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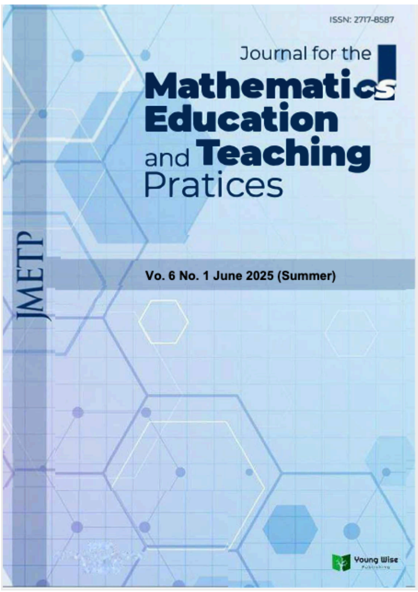

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Math Teaching Practice

Explorations with patty paper focusing on polygons: properties and area measurements

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Abstract

Doing mathematics is not an activity such as applying the rules or methods. Doing mathematics is developing methods to solve problems testing whether the answers given are meaningful or not and being able to model the work of doing mathematics in the real world as accurately as possible (Van de Walle, Karp, & Bay-Williams, 2013). Doing mathematics is a prerequisite for students to recognize meaningful mathematics. Meaningful mathematics can be discovered by students in many different ways and methods. One of these ways paper folding activities. Paper folding is an engaging and educational way to "do math." Therefore, an instructional activity was developed in order to let students and teachers see and appreciate the educational function of these activities. Patty paper folding activities were carried out to explore the properties and areas of polygons. For this purpose, "Patty Paper Folding Activity in the Focus of Polygons" (PFA) and "Patty Paper Folding Activity Opinion Form" (PFO) were administrated. The study was held with six graduate students who were also mathematics teachers studying at the Department of Mathematics Education of a state university. PFA was held for 4 hours. The results of the study showed that the patty paper folding activity is a useful and functional tool that can be used in teaching polygons. After the implementation of the activity, it was seen that teachers generally did not have difficulties with the patty paper folding activities; they only needed the researcher's guidance in using the initial assumptions of patty paper and emphasized that the planning of folding activities should be detailed; the order of instruction is important and more time should be given for activity tasks. All of the teachers stated that the patty paper folding activities were beneficial and they willingly made the applications in discovery process. They stated that they intend to use these activities in their classrooms.

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Introduction

"What does it mean to know a topic in mathematics?" The answer lies beyond applying the rule/formula or method related to that topic. Most mathematics courses are presented to students with a limited understanding that will not allow them to know what they learn. In order for students to know in mathematics, they must be given the opportunity to do mathematics. Doing mathematics means developing methods to solve problems beyond solving a bunch of examples by repeating the methods explained by the teacher, applying them, testing whether the answers given are meaningful by trying to see if these methods lead to a result, and being able to model the work of doing mathematics in the real world as accurately as possible. This begins with providing students with activities to think about and creating

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environments where they can share their thoughts (Van de Walle et al., 2003). When we think of doing mathematics, we think of learning through discovery. Learning through discovery was introduced in the 1960's by Bruner, but Piaget's (Cognitive Development Theory), Vygotsky's (Social Development Theory) is a closely related learning model. According to him, learning by discovery is new knowledge from previous knowledge and active experiences knowledge-building, is a teaching approach and reflects a constructivist perspective. Bruner (1961) focused on the intellectual intelligence of the individual and built four foundations from the experience of learning through discoveries. Accordingly, the first was the "Intellectual potency". After a series of studies, he concluded that discovery in learning makes the learner to be constructive, to organize what he encounters not only to discover order and relationality, but also to organize the knowledge in a good way and is a necessary condition for learning a variety of problem-solving techniques. Secondly, "Intrinsic and extrinsic motives" take place. To free the child from immediate control of environmental rewards and punishments is important. The child must be put in a position to experience success and failure not as a reward and punishment. Thirdly, "Learning the heuristics of discovery" is important. This is the process of trying to find out something in a process of problem solving that converts a puzzle form into a problem that can be solved in a way that gets the child where she wants to be. It is a way that recasts the difficulty into a form that one knows how to work with. And the fourth is "Conservation of memory", that is not storage, but retrieval. "*In sum, the very attitudes and activities that characteize "figuring out" or "discovering" things for oneself also seem to have the effect of making material more accesible in memory*" (Bruner, 1961, p. 9). This constructivist perspective, combined with learning by discovery, paved the way for the development of the concept of meaningful learning in mathematics education. In fact, the "Meaning Theory" introduced by Brownell led the way to the full recognition of the value of the childrens' experiences and studied on how to make mathematics meaningfull (Viewer & Suydam, 1972). Brownell (1954) in his investigations about the revolution searched for a better organization of content for better methods of teaching also emphasized learning through discovery. According to Brownell (1954), to be intelligent children had to see sense in mathematics and for this, instruction had to be meaningful organized around the ideas and relations inherent in arithmetic as mathematics. Besides, he added: "*They must also have experiences in using the arithmetic they learn in ways that are significant to them at the time of learning, and this requirement makes it necessary to build arithmetic into the structure of living itself* (Brownell, 1954, p. 5). The modern mathematics movement at that time sharpened and extended a focus on mathematical meaning and meaning has began to be recognized as a vital component in teaching and learning process (Viewer & Suydam, 1972). The emphasis on experiences and heuristic learning in teaching led to the inclusion of the use of manipulatives in teaching. Kennedy (1986) in his study mentioned that many teachers began to believe strongly in the benefits of manipulative materials in building a foundation for students' mathematical concepts. He said that in the constructivist point of view of Piaget and Skemp; mental images and abstract ideas of students are based on their experiences and manipulative materials are learnings aids in all four stages of constructivist theory. So, students learning with manipulatives have clearer mental images and represent their abstract ideas better than those who don't. Dienes also holds the idea that students should use manipulatives in learning. Learning theories advocate that student whose mathematical experiences grounded in manipulative experiences are likely to bridge the gap between the world in which they live and the abstract world of mathematics (Kennedy, 1986). From this perspective, paper folding is one of the most suitable manipulative materials for discovery learning and provides a suitable learning environment for doing mathematics which is embedded in the persoective of constructivist theory. Paper folding is not a new method or way of thinking and many researchers studied to make links and relationships between thking geometry and folding papers. T. Sundara Row in his writings about the geometric exercise in paper folding in 1893; mentioned that paper folding was a way to afford mathematical recreation to all people in an attractive and economical form. This geometry was not rigidly bounded to folding as the Euclidean Geometry was to compass and ruler study, his work showed that many relationships could be illustrated without ruler and compass by using papers (Row, 1917). At the same time, Wiener's folded paper models were exhibited in his manuscripts and formed an operative geometry of folding (Friedman, 2016). Serra (1994) after his geometry studies (in his book of discovering geometry); introduced Patty Paper Geometry with open investigations to encourage students to explore their own methods of discoveries by using another form of paper folding. Haga (2008) later in his book: “Origamics” showed several ways of folding with explorations. He exhibited the art of origami (paper folding) and its manipulative and experimental nature with relating it to Euclidean Geometry via axioms and theorems. So, linking geometry and folding has a history and has no doubt that focuses on exploration and learning by discovery.

Paper folding activities provide students with both enjoyment and the opportunity to make discoveries, justifications, and hypotheses by thinking about them. It is an engaging way to add realism and interest to mathematics teaching and to add active experiences is through paper folding. Creating lines by folding on a sheet of paper is a simple way to demonstrate and explore the relationships between lines and angles. Once a relationship is found by folding paper, formal explanations about the subject do not seem as difficult to students. Therefore, paper folding not only makes learning mathematics easier, but it also develops understanding and appreciation of mathematics and provides the experiential foundation necessary for further learning. (Johnson, 1957, Olson, 1975). Paper folding is an engaging and educational way to “do math”. Paper folding is frequently used in geometry instruction to provide students with a visual representation of geometric concepts such as shapes, properties of shapes, congruence, similarity, and symmetry, to establish cross-cultural connections to mathematical ideas, and to support the development of spatial perception, and the activities discussed align with the National Council of Teachers of Mathematics (NCTM, 2000) in geometry standards (Robichaux & Rodrigue, 2003). Geometry standards emphasize that students from preschool through high school analyze the properties of two- and three-dimensional shapes, develop arguments about the relationships between these properties, and use geometric modeling, spatial reasoning, and visualization to solve problems (NCTM, 2000, p. 41).

Paper folding activities are also an important method for students to comprehend spatial relationships and develop spatial skills (Boakes, 2008). The connection between paper folding and geometry is easy for students or teachers to discover. Because folds and edges represent lines; angles are formed by their intersections. The intersections of folds with themselves form points. Therefore, due to its manipulative and experiential nature, paper folding always has the potential to be an effective content for learning and teaching geometry (Haga, 2008). Patty paper folding is a type of paper folding and, in addition to paper folding, it brings many advantages in teaching geometry

Literature Review

Patty papers used for geometric explorations are light, waxy and transparent square papers. They are used in uncooked hamburger patties, which are called “meatball papers” in restaurants. Serra (1994) states in his book “Patty Paper Geometry” that many of the geometric rules and formulas and the properties of geometric shapes can be discovered by folding patty paper. In this study, baking papers sold in markets were used as “patty papers” and it is recommended to be used in classroom applications because they are more easily accessible and economical. In addition, it should be cut into squares. Any property discovered using compass and ruler can be discovered by folding patty papers and useful for discovering most of the properties in school geometry (Serra, 1994). Thanks to its transparency, patty papers can be used to measure and compare lengths and angles. A triangle equivalent to a triangle can be drawn by placing one patty paper on top of another; the interior angles of a triangle (obtuse, acute, etc.) can be examined by taking advantage of the property that one corner of patty paper is 90 degrees (initial assumpsions); or many different examinations and research activities can be carried out such as creating angle bisectors and side bisectors by making different folds. Many rules and formulas can be proven with patty paper folding activities.

Many studies (Adom &Adu, 2020; Aksoy& Işıksal Bostan, 2024; Bornasal, Sulatra, Gasapo, & Gasapo, 2021; Febriani, Susanti, & Hapizah, 2023; Patkin & Canner, 2010; Subaar, Asechoma, Asigri, Alebna, & Adams, 2010) revealed the positive effects of paper folding activities based instruction on students improvement in an experimental manner.

Adom & Adu (2020) investigated the use of paper folding on the performance of 9th graders on the focus of learning fractions. Quasi-experimental design was used, and they found that there was a significant difference between pretest and post test scores. In other words, instruction with paper folding as a manipulative material had a positive impact on learners’ academic performance in fractions. Similarly, Febriani et al. (2023) conducted their studies on fractions at different grade level. They aimed to evaluate a learning pathway's effect with paper folding activities on fourth graders' comprehension of fractions, with a focus on their comparison and sequencing. According to the findings of the study, it was revealed that student learning trajectory encompassed three principal activities. Initially, students used folding and gluing of paper to discern fraction values. Subsequently, they engaged in coloring and illustrating folds for fraction comparison. The final activity involved drawing, coloring boxes, and fraction comparison and sequencing. Students showed proficiency in understanding and determining fraction values and comparing them yet struggled with ordering certain fractions. The structured learning path facilitated students' understanding of basic fraction concepts, especially in comparing them. Aksoy & Işıksal Bostan (2024) investigated the effect of a paper folding activity on sixth-grade students' concept definitions and concept images of parallelism and perpendicularity. The study investigated how the concept definition and concept images changed after the paper folding activity. The findings revealed that the paper folding activity had a significant positive effect on students' concept definitions and concept images. In addition, the interviews after pre and post tests indicated that the students' personal concept definitions of parallelism and perpendicularity of two lines/line segments began to match the formal concept definitions of these concepts after the paper folding activity. Bornasal et al. (2021) investigated the effect of paper folding (origami) instruction in teaching 8th graders geometry. They used a quasi experimental model and found that both groups achieved better performance through paper folding and non-paper folding instruction. However, the experimental group recorded higher mathematics performance compared to the control group. They concluded that paper folding instruction promoted more effective learning in geometry. Patkin and Canner (2010) explored the extent of influence of learning abstract terms, such as "special segments of triangles", using illustrations of paper folding (of 8th grades). They found that students improved their definition capability and their comprehension of terms learnt in this way, demonstrating a change for the better in their attitude towards the geometry studies. Subaar et al. (2010) in their experimental study aimed to reveal if there was a difference in the performance of students who were taught using sets of objects (sets model) with paper folding activities and withour using them; to solve word problems involving addition and subtraction of proper fractions. The students' level of performance had improved drastically with the help of paper folding method. In conclusion, paper folding activities helped students to appreciate word problems involving addition and subtraction of proper fractions.

There were also a considerable number of studies which have also described the positive effects of folding activities on learning in differents design and methods manner. Robichaux & Rodrigue (2003) in their study reported that teachers who have used origami in their lessons said that students' understandings of the concepts incresed and they were well motivated. The level of understandings of the concepts were also evidents in students' writings. At the end of the foldings, students were able to name all the shapes and able to understand the properties. Gürbüz, Ağsu & Güler (2018) investigted habits of mind of 11th grade students while they were using paper folding activities. Ther searched for the potential of paper folding to improve students' geometric thinking skills and to enhance their achievement in national exams. They concluded that the students were able to reach solutions more easily by concretizing the intangible questions through paper folding. The students were able to comprehend the fact that the main components of triangles didn't change; that was, they were preserved and the students' thinking processes have improved in the study. They have permanently maintained the indicators of geometric habits of mind they have gained. Demirci & Çontay (2023) designed a paper folding activity task, which involved reaching the Pythagorean Theorem with a series of steps. The task was conducted in order to reach deductive reasoning and logical inference. Besides, it was aimed to examine the effectiveness of the task and to share the patty paper folding task with the teachers. The patty paper folding activity task was carried out for a total of 6 lesson hours for three weeks. According to the results of the study, it was revealed that the students did not have difficulty while folding, while they had difficulties in performing algebraic operations and expressing the concepts mathematically. According to the findings, the patty paper folding task helped students understand why the theorem was true and contributed to meaningful learning in terms of being a explanatory proof. Empson & Turner (2006) investigated students' solutions to folding tasks, which involved predicting the number of equal parts created by a succession of given folds and determining a sequence of folds to create a given number of equal parts. Analyzing a combination of cross-sectional data and case studies from standardized clinical interviews, they found first, third and fiftht grade students were most successful at coordinating folding sequences with multiplicative thinking when they used a conceptualization of doubling based upon recursion. This conceptualization tended to generate more sophisticated solutions. Friedman and Rittberg (2021) searched for the ways in which paper folding constituted a mathematical practice and prompted common mathematical activities. They presented the paper folding as a material reasoning practice. They concluded that mathematical paper folding is a reasoning practice which helps understanding of the of mathematical proof. Morye (2025) investigated the symbiotic relationship between mathematics and origami. He explored the utility of origami in education while finding out how origami could become an effective way of teaching methods of geometry because of its experiential nature. Galicha & Lazaro (2022) aimed to studied the level of acceptability of a research-based supplementary learning material in geometry for six grade students. The mentioned paper folding based material was found a useful learning aid. It was found that the paper folding material possessed adequacy, clarity, suitability and usefulness. Van Wijk, Bos, Shvarts and Doorman (2023) investigated what reasons did teachers from France and Germany report for implementing mathematical folding activities in authentic classroom situations. Study showed that the reasons were “to activate students by letting them manipulate paper” and “to visualise mathematics”. Teachers stated that folding allowed for dynamic representations that supported the transition from informal to formal mathematics and the practice of skills. They memorized these findings n two classifications: “to activate students by providing folding tasks” and “to grow mathematical understanding by folding”. Boakes (2008) discussed how the use of Origami-mathematics lessons implemented into a geometry unit impacted students’ math and spatial skills. She intended to provide with a perspective on using Origami in the mathematics classroom and concluded that the act of folding an Origami model holded great potential in the classroom in many ways like growing spatial abilites and knowing concepts, or engaging students in mathematics in a more exiting way. She reported that origami-mathematics lessons blended a variety of approaches to instruction in a way that students benefited mathematically.

Puspose of the Instructional Activity

The purpose of this study is to present an instructional activity that will demonstrate the use of patty paper activities that can be used for students to “do mathematics” for meaningful learning” in classroom applications. For this purpose, it was deemed appropriate to test the application that will be designed for students on teachers. For this purpose, it was planned to reveal whether this activity was useful under the guidance of teachers and through their eyes and to provide inferences about the education they will provide to their students in the future. In short, this study aims to test the teaching practice designed with patty paper activities on teachers and to provide inferences about patty paper activities for both teachers and researchers.

Focus of the Instructional Activity

The most effective method of classroom teaching in patty paper geometry is cooperative learning, and the best group structure is recommended as pair-sharing. Here, students are divided into groups of four and divided into two pairs. While one student from each pair does the joint folding, the other student reads the instructions. Then the pairs compare their results with the other pair in the group of four. Each pair creates and shares their assumptions. For the next research, the students who fold and the students who read the rules switch places. This cooperative group structure helps all students share the excitement of learning by discovering. Therefore, it helps students reduce their mathematical anxiety and provides permanent learning through discovery (Serra, 1994).

This study was conducted with teachers who were graduate students at the same time; it was thought that working with teachers in pairs would support them to teach in a double-shared structure in their classes. Patty paper explorations in this study were carried out with the focus on polygons. Focus on polygons has existed since the Babylonian and Egyptian mathematical writings, which are considered the first mathematics. So much so that; geometric shapes have been used since ancient times. The need to clarify the land boundaries with the overflow of the Nile River in ancient Egypt led to the necessity of calculating the areas of polygons with simple measurements. It later became systematic with Greek mathematics and has become a subject that we still encounter today in architecture, geography, art, nature and many other fields (Ulusoy, 2022). When the middle school mathematics curriculum of the Republican period is examined (MEB, 2024), it is seen that the concept of polygon is one of the most intensively included concepts in the curriculum. Polygons have been heavily included in all programs since 1938 (Yavuzsoy Köse & Özen Ünal, 2020). Therefore, it was deemed important to examine the subject of polygons, which is frequently included in middle school curriculum, with the focus of using patty paper activities.

Teachers' Readiness for Folding Patty Paper

Teachers who participated in the study voluntarily were students studying in master's program of the mathematics education department of the university where the researcher studied. The participants had previously participated in the TUBITAK 4004 project in which the author of the study was an educator and, in a workshop, organized by the author at an international congress on mathematics education, and they were familiar with the activities of folding patty papers on different subjects. Therefore, it was thought that they had relevant prior learning.

Information About the Implementation

This study was carried out in 4 lesson hours with 5 female and 1 male mathematics teachers who were master's students in the Department of Mathematics Education of a university in the Aegean Region. In the selection of the participants in the study, it was desired to study with mathematics teachers who were familiar with folding patty papers. As mentioned above, they were familiar to the paper folding activities. In addition, the fact that the teachers were also postgraduate students was effective in their appreciation of the effectiveness and their willingness. From this point of view, although it seems that the purposive sampling method was adopted in the sample selection, the fact that the teachers were graduate students in the author's department played a role in this selection. From this point of view, it can also be considered that it is close to the convenience sampling method. However, from both perspectives, this study sought to meet a variety of interests, multiple purposes and needs; therefore, hybrid sampling, one of the qualitative research sampling methods, was used (Baltacı, 2018). The data collection process was carried out in the mathematics classroom in a suitable environment in the main mathematics department. Since the participants were familiar with folding patty papers, they were not given information about how to use it. The purpose of the study was stated, and the study was carried out smoothly. The activities were carried out in pairs for the discovery of patty paper geometry. Patty Paper Folding Activity Focusing on Polygons (PFA) was conducted. Teachers were given a task with written instructions on what to do and they were also given additional explanations via powerpoint when needed in order to guide the teachers while exploring the properties. Teachers were in contact at every stage of the activity. The teachers first discussed the folds they made within the framework of learning by discovery, shared their ideas among themselves (in groups of two; in pairs), then each group individually explained to the other groups how they made the folds at each step. How these folds and discoveries could be made from the students' point of view was discussed and different ideas were shared with other groups. They were enabled to take part in an independent discovery process but were also guided. In this way, they were able to gain different experiences in the discovery processes they would carry out with their students and discuss how to build new knowledge on their students' old knowledge. They were encouraged to discuss to what extent and from which knowledge to start, thus enabling them to develop intuition about how to scaffold their stundets' knowledge. During the implementation, the teachers and their actions were audio and video recorded, and the folds they constructed were photographed. At the same time, observations were made, and notes were taken. In the light of all these data, the application was analyzed with a descriptive approach. Observation notes were analyzed in detail; POF was analyzed by considering common statements. This study focuses on the extent to which teachers perform multiplication rather than their performance and whether the PFA works effectively.

