperative to recommend as follows:

- Teachers should adopt peer and cross-age tutoring in teaching Mathematics to enhance interest and boost the self-esteem of gifted students in Mathematics instruction.
- Mathematics teachers should be trained in the use of appropriate techniques to organize peer and cross-age tutoring for effective classroom delivery to gifted students in an inclusive setting.

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Research Article

# Current trend of mathematic anxiety research and publication: a bibliometric analysis

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**Abstract**

People often feel anxious, frustrated and have negative attitudes towards mathematics because mathematics is often considered a difficult subject because the characteristics of mathematics are abstract, logical, systematic and full of confusing symbols and formulas. There is a need to address mathematic anxiety either through therapy, modification of learning methods, and so on to. Several research and publications related to mathematic anxiety has been done. However, until now, no definite solution has been found to overcome mathematic anxiety Therefore, this study was made to find out what is the current trend on research and publications related to mathematic anxiety in the last ten years. Data for this study is gathered from Scopus index. Analysis for this study was carried out using the bibliometric analysis method. Results of this study shows that Indonesia is the country with the highest number of publications related to mathematic anxiety. We also found that mathematic anxiety keyword has direct link to mathematics learning. Also, there are some topics that are less explored, for example the topic of therapy and the role of mathematics teachers which could be the focus of future research.

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## Introduction

Mathematics is frequently seen as a difficult topic since its traits are abstract, logical, systematic, and full of perplexing symbols and formulas. (Santri et al., 2017). People frequently experience anxiety, frustration, and have bad attitudes toward mathematics. Feelings of anxiety usually rising when they have difficulty solving questions or during exams. (Doruk et al., 2015)

According to Luttenberger et al. (2018), math anxiety is defined as a fear and heightened psychological activity when interacting with math, such as when manipulating numbers, solving math problems, or confronted with evaluative situations (tests/exams) related to math. Also, definition from one of the latest researches defines mathematic anxiety as anxiety felt by individuals when they solve mathematical problems (Choi-Koh & Ryoo, 2019). It can be said that math anxiety is a feeling of anxiety, pressure, worry, dislike, or fear of everything related to mathematics.

Mathematic anxiety has primarily been studied in educational settings, with little connection to clinical research on anxiety disorders. The Diagnostic and Statistical Manual of Mental Disorders (DSM) and the International Classification of Diseases (ICD) are two diagnostic systems for mental disorders that categorize mathematic anxiety

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under general anxiety section. But according to Paechter et al. (2017) mathematic anxiety is distinct from other subjects' anxieties or general test anxiety; for example, research on anxiety in related subjects such as math and statistics shows that, to a large extent, math anxiety and statistics anxiety are independent of each other and have different effects on learners.

People who suffer from math anxiety may find that the fear of doing math overwhelms their working memory. This is the region of the brain that stores small amounts of information needed to complete a task (Sokolowski & Ansari, 2017). Therefore, someone who experiences mathematic anxiety will have poor performance in math (Zhang et al., 2019).

Several research and publications related to mathematic anxiety has been done. However, until now, no definite solution has been found to overcome mathematic anxiety (Ramirez et al., 2018). Therefore, this study was made to find out how far the development of research and publications related to mathematic anxiety in the last ten years.

### Problem of Research

Based on the information that has been presented previously, the problem of this research is obtained as follows:

- What is the current trend of research and publications related to mathematic anxiety if reviewed using bibliometric analysis?

## Methods

### Research Model

This research was carried out through online data collection on June 12, 2023. The data was taken from Scopus document search results. The data taken is document data both from conference papers, scientific articles, and books. The documents used in this research are limited to a maximum of ten years back to maintain the relevance of this research.


**Figure 1.** Research method flow chart

### Data Analysis

Analysis for this study was carried out using the bibliometric analysis method. Bibliometric analysis is a popular and rigorous method for exploring and analyzing large amounts of scientific data (Donthu et al., 2021). It allows us to unpack the evolutionary nuances of a specific field while shedding light on its emerging areas. Bibliometric analysis is frequently used with network visualization software, either by using command-based software like the Bibliometric package in R.(Aria & Cuccurullo, 2017) or by using graphical user interface-based software like VOSviewer (van Eck & Waltman, 2014).

Data from Scopus search findings then exported into a file with the RIS (Research Information Systems) format. Then RIS file is imported into the VOSviewer software. VOSviewer version 1.6.19 was used for this study. VOSviewer is used to generate generate keyword maps based on existing keyword networks or relationships. The keyword frequency is set as desired when creating the bibliometric map, and irrelevant or less relevant keywords are removed.

## Results

The initial data found contained 58 relevant documents. After filtering with search criteria based on title, abstract, keyword "mathematic anxiety" and the maximum published year is the last ten years, 13 documents are found that meet the criteria.

**Table 1.** Document related to "mathematic anxiety" and their author

| No | Cite | Topic |
|---|---|---|
| 1 | Zhou et al. (2022) | The connections between problematic smartphone use, math anxiety, learning interest, and achievement |
| 2 | Collado-Soler et al. (2022) | Primary school students' motivation and anxiety toward mathematical learning |
| 3 | Almo et al. (2022) | Collaborative mathematics game experience using seven spell and peer tutoring |
| 4 | Dondio et al. (2021) | Literature review about possible games that reduce mathematic anxiety |
| 5 | Dewi & Royanto (2020) | Elementary student's metacognitive strategy for completing arithmetic word problems |
| 6 | Ifdil et al. (2019) | Chromotherapy as an alternate treatment for primary school kids' maths anxiety |
| 7 | Choi-Koh & Ryoo (2019) | Measurement of mathematic anxiety |
| 8 | Bjälkebring (2019) | Study about forms of teaching and learning that can help students with mathematic anxiety at university |
| 9 | Ardi et al. (2019) | Using rasch measurement investigated elementary students' learning difficulties in mathematics based on students' mathematic anxiety, mathematics self-efficacy, and value beliefs. |
| 10 | Daharnis et al. (2019) | Study about mathematic anxiety among prospective elementary school teachers and their treatment |
| 11 | Esa & Mohamed (2017) | A study of students' learning styles and mathematics anxiety |
| 12 | Yildirim & Gurbuz (2017) | An examination of elementary school teachers' maths concern in relation to many variables |
| 13 | Doruk et al. (2015) | The investigation of vocational school students' mathematic anxiety in terms of learning style and multiple intelligence |

These documents are then grouped according to the type of document and three groups of document types are obtained. Here are the three categories of documents.

**Table 2.** Grouping according to document type

| No | Document Type | Total Document(s) |
|---|---|---|
| 1 | Conference Paper | 6 |
| 2 | Article | 5 |
| 3 | Book Chapter | 2 |

Table 2 shows that there are six documents in the form of conference papers related to mathematic anxiety. A total of five documents are in the form of articles and two documents are in the form of book chapters.