During the implementation, Square-cut patty papers sold in markets were used and compasses and rulers were not used for any measurement purposes (Any property that can be discovered using compasses and rulers can be discovered by folding the patty papers (Serra, 1994). Compasses were used to draw circles, and rulers were used to mark the folds properly. In this study, "Patty Paper Folding Activity Focusing on Polygons" (PFA) (Figure 1) and "Patty Paper Folding Activity Opinion Form" (POF) were applied. PFA was applied to teachers in groups of two. PFA consisted of two parts. In the first part, discovering the properties of certain polygons were focused, and in the second part, calculating the area measurements of the polygons whose properties were discovered took place. In the first part, folding activities related to triangles and their properties, auxiliary elements of triangles were carried out and Euler Line was formed. Then, the properties of parallelogram, rhombus, deltoid, rectangle and square were discovered. In the second part, the area formulas of triangle, rectangle, parallelogram and trapezoid were obtained through patty paper folding activities. (Figure 1)

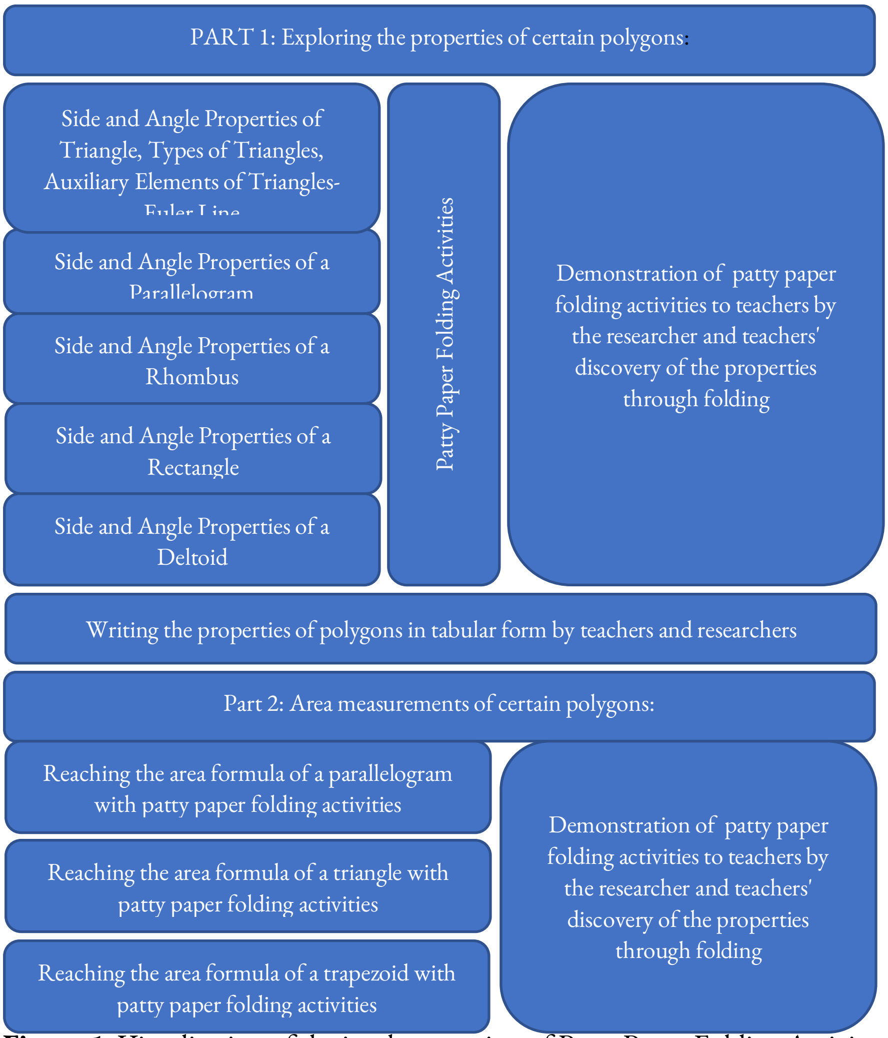

Figure 1. Visualization of the implementation of Patty Paper Folding Activity

After the implementation, POF was directed in a written form to determine the teachers' opinions about the activity. Questions in the POF were as follows:

- Do you find the patty paper folding activities useful? Why? Please explain in detail.
- Would you consider applying patty paper folding activities in your own classrooms? Please explain.
- Can patty paper folding activities be adapted to every math topic? Why?
- Would you recommend patty paper folding activities to your fellow math teachers?
- What areas do you think need improvement in the activities we conducted?
- What part/topic of the wax paper folding activities did you like most or find most useful?
- Is there anything else you would like to add? Please write and explain if applicable.

Limitations

Due to time constraints, there were limitations in the detail of information obtained from teachers. Not all teachers have the same knowledge and skill levels and pegadagogical competencies. While these differences were useful in terms of revealing the different implementations of PFA, on the other hand, imbalances in terms of time and implementation may have occurred between one group of teachers with different lower levels of pedagogical content knowledge and the other group. For this reason, each teacher's way of implementing this activity on their students may differ. Therefore, the learning outcomes for each teacher in this study may differ.

Implementation

In the first part of the PFA, teachers discussed how they could introduce the angle and side properties of triangles to students using patty paper. In this way, the initial assumptions of patty paper (patty paper being square, therefore a corner angle measure being 90º; all side lengths being equal and these side lengths being usable in measuring) were discussed with the teachers. Then, the question; "How do we fold triangles according to their angles and sides?" was asked to the teachers. All groups made triangle folds according to their angles and sides. One group first started folding with isosceles right triangles from special triangles; when asked why they started with a special triangle, they answered "we thought it would be easier to find". From here, it can be concluded that folding behaviors about reaching generalizations are related to starting from special situations. This group stated that they had difficulty in creating an equilateral triangle.

They used the expression "We though about creaing triangles by forming a hexagon and using a square." It was thought that the reason for this problem was that the teachers did not use the initial assumptions of the patty paper. So much so that; the other two groups did not use the initial assumptions in the same way. The second group stated that they started by folding a scalene triangle because they could compose the most irregular and random ones by acting with the behavior of reaching the correct generalization compared to the first group. They formed a scalene triangle with three different foldings, then they said that they composed the axis of symmetry by first determining a line and then lowering the isosceles triangle perpendicular to that line from a point outside it and obtained a triangle with a third folding (the exercise of lowering a perpendicular from a point outside a line to that line was experienced with the group before. For this, while folding a line; folding another line from a point outside it to that line, it is necessary to pay attention to the overlapping of the lines in hand. In this way, the right angle is divided into two to form a right angle and this right angle is formed by the line folded from that point to the line). For the relevant folding, see. (Çontay, 2018). Teachers stated that they used the height of the isosceles right triangle as the axis of symmetry. They stated that they constructed the equilateral triangle by forming three axes of symmetry in a similar manner after they started folding the isosceles triangle (Figure 2).

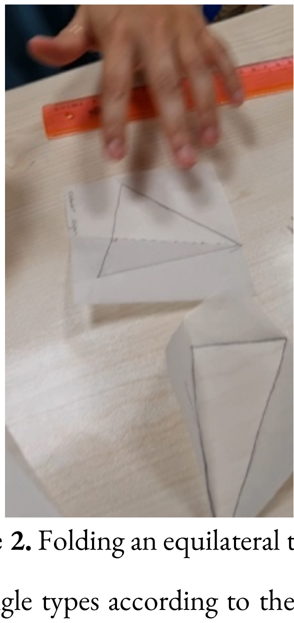

Figure 2. Folding an equilateral triangle

While these two groups started with triangle types according to their sides, the other group started with triangle foldings according to their angles. First, they formed the right triangle by using the folding of the perpendicular distance of a point to a line; then they formed the right triangle. They told how they formed the acute-angled triangle with the following expression:

> "*We drew a line segment with an angle smaller than the right angle, but we determined the right angle of this one in case it was obtuse or right angle, by folding it over and over again, to draw an angle smaller than this, we drew a narrower angle like this, all of them were acute, by testing the right angle, it turned out to be narrow when it was smaller than that, we did the same thing for the width, this time we chose an angle wider than the right angle, when*

we combined it, we didn't feel the need to test it because since it was an obtuse angle, it is certain that both of them would be acute."

The researcher stated that it would be easier for teachers to act with their initial assumptions after this stage. The researcher reminded them that they could use two sheets of patty papers with their transparency and benefit from their squareness. After this reminder, the teachers were able to easily identify right-angled, acute-angled and obtuse-angled triangles by placing one sheet of patty paper on top of the other (Figure 3).

Figure 3. Using the transparency Property of patty paper

All groups used the initial assumptions of the patty paper and folded the triangles by varying them according to both their sides and angles. Meanwhile, the second group developed a new strategy while finding the equilateral triangle according to its angles. They formed an angle of an equilateral triangle by folding one corner into three without measuring and using the transparency feature of patty paper, then they showed the equality of the other two sides by folding the angle measurements of 60° from the fold lines of the equality, using the axis of symmetry (Figure 4).

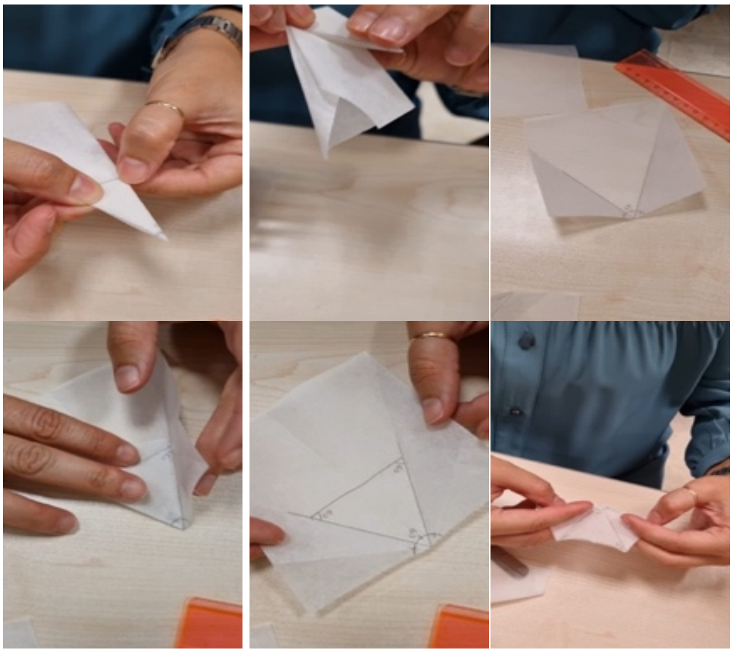

Figure 4. Folding an equilateral triangle by taking advantage of the transparency and squarenedd properties of patty paper

In the next folding activity of PFA, teachers were asked to find the auxiliary elements of the triangle (perpendicular bisectors, angle bisectors, medians and altitudes and their intersection points). For this, the same triangle would be used, so they were asked to fold and draw four identical, acute-angled wide scalene triangles. Teachers folded the perpendiculars of the triangle in groups of two. One group measured the distance from the intersection point to one corner of the triangle with a second piece of patty paper and placed this second piece of patty paper on top of the first one, measuring this length from the intersection point of the perpendiculars of the triangle to the other corners, and they saw that the lengths were the same (Figure 5).

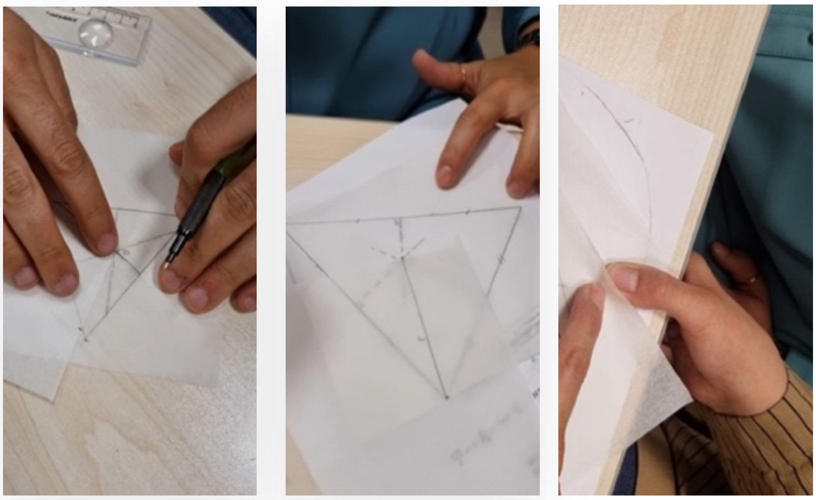

Figure 5. Finding the intersecion point of the perpendicular midpoints of a triangle

The second group also made the same measurement and stated that these were radius measurements because they were the same lengths. Thus, they concluded that the intersection point of the triangle's perpendiculars was the center of the circumscribed circle. The teachers were asked how they could teach this to students if they wanted to. (It was said that although the teachers could see that the measured length was the same as the radius, the students would not be able to see this, so the teachers were asked to assume that the students knew the definition of a circle). The first group performed measurements using a ruler; it was then reminded that there was no need to measure with a ruler when using patty paper. Teachers placed the pointed end of the compasses at the intersection of the perpendicular bisectors, continued by measuring the distances to the vertices of the triangle, and constructed the circle (Figure 6).

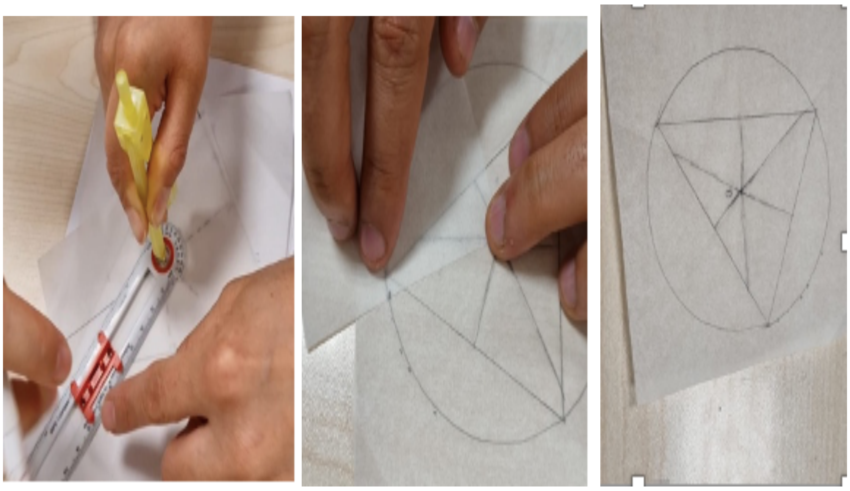

Figure 6. Finding the center of the circumcircle of a triangle

Teachers formed the points of intersection of the angle bisectors. One group folded a perpendicular from the point of intersection of the angle bisectors to one of the sides of the triangle without using a compass (Çontay, 2018); they marked the cases where this length was the same on the other sides (Figure 7).

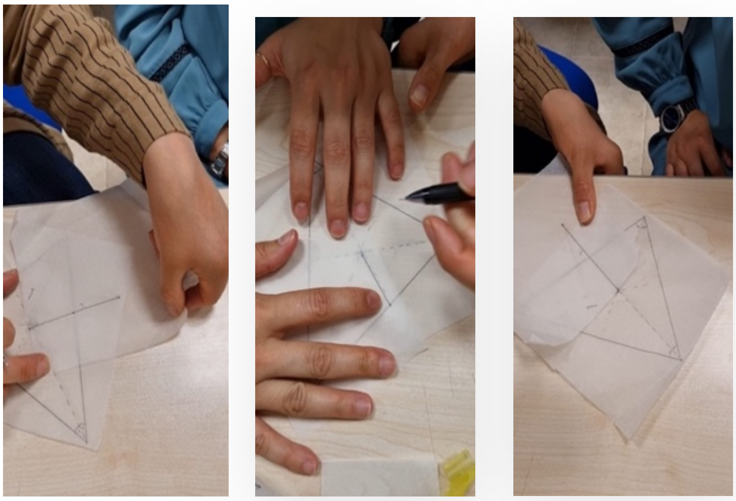

Figure 7. Finding the intersection point of the angle bisectors of a triangle

The other two groups reached the radius measurement by marking with compasses. They wrote their assumptions by saying that this circle is an inscribed circle (Figure 8).

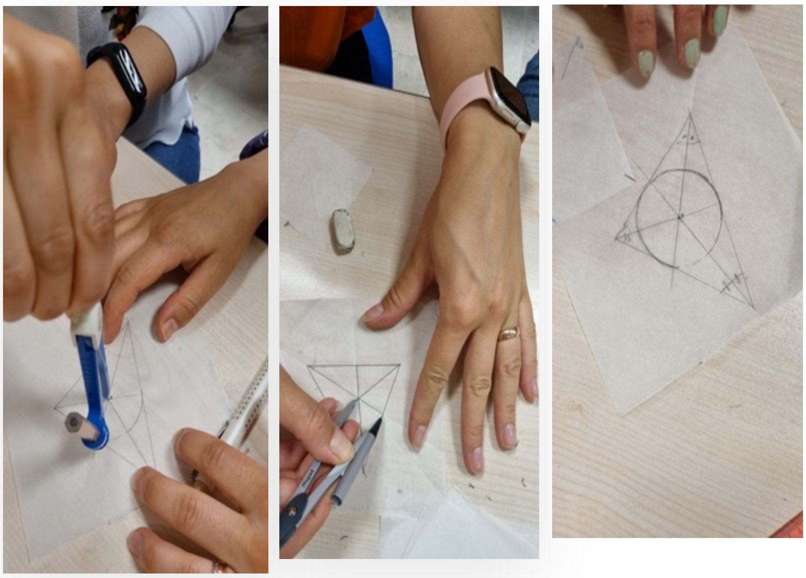

Figure 8. Finding the inscribed circle of a triangle

In the next stage, the teachers folded the medians of the triangles drawn identically to the other triangles on another sheet of patty paper and found their cutting points. After this, the teachers were given a thicker cardboard on which to copy the patty paper. They marked the same triangle and its medians on this cardboard and the teachers were asked how they could find the center of barycentre of the triangle (with the pencils in their hands). Thereupon, the teachers determined that the center of barycentre of the triangle they copied was balanced by placing the barycentre on the tip of the pencil. The teachers were asked how they would find the ratio in which the medians divided barycentre. All groups marked the length of the lower part of the barycentre on the patty paper with the help of another sheet of patty paper and wrote their assumptions by measuring that the upper part was twice the lower part. Here, the transparency property of the patty paper helped them make the measurement (Figure 9).