**Author**

This research is complemented by data on the authors of documents related to mathematic anxiety. We present this data so that readers can find out the development of documents on the topic of mathematic anxiety through author data. Based on the documents collected, we found the top 10 author names that most researched mathematic anxiety.

**Table 3.** Authors of mathematic anxiety document publication

| No | Author | Total Document(s) |
|---|---|---|
| 1 | Afdal, A. | 3 |
| 2 | Ardi, Z. | 3 |

| No | Author | Total Document(s) |
|---|---|---|
| 3 | Ifdil, I. | 3 |
| 4 | Alizamar, A. | 2 |
| 5 | Daharnis, D. | 2 |
| 6 | Dondio, P. | 2 |
| 7 | Erwinda, L. | 2 |
| 8 | Fadli, R.P. | 2 |
| 9 | Rangka, I.B. | 2 |
| 10 | Refnadi, R. | 2 |

Table 3 shows that Afdal, A., Ardi, Z. and Ifdil, I. are the authors who have the most research related to mathematic anxiety with each author producing 3 documents. Then Alizamar, A., Daharnis, D., Dondio, P., Erwinda, L., Fadli, R.P., Rangka, I.B. and Refnadi, R. each produced 2 document works.

**Subject Area**

Mathematic anxiety is examined in various subject areas, but there are five subject areas that are most related. The five subject areas are shown in the following table.

**Table 4.** Subject area with the publication topic of mathematic anxiety

| No. | Subject Area | Total Document(s) | Percentage (%) |
|---|---|---|---|
| 1 | Social Sciences | 7 | 36,84 |
| 2 | Physics and Astronomy | 4 | 21,05 |
| 3 | Computer Science | 3 | 15,79 |
| 4 | Psychology | 3 | 15,79 |
| 5 | Arts and Humanities | 2 | 10,53 |

Based on table 4, it can be seen that social science is the largest subject area that is most related to the topic of mathematic anxiety. This is followed by physics and astronomy in second place, computer science in third place, psychology in fourth place and finally arts and humanities in the fifth place.

**Country/territory**

Based on the data obtained, it can be seen which countries are actively publishing documents related to mathematic anxiety. There are nine countries that have publications related to mathematic anxiety which are summarized in the following table.

**Table 5.** Countries that actively carry out publications about mathematic anxiety

| No. | Country/territory | Total Document(s) |
|---|---|---|
| 1 | Indonesia | 4 |
| 2 | Ireland | 2 |
| 3 | Turkey | 2 |
| 4 | China | 1 |
| 5 | Malaysia | 1 |
| 6 | South Korea | 1 |
| 7 | Spain | 1 |
| 8 | Sweden | 1 |
| 9 | United States | 1 |

Based on table 5, it can be seen that the topic of mathematic anxiety is most researched in Indonesia with four documents produced. Ireland and Turkey produced two documents each. China, Malaysia, South Korea, Spain, Sweden, and United States each contributed 1 document.

**Data Mapping**

Data retrieved from Scopus related to mathematic anxiety resulted in 13 documents between 2014 and 2023. The data was then exported into RIS format and imported into VOSviewer for analysis and visualization. Six main keywords were found and there were nine relationships with a total relationship strength of 11 from the visualized data. Two clusters were found from the six main keywords generated. The first cluster contains four main keywords, namely learning systems, mathematics anxiety, mathematics achievement, and mathematics learning. The second cluster contains two main keywords, namely elementary schools and primary schools.

**Network Visualization**

The network visualization shows the relationship between keywords that are interrelated, and the total strength of a keyword used in research related to mathematic anxiety. Based on the existing data, the following visualization results are obtained.

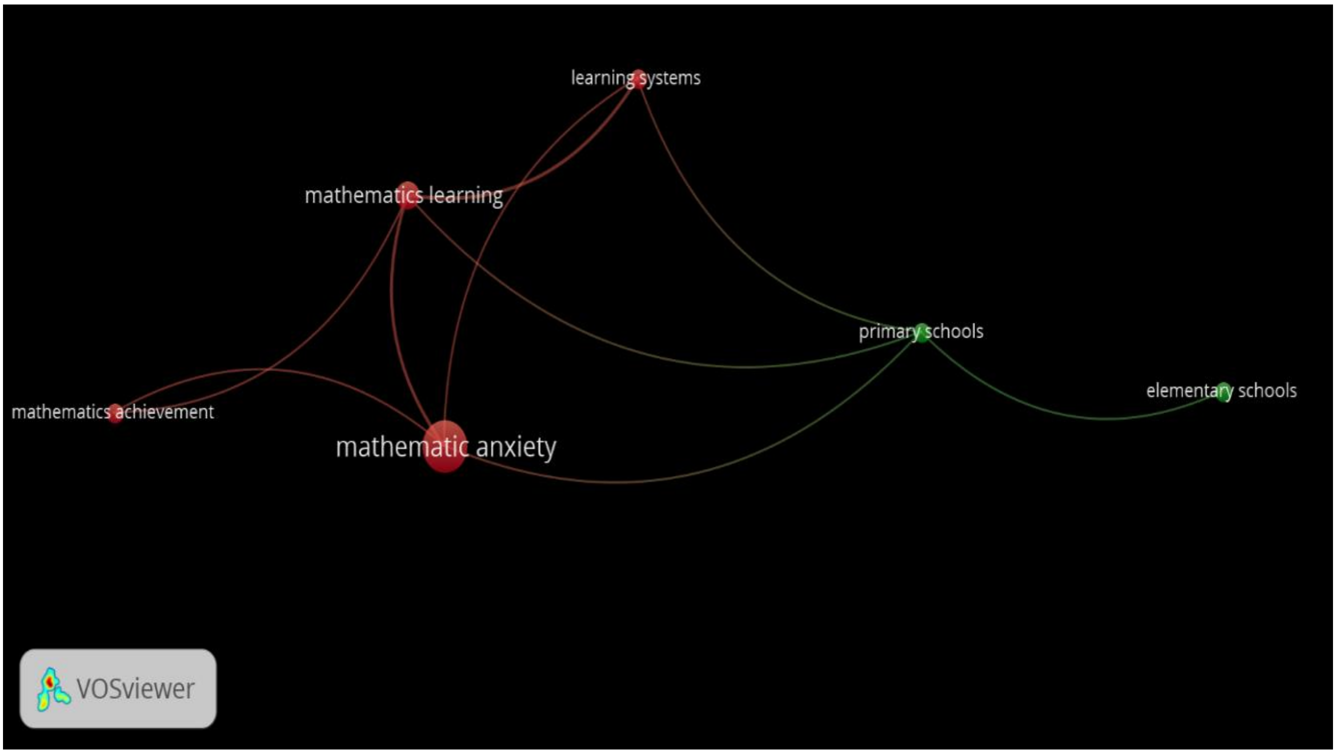


**Figure 2.** Network visualization

The visualization results in Figure 2 show that there are six main keywords found. The keywords in the network visualization are displayed into two clusters. The first cluster is shown in red which contains four keywords namely mathematic anxiety, mathematics learning, mathematics achievement, and learning systems. Then the second cluster is shown in green which includes two main keywords namely primary schools and elementary schools. Each keyword has a relationship with other keywords which can be seen in the following table.

**Table 6.** Keywords total link and link strength

| No | Cluster | Keyword | Link | Link strength |
|---|---|---|---|---|
| 1 | 1 | Mathematic anxiety | 4 | 5 |
| 2 | | Mathematics learning | 4 | 6 |
| 3 | | Learning systems | 3 | 4 |
| 4 | | Mathematics achievement | 2 | 2 |
| 5 | 2 | Primary schools | 4 | 4 |
| 6 | | Elementary schools | 1 | 1 |

**Overlay Visualization**

Overlay visualization is used to illustrate the distribution of document years from the data that has been retrieved with the keywords displayed differentiated into dark and light. Overlay visualization can also explain the average document year of a keyword. The distribution of documents by year is divided into five main scales, so that the following visualization is obtained.

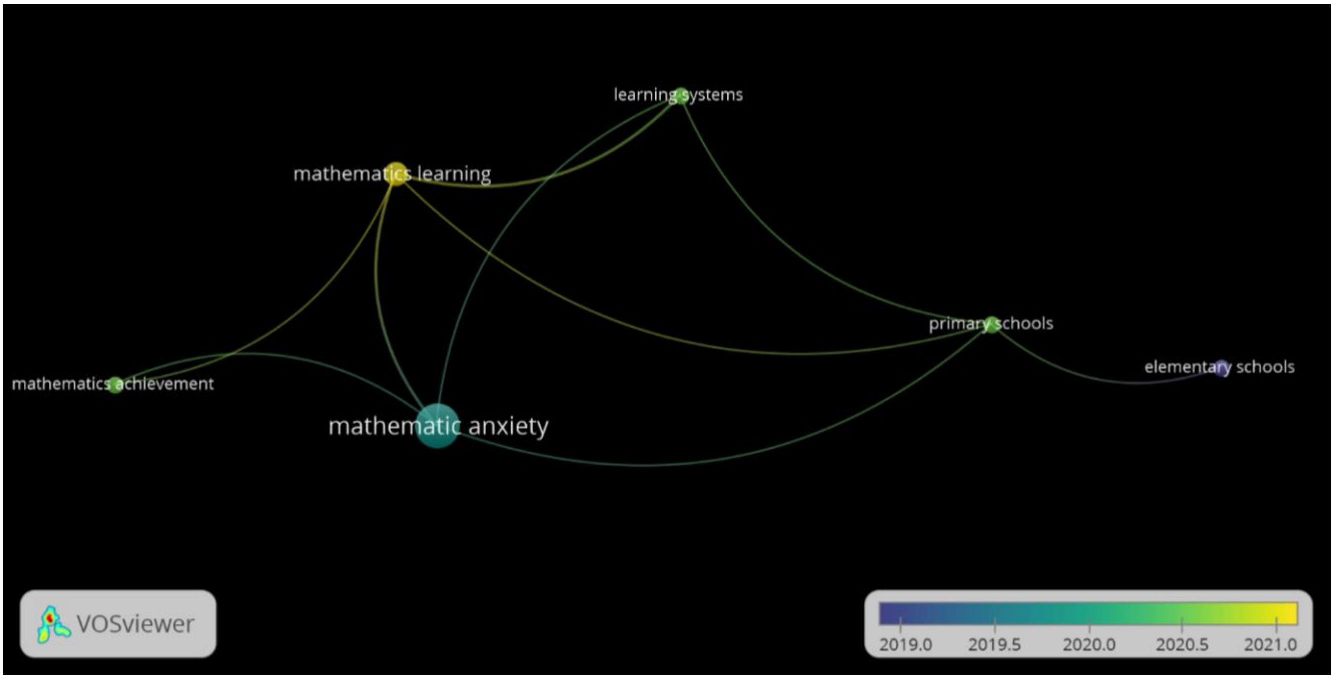


**Figure 3.** Overlay visualization

Figure 3 shows the result of overlay visualization that illustrates keywords with different color spheres, ranging from dark to light. Keywords with dark colors are elementary schools as the oldest document from the visualized data, which is 2019. Then the light blue keyword is mathematic anxiety which was visualized in the middle of 2019. Keywords primary schools, mathematics achievement, and learning systems are visualized in green indicating 2020. Finally, mathematics learning is visualized in yellow which indicates 2021. The document data can also be illustrated through the average annual document publication described in the following table.

**Table 7.** Average keywords publication year

| No | Keyword | Year | Average Publication Year |
|---|---|---|---|
| 1 | Primary schools | 2019 | 2019.00 |
| 2 | Mathematics anxiety | 2019 | 2019.83 |
| 3 | Learning systems | 2020 | 2020.50 |
| 4 | Mathematics achievement | 2020 | 2020.50 |
| 5 | Primary schools | 2020 | 2020.50 |
| 6 | Machine learning | 2021 | 2021.00 |

**Density Visualization**

Density visualization describes the distribution of documents through the two most visible colors, namely dark blue and yellow. Based on the research data, the density visualization is obtained as follows.