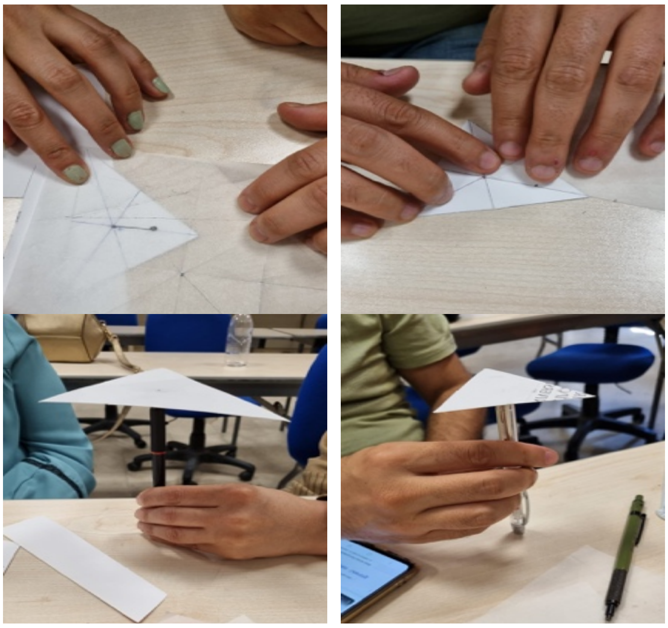

Figure 9. Discovery of properties related to the intersection point of the medians of a triangle

Then the teachers were asked to copy the same triangle onto another piece of patty paper and fold the heights of the same triangle to find the intersection point of the heights. The teachers reached the heights by finding the shortest distance from a point to a line. They folded the height from the point they took at one corner of the triangle to the opposite side of the triangle as the shortest distance; while doing this, they paid attention to the fact that the opposite side coincided while folding and formed a right angle (Figure 10).

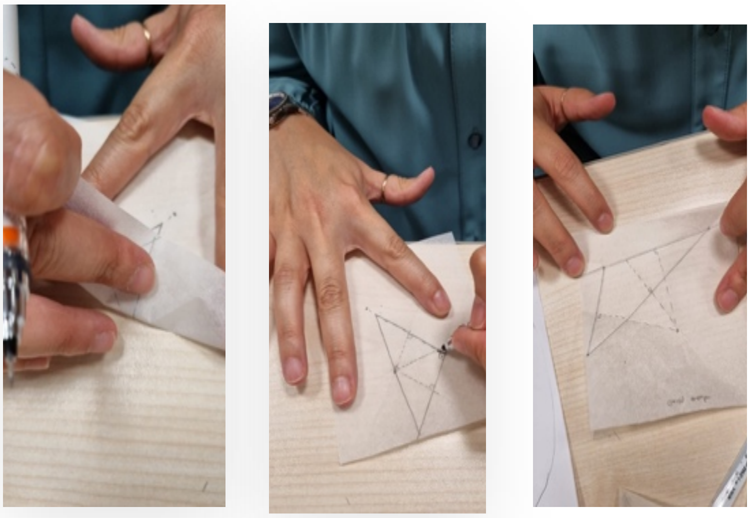

Figure 10. Finding the intersection point of the heights of a triangle

The teachers were given time to find the Euler line. When the teachers were asked what the Euler line was, they stated that they did not know. The teachers were asked to put the identical triangles on top of each other and it was stated that the points they found (three out of four points) had a property. Due to the transparency property of the patty paper, all groups concluded that the intersection points of the medians, medians and altitudes could be on a side-by-side line. All groups determined the Euler line (Figure 11).

Figure 11. Constructing the Euler Line

After the foldings and measurements related to triangles were completed, folding activities were carried out with the teachers to discover the properties of certain quadrilaterals. The groups were given two rulers, one thick and one thinner than the other, to fold the properties related to rhombuses and parallelograms. First, the teachers were asked to draw a rhombus and a parallelogram using these rulers (assuming that both sides of the rulers are parallel to each other). The groups drew the rhombus using a single ruler and the parallelogram using two rulers (Figure 12). Later, the teachers were asked to define the rhombus and parallelogram, and patty paper activities were conducted on their properties. The groups revealed the properties of the rhombus and parallelogram by folding patty paper along their diagonals, sides, and angles and performing measurements.

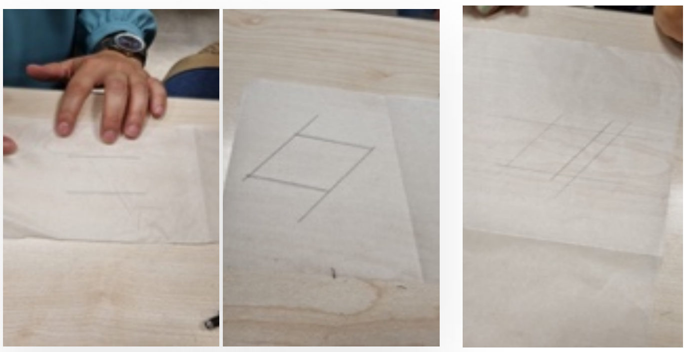

Figure 12. Constructing the rhombus and parralellogram

They folded the rhombus along its diagonals and then demonstrated in various ways that these diagonals bisect each other perpendicularly. For example, the second group folded the diagonals twice to form a right triangle, showing that the rhombus is composed of four right triangles. Another group divided the rhombus into two equal parts, dropped a perpendicular from the vertex to the diagonal, and showed that this line coincided with the diagonal.
In the case of the parallelogram, they showed that the diagonals do not intersect perpendicularly, are of different lengths, but still bisect each other. They folded the diagonals, then placed another piece of patty paper over the existing one to compare the lengths, demonstrating that the diagonals bisect each other (Figure 13).

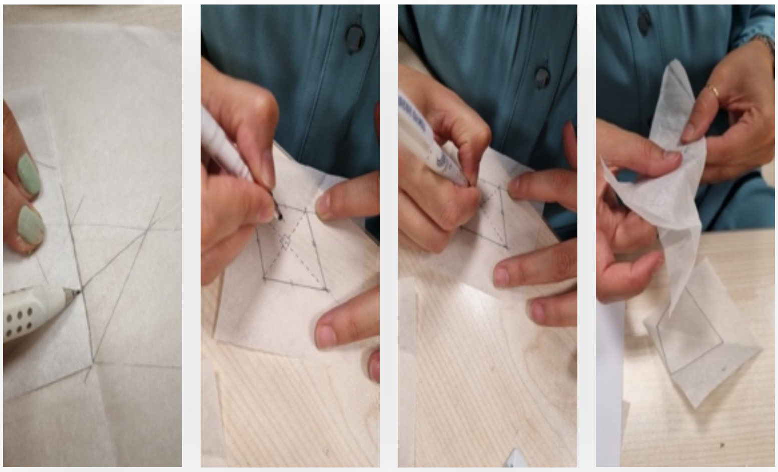

Figure 13. Diagonal properties of rhombus and parralellogram

Teachers have discovered properties such as the sum of adjacent angles of a rhombus and parallelogram being supplementary, and the opposite angles being equal, by measuring with another piece of patty paper (Figure 14).

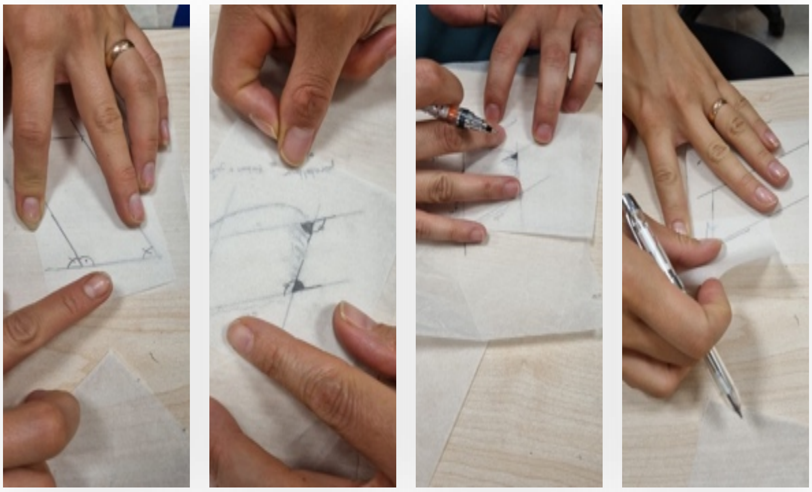

Figure 14. Angle properties of parralelogram and rhombus

Later, the teachers made folds to explore the properties of the rectangle. First, they used a ruler to form the parallel long sides; then, they folded perpendicular lines to the parallel sides of the long sides (when they folded the other long sides on top of each other, a right angle was formed, dividing the shape exactly in half (Figure 15).

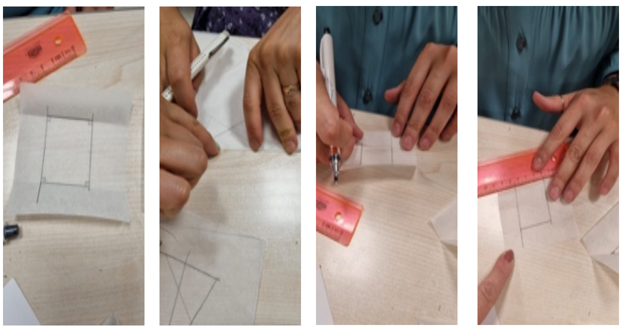

Figure 15. Discovery of the properties of the rectangle

In the next step, the teachers proceeded with the researcher's statement, "Let's form a quadrilateral with two pairs of equal adjacent sides," to form the deltoid. The teachers, using an experimental approach, made observations about the angles, and with another piece of patty paper, they discovered the angle bisectors and perpendicularity conditions. They also discovered the equality of the angle measures using the patty paper again (Figure 16).

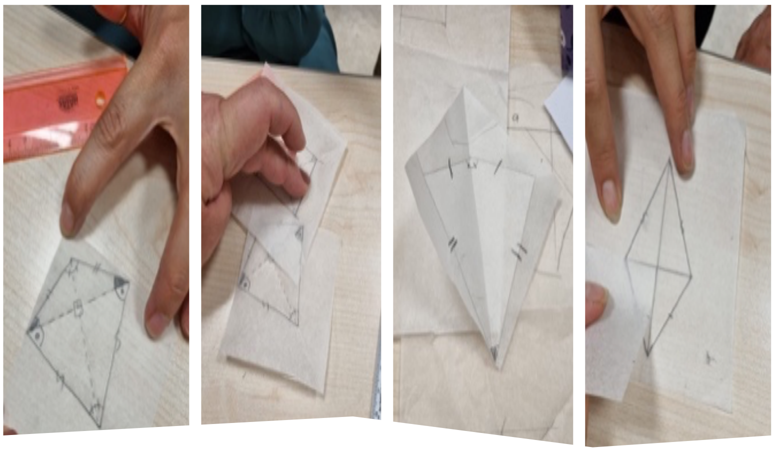

Figure 1. Discovery of deltoid and its properties

After this stage, the second part of the lesson plan shifted to area measurements. First, teachers were asked how they taught the area of a rectangle to their students. Teachers explained the reason why the formula for the area of a rectangle long side times short side with expressions is such as "*repeated sum of width and length*" and "*repeated sum of length and width*"; "*sum of unit squares*", "*covering*". Then, teachers were asked to reach the formula for the area of a parallelogram based on this. All groups drew the area of the triangle they created based on the short side inside the parallelogram on another piece of patty paper and moved it to the side where the other short side was and showed that the area did not change by creating a rectangle. Thus, they reached the conclusion that the area of a parallelogram with the same base length and height is the same as the area of a rectangle (Figure 17).

Figure 17. Finding the area formula of a parralellogram

In the next step, the teachers were asked to use the area of the parallelogram to derive the area of the triangle and to start by folding the triangle onto patty paper. Following this, the teachers copied an acute-angled triangle onto another piece of patty paper, flipped it, and joined the two halves to form a parallelogram. They easily demonstrated that the area of the parallelogram is twice the area of the triangle (Figure 18).

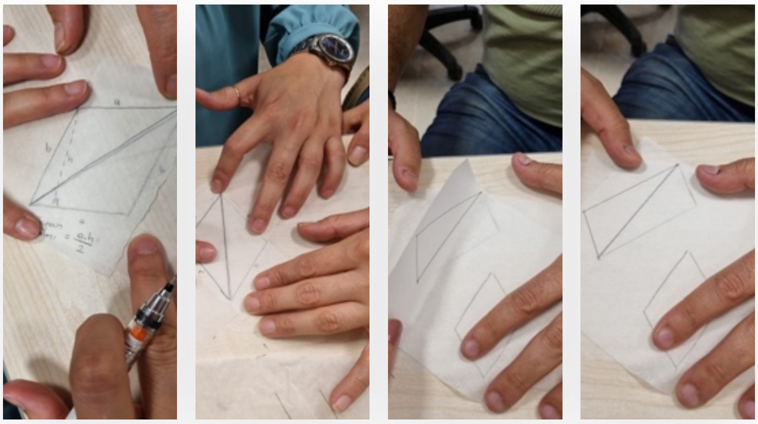

Figure 18. Finding the area formula of a triangle

In the final stage, the teachers were asked to use the formula for the area of the parallelogram to derive the formula for the area of the trapezoid. The teachers were asked to draw a trapezoid. All the groups, in a similar manner, copied the trapezoid onto another piece of patty paper and placed it by flipping it next to the original trapezoid. They easily observed that a parallelogram was formed and concluded that the area of the trapezoid is half the area of the parallelogram, thus deriving the formula for the area of the trapezoid.

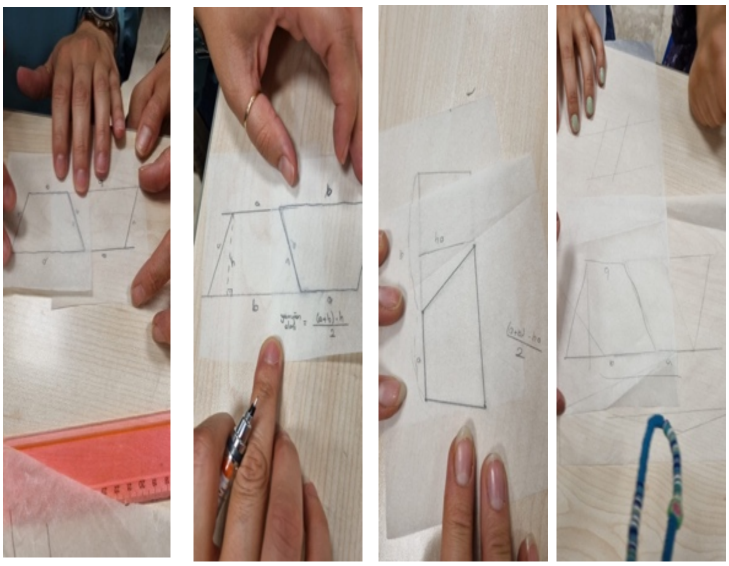

Figure 19. Finding the area formula of a trapezoid

Conclusion and Recommendations

In this study, patty paper folding activities (PFA) were conducted with mathematics teachers, focusing on the properties of specific polygons and area measurements. In general, it was observed that the teachers displayed a positive attitude during these activities, eagerly performed the folds, and enjoyed the discovery process. The fact that teachers find folding activities useful is an important factor in the implementation of these practices in their classrooms. Parallel to this assumption; Robichaux & Rodrigue (2003) concluded that teachers who have used origami in their lessons mentioned that students' understandings of the concepts incresed and they were well motivated. Van Wijk et al. (2023) in their study said that teachers who participated their study reported that folding allowed for dynamic representations that supported the transition from informal to formal mathematics and the practice of skills.

The activities were carried out in pairs for the discovery of patty paper geometry. It is suggested that these activities be conducted in pairs with students, as recommended by Serra (1994), through shared pairing. For future studies, it is recommended to conduct these activities with middle and high school students and, if possible, with larger samples. Since the activities were conducted with teachers, there was no need for shared pairing. It was determined that the teachers needed support to use the initial assumptions of the patty paper. For example, they were asked to start the folds by assuming the patty paper was a square, but from the first folds onwards, even though they knew that one of the corners of the patty paper formed a 90° angle or that the sides were equal, they preferred more complicated methods to discover these properties. For instance, instead of using a corner to form a right angle, they chose to fold perpendicularly by considering the distance of a point from a line. Later, with the researcher's guidance, they began using the properties of the patty paper more effectively. For example, after guidance, they made folds to form a right angle by placing one piece of patty paper over another, utilizing the transparency property of the patty paper. The reason the teachers did not immediately use these properties at the beginning is believed to stem from their limited experience with making assumptions in classroom teaching. Therefore, it is recommended that preparatory activities emphasizing the use of these properties be conducted at the beginning of patty paper folding activities.

It was observed that as the teachers gained experience in the PFA, they became more comfortable with discovering properties and measuring areas. Therefore, it is recommended that patty paper folding activities can be conducted frequently with students. As students gain experience, they will be more comfortable discovering properties and making assumptions. The teachers stated that they were not familiar with Euler's line. Since Euler's line is the intersection of the altitudes, medians, and heights of a triangle, it is considered beneficial to apply this activity at the 8th grade level. This way, students will have the opportunity to explore the auxiliary elements and properties of a triangle, leading to more lasting learning. In this study, the time allocated for PFA was limited for all polygon properties and area measurements. It is recommended that practitioners plan with wider intervals and provide longer time frames.

After the PFA was conducted with the teachers, the "Patty Paper Folding Activity Opinion Form" (PAF) was administered. The teachers responded in writing in the same classroom setting. All the teachers gave a positive response to the first question. Three of the teachers emphasized the transparency property. One teacher found it effective for seeing the reference measurements, while the other two teachers stated that the folds were drawable due to the transparency, allowing the activity to progress. They also mentioned that this helped make the concepts of perpendicularity, parallelism, and equivalence clearer, and the activity could continue by stacking the patty papers due to transparency. Two teachers mentioned that the idea of "*learning through discovery*" emerged as a result of the activities, and two other teachers stated that learning types such as "*permanent learning*," "*discovery learning*," and "*learning by doing*" were formed through these activities. Additionally, they described the activities as "*functional*," "*highly efficient*," and "*useful*". All the teachers gave a positive response to the second question about whether they would apply the patty paper folding activities in their own classrooms. One teacher mentioned that they already used patty paper folding activities in some classroom activities and would use them more frequently in the future. They explained this as "*the opportunity to achieve the highest outcomes with inexpensive materials*." Another teacher stated that they would use it as "*pre-discovery*" in some classes. When asked why, they responded with the phrase "*to encourage fruitful mathematical dialogues*."

The other four teachers indicated that they were considering using patty paper, explaining their reasons with phrases such as "*a material that provides learning by doing in a fun way*," "*stimulates prior knowledge*," "*accessible, economical*," "*promotes learning by doing*," "*increases interest in the lesson*," and "*enhances retention*." One teacher emphasized that it was important to check whether students folded correctly. Another teacher stated, "*I was only using it in the baking tray, but now it will be in the classroom too.*" In response to the third question about whether the activity could be adapted to any math topic, most teachers gave a positive answer. Four teachers specifically thought it could be more adaptable in the field of geometry, while one teacher pointed out, "*Although it can be adapted to any math topic, how functional it* *would be is confusing; the teacher must definitely do their own preparation before the lesson*," highlighting the need for the teacher to be prepared and patient. A teacher who said it could be adapted to geometry found it applicable in terms of providing visual representation, while another teacher stated that it could not be applied to topics like probability, but it could be applied to subjects such as fractions, with about 90 % adaptability. All the teachers gave a positive response to the fourth question about whether they would recommend patty paper folding activities to their fellow math teachers. When discussing areas for improvement, the teachers suggested that the process should be planned in detail, that instructions should be prepared, and that the sequence of topics covered was important. Similar to these expressions, Van Vijk et al. (2023) concluded that mathematics teachers find paper folding activities useful "to activate students by providing folding tasks' and 'to grow mathematical understanding by folding'.

In addition, the teachers agreed that students should be given enough time, that guidance should be done well, and that groups should consist of two-three people. One teacher also pointed out that there was an issue when copying some drawings and suggested that a material with less chance of slipping would be better. In response to the question, "*What part/topic of the patty paper folding activity did you find most useful or enjoyable*?" one teacher found Euler's line surprising, while two teachers found the topic of area in quadrilaterals very useful. They stated that it would increase retention and reduce misconceptions. Two teachers expressed that they found it useful for students to make assumptions. Two teachers mentioned that patty paper was a versatile material, as it replaced tools like rulers and protractors. One teacher talked about the benefits of transparency and usability with minimal materials. They said that the properties of geometric shapes became naturally visible with the patty paper. One teacher found the process of forming congruent shapes by stacking them to be useful. When asked if they had anything else to add, three responses were given. One teacher stated that the activity was enjoyable and instructive and that they were happy to participate. Another teacher reiterated that in-class time management and attention to groups should be handled carefully during the folding activity. One teacher emphasized the necessity of being well-prepared to manage the process. So, there were limitations in the details of the information received from the teachers due to time constraints. More efficiency can be obtained in more time. It is recommended to conduct proctracted studies.

Overall, it can be said that the patty paper folding activities conducted with the teachers were useful. It is recommended that teachers carry out patty paper folding activities in their classrooms. This activity, which was found useful by the teachers, is recommended to be practiced on students. Indeed, many studies (Adom &Adu, 2020; Aksoy& Işıksal Bostan, 2024; Bornasal et al., 2021; Boakes, 2008; Galicha & Lazaro, 2022; Demirci & Çontay, 2023; Empson & Turner, 2006; Friedman & Rittberg, 2021; Gürbüz et al., 2018; Febriani, et al., 2023; Morye, 2025; Patkin & Canner, 2010; Robichaux & Rodrigue, 2003; Subaar, et al., , 2010; Van Wijk et al., 2023) revealed the positive effects of paper folding activities based instruction on students improvement. In this sense, experimental studies are recommended. As a continuation of the explorations with patty paper in the focus of polygons, studies in which the hierarchical properties of polygons will be discussed in an experimental manner are recommended.

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Research Article

Mathematics novice teacher readiness in implementing 21st century learning

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Abstract

Teachers have a vital role in teaching 21st century skills to students. This study focuses on analyzing the readiness of novice teachers with less than five years of teaching experience in implementing learning to stimulate critical thinking skills after references of implementing this skill was given to them. This research was descriptive research with a sample of 23 novice mathematics teachers in Mataram City, Indonesia. The instrument used was a critical thinking skill test, questionnaire, and interview guidelines. The results of the study showed that after given the study references and discussion was conducted, more than 95% of novice teachers were ready enough to teach critical thinking in class. In the view of duration of teaching service year, more than 50% of novice teacher were in high level readiness to teach critical thinking in all category of year service after the intervention conducted. It was recommended to provide the teacher with references about implementing critical thinking in class to maintain teacher professional quality in teaching.

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Introduction

Education has to provide the students with skills that beneficial for life. Many national and international educational institution have put agenda for their student skill output. Among the multiple skills essential for students, "21st century skills" have been advocated for success in both professional and academic fields. (Ball et al., 2016).

Kennedy & Odell (2014) characterize 21st-century skills as critical thinking, global awareness, creativity, technology proficiency, and media literacy. Align with that, Motallebzadeh et al. (2018) also agree that these skills also including critical thinking and creativity, aside from other competencies in problem solving, collaboration, and digital literacy. Other scholar abbrev this 21st century skill as "4Cs" (creativity, critical thinking, communication, and collaboration) (Thornhill-Miller et al., 2023). All the studies mention critical thinking as key component of this century skill.

Numerous well-known definitions of critical thinking were mentioned by scholars. One of them is the definition of critical thinking as reflective thinking which reasonable and focused on take the decision on what to believe or do (Ennis, 2018). Specifically in mathematics, critical thinking is defined as the skill in combining knowledge, provide reasons, and methods of cognitive to reflectively make generalization, prove, and assessment unexpected mathematical issues (Fidaus

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et al., 2015). Critical thinking contains some primary competencies include interpretation, analysis, inference, evaluation, explanation, and self-regulation (Facione, 2015).

The ability to think critically could be developed from processes which facilitating that skill. One of that process is from active learning provided by school teacher so that students can experience solving problems from diverse perspectives and deal with complicated issues in society. Thus, teacher competence in teaching critical thinking become important to support student.

Teacher readiness in teaching critical thinking started in their university classes. Previous studies from Ronfeldt & Reininger (2012) propose that if student teachers are able to learn a clear set of measurable, essential, current, and cohesive teacher competencies during their initial teacher education, it is truly possible to assess their level of preparation for the job. This implies the exposure to one of this 21st century skill in educational institution become crucial. The studies by Kirbas & Bulut (2024) found that , teacher candidates who have heard about this 21st century talent before have better competencies than those who haven't, making it imperative that educational institutions expose their students to it. In line with that, Aizenkot & David (2023) explained that mastery of 21st century abilities was stronger in advanced years as opposed to freshman years suggests that students get better at this competency the more they study about it.

Numerous studies have focused on exploring knowledge of prospective mathematics teacher about critical thinking. In Saudi Arabia, there was a study revealing that many teacher students had inadequate knowledge about critical thinking skills although they believed this skill important for their future student (Gashan, 2015). Furthermore, study from As'ari et al. (2017) in one of Indonesia's cities revealed that the majority of prospective mathematics teachers' critical thinking dispositions are at the level of non-critical thinker yet, only few of them are at the emergent critical thinker and in level of developing critical thinker. This study supported by study from Siahaan et al. (2023) who did literature review about several studies in critical thinking area and concluded that critical thinking skills of pre-service teacher education teachers Mathematics in Indonesia is still categorized as low according to results of the tests given.

Ismail et al. (2022) revealed that critical thinking skill has enhanced the pedagogical quality of teachers, especially in meeting the needs of 21st century learning in the classroom. However, according to our preliminary study, more than 50% of teacher reported that they had limited teaching resources about implementing critical thinking lesson in class and attending the training about teaching critical thinking. Therefore, providing them the suitable references and training are necessary for prospective teacher as well as novice teacher to enhance their professionals. Online training has been proved as one of the effective teaching methods for training teacher and prospective teacher critical thinking (Sutoyo et al., 2023).

As critical thinking becomes pivotal for student and also affect the pedagogical quality of teacher, the investigation of the critical thinking of novice teacher which have been teaching less than five year is necessary to conduct. This study focused on exploring the novice teacher readiness in teaching critical thinking after they study about critical thinking in learning resource provided by us. We conceptualize their preparedness in terms of their knowledge of teaching critical thinking, as this knowledge serves as the foundation for effectively implementing instructional practices aimed at fostering critical thinking skill. The research question formulated for this study is as follows 1) how novice teacher readiness in teaching critical thinking in the terms of their knowledge of teaching critical thinking? 2) how novice teacher readiness in teaching critical thinking according to their year of services?

Method

Research Model

This descriptive research explored novice teacher readiness in teaching critical thinking. The population was novice teacher in Mataram City, Indonesia with less than 5 years teaching experience. The prospective teachers who have done the teaching practice at school for more than 6 months were included in this population since they already have the experience teaching the real students at school. The sample was 23 people choose by purposive sampling.

The flowchart of our research is presented in Figure 1. In Figure 1, this research was started by constructing the research instruments, namely test to measure teacher readiness in teaching critical thinking, questionnaire, and interview guidelines, and then validating them. After that, we choose the sample for the research. Data collecting was done in 4 weeks. In first two weeks, we asked the novice teacher to study independently using our learning resource. The learning resource was from our previous research in Sarjana et al. (2024). In the third week, we conducted one session of blinded learning to discuss about how to teach critical thinking skill in class. In fourth week, the test to measure teacher readiness in teaching critical thinking was conducted. The teacher also filled the questionnaire about their background, namely the duration of teaching at school, the professional certification as teacher, and the frequency joining the workshop about critical thinking skill.

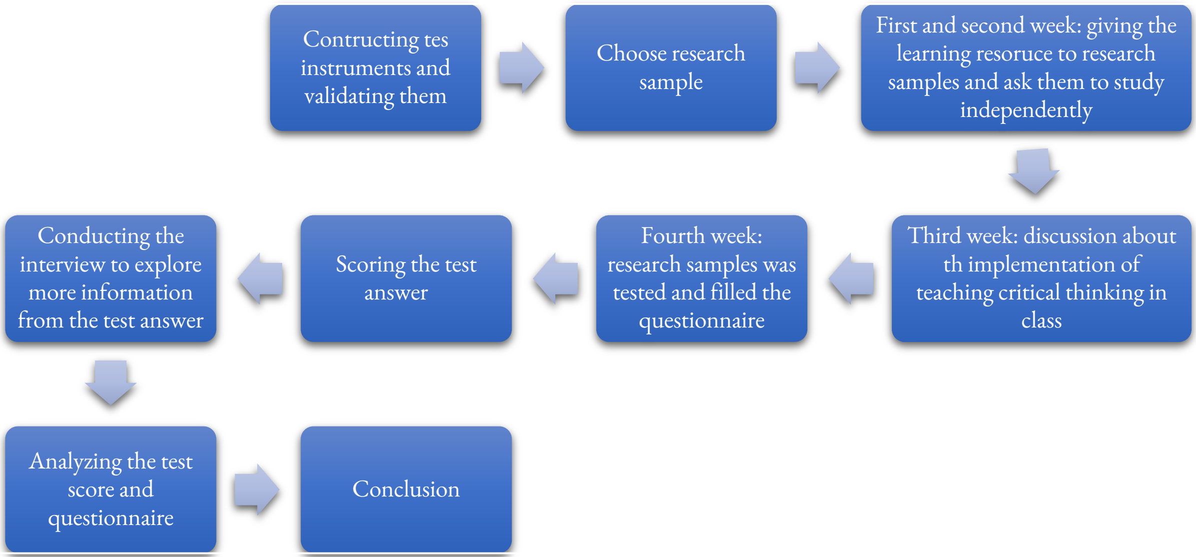


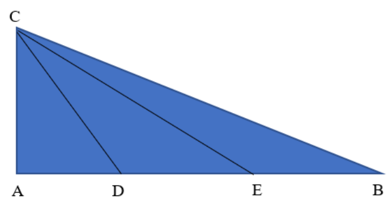
Figure 1. Flowchart of the reseach

Data Collection Tools

The instrument used in this research was test to measure teacher readiness in teaching critical thinking, questionnaire, and interview guidelines.

Test to Measure Teacher Readiness in Teaching Critical Thinking

ΔABC with right angle at point A. There are such points D and E that $|\overline{AD}| = |\overline{DT}| = |\overline{EB}|$. It is also known that $|\overline{CD}|= \sqrt{221}$ cm and $|\overline{CE}| = \sqrt{521}$ cm as shown in the following picture:



a) Determine the area ΔABC
b) Develop questions that build **critical** thinking skills from each step of completion!
c) Identify each of these questions on b) based on the table of **critical** thinking indicators and give the explanation!

Figure 2. The questions in test to measure the readiness of novice teacher tot each critical thinking skill

Figure 2 presents the questions for the research subject. According to Figure 2, there are 3 question items in the test. The command in the first question is to solve problems related to the area of a triangle, in the second question develop questions that build critical thinking skills, and in the third question identify every indicator of critical thinking in each question that has been created in the second question. The indicator we asked teacher to analyze was critical thinking indicator from Facione (2020), namely interpretation, analysis, inference, evaluation, explanation, and self-regulation as this indicator is suitable for mathematics test.

Test validation was carried out by piloting the test on 23 prospective teachers. The validation results were then analyzed using the Pearson Correlation test. The level of validity was determined based on the criteria outlined in Table 1. Meanwhile, the reliability was analyzed using Cronbach's Alpha test, with the level of reliability determined according to the criteria presented in Table 2.

Table 1. Criteria of Correlation Coefficient Validity (Sugiyono, 2013)

Correlation Coefficient	**Interpretation**
$0{,}90 \le r_{XY} \le 1{,}00$	Very High Validity
$0{,}70 \le r_{XY} < 0{,}90$	High Validity
$0{,}40 \le r_{XY} < 0{,}70$	Moderate Validity
$0{,}20 \le r_{XY} < 0{,}40$	Low Validity
$0{,}00 \le r_{XY} < 0{,}20$	Very Low Validity
$r_{XY} < 0{,}00$	Not Valid

Table 2. Classification of Alpha Cronbach Score (George & Mallery, 2003)

Alpha Cronbach Score	**Classification of Reliability**
$\alpha < 0{,}5$	Unacceptable
$0{,}5 \le \alpha < 0{,}6$	Poor
$0{,}6 \le \alpha < 0{,}7$	Questionable
$0{,}7 \le \alpha < 0{,}8$	Acceptable
$0{,}8 \le \alpha < 0{,}9$	Good
$0{,}9 \le \alpha < 1{,}0$	Excellent

Table 3. Piloting result of test to measure teacher readiness in teaching critical thinking

Question Number	1	2	3
Validity			
r_{xy}	0,543	0,504	0,664
Interpretation	Valid	Valid	Valid
Reliability			
$\boldsymbol{\alpha}$ alfa cronbach		0,773	
Interpretation		Acceptable	

The piloting result is presented on Table 3. According to Table 3, all the question are valid in moderate level and also reliable in acceptable evel. It indicated that the test was able to be used to measure the readiness of novice teacher to teach critical thinking.

The evaluation criteria used for scoring the test are presented in Table 4, providing a structured framework to ensure consistency and objectivity in the assessment process.

Table 4. Rubric for scoring the test

Question Number	Indicator	Answer Criteria	Score
1.	Novice teachers are able to solve problems related to the area of a triangle	No answer	0
		The novice teacher approaches problem-solving by applying a procedural method, following step-by-step instructions or established algorithms to reach a solution	0-18
		Novice teacher give conclusion about the conclusion about the area of triangle	0-2

Question Number	Indicator	Answer Criteria	Score
		Total score for first question	**0-20**
2.	Novice teachers are able to develop questions that build critical thinking skills	No answer	0
		Novice teachers employ clear guiding questions to support students in systematically working through a problem until they reach final answer for triangle area	0-10
		Novice teachers provide minimum five questions as the representation of critical thinking indicators.	0-5
		Total score for second question	**0-15**
3.	Novice teachers are able to identify every indicator of critical thinking in each question that has been created in the second question	No answer	0
		Novice teachers identify that has been created in the second question into critical thinking indicator	0-10
		The novice teacher provides explanations for each indicator they referenced in relation to the questions developed in the second question	0-20
		Total score for third question	**0-30**
		Range for total score	**0-65**

The score obtained then converted into the scale 0-100 using formula in 1 below

$$Final\ score = \frac{total\ score}{65} \times 100 \qquad (1)$$

The final score were interpreted to analyze the level of novice teacher in teaching critical thinking using Table 5.

Table 5. The formula to classify module's category (Widoyoko, 2016).

Formula	**Classification**
$\boldsymbol{X > \bar{X}_i + 1,8 \times sb_i \iff X > 79,88}$	Very High
$\boldsymbol{\bar{X}_i + 0,6 \times sb_i < X \leq \bar{X}_i + 1,8 \times sb_i \iff 59,96 < X \leq 79,88}$	High
$\boldsymbol{\bar{X}_i - 0,6 \times sb_i < X \leq \bar{X}_i + 0,6 \times sb_i \iff 40,04 < X \leq 59,96}$	Enough
$\boldsymbol{\bar{X}_i - 1,8 \times sb_i < X \leq \bar{X}_i - 0,6 \times sb_i \iff 20,12 < X \leq 40,04}$	Low
$\boldsymbol{X \leq \bar{X}_i - 1,8 \times sb_i \iff X \leq 20,12}$	Very Low

Note:

$\bar{X}_i$ = $^1/_2$ (maximal ideal score + minimal ideal score) = ½ $\times$ $(100 + 0) = 50$

sb_i = $^1/_6$ (maximal ideal score - minimal ideal score) = $\frac{1}{6} \times (100 - 0) = 16{,}67$

X = the sum of the score

Questionnaire

The questionnaire collect the information about a) the novice duration of teaching at school in four categories, namely 0-1 year, 1-3 years, and 3-5 years, b) the professional certification as teacher categorized as have the certification or not, c) the frequency joining the workshop about critical thinking skill in Likert scale 1 to 5, d) how many times they had teaching ti foster student critical thinking, e) the open question about challenge in implementing learning activities that effectively stimulate critical thinking if the novice teachers had implemented it before, and f) other questions about the effectiveness about teaching resources from our team.

As the questionnaire contain open question, we validate the questionnaire validity using face validity from two lecturer from mathematics education department. The validators validated the test in three aspects, namely content, construct, and language. The validation score was analyzed using Aiken V with formula in 2 (Aiken, 1980).

$$V = \frac{\sum s}{n(c-1)} \quad (2)$$
$$S = \mathrm{r} - l_0$$

Information:
V : Aiken content validity index
n : A lot of ratings
l_0 : lowest rating figure
c : highest rating
R : the number given by the assessor

(Arikunto, 2010) revealed that the instrument will be considered valid if the results of the validity index meet the values in Table 6 as follows:

Table 6. Criteria for the validity of research instruments

Value	**Criterion**
0,81-1,00	Very high validity
0,61-0,80	High validity
0,41-0,60	Moderate validity
0, 21-0,41	Low validity
0,00-0, 20	Very low validity

The result of questionnaire validity is presented in Table 7.

Table 7. The validity of questionnaire item

Item Number	Validator Score		S1	S2	S	n(c-1)	V	Interpretation
	1	2						
Item 1	5	5	4	4	8	8	1	Very high validity
Item 2	5	5	4	4	8	8	1	Very high validity
Item 3	5	5	4	4	8	8	1	Very high validity
Item 4	5	5	4	4	8	8	1	Very high validity
Item 5	4	4	3	3	6	8	0.75	High validity
Item 6	4	4	3	3	6	8	0.75	High validity
Item 7	5	4	4	3	7	8	0.875	Very high validity
Item 8	4	4	3	3	6	8	0.75	High validity
Item 9	5	5	4	4	8	8	1	Very high validity
Item 10	5	5	4	4	8	8	1	Very high validity

In Table 7, seven items (Items 1–4, 7, 9, and 10) achieved a coefficient of 0.875 or higher, indicating a high degree of expert agreement. The remaining three items (Items 5, 6, and 8) had V values of 0.75, which still fall within an acceptable range. Overall, the results suggest that the instrument demonstrates strong content validity and is suitable for further application.

Interview Guidelines

The interview was also conducted to explore the participant answer if their test answer were not clear enough. We validated this instrument using face validity with two validators in their capacity as mathematics education lecturer and analyze it using V Aiken. Table 8 elucidated the validity result of every item in the instrument.

Table 8. The Validity Result for Interview Guidelines Item

Item Number	Validator Score		S1	S2	S	n(c-1)	V	Interpretation
	1	2						
Item 1	5	5	4	4	8	8	1	Very high validity
Item 2	5	5	4	4	8	8	1	Very high validity
Item 3	5	5	4	4	8	8	1	Very high validity
Item 4	5	5	4	4	8	8	1	Very high validity
Item 5	5	4	4	3	7	8	0.875	Very high validity

According to Table 8, all the items reach coefficient above 0,8 which indicate that every item have very high validity and can be employ to further implementation.