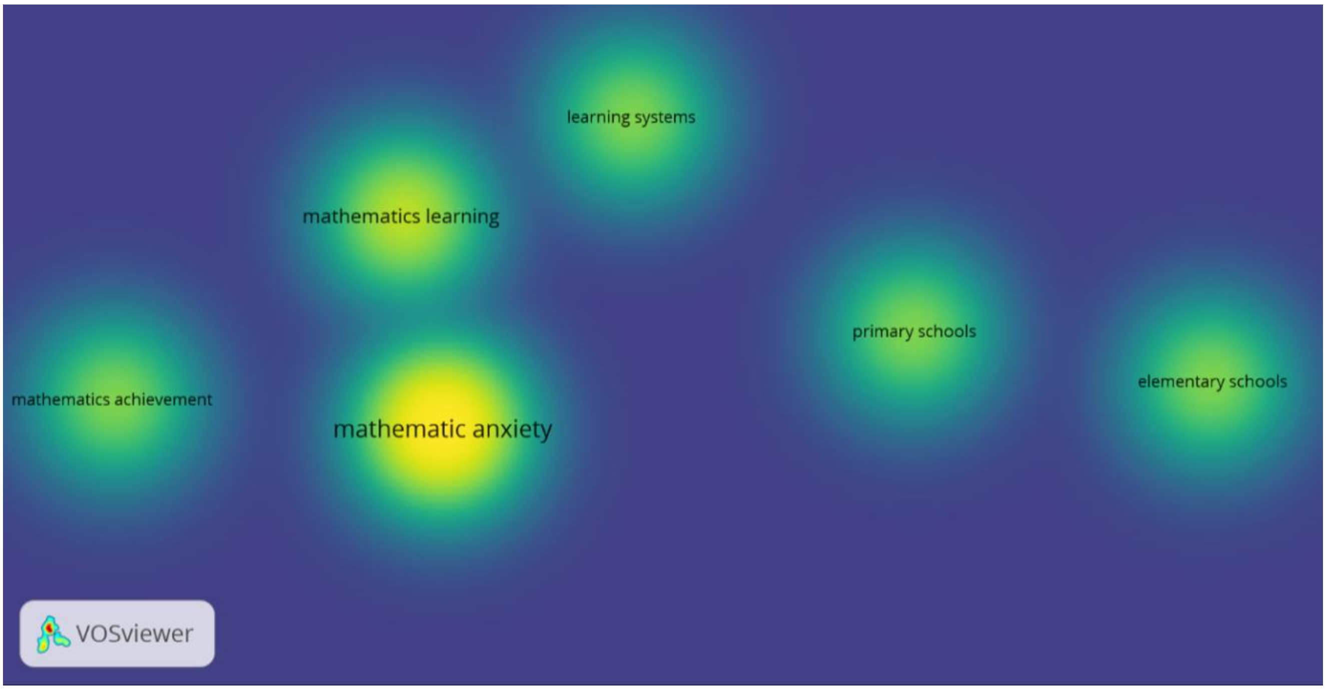


**Figure 4.** Density visualization

The visualization results in figure 4 show that the most used keyword is mathematic anxiety and then followed by mathematics learning. Keywords mathematics achievement, learning systems, primary schools, and elementary schools are visualized with similar colors.

## Discussion and Conclusion

According to table 5, Indonesia is the country with the most publications related to mathematic anxiety. In line with this, we found an interesting fact. Several studies were conducted in order to assess the mathematical ability of students in various countries around the world, one of which was the Trends in International Mathematics and Science Study (TIMSS). The studies found that average mathematics achievement score of Indonesian students is categorized as Low International Benchmark. This achievement indicates that on average Indonesian students are only able to recognize a number of basic facts but have not been able to communicate, relate various topics, let alone apply complex and (Hadi & Novaliyosi, 2019). Also, according to (Udil et al., 2017) high mathematics anxiety can cause the lack of students' metacognition process while solving the mathematics problem. Based on this information, it seems that Indonesia is actively trying to solve the problems related to mathematic anxiety.

According to 2 and 3 we see that mathematics anxiety has a direct link with mathematics learning and both of them has larger circle diameter than other keywords. The darker the yellow color, the larger the diameter of the circle, the more frequent research on these topics increases, but if the color fades, blending into the background, the number of studies about these topics will decrease (Nandiyanto & Al Husaeni, 2021). Therefore, research that modifies mathematics learning technique might be a trend for future research.

Density visualization as shown in figure 4 gives us bright heatmap about mathematic anxiety and mathematics learning, but no keywords shown about the therapy/method for overcoming mathematic anxiety. Also, we didn't find any heatmap about teachers or educators even some of documents that were used in this study also related with topic such as teacher or educator. According to Udil et al (2017) it's important for the teachers to consider students mathematics anxiety to design and plan better mathematics learning. This phenomenon may be due to the lack of research on those topics. These findings are expected to be used as input and to provide insight into new areas of mathematical anxiety research.

## Recommendations

This research has explored information related to publications on mathematic anxiety. From the results of this study, it can be seen that there are some topics that are less explored, for example the topic of therapy, the role of mathematics teachers and mathematics learning. Future research may be able to contribute to developing these topics

## Limitations of Study

One of the limitations of this study is that the source of data collection only takes from publications listed on Scopus. There are some alternative sites that may be able to provide more data, such as Web of Science, PubMed, and ScienceDirect.

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All authors contributed equally to the study. The authors declare that there is no conflict of interest. Ethical rules were followed in all processes of the study.

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**Research Article**

# The use of interdisciplinary approach in geometry teaching: The example of Arab-Islamic civilization

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**Abstract**

This paper presents the important topic of using photographic images connected with tradition and art as enrichment items in geometry classes, on subjects relevant in the context of Arab-Islamic civilization. In addition, the study presents the use of photographic images (photographs) of mathematical objects in lessons on geometric thought, as a means of increasing students' involvement and enhancing their learning experience. Examples of photographs taken from Arab-Islamic civilization are presented, suggested for being especially appropriate for teaching geometry to Arab students at an Arab teacher training college. The study involved 50 participants: Twenty-four teachers in the academic retraining program, to become mathematics and computer sciences teachers; 26 regular students in their fourth year, majoring in mathematics and computer sciences The teachers and students participated in the compulsory course: Arab-Islamic Civilization at an Arab teachers training college. This group was chosen since they are designated to become math teachers and serve as agents of change in schools. In this study we used a questionnaire with one version for both groups of participants – teachers in retraining and regular students in their fourth year of study in the teaching of mathematics and computer sciences. The first two question are intended to provide initial personal information (profession, and math teaching experience) and the Questions 3-13 are open questions providing information with regard to teachers' and students' positions on the study topic. At the end of the questionnaire, the participants were asked to make additional comments. The paper include a model of a mathematics lesson including a mathematical photograph connected with Arab-Islamic civilization and Examples of pictures and an example of a lesson plan designed in accordance with the proposed model. The analysis of the findings connected with the questions that appeared in the opinion questionnaire, and divided in to seven categories. The findings of this study indicate positive attitudes of teachers and students towards integration of teaching methods including the use of photographic images in teaching in general and geometry in particular. The study discussed in this paper may contribute to teachers' and teacher cadets' professional development in making geometry classes in the mathematics discipline more experiential, and thus reducing anxiety and fear of mathematics.

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## Introduction

In this day and age of advanced technology, we are witness to significant change—as in the training of teachers and teacher cadets in the colleges, expressed in integration of a variety of innovative methods and ways of teaching, as well as new technological tools—as an integral part of realizing significant, experiential learning in the framework of the *Ofek*

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*Hadash* ("New Horizon") and *Oz LeTmura* ("Courage to Change") educational reforms. As opposed to these, we are also witness to revulsion from the traditional frontal method.

A great deal of research literature exists on the anchoring of mathematics instruction in students' cultural contexts (Katzaf, 2004, 2006; Mussarwa, 2012; Daher, 2004). Many philosophers touched on the importance of photographic images and integration of pictures in the teaching of various academic disciplines. The most ancient and commonly used saying: "A picture is worth a thousand words" is attributed to the famous Chinese philosopher Confucius. Philosopher Régis Debray offered his perspective too, in an attempt to define a picture: "A picture contains five billion meanings and translations, as the number of living things in the universe". As opposed to them, Barnes (2014) noted that Aristotle did not describe what a picture is in his opinion, but rather saw the practical aspect of using them, saying that "thinking is not possible without the use of pictures in teaching".

Williams (1963) in his book *Learning from Pictures* noted 31 reasons to learn with pictures, the main ones being: Developing engagement between teacher and student and among the students themselves, structuring knowledge, and assisting in understanding the subject material. As opposed to Williams, Roland Barth (1980), from the definitive aspect, claimed in his book *Thoughts on Photography* that photography derives from the source of theater, with the photographer conducting a kind of play for the spectators – to amaze them, arouse their interest, and shock them. Contrary to the previous claim, several years later, Barth (2002) noted in his article that in his view, photography provides us with reality itself.

This paper deals with the important topic of using photographic images connected with tradition and art as enrichment items in geometry classes, on subjects relevant in the context of Arab-Islamic civilization. In addition, the study suggests the use of photographic images (photographs) of mathematical objects in mathematics lessons as a means of improving students' engagement and learning. When teaching mathematics, we are interested in showing its presence in various aspects of life, and its presence in cultural treasures may be relevant from both the mathematical and cultural aspects. Examples of photographs taken from Arab-Islamic civilization are presented, suggested for being especially appropriate for teaching mathematics to Arab students.

The photography picture taken with a camera is considered a source of knowledge, providing information from daily life (Ismaili and Awatef, 2011). Learning with pictures can spur development of engagement between teacher and student and among the students themselves and assist in structuring knowledge and understanding the subject material. The use of photography in a cultural context is considered an effective means of teaching mathematics in general, and geometry in particular. Practicing teachers and teaching cadets constitute a relevant group for training and guidance in integrating innovative and challenging means of teaching mathematics to achieve significant learning. Both groups see the importance of having an elaborate toolbox to assist them in delivering mathematical content to students in an interesting and experiential way, mainly in teaching geometry. In the mathematics textbooks and curricula there is no mention of integrating pictures or photographs in the teaching of math in general and geometry in particular. Williams (1963) in his book *Learning from Pictures* notes thirty-one reasons to teach with pictures, the main ones being: Developing engagement between teacher and student and among the students themselves and structuring knowledge assist in understanding the subject material. As opposed to Williams, Barth (1980), in his book *Thoughts on Photography*, claims that photography derives from the source of theater, with the photographer conducting a kind of play for the spectators – to amaze them, arouse their interest, and shock them. In addition, in his second book: *The Photographic Message*, Barth (2002) notes that in his view photography provides us with reality itself.

Geometry is a central area in the study of mathematics (Lester, 2007). From a young age, children encounter geometrical shapes (Ministry of Education, 2006). In addition, geometry is considered by students to be one of the most difficult areas of mathematics (Hofer, 1981).

Studies focused on identifying students' difficulties in geometry indicate the need to raise teachers' awareness of them and delve deeper into students' understanding in order to preclude problematic situations (Gal, 2011). One way of dealing with these difficulties is contextual learning (Verner & Maor, 2005). Context problems are defined as problems dealing in situations close to the student's world and culture (Gravemeijer & Doorman, 1999). Researchers in mathematical education recommend connecting the teaching of mathematics to the student's culture (Orey & Rosa, 2007). This can be achieved through an ethnomathematical approach, i.e., teaching mathematics within a cultural and ethnic context, integrating mathematical problems associated with different civilizations or the student's culture, thus making the subject of mathematics more interesting and the mathematical content easier to learn.

Katzaf (2006) notes in her articles that teachers who experienced the integration of ethnomatematics in mathematics lessons became convinced that recognizing mathematics as part of civilization nurtures a positive attitude towards mathematics and constitutes a bridge between the mathematics class and the outside world. In addition, Katzaf (2004) stresses in her article the significance and contribution of humanistic mathematics. Similarly, Mussarwa (2012) noted in her study that the students who participated in the study reported that their experience in studying and teaching geometry in a cultural context increased their awareness of the importance of cultural aspects in teaching geometry.

Edri and Movshovitz-Hadar (2014) noted in their study that education for personal and social values may be integrated in teaching mathematics, without compromising students' achievement and teaching of the curriculum. This requires preparation of suitable teaching materials and training teachers and teaching cadets in using them.

Daher (2004) relates to several advantages of integrating the history of Islamic civilization in teaching mathematics. Giving the mathematics subject meaning reduces students' anxiety over learning mathematics, spurs and encourages them in solving challenging mathematical problems, and strengthens their connection and affinity to figures that have greatly contributed to advancing the teaching of mathematics by developing various methods and ways of solution. In addition, Daher notes in his article that one of the factors encouraging integration of photographic images in teaching mathematics in general and geometry in particular is teachers' lack of knowledge in delivering lessons and building study units. In addition, the textbooks and curricula do not relate to integration of this tool in teaching mathematics.

To the best of our knowledge, no studies addressing the use of photographic images in teaching geometry at schools have been done yet.

The present study discusses the importance of the use of photographs or pictures as a tool in teaching geometry in a cultural context in general, and in the context of Arab-Islamic civilization in particular. The paper will present the stages of development of geometry lessons integrating the use of photographic images and relate to participants' responses. In addition, an example of a lesson and two additional pictures from Islamic civilization suitable for a geometry lesson.
The paper includes the following components: A. The model of a mathematics lesson including a mathematical photograph connected with Arab-Islamic civilization. B. Examples of pictures and an example of a lesson plan designed in accordance with the proposed model: integrating mathematical pictures and photographs on topics such as the "the box", "shapes on planes and in space", geometrical forms and symmetry". Activity in the lessons integrates learning the definition of the concepts "the box", "shapes on planes and in space", geometrical forms and symmetry".

It should be noted that all lesson plans include full integration of questions relating to historic, cultural, and religious content, and questions relating to mathematical content.

**The Kaaba**

**Photo 1.** The Kaaba

*Target audience*: sixth grade classes

*Lesson objectives*:

- Create an encounter with geometry by presenting pictures from Arab-Islamic civilization.

- Become familiar with the holiest and most important site of worship in Arab-Islamic religion and civilization.
- Learning the concept "box" through a photographic image associated with Islamic structures.
- Experience in calculating the volume and surface of the box.

*Opening the lesson*

*Class discussion on the question*: What is a body in general, what is a "box" in particular, and how the teacher, and how does the teacher connect the subject of the lesson to historical, religious, and cultural content.

The teacher directs the students to search for knowledge in databases, such as the description of a box or cube, or concepts associated with the box structure such as sides, corners, vertices, and diagonals, as well as the properties of the box, calculation of the area of a rectangle, square, and volume of a box.

First possible question, in pairs: Students, try and have a discussion on geometry studies. In this case, "a box", as part of Arab-Islamic history.

*Another possible question*: Students, try to imagine the period and life of Arab-Muslims in this era. What, in your opinion, were the uses of mathematical knowledge?

*Lesson structure*

Learning in the lesson can be done in several ways:

A. Self-study – each student receives a page of questions for self-study, relating to finding relevant information on the lesson topic.

B. Work in pairs – each pair will try to work together in solving the questions raised in class.

C. Class discussion – the teacher conducts a symposium with the whole class participating, and everything said in the discussion is written on the board.

The choice of the way of study depends on the composition of the class and lesson objectives set by the teacher.