Result and Discussion

Novice Teacher Readiness to Teach Critical Thinking in The Terms of Their Knowledge

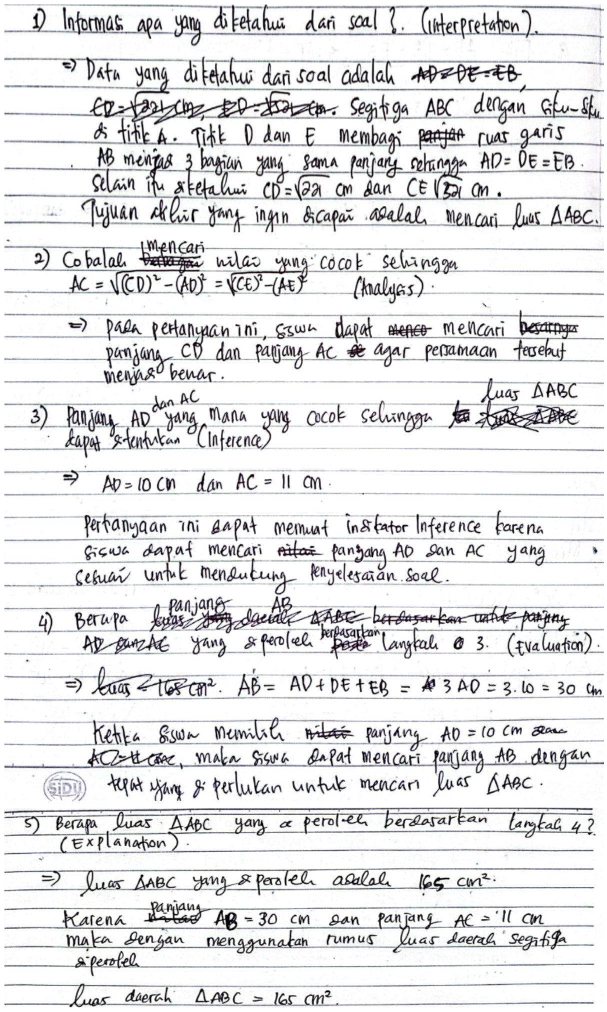
There were 100% of teachers who could answer the first question about solving problem about triangle area, but in the second and third questions related to knowledge of the implementation of critical thinking, some teachers could not fully explain it

Table 9. The Test Score Result of Second and Third Question

Category	Quantity of Novice Teacher(s)	Percentage (%)
Very High	10	43
High	7	30
Moderate	5	22
Low	1	4

Table 9 presents the test score result of second and third questions. According to that table, 22 of 23 novice teachers or 95,6 % of them were ready enough to teach critical thinking. Only one teacher who was in low category. This indicates that majority of novice teachers were ready to implementing critical thinking learning to their student after intervention involving giving learning resources and discussion given by us. This finding in line with research from Rusdin (2018) and Qasserras & Qasserras (2023), suggesting that teachers generally recognize the importance of critical thinking skill and demonstrate high readiness to implement 21st-century learning approaches.

Figure 2 elucidates one of the novice teacher answers whom his readiness categorized as high.



1) Informasi apa yang diketahui dari soal? (Interpretation).

=> Data yang diketahui dari soal adalah ~~AD=DE=EB~~, ~~CD=√221 cm, ED=√321 cm~~. Segitiga ABC dengan siku-siku di titik A. Titik D dan E membagi ~~panjang~~ ruas garis AB menjadi 3 bagian yang sama panjang sehingga AD=DE=EB. Selain itu diketahui CD=√221 cm dan CE √321 cm. Tujuan akhir yang ingin dicapai adalah mencari luas ΔABC.

2) Cobalah mencari nilai yang cocok sehingga $AC = \sqrt{(CD)^2-(AD)^2} = \sqrt{(CE)^2-(AE)^2}$ (Analysis).

=> Pada pertanyaan ini, siswa dapat mencari panjang CD dan panjang AC agar persamaan tersebut menjadi benar.

3) Panjang AD dan AC yang mana yang cocok sehingga luas ΔABC dapat ditentukan (Inference)

=> AD = 10 cm dan AC = 11 cm.

Pertanyaan ini dapat memuat indikator Inference karena siswa dapat mencari ~~nilai~~ panjang AD dan AC yang sesuai untuk mendukung penyelesaian soal.

4) Berapa panjang AB yang diperoleh berdasarkan langkah 3. (Evaluation).

=> ~~luas = 165 cm²~~. AB = AD + DE + EB = 3 AD = 3 . 10 = 30 cm

Ketika siswa memilih ~~nilai~~ panjang AD = 10 cm, maka siswa dapat mencari panjang AB dengan tepat yang diperlukan untuk mencari luas ΔABC.

5) Berapa luas ΔABC yang diperoleh berdasarkan langkah 4? (Explanation).

=> luas ΔABC yang diperoleh adalah 165 cm².

Karena panjang AB = 30 cm dan panjang AC = 11 cm maka dengan menggunakan rumus luas daerah segitiga diperoleh

luas daerah ΔABC = 165 cm².

1. What kind information which we get from the problem given? (interpretation)
   ➔ The information from the problem given are Triangle ABC, the right triangle is at point A. Point D and E divide line segment AB into three parts with the same length, so that AD=DE=EB. Furthermore, given that CD=$\sqrt{221}$ cm and CE=$\sqrt{321}$ cm. The purpose is to count the area of triangle ABC.

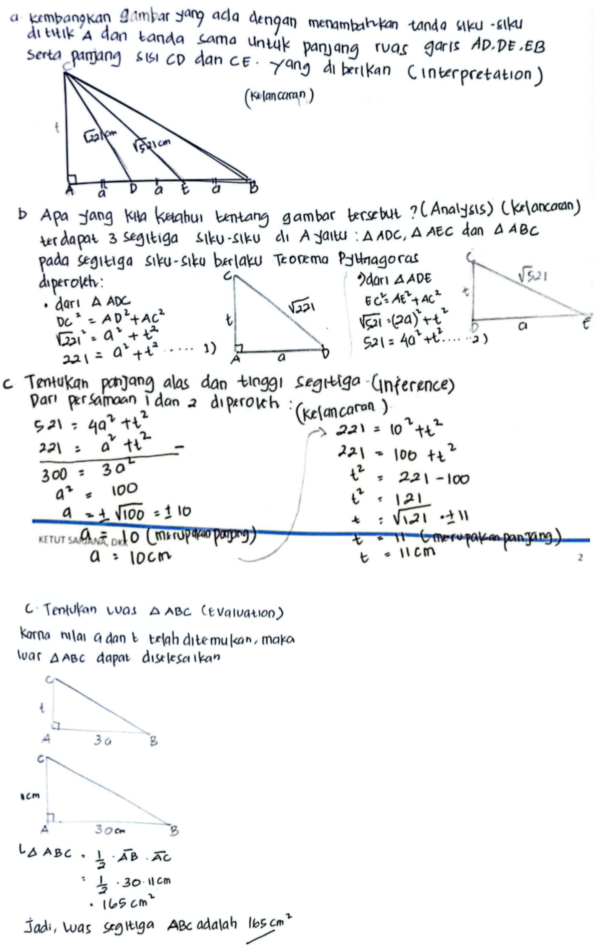
2. Try to find the equation from information given so that $AC = \sqrt{CD^2 - AD^2} = \sqrt{CE^2 - AE^2}$ (Analysis)
   ➔ In this question, student can search for the length of line segment CD and AC so that the equation is true.
3. Which length of AD and AC that is right, so that we can calculate the area of triangle ABC. (Inference)
   ➔ $AD = 10\ cm$ and $AC = 11\ cm$
   This question contains inference because student can decide the length of AD and AC which suitable to solve the problem.
4. What is the length of AB that you got according to question number 3? (Evaluation)
   ➔ $AB = AD + DE + EB = 3\ AD = 3\ .\ 10 = 30$ cm
   When student choose the length of AD = 10 cm, student can evaluate the suitable length of AB in order to evaluate the area of triangle ABC.
5. Evaluate the area of triangle ABC from question number four! (Explanation)
   ➔ The area of ABC is 165 cm^2

   Since the length of $AB = 30$ cm and the length of $AC = 11$ cm, from the area of triangle formula we obtain the area of triangle ABC equal 165 cm^2.

Figure 2. Example of novice teacher answer with high readiness

We asked the participant to answer three questions, the second question develop questions that build critical thinking skills, and in the third question identify every indicator of critical thinking in each question that has been created in the second question. According to Figure 2, the novice teacher is able to emerge five sub questions, each of them contained the indicator of critical thinking, namely interpretation, analysis, inference, evaluation and explanation. The participant also explains the reason why each question contains certain indicator. For example, in the sub-question number three, the novice teacher asked "Which length of $\overline{AD}$ and $\overline{AC}$ that is right, so that we can calculate the area of triangle ABC" and categorizing that sub question to measure the inference as the student will be facilitated to decide the length of $\overline{AD}$ and $\overline{AC}$ which suitable to solve the problem from the equations which they obtain from sub question number three.

Figure 3 demonstrated the example of novice teacher answer with moderate level in readiness.



a. Add the right angle in point A and the sign that the $\overline{AD}$, $\overline{DE}$, and $\overline{EB}$ have the same length and $\overline{CD}$ and $\overline{CE}$ also have the same length. (Interpretation)

b. What kind of information that we know from the picture? (Analysis).
There are three right angle triangles with right angle in point A, namely triangle ADC, triangle AEC, and triangle ABC. In right angle triangle the Phytagorean formula can be use in three triangles.
From triangle ADC

$$DC^2 = AD^2 + AC^2$$
$$\left(\sqrt{221}\right)^2 = a^2 + t^2$$
$$221 = a^2 + t^2 \text{ .... 1)}$$

From triangle ADE

$$EC^2 = AE^2 + AC^2$$
$$\left(\sqrt{521}\right)^2 = (2a)^2 + t^2$$
$$521 = 4a^2 + t^2 \text{ .... 2)}$$

c. Evaluate the length of base and altitude of triangle (Inference)

$521 = 4a^2 + t^2$
$221 = a^2 + t^2$
---------------------- -
$300 = 3a^2$
$a^2 = 100$
$a = \pm\sqrt{100} = \pm 10$
$a = 10$ cm

d. Calculate the area of triangle ABC (Evaluation)
As the value of a and t had found, so the area of ABC can be solved.

$$L_{\Delta ABC} = \frac{1}{2} . \overline{AB} . \overline{AC}$$
$$= \frac{1}{2} . 30 . 11$$
$$= 165\ cm^2$$

So, the area of ΔABC is $165\ cm^2$.

Figure 3. Example of novice teacher answer with moderate readiness

According to Figure 3, teachers with moderate level of readiness were able to emerged the sub question of critical thinking indicator. Different with novice teacher with high readiness who could emerged all the critial thinking indicator, moderate level teacher just revealed four indicators of critical thinking. In Figure 3, they had subquestion in interpretation, analysis, inference, and evaluation, but not in indicator of explanation. Besides, they also could not give the reasoning about why certain subquestiin represented certain indicator. According to interview, those novice teacher had difficulties in developing and differentiate the sub-question for crtitical thinking indicator in inference, evaluation, and explanation.

How Novice Teacher Readiness in Teaching Critical Thinking According to Their Year of Services

We also explored novice teacher readiness in term of how long they had been teaching at school. Table 10 presents the result.

Table 10. Years of service and categories of teacher readiness in implementing critical thinking learning

Year of Service	Level of Readiness and Quantity of Novice Teacher(s)				Total
	Very High	High	Enough	Low	
3-5 years	2 (40%)	2 (40%)	1(20%)	0	5
1 until <3 years	3 (42,8%)	1 (14,3%)	2(28,6%)	1(14,3%)	7
0 until <1 year	5(45,6%)	4(36,4%)	2(18,2%)	0	11
Total	11	7	5	1	23

Table 10 shows the relationship between teachers' years of service and their readiness in implementation critical thinking learning. After intervention from us in giving learning resource and discussion, 22/23 participants were in stage of ready enough to implement critical thinking lesson for their student. There are 20% of teachers with 3-5 years of service, 28.6% of teachers with 1 until <3 years of service, and 18.2% of teachers with 0 until <1 years has knowledge in the enough category. In all category of year service, more than 50% of novice teacher were ready to teaching critical thinking.

The findings indicate a complex relationship between teaching experience and readiness to implement critical thinking learning. Interestingly, the highest levels of readiness were found among teachers with less than one year of service. According to interview, this is due to their recent exposure to current pedagogical frameworks in teacher training programs in college. The novice teacher with less than one year experience tend to had bachelor thesis with the topic about critical thinking or other skill for 21st century learning. It provided them with more opportunity to learn about this skill. As this research only explore the readiness in knowledge side, they will superior another teacher with longer experience. This finding aligns with study from (Kleickman et al., 2013) suggesting that in some cases, novice teachers who have recently completed rigorous teacher preparation programs may demonstrate more current knowledge of subject matter and innovative instructional methods than their more experienced counterparts, whose professional development may have stagnated over time. This discrepancy is especially apparent in rapidly evolving educational fields, such as critical thinking, where recent graduates are often more attuned to contemporary pedagogical approaches.

However, this early readiness appears to decline or plateau over time. This is particularly due to the absence of sustained professional development opportunities. Sustained engagement professional development is recognized as one of key factor in enhancing teaching competencies and fostering instructional innovation (Darling-Hammond et al., 2017; Desimone, 2009). However, findings from the questionnaire reveal that 57% of participating teachers reported having never received formal training, attended seminars, or participated in webinars specifically focused on the pedagogy of critical thinking. This suggests a significant professional development gap, which likely hinders teachers' ability to deepen their instructional approaches over time. The lack of professional development becomes the major obstacles that may prevent instructors from using this pedagogical element in their instructional practices (Qasserras & Qasserras, 2023).

Interestingly, the only teacher categorized as "Low" in readiness was from the 1 to <3 years of experience group. According to interview, that teacher have different opinion on the meaning of the question given by us. In contrast, during the interview, the teacher could give enough explanation about the sub question need to accommodate five indicators of critical thinking and the reason why those question suitable for each indicator.

The significant improvement following the intervention underscores the value of structured professional development, even when delivered in a relatively short and focused format. Access to relevant resources, combined with peer discussion and reflection, can markedly enhance teacher confidence and readiness. This supports previous literature emphasizing that ongoing, collaborative, and practice-oriented training is essential for equipping teachers with the necessary skills to promote higher-order thinking in students (Brookfield, 2012; Desimone, 2009; Guskey, 2002). This shows that novice mathematics teachers need or need to take part in training activities related to critical thinking learning on a regular basis to maintain their quality in teaching.

Conclusion and Recommendations

The results of the study showed that after given the study references and discussion was conducted, more than 95.6% of novice teachers were ready enough to teach critical thinking in class. In all category of year service, more than 50% of novice teacher were ready to teaching critical thinking and the highest levels of readiness were found among teachers with less than one year of service. According to the conclusion, 90% of the novice teacher were in category of ready to implementing critical thinking in their lesson after we provide them the learning resource, yet more than a half of them never attending critical thinking training before. Our preliminary study also indicates that more than half of teacher had difficulty in finding references for critical thinking implementation suiting their context. Hence, we suggested to provide the teacher with suitable references for stimulating student critical thinking skill and teacher need to engaged in sustainable professional development along their career.

Limitations of Study

In this research, novice teacher readiness was measured according their knowledge in critical thinking concept and their skill in providing question to stimulate student's critical thinking. We suggested to explore the novice teacher readiness also from their skill in practicing the skill in questioning in real class with their student and managing the class when conducting the lesson.

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Research Article

The relationship between mathematics anxiety and mathematics learning level

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Abstract

One of the most important factors in mathematics success is mathematics anxiety. For a long time, mathematics education researchers have focused on this. In this study, the differentiation status of mathematics learning levels and gender variables in terms of mathematics anxiety was examined. This differentiation examined internal factors: Mathematics ability, Beliefs regarding mathematics, and external factors; Parents, Teachers, Social Beliefs, and Social Effects. The survey research type, one of the quantitative research types, was used in this research. Two hundred and forty high school students were determined as participants in this study. The mathematics learning levels of these students were low (87 students), medium (116 students), and advanced (37 students). The mathematics anxiety scale was used as the data collection tool. As a result of the research, it was determined that there was no differentiation in all the mathematics learning levels, internal and external factors (p>.05), however, it was determined that there was a differentiation in some items of the scale (p<.05). However, it was determined that there was a differentiation in some of the internal factors (Parents, Teachers) in the gender variable (p<.05). It may be recommended that this research be conducted with different education levels and a wider range of participants in the future.

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Introduction

Since the 70th of the previous century, mathematical education researchers have investigated various aspects of mathematical anxiety (MA). MA is defined as feelings of tension and anxiety when doing mathematics (Richardson and Suinn, 1972; Paechter et al., 2017; Luttenberger et al., 2018), as a negative cognitive reaction from someone when he or she deals with mathematics (Holmes, 1991) , or as a discomfort situation that occurs in response to situations involving mathematical tasks which are believed to threaten self-esteem (Trujillo & Hadfield, 1999). Generally, Anindyarini & Supahar (2019) summarized all these definitions by suggesting that MA is an emotional response generated by students toward mathematics.

A bulk of recent mathematics education research investigated the term MA from different views (Zhang, Zhao and Kong, 2019). Some research investigated the relationship between MA and mathematics performance (Olmez and Ozel, 2012; Necka et al., 2015; Lukowski et al., 2016; Justicia-Galiano et al., 2017). Beilock and Willingham (2014) suggested that there is a strong MA performance link in mathematical reasoning, while Harari et al. (2013) reported that there is a weak link between MA and basic computational skills.

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Many scholars investigated the impact of MA over time, in other words, they reported on this impact over grade level, i.e. age (Ramirez et al., 2013; Vukovic et al., 2013; Maloney et al., 2015). It was reported that as the difficulty of math learning increases with age the MA might also increase (Olmez and Ozel, 2012). Zhang and colleagues (2019) reported that grade level might modulate the MA performance link. However, it should be emphasized that the relationship between MA and mathematics learning level of the same age was not investigated.

Furthermore, a variety of studies have shown that gender moderates math anxiety. For example, some studies have suggested that females may have stronger math anxiety than males (Maloney et al., 2012). However, other studies report that girls did not experience higher levels of MA than boys during math content learning or test (Goetz et al., 2013).

Considering these claims, the current study aims to investigate the relationship between several factors of MA and mathematics learning levels of the same age (i.e. the same grade but learning different mathematics levels), and gender.

The importance of this study is hidden in adding to the literature some new results referring to the same age but different mathematics learning levels.

Theoretical Background and Literature Review

Mathematics anxiety is a worldwide phenomenon. Throughout the history of research in this field, many researchers have addressed the extent of this phenomenon in different countries. Blazer (2011) addressed this trend in the United States, claiming that 93% of adults there have some degree of MA. In addition, other researchers reported that 17% of adults there have high levels of MA (Ashcraft & Moore, 2009). In 2013, the Organization for Economic Co-operation and Development (OECD) reported that 59% of students aged 15-16 showed a tendency towards MA, about 33% of these students conveyed MA when handling math homework, and another 31% of these students felt nervous when handling a math problem.

Many scholars investigated the determinants or factors of MA. Brewster and Miller (2020) suggested four MA determinants: cognitive/affective, social, genetic, and missed opportunity. They clarified the interconnection between the cognitive and affective domains in mathematical anxious individuals by claiming that 'while mathematically anxious individuals experience cognitive and physiological symptoms, they also experience the emotional symptoms of worry that heighten the anxiety' (Brewster et al., 2020, p. 5). Moreover, it is worthy to mention that Skagerlund and colleagues (2019) argued that the impact of MA on mathematical activities is mediated by working memory which was defined by Baddeley (1992) as "a brain system that provides temporary storage and manipulation of the information necessary for such complex cognitive tasks as language comprehension, learning, and reasoning". At this point, it is important to mention that Sweller (2023) claimed that teaching methods may cause memory overload because working memory has limited capacity. This claim of Sweller crosses the Cognitive Load Theory that deals with the learning of complex cognitive tasks, where learners may be overwhelmed by the number of interactive information elements that must be processed simultaneously before meaningful learning can begin (Sayed, 2021; Sweller, 2023).

Back to the determinants according to (Brewster et al., 2020), the social factor of MA includes some parameters such as societal beliefs, cultural influences, and gender issues. Brewster et al. (2020) also suggested that there is a direct link between MA and human genetics challenges; furthermore, they bring to mind the factor 'missed opportunity' that include individuals who score high in academic areas but show evidence of MA, they are unable to perform well in mathematics because they have not had the opportunity to learn the basic mathematics knowledge required to continue to learn higher levels of mathematics, e.g. it can refer to poor mathematics instructions that may focus on mathematics teachers. In 2021, Vargas referred to two main factors for math anxiety: internal factors and external factors. The inner factors are determined by the person's characteristics, such as neurobiological factors, genes, gender, and age. While the external factors are determined by the environment, culture, and nationality, e.g. parents and teachers are external factors. Later, Rada and Luceitto (2022) reported that there are confounding factors related to MA such as: a person's innate tendency to solve mathematical problems, the effects of previous performance in mathematics, as well as environmental influences such as educational systems, family relationships and resources, as well as society had perspectives.

Following these factors, to describe MA we refer to some MA theories or models, the three most widely used theories are: i) 'Deficit Theory', this theory claimed that a poor performance in mathematics leads to MA (Tobias, 1986); ii)

'Debilitating Theory' which is based on the premise that MA will reduce performance in mathematics (Carey, Hill, Devine, and Szücs, 2016), it should be mentioned that this theory may cross the internal factor "student characteristics", e.g. if a mathematics teacher calls a student with MA to respond to a mathematics question as part of the lesson, the student may experience adverse physiological symptoms. iii) Carey and colleagues (2016) also suggested a bidirectional relationship between MA and mathematics performance (The Reciprocal Theory); this relationship between MA and mathematics performance can influence one another. Besides, they suggested the 'social cognitive theory' which claims that MA affects the individual's cognitive abilities and the physiological symptoms (Carey et al., 2016).

The factor gender has been investigated by several researchers who have reported that math anxiety in secondary and post-secondary education almost always finds higher levels of MA in females than in males (Bieg et al., 2015; Dowker et al., 2016; Luttenberger et al., 2018). This may be explained by societal stereotypes (i.e. mathematics females' abilities are lower than males' abilities). Interestingly, gender differences in math anxiety were widest in countries that have comparatively low levels of math anxiety (Lee, 2009). A negate research reported on higher numerical anxiety in males than in females (Baloğlu and Koçak, 2006).

Aim of the Study

The current study aimed to identify the inner and external factors that can cause mathematical anxiety (MA) among high school students in different mathematics learning levels (low, moderate, and advanced mathematical learning levels in high school). Moreover, a secondary aim was to identify gender differences in mathematical anxiety.

Method

Research Model

This study is quantitative according to the definition of (Creswell, 2009, p. 22): "*quantitative research is a means for testing objective theories by examining the relationship among variables. These variables, in turn, can be measured typically on instruments so that numbered data can be analyzed using statistical procedures*." Moreover, it is non-experiment correlational research (Khaldi, 2017); it examines the relationship between the students' mathematical learning level and their MA.

Participants

Two hundred and forty high school students who studied in 11th grade participated in this study. They were distributed according to their mathematics learning level: 87 low-level, 116 moderate-level, and 37 advanced-level. All the participants studied in the same high school, which is considered a developed Arab school in the north of Israel. The total number of students in this school is 682 (237 studied in 10th grade, 247 in 11th grade, and 223 in 12th grade). There are 344 males (51%) (Distributed as 131, 93, and 107 students according to the 10th, 11th, and 12th grades, respectively); and there are 338 females (49%), which are distributed as 106, 154, and 116 according to the grade level, respectively. It is important to mention that: 1) this school was chosen because of its characteristics (including many students, and several specialties like physics, chemistry, biology, biotechnology, history, and more); 2) in the north zone of Israel 4 Arab towns include about 1800 high school students, the portion population of each town is about 25%, and the calculated sample size was 245 students; 3) moreover, the socioeconomic level of the students is about moderate, which may represent any other high school of the Arab community in Israel. The 11th grade was chosen because it includes the highest number of students, who are distributed into the three levels of mathematics learning, different specialties, and three mathematics-learning levels (low, moderate, and advanced). Although the 12th graders had three mathematics-learning levels it was decided not to include them in this research because they were at the end of their high school learning and had a lot of time to learn at home; in addition, all 10th graders learned the same mathematics learning level, thus we did not include them in the current study.

Data Collection Tools

A questionnaire that was based partially on the Abbreviated Math Anxiety Scale (AMAS) was constructed to examine the aim of the current study. Hopko and others (Hopko, Mahadevan, Bare, & Hunt, 2003) developed the AMAS questionnaire, which included 12 Items related to several aspects of coping with mathematics. In the current study, the

AMAS questionnaire was extended to 26 items covering two main factors: internal and external. The internal factors that may cause MA are focused on the student (e.g. student's self-confidence, student's fear of failure, etc.); while the external factors that may cause MA are related to other factors like parents, teachers, society, mathematical content, and others.

The AMAS questionnaire was validated by Hopko and colleagues (2003). In addition, the derived questionnaire for the current study was validated by two mathematics education scholars.

The whole reliability of the questionnaire was 0.88. Table 1, details the distribution and the reliability of the items according to the inner and external factors.

Table 1. Mathematical anxiety questionnaire reliability.

Type		Reliability- inner α Cronbach	Number of Items	Examples
Internal factors	*Students*	0.809	8	The claim "In order for a student to improve his thinking he must succeed in mathematics" effects math stress.
External factors	*Parents*	0.892	2	The claim "Parents are only satisfied when their children get high grades in math" effects math stress.
	Teachers		8	The claim "Math teachers do not understand their students' difficulties" effects math stress.
	Social Beliefs		6	The claim "Teachers believe that advanced-level student is smart" effects math stress
	Social Effect		2	Competitiveness in the mathematics study group, effects math stress.
Total Reliability- α Cronbach		0.88		

The internal factor had two sub-categories: four items related to the student's beliefs in his/her mathematics ability, and other items related to the student's beliefs regarding mathematics. The external factors related to teachers had two sub-categories: four items related to the interaction between student and teacher, and four other items related to students' beliefs regarding the mathematics teacher characterization. In addition, the external factor related to student's social beliefs also included two sub-categories: two items related to the student's beliefs in society's attitudes to the math student's level and other four items related to the student's beliefs about the system's attitude to mathematics (such as universities, ministry of education, etc.)

Students were asked to answer the questions using a 4-point Likert scale: 1= "does not affect math stress at all", 2= "has a little effect on math stress", 3= "largely influential on math stress", and 4 = "affects math stress to a very large extent". The higher the score, the higher the student's math stress.

To see an overall picture, the subjects' responses to the various MA items (or factors) must be grouped into one dataset representing them all. First, the subjects' responses to the various MA items were averaged into one data point (between 1 and 4), then each MA factors' items (internal and external, and all their sub-categories) were averaged into one data point (between 1 and 4) that represents them all. This composite variable is called a score that consists of data averaging across various factors. Moreover, to interpret the weighted average MA scores of the participants, the scale 1-4 was divided into 4 equal parts determined as:

Math Anxiety Score	1-1.75	1.76-2.5	2.51-3.25	3.26-4
Interpretation	*Low Anxiety*	*Moderate Anxiety*	*High Anxiety*	*Very High Anxiety*

Results

Characterization of math anxiety factors

Table 2 details the distribution of the participants' answers according to the different MA factors.

The total math anxiety mean for all participants was 2.36(SD=1.101), which is interpreted as moderate MA. Table 2 details the math anxiety score, range, and average MA interpretation, according to factors. The highest item mean was 2.68(SD=1.169), interpreted as high MA; and related to the item "*The admission requirement for some faculties at the university is 'a high grade in mathematics'* " which is within the external factor of social beliefs, then the item "*Parents' expectations from their children to succeed in mathematics*" got the following highest mean 2.65(SD=1.133), interpreted as high MA. The highest internal factor 2.56(SD=1.13), interpreted as high MA was the item " *Mathematics exams affect mathematics stress*."

Table 2. Mathematics anxiety scores by factors.

Factor type	Category	Sub-category	Range	Average	MA interpretation
Internal	*Student*	*Mathematics ability*	2.14(1.08)-2.4(1.07)	2.265	low
		Beliefs regarding mathematics	2.45(1.11)-2.56(1.13)	2.51	high
External	*Parents*		2.46(1.11)-2.65(1.13)	2.555	high
	Teachers	*Student-teacher interaction*	1.94(1.12)-2.46(1.15)	2.215	moderate
		Mathematics Teacher Characterization	2.06(1.05)-2.3(1.15)	2.21	moderate
	Social Beliefs	*Society's Attitudes to student's math Level*	2.42(1.15)-2.51(1.18)	2.465	moderate
		System's attitudes to mathematics	2.37(1.11)-2.68(1.17)	2.48	moderate
	Social Effect		2.28(1.14)-2.51(1.18)	2.325	moderate

Mathematical Anxiety factors according to Mathematics Learning Levels

One-way ANOVA was performed to examine the differences between different mathematics learning levels due to MA factors. Table 3 indicates that there are no differences in the total internal factor and sub-external factors.

Table 3. One-way ANOVA – total mathematics anxiety according to mathematics learning level.

Factor		Mathematics Learning Level –Total Mean: *Low (n=87)*	*Moderate (n=116) **	*Advanced (n=37)*	F	df (between/within)	p	η^2
Internal		2.32 (moderate)	2.46 (moderate)	2.31 (moderate)	1.186	2, 237	ns**	0.072
External	*Parents*	2.52 (high)	2.60 (high)	2.50 (high)	0.266	2, 237	ns**	0.096
	Teachers	2.20 (moderate)	2.27 (moderate)	2.10 (moderate)	0.976	2, 237	ns**	0.034
	Social Beliefs	2.41 (moderate)	2.54 (high)	2.44 (moderate)	0.582	2, 237	ns**	0.058
	Social Effect	2.26 (moderate)	2.43 (moderate)	2.20 (moderate)	1.360	2, 237	ns**	0.077
	Total MA	2.317 (moderate)	2.431 (moderate)	2.283 (moderate)	0.645	2.237	ns**	0.050

(*) Eleven students did not fill out the questionnaire, thus we did not include them in the analysis. (**) ns means p>0.05.

Although Table 3 indicates that there are no differences in each total MA category (internal/external) between the different mathematics learning levels, further one-way ANOVA was conducted to compare the effect of math learning level on each MA item. It was found that there were significant effects at the level p<.05 for three internal factors, all related to the internal sub-category "*student's beliefs regarding mathematics*", other three external factors, two of them related to the teacher sub-category "*mathematics teacher characterization*", and one that relates to social beliefs (*System's attitudes* to mathematics), Table 4 details those findings.

Table 4. One-way ANOVA –mathematics anxiety per item according to mathematics learning level.

Factor/item affects math stress			Mathematics Learning Level –Total Mean: *Low (n=87)*	*Moderate (n=116)**	*Advanced (n=37)*	F	df (between/ within)	p	η^2
Internal		*1. Math exams influence math stress*	2.33 (moderate)	2.64 (high)	2.79 (high)	3.061	2, 237	<.05	0.17
		2. Complex mathematics problems affect math stress	2.49 (moderate)	2.64 (high)	2.15 (moderate)	3.849	2, 237	<.05	0.16
		3. Un-understanding the mathematics teacher's explanation affects math stress	2.00 (moderate)	2.39 (moderate)	2.04 (moderate)	3.669	2, 237	<.05	0.18
External	*Teacher*	*4. The attitude that a math teacher is a content provider affects math stress*	2.43 (moderate)	2.86 (high)	2.73 (high)	3.376	2, 237	<.05	0.18
		5. The impact of a math teacher's expertise in preparing students for the matriculation	2.22 (moderate)	2.47 (moderate)	1.96 (moderate)	3.749	2, 237	<.05	0.18

	exam affects math stress							
Social Beliefs	*6. An admission requirement of "high grade in math" at some faculties at the university affects math stress*	2.43 (moderate)	2.86 (high)	2.73 (high)	3.376	2, 237	<.05	0.19

The findings in Table 4 indicate that the moderate level had the highest means for five items, and the advanced level had the highest means for the item related to math exams.

In addition, to reach more accurate findings, a t-test of unequal variance was conducted to find the differences between low and advanced levels for items that showed a large mean difference between these two levels. It was found that the mean of the item "*Math exams influence math stress"* was 2.33 (1.011) and 2.79(1.021) for low and advanced levels, respectively, $t(133)=-2.172$, $p<.05$, Cohen's d = 0.4528. The averages of the item "*The claim 'A math teacher explains ideas only once' affects math stress*" were 1.99(.67) and 2.4(1.00) for low and advanced levels, respectively, $t(133)=-2.189$, $p<.05$, Cohen's d = 0.4817. The item "*The claim 'Math is a subject that develops thinking' affects math stress*" had averages of 2.49(1.011) and 2.08(.89) low and advanced level, respectively, $t(133)=2.025$, $p<.05$, Cohen's d = 0.4305. And finally, the item "*The claim 'Students' broadcast that a certain student is displaying the classroom' affects math stress*" had averages 2.38(1.011) and 1.96(.97) for low and advanced levels, respectively, $t(133)=2.093$, $p<.05$, Cohen's d = 0.4239. However, in all the other items, there were no significant differences between these two levels. It is important to emphasize that items with significant and moderate size effects ($0.4305<d<0.4817$, almost 0.5) related to the mathematics exam, mathematics as content, and beliefs regarding teachers and peers in the mathematics classroom.

Mathematical Anxiety Factors According to Gender

The findings indicated that females had more MA in internal factors and some external factors than males. Table -5 details these differences.

Table 5. Mathematics Anxiety gender differences per factor (t-test)

Factor		Mathematics Learning Level – Total Mean		t	df	p	Cohen's d
		males	*females*				
Internal		2.237 (moderate)	2.466 (moderate)	-2.447	238	<.05	0.225
External	*Parents*	2.478 (moderate)	2.60 (high)	-0.948	238	ns*	0.106
	Teachers	2.091 (moderate)	2.283 (moderate)	-1.949	238	ns*	0.171
	Social Beliefs	2.335 (moderate)	2.555 (high)	-1.990	238	<.05	0.192
	Social Effect	2.056 (moderate)	2.480 (moderate)	-3.592	238	<.01	0.372
Total MA		2.22 (moderate)	2.442 (moderate)	-1.967	238	<0.05	0.197

(*) ns means p> 0.05

Further statistical analysis was performed to examine the MA differences per item between males and females. The relevant items were categorized into three basic categories: One internal factor, 'Exam-Teacher-Peer (ETP) influence', and two external factors, 'Beliefs and Perceptions' and 'Expectations'.

Internal factors 'ETP influence'

The means of the item "*Math exams influence math stress*" were 2.34 (1.111) and 2.69(1.001) for males and females, respectively, t(238)=-2.351, p<.05, Cohen's d = 0.331. And the means of the item "*The belief that mathematics teachers don't understand their students' difficulties affects math stress*" were 1.97(0.667) and 2.56(1.122) for males and females, respectively, t(238)=-4.144, p<.01, Cohen's d =0.639. Finally, the item "*The claim 'Students' broadcast that a certain student is displaying the classroom' affects math stress*" had means 2.00(0.891) and 2.44(1.12) for males and females, respectively, t(238)=-2. 971, p<.05, Cohen's d =0.435. There are significant gender differences for each 'peer influence' item, and the Cohen's d effect size for each 'peer influence' item is at least approaching moderate effect size (0.331), and it reaches large effect size (0.639), which indicates a relatively strong gender effect (in favour to females).

External factors 'Beliefs and perceptions'

Some items related to parents and others were related to achievements. On the one hand, the means of the 'parents' related item "*Parents' expectations for their son's/daughter's success in mathematics affect math stress*" were 2.46(1.067) and 2.76(1.02) for males and females, respectively, t(238)=-2.001, p<.05, Cohen's d =0.287, which indicates that there is a small effect size for 'parents' expectations'. On the other hand, there were significant gender differences for three 'achievements' items: The means of the item "*The claim that a student who succeeds in mathematics is smart affects math stress*" were 2.15(1.007) and 2.58(1.012) for males and females respectively, t(238)=-2.892, p<.05, Cohen's d =0.426. The means of the item "*It is important to the student to have high achievements in mathematics affects math stress*" were 2.20(1.011) and 2.51(1.002) for males and females respectively, t(238)=-2.169, p<.05, Cohen's d =0.308. The means of the item "*Competition in math class affects math stress*" are 2.11(0.987) and 2.52(1.12) for males and females, respectively, t(238)=-2.468, p<.05, Cohen's d =0.388. These results show gender significant differences for these items, and Cohen's d effect sizes were about medium (0.308<d<0.426).

External factors 'Expectation'

The means of the item "*The students' belief that math teachers will be disappointed if their students do not succeed in math matriculation affects math stress*" were 2.25(1.012) and 2.59(1.011) for males and females, respectively, t(238)=-2.247, p<.05, Cohen's d =0.336. The item "the claim that '*the belief that students who learn for advanced math level are smart' affects math stress*" had means 2.30(1.11) and 2.63(1.21) for males and females, respectively, t(238)= 2.083, p<.050.284, Cohen's d =0.284. Cohen's d for these items were between (0.284~0.3<d<0.336), meaning that the effect sizes of the 'expectation' items range between small and moderate.

Conclusion and Discussion

The main aim of the current study was to examine the MA factors among eleventh graders from different mathematics learning levels (low, moderate, and advanced levels). Besides, the study also examined the MA factors among males and females. Several mathematics education scholars investigated the MA factors. Some scholars reported on four factors: cognitive factor, psychological factor, physical factor, and environmental factor (Fennema & Sherman, 1976); others identified eight MA factors: lack of self-confidence, interest and study habits, teacher factors, fear of failure, pressure from parents and peers, pressured quizzes and tests, poor skills in analysis and abstract math concepts (Estonanto & Dio, 2019). In the current study, we identified internal and external factors. The internal factor is based on factors that relate to the student (e.g., cognitive/psychological/interest and study habits, etc.). In this study, the internal factor has two sub-categories *Mathematics ability* and *Beliefs regarding mathematics.* The external factor has four main sub-categories: *parents, teachers, social beliefs,* and *social effects.* It is important to emphasize that these sub-categories match factors of previous studies (Fennema & Sherman, 1976; Estonanto & Dio, 2019).

The findings indicated that for the whole sample, the MA inner factors related to the student's *Mathematics ability* and *Beliefs regarding mathematics* were low and high, respectively. While the external factor for each sub-category was moderate, except for the "*parents*" sub-category, which was high. These findings are supported by the findings of (Estonanto & Dio, 2019) who reported that the major factors that caused MA of the student were the difficulty of abstract mathematical concept (in our scale it is the internal factor *student's beliefs regarding mathematics*), teachers' factor, and poor comprehension and analytical skills of students. Moreover, the finding, that relates to the "*parents*" external sub-category is supported by (Casad et al, 2015) who suggested that the parents' MA affects their sons or daughters. The items in our study related to the student's *Mathematics ability* and *Beliefs regarding mathematics* may indicate that the student's mathematics ability and performance are poor. Thus, we may explain these findings by the claim " People who think that they are bad at mathematics are more likely to anxious" (Dowker et al., 2016, p. 3), this claim relates to the deficit theory that states that poorness in mathematics performance leads to increasing in MA (Tobias, 1986). The finding related to the level of the external factor "parents" is explained according to previous research that suggested that there is a great relationship between the student's self-esteem and mathematical confidence and between the parents' perceptions and expectations (Rossnan, 2006).