*Description of the activity*

After an opening discussion, the students should be told that the topic of the lesson is "the box". The students shall work in four different groups, and attempt to solve the questions proposed in two stages:

*Stage one*: The first two groups will receive questions relating to the historical-religious content, and the other two groups will receive questions relating to the mathematical content.

*Stage two*: The first two groups will switch with the other two in solving the mathematical questions. The other two groups will attempt to solve the questions relating to the historical-religious content.

It should be noted that after solving the questions, the two groups will write the answers on Bristol boards for presentation, discussion, and summary of the lesson.

The questions presented and proposed to the students in connection with the picture:

*Questions relating to the historical-religious content*:

- Why is the Kaaba considered the holiest and most important building in the world for Muslims?
- Where in the Kaaba located?
- According to the Koran and Islamic tradition, who built the Kaaba?

Describe the Kaaba structure.

- How is the Haj (pilgrimage) precept connected with the Kaaba?

Questions relating to the mathematical content:

Search the sources and note what the body of the Kaaba is.

Describe the body you have noted in question A.

According to the body you noted in Question A, answer the following questions:

- How many sides does the body have?
- How many vertices does the body have?
- What are the body's sides? Squares, rectangles, triangles, etc.?
- How many diagonals does the body have?
- How many edges does the body have?

- What types of polygons are obtained by spreading the body?

What is required in order to calculate the volume of the body?
Search the sources to find the real dimensions of the Kaaba and calculate its volume accordingly.
Those responsible for the Kaaba structure decided to renew its exterior paint coating (without the roof). Each square meter ($m^2$) requires 15 liters of paint, and the cost of each liter is 90 Saudi riyals.
Calculate how many liters of paint are required to paint the external walls.
Calculate the total costs of painting the exterior walls of the Kaaba.

**Summary of the lesson**

In the concluding discussion, each group shall choose a representative on its behalf to hang the Bristol board it prepared for conclusion and general discussion in class. It is recommended to ask the students whether they enjoyed the lesson, and have each student tell about something new he learned from the lesson.
Testimonies and opinions of teachers and students exposed to the practice

**Pictures proposed for preparing the lesson plan in geometry**

**Photo 2.** Some examples for lesson

**A model of the development stages including pictures or photographs**

**Choosing the picture**

At this stage, the teachers choose a picture with a cultural context, with which they can construct an interesting and enjoyable geometry lesson in line with the curriculum that will lead to an understanding of geometric content.

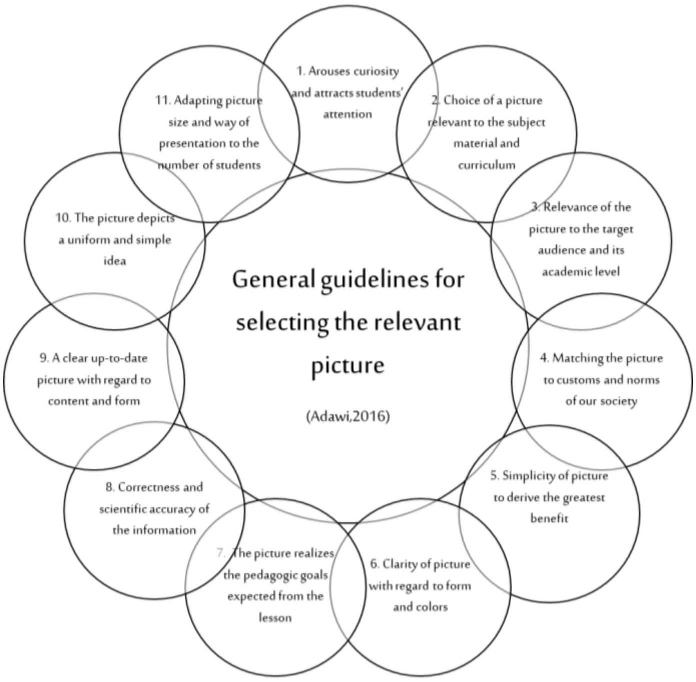


**Illustration 1:** General guidelines for choosing a picture

This diagram includes general guidelines that direct teachers and teaching cadets in choosing a relevant picture and integrating it in geometry teaching.

- Brief information on the picture: A short explanation on the picture, both content and form.
- Link between the picture and the syllabus/curriculum: Relevance of the picture to the course syllabus and curriculum, mainly in geometry.
- Expectations: Teacher's expectations – does the picture chosen for the lesson arouse the students' emotions and desire to learn (emotion and morality/ethics).
- Way of presentation: Whether to present the picture before or after learning the topic.
- Students' reactions: Give the students the opportunity to comment after allocating the lesson's activity, reading the questions, and presenting the picture.
- Opening questions to arouse curiosity and for illustration: After presenting the picture, the teacher presents the opening activity for curiosity and brainstorming, such as a quiz, a puzzle, questions, a map of concepts, etc.

**The planning and building stage of the lesson**

"First steps": As a first step, the students should be given a brief explanation of the subject and its goals. In addition, the teacher should arouse students' motivation by presenting guiding questions, associations, a quiz, or an educational puzzle. The teacher should also explain the stages of the lesson's structure, and how learning will be conducted (self-study, work in groups, class discussion).

**Performing the lesson in class**

The class structure is as follows:

- After the lesson is started, the students may be divided into work groups according to the class composition.
- Allocation of tasks among the various groups for experience.
- The students shall solve the task questions.
- The teacher will go from one group to another to verify group activity and performance of the tasks.
- A summary of the answers to the questions shall be drawn on a Bristol board by the various groups.
- A representative from each group shall hang the Bristol on the board for presentation purposes and a general class discussion.
- The students shall tell of their experience in solving the tasks and anything new they learned from the lesson.

**Reflection stage**

After the execution stage, teachers and students were asked to reflect on the process they went through. They are asked to discuss the process of finding the picture or photograph relevant to the lesson, the target audience, and the curriculum and then connect the depicted information with the subject matter and share impressions such as difficulty of execution, contribution, efficiency and effectiveness, interest, and enjoyment.

The teachers and students performed and submitted the preliminary stage, the execution stage, and the reflection stage. In addition, some of the participants gave a presentation on building a though-provoking lesson according to the stages and guidelines required for assessment for learning (AFL).

**Aim of the Study**

This study examines the positions and outlooks of teachers in retraining and fourth year students majoring in mathematics and the computer sciences in one of the Arab teacher training colleges, toward the use of photographic images in teaching geometry in the context of Arab-Islamic civilization as part of the course: Arab-Islamic Civilization.

## Method

**Research Model**

The following is a qualitative study. The choice of the qualitative method is derived from the need to explain positions from the various personal perspectives of the participants themselves. The qualitative research method enables openness and the ability to clearly understand the opinions of teachers and students on realization of significant learning through integration of photographic images in teaching (Tsabar Ben-Yehoshua, 1999; Shkedi, 2003)

**Participant**

Twenty-four teachers in the academic retraining program, to become mathematics and computer sciences teachers, and 26 regular students in their fourth year, majoring in mathematics and computer sciences. The teachers and students participated in the compulsory course: Arab-Islamic Civilization at an Arab education and teacher training college.

The teachers in retraining are actual teachers with 5-10 years of teaching experience, some of whom teach mathematics in grades 3-6 and other 7-8, and the purpose of their studies is to train them to teach mathematics. At the end of their studies, they receive a teaching certificate in mathematics, recognized by the Ministry of Education. The students in the regular track are in their fourth year of study towards receiving a B.Ed., majoring in mathematics and computer sciences.

**Data Collection Tools**

In this study we used a questionnaire with one version for both groups of participants – teachers in retraining and regular students in their fourth year of study in the teaching of mathematics and computer sciences. The first two question are intended to provide initial personal information (profession, and math teaching experience):

Q1. How many years have you been teaching the mathematics subject?

Q2. How long have you been teaching at your present school?

Questions 3-13 are open questions providing information with regard to teachers' and students' positions on the study topic. At the end of the questionnaire, the participants were asked to make additional comments:

Q3. What does the expression "integrating a photographic image as a teaching or learning method" mean to you?

Q4. To what degree are pictures as a teaching method in mathematics used in the school in which you teach in general, and in the classes you teach, in particular? Provide examples.

Q5. Describe the difficulties you've encountered, as a teacher in the retraining program and as a regular student in the college, in choosing the appropriate picture for preparing a lesson for thought in geometry.

Q6. Why did you prefer that particular picture over the others?

Q7. In your opinion, what is the educational benefit of using photographic pictures in teaching geometry?

Q8. What benefit does the use of photographic images in teaching geometry have in the development of your professional career.

Q9. In your opinion, what is the benefit of this method for the students?

Q10. Define what the "significant teacher" is, from your point of view.

Q11. Do you see yourself as a creative teacher in the use of photographic images as a means of teaching geometry? Explain your opinion.

Q12. Would you recommend to your colleagues to integrate photographic pictures in teaching geometry in your school?

Q13. Evaluate the lecturer in the course: Arab-Islamic Civilization, as far as delivery of the material and use alternative teaching methods.

Any further comments you might like to add:The questions in the questionnaire are open and are intended to enable a deepening of understanding with regard to participants' answers based on the qualitative methodology. During the intensive one-week trial period, observations in classes were conducted in addition to interviews with some of the participants in the study.

**Implementation of the Course**

The participants in the study were asked to construct a 45-minute lesson in geometric thought through a practical assignment. It was made clear to both teachers and students that the data collected shall remain confidential and be used for research purposes only. In addition, they received a detailed explanation on the importance of the study and that its purpose is to instruct them in integration of photographic images in teaching geometry. All of the teachers and students agreed to participate in the study and were invited in small groups for individual meetings. It was explained to them that the purpose of the study is effective use of photographic images in teaching geometry. The participants prepared the practical assignment according the previously explained stages.

## Results

The findings connected with the teachers' and students' positions regarding integration of photographic images in teaching geometry were obtained from the researchers' assessment of assignments execution, observations in the students' lessons during the week-long practicum the students carried out at schools, from reflection, and the opinion questionnaire.

The findings attest to positive attitudes toward the use of photographic images as a tool for significant learning. The students viewed the assignment as one that serves their professional development in teaching and assists them in effective delivery of classes. This despite the difficulties the students had to deal with in choosing a culturally relevant picture in optimal compliance with all the guidelines. With regard to professional experience, a close correlation was found between professional experience and knowledge of the teaching field, and preference for the use of photographic images as a didactic tool.

The teachers in the retraining program admit that preparation for lessons in which photographic images are used is different from preparation for lessons according to the frontal method. In their opinion, teaching that includes the use of photographic images assists in conveying the subject matter visually, which contributes to comprehension. Furthermore, they admit and say that Confucius was indeed right when he said: "A picture is worth a thousand words". The use of pictures makes learning more interesting and contributes to social involvement and value orientation. In light of the above, it should be noted that some of the students' feedback was received after delivering a geometry lesson by this teaching method during the intensive week-long practicum (a week that all students spend performing practical work at schools).

Both teachers and students related, among other things, to the difficulties they encountered while performing the assignment. The most prominent difficulty was choosing the picture relevant to the studied material, its goals, the curriculum, the target population, and how to construct a lesson accordingly. Nevertheless, despite the difficulties, both teachers and students saw the assignment as a challenging task that serves their professional development in teaching and assists in the successful delivery of lessons. What's more, the photographic images chosen were within their own world, civilization, heritage, and religion.

In light of the experience in constructing lessons based on pictures, the students' awareness, and openness regarding the use pictures in teaching have increased, not just in mathematics and computer sciences but in other disciplines as well such as history, religion, and more. Therefore, they recommend to their colleagues, teachers at schools, to employ this teaching method both due to its effectiveness and since it makes lessons more successful, experiential, and interesting.

The following is an analysis of the findings connected with the questions that appeared in the opinion questionnaire, according to the following categories:

### Difficulties of understanding the practical assignment

An examination of several participants' comments gives rise to several difficulties: In the following are several comments:

> *"At first it was hard to choose a picture relevant to the lesson, its goals, and the target audience. To what degree will I succeed in choosing a suitable picture, that will encourage the students to be more active and significant in the lesson and make the lesson significant and interesting."*

> *"I didn't have any prior knowledge of the importance of using pictures in teaching geometry, as a means of achieving significant learning. I felt confused. The fact that it was my first attempt to construct a lesson for thinking in order to achieve the educational goals; however, the guiding lecturer's guidance and instruction helped in preparing the lesson."*

*"When proceeding to choose a picture, all kinds of questions and dilemmas arose with regard to delivering the lesson for 45 minutes. However, after meeting with the guiding lecturer for guidance and consulting I found the will and courage to succeed and overcome the difficulties."*

*"The difficulty I encountered was the fact that the course's connection with Arab-Islamic civilization, and my being of a different faith. However, the beauty of the pictures chosen from the world of pure Islamic art enabled me to overcome this difficulty, by accepting the other's civilization."*

**Type of picture and reasons for choosing it**

Forty-five of the participants chose pictures with an affinity to their identity and civilization and to clear social values. For example, the Kaaba in Mecca, the Taj Mahal in India, the pyramids in Egypt, the Dome of the Rock, the Hisham Star in Jericho, and more.

Participants noted several reasons for choosing a picture:

- The choice of picture is connected with the spiritual-moral aspect, holy sites, and pure Arab-Islamic art.
- The choice of picture derives from appreciation of Arab-Islamic art and its holy sites.
- The chosen picture symbolizes a value held in esteem by the student or civilization, such as respect for art, progress, modesty, equality, accepting the 'other', and more.
- The chosen picture strengthens the connection to a religion, or affinity to the other's civilization and heritage.