The findings also showed that there were no significant differences between the total level of MA, internal MA factors and external MA (including each external MA sub-category) and the mathematics learning level. Moreover, all MA levels were moderate. These results add new information to previous research (Erdem, 2017) that investigated MA as the grade/age increases and determined that MA is in 5th grade at the least and in 10th grade at the most. Moreover, Erdem (2017) reported "it cannot be said anything clearly about the fact that math anxiety increases or decreases as the grade/age increases". The current study found that there were no differences in MA among students in the same age group (11th grade), this result does not contradict previous research because it investigated the same grade (11th grade). Our study was performed at the last trimester of the academic year, which means that our participants are approaching the end of their high school studies (remaining about one trimester in 12th grade), and they are getting closer to integrating into future academic studies. Thus, we may claim that this result may reinforce the study of Antonio (2023) who found that Bachelor of Science in Mathematics students possess the same level of MA, regardless of their age. Although this was the result, the attention should be directed toward our result concerning the differences between the different mathematics learning levels per MA item. The advanced mathematics learning level had a high score of MA and the highest compared to the low and moderate learning levels for the item "*Math exams influence math stress*". It is important to highlight that this item pertains to future academic progress, as students at advanced learning stages typically plan to pursue higher education. To successfully integrate into academic institutions, they must achieve strong grades in mathematics exams, which are essential for their admission and success in academic programs. Moreover, it should be noted that the moderate learning level was significantly higher than the low and advanced learning levels, in the two internal items "*Complex mathematics problems affect math stress*" and "*Un-understanding the mathematics teacher's explanation affects math stress*"; and in three external MA items "*The attitudes that 'a math teacher is a content provider' affects math stress*", "*The impact of a math teacher's expertise in preparing students for the matriculation exam affects math stress*" and " *An admission requirement of 'high grade un math' at some faculties at the university affects math stress*". These differences can be explained as follows: since these items are related to the characteristics of mathematics and its exams, and since moderate-level students generally aim to pursue academic studies, it can be assumed that they strive to achieve high grades in mathematics to secure admission to future academic programs, thus they tend to develop mathematics anxiety. Moreover, these reasons may explain the differences between low and advanced levels in some item that relates to mathematics and mathematics teachers' characteristics. All these findings may be explained by the 'Reciprocal Theory' that offers bidirectional relationship between MA and mathematics performance (Carely et al., 2015) and by the Cognitive Load Theory (Sweller, 2023). In our study, the participants included three different mathematics learning levels (low, moderate, and high levels). It is obvious that the mathematics learning levels differ in the complexity of the mathematical tasks. The mathematical contents, problems, and tasks are ordered according to the mathematical learning level, i.e., easy, difficult, and more difficult. Further, we may assume that students at different levels have different working memory capacities, also ordered according to the student's mathematics learning level. In other words, the working memory capacities of the moderate mathematics learning level are larger than those of the high mathematics learning level. This evidence shows that students at a high mathematics learning level have a higher mathematics performance than those in moderate and low learning mathematics levels. Thus, those students at a high learning level have better

mathematical performance and abilities, following the cognitive load theory and the reciprocal theory, the moderate mathematics learning level students have a higher MA level than the high learning level.
In addition, the current study indicated that there were gender significant differences in internal MA factors and external MA, it was concluded that the females developed higher MA than males, this result is supported by previous research (Anatonio, 2023; Cipora et al., 2022; Barroso et al., 2021; Spelke, 2005). It is worth to mention that the innovation in the current study is that females' MA was interpretated as high in items that relate to parents, beliefs and mathematics characteristics.

Recommendations

The current study emphasized the power of the inner and external factors of MA among different learning levels and genders. The researcher recommends integrating these insights into the training of mathematics teachers' programs, and to expose both in-service and pre-service mathematics teachers to the different inner and external MA and the power of each factor for different learning levels and genders. Understanding these factors may encourage educators to look for treatments for this phenomenon.

Limitations of the Study

Some limitations of the conducted study should be the research sample, which might be insufficiently representative, but it still offers an insight into the studied issue. Future research is needed to test whether the results of this study are invariant for larger samples that include different societies, different ages (10th, 11th, and 12th grades), and different zones of the country; moreover, different countries.

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Research Article

Overcoming the intuitive rule of three: improving the false direct and inverse proportion[1]

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Abstract

This article analyzes the teaching of the rule of three and its application in solving mathematical problems involving magnitudes. It is also called direct and inverse proportion. It questions traditional didactics, which rely on intuition, arguing that they omit mathematical justification and lead to conceptual errors. It is proposed an alternative method based on the "First Algebra of Magnitudes", introducing a more rigorous and logical approach. The study highlights how current teaching methods fail to adequately operate physical magnitudes by deleting the magnitudes of operations, leading to the "arithmetization of physics". Traditional resolution methods (unit reduction, proportions, and practical methods) are criticized for their lack of logical foundation. The proposed new method structures reasoning through the algebraic formalization of magnitudes and demonstrates its effectiveness with an empirical study conducted on 97 students. A compound rule of three problem is presented, and different approaches are compared, concluding that the method based on the First Algebra of Magnitudes is more precise and logical. The article emphasizes the need to rethink the teaching of proportionality in mathematics to avoid mechanical and intuitive procedures and promote learning based on deduction and a deep understanding of the relationships between magnitudes.

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Introduction

The rule of three has always been a problem in teaching for student learning. Throughout my teaching career at the university level I have observed the common confusion among students regarding the correct approach to proportionality arithmetic and magnitudes according to the different methodologies taught when the problem poses different dimensions, leading to solutions that do not make logical sense, an issue demonstrated in the present statistical study.

Martínez, Muñoz and Oller (2015, p. 104) point out that “the general practice when presenting resolution methods is the complete absence of justifications. Most of the texts limit themselves to giving a series of instructions to outline the steps to follow for the construction of the solution”, something I am in agreement with and which is precisely the reason that led to this research.

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Mathematics is a "deductive science that studies the properties of abstract entities, such as numbers, geometric figures or symbols, and their relationships", a definition given in the fifth meaning of the Dictionary of the Royal Spanish Academy. These properties are precisely the soul of mathematical science, as it requires progressing step by step, with logic, moving from the universal to the particular, demonstrating at each step the properties on which it is based, leaving no room for mere intuition.

Royal Decree 217/2022, of March 29, which establishes the organization and minimum teachings of Compulsory Secondary Education in Spain, states in Annex II regarding mathematics:

> *Reasoning, argumentation, modeling, knowledge of space and time, decision-making, prediction and control of uncertainty or the correct use of digital technology are characteristics of mathematics [...] it is important to develop in students the basic tools and knowledge of mathematics that enable them to navigate successfully in personal, academic and scientific contexts as well as in social and professional environments.*
>
> *[...]*
>
> *Research in didactics has shown that performance in mathematics can be improved if prejudices are challenged and positive emotions towards mathematics are developed. [...] Solving problems is not only a goal of learning mathematics, but is also one of the main ways to learn mathematics. Problem-solving involves processes such as interpretation, translation into mathematical language, application of mathematical strategies, evaluation of the process, and verification of the validity of solutions.*
>
> *[...]*
>
> *The connection between mathematics and other subjects should not be limited to concepts, but should extend to procedures and attitudes, so that basic mathematical knowledge can be transferred and applied to other subjects and contexts.*

As we will see in the following sections, this has been precisely the shortcoming observed with the so-called "compound rule of three", which cannot be referred to as "proportionality of magnitudes" or "direct and inverse proportion", at least not with the approach given in the current bibliography. This is because the rule of three with magnitudes is not simply mathematical; it goes beyond that, belonging to the field of applied mathematics from the very moment the magnitudes or dimensional elements are added, thus encroaching into another discipline such as physics.

At this point, it should be noted that the confusion surrounding the rule of three with magnitudes is not due to a mathematical error, as it is not purely numerical, but rather pertains to physics. Mochón (2012) has identified a problem with the rule of three, blaming it on its mechanical teaching in place of proportion, proposing very interesting didactic methods; however, all of them are characterized by the absence of the magnitudes themselves.

The observed problem is that operations involving physical magnitudes often omit the magnitudes themselves, leading to a significant issue: the arithmetization of physics. This has been identified by the author Arnaiz (2017, p. 25):

> *There is thus a gap yet to be resolved in the operations with physical magnitudes, which causes the proliferation of diverse and contradictory opinions regarding their nature and formulation, discussions that could be settled simply by defining the appropriate composition laws. A group of authors, such as Tolman, attribute to the symbols of dimensional expressions a certain impenetrable or mystical character and consider that "the true essence of magnitudes, from a physical point of view, is represented by their dimensional formula" (Physics Review, 1917, p. 25). This hypothesis does not seem to be true, because it would imply that such disparate magnitudes as the moment of a force and its work, both of which can be expressed as "newton×meter", are essentially manifestations of the same magnitude, energy, which clearly seems unreasonable, as we justify in section XXVI of the First Algebra of Magnitudes.*
>
> *Great authors such as Planck indicate that "it is as meaningless to speak of the 'real' dimension of a magnitude as of the 'real' name of an object", which would imply that physical magnitudes should be concealed from understanding.*

He continues stating (p. 27):

> *All authors versed in dimensional analysis take for granted that unit abbreviations operate with the same algebra as abstract numbers, and based on this implicit and unjustified assumption, they develop their respective theories, which completely omit any specific algebra for magnitudes.* ***The same occurs in the educational field, where the philosophical problems related to magnitudes and their composition laws are overlooked as if they did***

> ***not exist, teaching concrete operations in an intuitive, subjective and arbitrary manner, leaving students, even without knowing it, with a residue of uncertainty*** *that taints all the knowledge acquired, due to this unresolved gap.*

This problem of the arithmetization of magnitudes, which can be summarized as performing operations with concrete numbers by simply omitting the magnitudes —thereby ignoring the influence of the dimensional element on the result and turning the physical operation into a merely arithmetic question (hence the term "arithmetization")— is extensively researched by Arnaiz (2017), who has not only identified the problem and studied its historical origins, but also proposed a solution that has proven to be the only applicable way to demonstrate something as basic as the rule of three with magnitudes, also known as "proportional magnitudes" or "direct and inverse proportion" —an inaccurate expression according to the didactic methodology employed since proportional applied is on the numbers, not to dimensional elements—.

By applying the *First Algebra of Magnitudes* to the proportional magnitudes, the reader will see that the current approach is meaningless, reducing everything to something much simpler and deductive, eliminating any hint of mere intuition referenced by Martínez, Muñoz and Oller (2015, p. 104).

The problem of the arithmetization of magnitudes with the current approach leads to issues such as the problems of estimating unattainable magnitudes posed by Albarracín and Gorgorio (2013).

To guide the reader toward a correct understanding of the practical didactic issues currently present in teaching, a practical problem will first be presented, outlining the different methodologies taught for its solution. Subsequently, the necessary concepts of the *First Algebra of Magnitudes* applicable to this topic will be introduced with the corresponding demonstrations, to finally apply them to the same problem statement.

Finally, the results of a statistical study will be presented, measuring the effectiveness in teaching each of the four methods (the three traditional ones and the proposed fourth method), evaluating the proportion of students who correctly reasoned a problem using each of the didactic methods. This statistical study was conducted among university students to first determine how many remember how to reason a problem correctly before teaching the different methods. This is without prejudice to the proposed subsequent research among secondary students outlined at the end of this article.

Literature review

Problems specific to the compound rule of three are those that involve more than two magnitudes. Problems with two magnitudes are simple.

The problem taken as a reference to contrast the main didactic methods with the fourth one proposed in this research involves four magnitudes, and its statement is as follows: If 5 men dig a 40-meters trench in 8 days working 6 hours a day, how many days will it take for 9 men working 8 hours a day to dig a 60-meter trench?

A problem with this context has been chosen for the arguments given by Martínez, Muñoz and Oller (2015, p. 105): "the predominant contexts in the problems [presented in school books] are those of 'production or consumption in a cooperative work framework' and 'economic or temporal costs of an activity'" with only Anaya's textbooks presenting "problems that involve magnitudes from Physics". However, it is considered that it is precisely with the magnitudes from this science where the true nature of applied mathematics concerning the content of the compound rule of three is observed, without prejudice to the proposed method being equally applicable to any other type of context.

Different resolution methods have been considered in this study: reduction to unity, proportions and practical method. These methods are applied complementarily or substitutely, and even in combination, across all the didactic mathematical bibliography consulted (Zuasti, 2022; González, 2020; García and Ortega, 2016; Mira, 2016; Vallejo and Fuentes, 2016; Didáctica de las ciencias para docentes de educación primaria, 2016; García and Resano, 2015; Segovia and Rico, 2015; Volera and Gazxtelu, 2014; Matemáticas 7, 2011; Gómez, 2006; Galdós, 2000; Postigo, 1998; Matemáticas. Regla de tres, 1982). The proportion method has not been observed independently in any school bibliography. However, it was considered relevant to include it in this research as it has been studied in research articles such as Gómez (2006).

An interesting study is the one proposed by Silvestre and da Ponte (2012), who suggest that before teaching direct proportion or the direct rule of three, students make use of simultaneous calculations with additions and multiplications with "rudimentary strategies" such as unit counting. They conclude with an observation about the student's ability to understand the relationship between variables and the problem's context. This last observation is very important because the system proposed in this article, in replacement of the purely intuitive methods taught, aims precisely to facilitate the algebraic representation of reality.
Quite interesting is the observation made by Ibáñez and Martínez (2020, p. 53), who state that none of the textbooks teaching compound proportions (referring to the compound rule of three) distinguish between quantities of magnitudes and numbers, an issue that has been anticipated here with the problem of the arithmetization of physics.

Theorical Framework

Before applying each of them, we must refer to two premises without demonstration on which they are based:
1. It is said that two magnitudes are directly proportional when, by multiplying one of them by a number, the other is multiplied by the same number, and by dividing one of them by a number, the other is divided by the same number (Grence, 2023, p. 27; Llanos Vaca et al., 2022, p. 131; Galdós, 2000, p.197; Carrillo et al., 2016, p. 157).
2. On the contrary, it is said that two magnitudes are inversely proportional when, by multiplying one of them by a number, the other is divided by the same number, and by dividing one of them by a number, the other is multiplied by the same number (Grence, 2023, p. 29; Llanos Vaca et al., 2022, p. 131; Carrillo et al., 2016, p. 157; Galdós, 2000, p. 198).

Reduction to unity method.

Following the same bibliography, "it is about finding the days it would take for 1 man working 1 hour a day to dig a 1-meter trench and then calculating the time it would take for 9 men working 8 hours a day to dig the 60-meter trench" (Galdós, 2000, p. 199). According to Zuasti (2022, p. 47), there are four magnitudes: the days of work, the daily working hours, the number of men, and the size of the trench, considering that "the best method is to reduce it to a simple proportionality problem".
To this end, it is proposed to multiply 5×8=40 days, which is the time it would take for one man working 6 hours a day to dig a 40-meter trench. By multiplying 40×6=240, we obtain the days it would take for one man working 1 hour a day to dig a 40-meter trench. Finally, they propose that by dividing 240÷40=6, we obtain the days it would take for one man working 1 hour a day to dig a 1-meter trench. By performing the operation

$$\frac{6}{9}=\frac{2}{3}$$

we get the days it would take 9 men working 1 hour a day to dig a 1-meter trench. By performing the operation

$$\frac{\frac{2}{3}}{8}=\frac{1}{12}$$

we obtain the portion of the day it would take for 9 men working 8 hours a day to dig a 1-meter trench. Finally, multiplying

$$\frac{1}{12}\times 60=5$$

we get the days it would take for 9 men working 8 hours a day to dig a 60-meter trench, thus solving the presented problem.
However, only the necessary operations to be performed are explained in the given terms; no general reasoning is provided that could apply to any problem of the same type.

Proportions method

Although this method was completely lost in the LOGSE system by combining with other methods (Martínez et al., 2015, p. 104), which is why it has been included in this study in its independent version, striving to remain faithful to the original symbolism[3].

It method involves organizing the different numerical data based on their magnitudes. Thus, in the first row, the data of the initial assumption are placed, and in a second row the data of the question where the unknown number is located:

Assumption	5 men	6 hours a day	40-meter	8 days
Question	9 men	8 hours a day	60-meter	x

The first step of this method consists of the decomposition into simple rules of three, so that, firstly, we have:

Assumption	5 men	8 days
Question	9 men	x

According to the explanation given for this methodology, "the number of men and the time spent are inversely proportional magnitudes since more men means less time and fewer men means the more time" (Galdós, 2000, p. 201). The following proportion is reached:

$$9:5::8:y$$

It can be observed that the proportion has been reduced to an operation with only two magnitudes, although these have been eliminated from the equation.

Similarly, it is said, we have:

Assumption	6 hours a day	y days
Question	8 hours a day	z

Again, it is noted that "the number of daily hours and the time used are inversely proportional magnitudes since more hours mean less time and fewer hours mean more time". Thus, obtaining:

$$8:6::y:z$$

The same criteria apply to the last proportion:

Assumption	40-meters	z days
Question	60-meters	x

In this case, it is stated that "the length of the trench and the time taken are directly proportional magnitudes", leading of the following proportion:

$$40:60::z:x$$

Finally, the obtained proportions are multiplied term by term:

$$\frac{9 \times 8 \times 40}{5 \times 6 \times 60} = \frac{8 \times y \times z}{y \times z \times x}$$

Solving the mathematical operation, we get:

$$x = \frac{8 \times 5 \times 6 \times 60}{9 \times 8 \times 40} = 5$$

Thus, the calculated solution is "5 days". As in the previous method, no general reasoning is carried out to attempt to prove it.

Practical method.

This method is explained in the following words by Galdós (2000, p. 202):

> *The assumption and the question are written and then each of the magnitudes is compared with the unknown number, assuming that the other magnitudes remain constant, to see if these magnitudes are directly or inversely proportional to the unknown number. The magnitudes that are directly proportional to the unknown number are marked with a + sign at the bottom and a – sign at the top, and the magnitudes that are inversely proportional to the unknown number are marked with a – sign at the bottom and a + sign at the top. Once this is done, the value of the unknown number will*

[3] The symbolism used in this method is specific to the original bibliography when dealing with proportions. That is, the symbol ":" corresponds arithmetically to division, and the symbol "::" to equality. This symbolism has been used since Bails, B. (1805), Galdós (2000, pp. 199-201), and Gómez Alfonso (2006, p. 58). However, it has not been observed in any recent editions of teaching materials for ESO, although it was considered interesting to maintain it in the study.

be equal to the known data of its kind, which is assumed to always carry a + sign, multiplied by all the quantities with a + sign and divided by all the quantities with a − sign. This is the fastest method for solving all types of rule of three problems.

Following these steps, our problem would be set up as follows:

	+	+	−	+
Assumption	5 men	6 hours a day	40-meter	8 days
Question	9 men	8 hours a day	60-meter	x
	−	−	+	−

Thus, Galdós (2000, p. 203) states:

As can be seen, the number of men and the time taken are inversely proportional magnitudes and, therefore, we put the + sign above the men and the − sign below. Similarly, the number of daily hours and the time taken are inversely proportional magnitudes, and, therefore, we put the + sign above the daily hours and the − sign below. Similarly, the length of the trench and the time taken are directly proportional magnitudes, and therefore, we put the − sign above the length and the + sign below.