**Contribution of the use of pictures to professional development**

Both groups participating in the study related to the same contribution, noting that the use of pictures in teaching geometry is considered a means of realizing significant learning. This tool increases students' interest in the class, more than the traditional frontal method. One of the teacher's noted in his evaluation: "This course gave me a great deal professionally, and familiarity with a new teaching aid that may be integrated in the teaching of geometry. Another teacher noted: "Without flattery, I can say that without this course I would not have become familiar with this tool." Another teacher said: "This course was of high quality and I really learned a great deal about how to use technological tools in conducting the lesson and delivering the subject matter in a different way, thus making the lesson more significant and experiential. Inclusion of pictures as a teaching aid assists in getting students' attention in class, arousing their senses, and increasing their motivation and desire to learn, more than the traditional frontal method."

**Contribution of the use of pictures to the class atmosphere**

From the participants' standpoint, it was found that choosing the relevant means of delivering the subject matter in geometry improves the class atmosphere, reduces discipline problems, and strengthens students' persistence in regular attendance in class. This method is characterized by the fact that it arouses the student's visual senses and spurs him to more focused and challenging learning, as opposed to traditional methods.

**Contribution of the use of pictures on the moral level**

in addition to the study content, this teaching method contributes to strengthening the student's social and human values; for example, cultural pride, sense of belonging, tolerance, and acceptance of the 'other'. Furthermore, it contributes to strengthening his identity and knowledge of his heritage.

**The creative teacher**

The participants were asked to define a significant teacher from their standpoint. The following definitions were given:

- The creative teacher is the one who can open his students' cognitive horizons, arouse active participation in class and cultivate positive traits among them, such as curiosity, diligence, and self-esteem.
- The creative teacher is the one who chooses the relevant means of conveying the material to students, and at the end of each lesson makes a personal assessment and reflection of the degree to which the lesson's goals were applied in the activity (before, during, and after). The participants added the following statement to this definition: "What counts is the quality of the teacher and not the method. A significant and successful teacher

is one who chooses the relevant means of delivering the material in practical terms and leads to discussion and students' involvement in class."

- The creative teacher is one who develops professionally and becomes more creative by preparing activity that encourages creativity among the students and applies high order learning strategies such as asking questions, solving problems, and more.

**The lecturer's contribution**

The participants noted that the lecturer successfully delivered the course: Arab-Islamic Civilization in an accessible way, by including technological tools in order to achieve significant learning, which contributed greatly to the participants' career development. Furthermore, the lecturer enriched their world with knowledge on Arab-Islamic civilization and its contribution to humanity. Furthermore, they noted that the teaching method of using pictures in teaching geometry is considered as a more effective means than the traditional frontal method.

## Conclusion

The findings of the study attest to the fact that the teaching method of using a photographic image associated with the people's civilization is considered as one of the most effective means of teaching geometry, and preferable to the traditional frontal method for the following reasons:

- Teachers in retraining and regular students in their fourth year of studies constitute a relevant group for training and guidance in integrating innovative and challenging means of teaching mathematics to achieve significant learning. This is because, on the one hand, retraining teachers seek to develop professionally and, on the other hand, regular students in their fourth year are supposed to begin teaching at schools. Both groups see the importance of having an elaborate toolbox to assist them in delivering the content to students in an interesting and experiential way, mainly in teaching geometry.

- The participants in the study reached the conclusion that the picture chosen as a means of realizing significant learning in teaching geometry should be relevant to the curriculum, the target audience, the subject matter, and the lesson objectives. In addition, they have learned that in order to make the lesson experiential, interesting, and comprehensible, the students need to be involved in the discussion about what they viewed, in order to raise their intrinsic motivation and spur them to active participation. The discussion will contribute to the students' development from the cognitive, emotional, social, and moral aspects.
- Both teachers in retraining and students in their fourth year of study attest to having gotten a golden opportunity to learn about integrating a visual element associated with their civilization. This has strengthened their belief in the effectiveness of using pictures in teaching in general, and in mathematics and geometry in particular.

During observation by college instructors in the lessons of the new teachers during the intensive one-week trial period at the college (in which all teachers and students prepare and deliver lessons), and from checking the lesson plans it is clear that the teachers and students have chosen to use pictures in their lessons. As a result, the students responded to it and demonstrated active participation in class. In addition, the students solved geometry problems as an experience and with pleasure.

The teachers in retraining and regular students in their fourth year of studies noted: "In order to integrate pictures in teaching and implement the method in schools, access to the following means is required:

- Computer rooms equipped with the necessary infrastructure, such as an internet network and various software programs, to enable searching for pictures in various databases and websites relevant to the subject matter.
- An interactive whiteboard and projectors for effective delivery of the lesson.
- A hall with proper conditions (air conditioning, suitable furniture, and a large screen) for proper delivery of the lessons.

From the participants' standpoint, the key to success in realization of significant learning is advance preparation and the degree of relevance of the picture as a means for the target audience, the curriculum, the lesson's objectives and content, and selection of the relevant work method through activity before presenting the topic (picture), during the lesson and at the end of dealing with the subject, with consideration for the students' motivation, their diversity, levels of thinking, functioning, presentation of questions, and creativity.

It should be noted that both teachers and students note that the various activities and tasks prepared while integrating the photographic image in teaching geometry contribute to their professional development. In addition, they note that from their point of view, this is the most suitable and effective way of delivering the content. What's more, they themselves chose pictures that expressed their cultural world. This process contributed to increasing the students' motivation, interest, and desire to learn.

The use of photographic images as a visual aid in teaching appeals to students of all ages – from kindergarten to high school. Even at academic institutions, pictures assist in realization of significant learning while making lessons experiential and not boring. The teacher, in addition to his role as guide, has become the mediator between the material and the student, directing and encouraging his students to learn differently while developing creative 'out of the box' thinking, as opposed to that practiced in the traditional frontal method.

It should be noted that participants recommend to their counterparts to integrate photographic images in teaching, as they have found this tool relevant in communicating messages and knowledge to students on a voluntary basis, without coercion. Lessons delivered by this means contribute to realization of significant learning and are unique in their conception of the subject as being integrated with other disciplines such as civilization, heritage, history, religion, art, architecture, languages, and more, conveyed in an enjoyable and experiential way.

From our point of view, the advantage of the use of pictures both as a cultural treasure and as a means of deepening the understanding of the place's value. The study of mathematics may deepen students' appreciation for cultural treasures (for example, appreciating the magnitude of the cultural achievement) and the cultural context can give real motivation to practice and deepen involvement in teaching mathematics.

## Recommendations

The model of using photographic images in teaching mathematics in general and geometry in particular can be used by teachers as an alternative tool for learning and teaching, creation of teaching units, and development of a teacher's training course for their professional development. Furthermore, we recommend that lecturers at academic institutions integrate this tool in their curricula, in the teaching of mathematics, as a means of introducing students to a successful, interesting, and experiential method that may increase students' motivation to study mathematics.

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