With all this, it must be inferred that the operation to be performed is, in days:

$$x = \frac{8 \times 60 \times 6 \times 5}{40 \times 8 \times 9} = 5$$

Proposed method of resolution based on the *First Algebra of Magnitudes*.

The topic of the rule of three with magnitudes is the first lesson in Applied Mathematics, as it is the first time that magnitudes are introduced to numbers. Therefore, it is the first moment we encounter the problem of the arithmetization of physics.

When the problem statement specifies quantities “5 men”, “6 hours a day”, “40 meters” and “8 days”, each of the four measurements is what is termed a “dyad”. Every dyad consists of two elements: the abstract number and the dimensional element or magnitude it accompanies, which gives a natural sense to the abstract number. As stated by Arnaiz (2017, p. 51): “In general, we have called measurement the quantity, extension or portion of a magnitude expressed in the form q U, as a symbol of the times q, a real number, that a unitary quantity U is present in a phenomenon, denoting q as the measure with the unit U of the magnitude included in the observed fact”.

Thus, in our case U=day and q=8, because as he continues, “this newborn entity that refers to physical measurement could also receive the mathematical name of, for example, concrete entity or physical dyad, and its elements will be called primary or measure q or $\bar{q}$, and unit U or secondary. The primary is the mathematical part of the dyad. The secondary is the physical or dimensional part.”

The problem arises when we want to operate with dyads, performing additions, subtractions, multiplications, divisions, etc., in such a way that we do not omit the dimensional element ─the magnitude─ of the dyad. The proposed solution is the one that gives its name to the book itself: the *First Algebra of Magnitudes*, where the first steps are taken for operations with magnitudes based on geometry, seeking a new symbology that differentiates it from the merely arithmetic.

The deduction of the proportionality of magnitudes based on the algebra of magnitudes on which the compound rule of three problems are based is carried out in the following sections.

Proportions with homogeneous dyads.

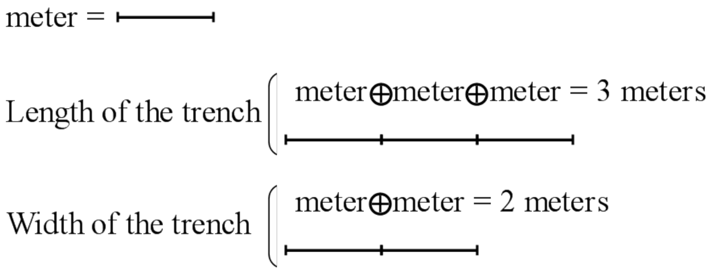
To be clear in the exposition, a simpler problem than the initial one will first be presented, based on the very definition of the rule of three according to Galdós (2000, p. 198): “the rule of three is the arithmetic operation that consists of determining the fourth term of a proportion knowing the other three”.

The problem that is presented is the following: a trench 3 meters long by 2 meters wide has been dug. Another one is to be made with sides proportional to those of the previous trench, with the longer side being 6 meters. How wide should it be for both sides to be proportional to those of the smaller trench?

Figure 1 shows two dyads: the first refers to the length of the trench, "3 meters"; and the second to the width, "2 meters". In *First Algebra of Magnitudes*, operations are performed geometrically by defining each magnitude according to a segment. Thus, "meter" is specified in a segment of a specific measurement. Thus, "meter" is represented by a segment of a specific measure. Since both dyads have "meter" as their dimensional element, the segments representing them will have the same measure.

Figure 1

Geometric representation of homogeneous dyads.



Note: First, the *meter* magnitude is defined by a fixed-length segment. Then, this segment is consecutively added for the length of the trench as many times as the number of meters it has, what is deffined by the abstract number. Since the trench is 3 *meters long*, the segment is added three times. The same applies to the 2 *meter width* of the trench. Since the length and width have different dimensions (a meter of length is not the same as a meter of width), the addition is performed separately.

If we analyze each dimension of the trench, the length is "3 meters," which geometrically would mean taking the "meter" segment and adding it three times, as shown in Figure 1. Symbolically, we cannot use the "+" sign because we are in the algebra of magnitudes with geometry, not in arithmetic, and we must clearly distinguish between both kind of algebra. Thus, respecting the symbology of the *First Algebra of Magnitudes*[4], the sign we will use is ⊕, resulting in:

$$meter \oplus meter \oplus meter = 1meter \oplus 1meter \oplus 1meter = (1+1+1)\ meters = 3\ meters$$

The problem consists of adding 3 more "meter" segments of length and calculating how many "meter" segments we need to add to the 2 we already have in width to maintain proportionality. Thus, the geometric algebraic operation with respect to the length of the trench will be:

$$3meters \oplus 3meters = (3+3)\ meters = 6\ meters$$

To answer the question, we can observe that the length, composed of 3 "meter" segments, has been added by the same amount:

$$3meters \oplus 3meters = 2 \circ (3meters) = 6\ meters$$

Again, the multiplication symbol known to all for operations with abstract numbers cannot be used in the same way for geometric multiplications of an abstract number by a dyad, so the symbol ○ is chosen following the *First Algebra of Magnitudes*. The same applies to division, whose symbol used as a geometric operation is //.

Thus, if we compare the length of the trench before and after the addition, we find the multiplier of the width of the trench to answer the question:

$$\frac{6\ meters}{\overline{3\ meters}} = \frac{6}{3} = 2$$

Therefore, 2 is the abstract multiplier of both sides of the trench to maintain proportionality between the two dyads. Note that dividing two homogeneous dyads results in an abstract number, as one magnitude is geometrically divided by itself. As Arnaiz (2017, p. 152) explains, algebraically:

$$a \circ (b\ magnitude) = c\ magnitude \rightarrow a = \ (c\ magnitude)//(b\ magnitude)$$

To facilitate the solution to the problem, given the measurements of the length and width of the trench before and after the addition, whose final width is unknown, we have:

[4] The symbology of the *First Algebra of Magnitudes* is developed throughout the entire volume as the various demonstrations are carried out. However, the author provides a synoptic diagram of all the signs used for the different operations, differentiating them according to whether they pertain to ordinary numerical algebra or dyadic algebra (Arnaiz, 2017, p. 220).

$$\frac{6\,meters}{3\,meters} = \frac{x\,meters}{2\,meters} \rightarrow \frac{6}{3} = \frac{x}{2} \rightarrow x = \frac{6}{3} \times 2 = 4$$

Let's look at it with another example: if 12 coats cost 360 euros, how much will 8 coats cost? We will pose the problem with magnitudes as:

$$\frac{12\,coats}{8\,coats} = \frac{360\,euros}{x\,euros} \rightarrow \frac{12}{8} = \frac{360}{x} \rightarrow x = \frac{360 \times 8}{12} = 240$$

But 240 for what exactly? To answer this question, we refer to the geometric proportion where we have the dyad with its magnitude, allowing us to complete the dyad of the consequent in the second geometric ratio, referring to 240 euros. Having explained the dyadic addition with two different examples, it is time to verify the generic property for its validity in any particular case.

Demonstration of proportions with homogeneous dyads

Given a numerical element "a" and its magnitude or dimensional element "$magnitude_a$" forming the dyad "$a\ magnitude_a$", if any abstract multiplier such as "x" is applied, the following geometric operation occurs:

$$a\ magnitude_a \rightarrow x{\circ}(a\ magnitude_a)$$

For the ratio of this addition to be proportional to the addition of another ratio of a different dyad such as "$b\ magnitude_b$", the multiplier must be the same:

$$b\ magnitude_b \rightarrow x{\circ}(b\ magnitude_b)$$

such that it will always be hold that:

$$\frac{x \circ (a\,magnitude_a)}{a\,magnitude_a} = \frac{x \circ (b\,magnitude_b)}{b\,magnitude_b} \rightarrow \frac{x \times a}{a} = \frac{x \times b}{b}$$

thus proving the property of proportions with homogeneous dyads, which can be stated as follows: for two ratios between homogeneous or additive dyads to be proportional, they must have the same abstract multiplier or quotient.

The reader will observe that this property nullifies one of the premises on which the rule of three with magnitudes is based, specifically the one that refers to the "direct proportionality between magnitudes".

Proportions with heterogeneous dyads

Continuing with the same type of approach, let's start with a simple example: if 6 workers complete a job in 20 days, how long will it take 8 workers to complete the same job?

In this case, the dyads we encounter have "workers" as one dimensional element and "days" as another. This means that the nature of the magnitudes is different, so their combination results in a third magnitude that can be defined as "work necessary to finish the job", meaning the relationship between both dyads is no longer additive, as in the previous problems.

This is what the *First Algebra of Magnitudes* refers to as "heterogeneous dyads", whose geometric treatment must be different in terms of the relationship between the two magnitudes.

Thus, like any other magnitude, "worker" and "day" will be geometrically defined by respective segments of equal or different lengths. How should both magnitudes be related? Through geometric multiplication.

In this way, defining the magnitude "worker" as a segment of a certain fixed length and the magnitude "day" as another segment of equal or different fixed length, multiplying both segments geometrically results an area that expresses the third magnitude directly related to the two previous ones: the composite unit in which the work necessary to complete the job will be measured. This is shown graphically in Figure 2.

To perform the geometric multiplication operation between two dyads, we cannot use the × or • signs used for abstract numbers, nor the ○ symbol defined in the *First Algebra of Magnitudes* for the product between an abstract number and a dyad. Thus, following the same bibliography, the chosen sign is $*$.

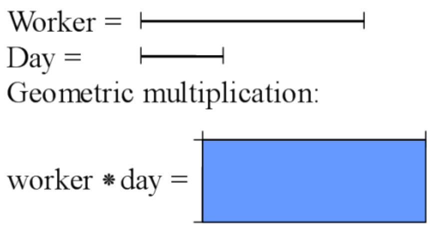


Figure 2. Graphic representation of heterogeneous dyads

Note: First, we define the fixed length of a segment for each magnitude. In this case, the *worker* magnitude will be a segment of equal or different length than the *day* magnitude. Since we are combining two magnitudes of different dimensions, the geometric operation to be performed is multiplication, which is symbolized according to the *First Algebra of Magnitudes* with the symbol ∗. The combination of these two magnitudes through geometric multiplication results in a new dyad, geometrically defined by the resulting surface. This is why they are heterogeneous dyads.

Therefore, completing the job in question requires 6 workers and 20 days of work. The geometric area obtained by multiplying both dyads is the "work needed to complete the job" (Figure 3), which symbolically will be:

$$6\ \textit{workers} * 20\ \textit{days} = \textit{work needed}$$

The question posed is: to finish that same job, or in other words, the work needed to complete the job with 8 workers, how many days will it take?

This implies that we are being presented with another geometric operation, but the result is the same:

$$8\ \textit{workers} * x\ \textit{days} = \textit{work needed}$$

Therefore:

$$6\ \textit{workers} * 20\ \textit{days} = 8\ \textit{workers} * x\ \textit{days}$$

If we operate separately on the numerical elements from the dimensional ones on the first side of the equation:

$$(6 \bullet 20) \circ \textit{workers} * \textit{days} = 8\ \textit{workers} * x\ \textit{days}$$

Solving geometrically for the dyad that is the unknown in the problem, we get:

$$\frac{(6 \bullet 20) \circ \ workers \ * \ days}{8\, workers} = \ x\ days$$

As explained earlier, "dividing two homogeneous dyads results in an abstract number, as a magnitude is geometrically divided by itself".

$$\frac{(6 \bullet 20) \ \circ \ days}{8} = \ x\ days$$

Separating on the first side of the equation and operating on the abstract numbers of the magnitude "day", we have:

$$\frac{6 \bullet 20}{8} \circ \ days = \ x\ days$$

Thus, it results in:

$$15\ \textit{days} = x\ \textit{days}$$

This answers the question, concluding that to complete the same work with 8 workers, 15 days are needed.

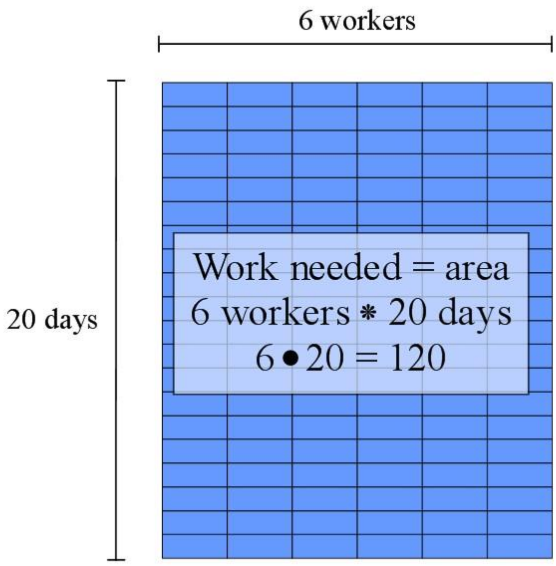


Figure 3. Graphical representation of the heterogeneous dyads in the example

Note: Once we have defined the segments of the two magnitudes separately and combined them as shown in Figure 2, the same operation can be performed after adding to each magnitude. In this way, if we add the *day* magnitude 20 times and the *worker* magnitude 6 times, resulting in two segments of size 6 *workers* and 20 *days*, and then combine the two resulting dyads through geometric multiplication, we obtain the third resulting heterogeneous dyad, defined by the expression 6 *workers* ∗ 20 *days*, which in this case can be defined as w*ork needed*. Geometrically, this new dyad is the surface.

As we have done in the case of proportions with homogeneous dyads, let's consider a second example of proportions with heterogeneous dyads: a car traveling at 140 kilometers per hour takes 4 hours to complete a trip. How long would it have taken if it had traveled at 112 kilometers per hour?

We observe that the given magnitudes are "speed" and "time", so both combined give rise to a third magnitude: "total distance traveled". Time is measured in this case in "hours", and speed in "kilometers per hour". By geometrically defining each of the two given magnitudes with a segment of a certain length, whether equal or different, their combination will result in an area that represents the distance traveled in kilometers. Thus, considering the data of the problem, the geometric operation will be:

$$Distance = 140\ kilometers\ per\ hour * 4\ hours$$

For the question posed, the distance (area of the geometric square obtained) being the same:

$$Distance = 112\ kilometers\ per\ hour * x\ hours$$

Since both geometric products are equal to the same area, we know their equality:

$$140\ kilometers\ per\ hour * 4\ hours = 112\ kilometers\ per\ hour * x\ hours$$

Operating with the established geometric symbolism:

$$\frac{140\ kilometers\ per\ hour}{\overline{112\ kilometers\ per\ hour}} * 4\ hours\ =\ x\ hours$$

Having both dyads in the geometric division with the same dimensional elements "kilometers per hour" in numerator and denominator and separating the numerical elements from the resulting dimensional:

$$\left(\frac{140}{112} \times 4\right) \circ\ hours =\ x\ hours$$

Thus, it resolves that:

$$5\ hours = x\ hours$$

Answering the question.

However, it is convenient to make a note here, as algebraic operations were performed with a magnitude (speed), which is itself part of another combination of dyads. Speed is measured in kilometers per hour as has been discussed in the problem; but this approach can be different if we consider that the speed is expressed as the distance traveled in a specific unit time period, so the assumption will be:

$$distance = \frac{140\ kilometres}{1\ hour} * 4\ hours$$

while the question will be:

$$distance = \frac{112\ kilometres}{1\ hour} * x\ hours$$

This leads to the following equality, which, when operating geometrically:

$$\frac{140\ km}{1\ hour} * 4\ hours = \frac{112\ km}{1\ hour} * x\ hours$$

$$(140 \bullet 4) \circ \frac{km * hours}{hour} = (112 \bullet x) \circ \frac{km * hours}{hour}$$

Since the combination of equal magnitudes results in an arithmetic equality, weobtain the same solution as the previous approach:

$$x = \frac{140 \bullet 4}{112} = 5$$

Demonstration of proportions with heterogeneous dyads

Continuing with the same methodology as with homogeneous dyads, let's demonstrate that proportions are met with heterogeneous dyads, establishing a new general property applicable to any particular case, as a deductive system that characterizes applied mathematics, in this case, physics.

Given two heterogeneous dyads such as "$a\ magnitude_a$" and "$b\ magnitude_b$", these give rise to a third "$c\ magnitude_c$" which is a combination of the two through geometric multiplication:

$$a\ magnitude_a * b\ magnitude_b = (a \bullet b) \circ magnitude_a * magnitude_b = c\ magnitude_c$$

such that the value of the combined dyad

$$(a \bullet b) \circ magnitude_a * magnitude_b$$

which is the area of the abstract rectangle formed, must remain invariant. When multiplying any of the two original dyads, which are the sides of this rectangle, the other will be divided, maintaining the geometric equality of the area.

Thus, in the previous general approach, let's suppose that the first dyad increases its value by an abstract multiplier such as "x". If the combined dyad (the area of the rectangle) remains invariant, the equality would be lost:

$$[x \circ (a\ magnitude_a)] * b\ magnitude_b \neq (a \bullet b) \circ magnitude_a * magnitude_b$$

To maintain this equality with the combined dyad being invariant, the second dyad will be altered, being divided by the same multiplier, which is symbolically expressed as:

$$[x \circ (a\ magnitude_a)] * [(1/x) \circ b\ magnitude_b] = (a \bullet b) \circ magnitude_a * magnitude_b$$

This is so because, if we operate the first member of the geometric equality:

$$[x \circ (a\ magnitude_a)] * [(1/x) \circ b\ magnitude_b] = (x \bullet a \bullet b/x) \circ magnitude_a * magnitude_b$$

It is not necessary to recall that a number divided by itself is the unit.

Thus, the proportion between heterogeneous dyads can be formulated as follows: given the combination between two heterogeneous dyads that result in a third combined dyad, when the first dyad is multiplied or divided by an abstract number, to maintain this equality with the combined dyad being invariant, the second dyad will be divided or multiplied, respectively, by the same abstract number.

Application of the proposed *First Algebra of Magnitudes* method to the given problem.

Let us recall the statement of the initial problem: if 5 men working 6 hours a day have dug a 40-meter trench in 8 days, how many days will it take for 9 men working 8 hours a day to dig a 60-meter trench?

In the given assumption, the following dyads are observed: "5 men", "6 hours a day", "8 days" and "40 meters". It is immediately observed that there is a homogeneous relationship between magnitudes "hours a day" and "days", which is "time", thus the dimensional element "days" is redundant of the "hours a day", making the numerical element the multiplier or abstract number. Therefore, the symbolic statement of this fact can be:

$$8 \circ (6\ hours\ a\ day) = 6 \circ (8\ days) = (6 \bullet 8) \circ hours$$

However, from the combination of "time" and the "men" working, a new magnitude arises, which is the "length of the trench" in question, representing the work done. Thus, geometrically, "time" and the "men" working will be defined by respective segments of certain lengths added as many times as the abstract multiplier establishes, and each resulting dyad will form one side of the area formed by geometric multiplication. Thus:

$$(6 \bullet 8) \circ \textit{hours} * 5\ \textit{men} = 40\ \textit{meters}$$

Similarly, for the symbolic representation of the question:

$$(8 \bullet x) \circ \textit{hours} * 9\ \textit{men} = 60\ \textit{meters}$$

It is observed that in this case, the area formed by the combination of the two dyads is not the same; however, they do have the same dimensional element, so if we reduce the combined dyad to the dimensional unit in each of the geometric equalities, we have in each case:

$$\frac{(6 \bullet 8) \circ hours * 5\, men}{40} = meters$$

$$\frac{(8 \bullet x) \circ hours * 9\, men}{60} = meters$$

Thus, we obtain the following equality:

$$\frac{(6 \bullet 8) \circ hours * 5\, men}{40} = \frac{(8 \bullet x) \circ hours * 9\, men}{60}$$

Operating geometrically distinguishing the abstract numbers from the magnitudes:

$$\left(\frac{6 \bullet 8 \bullet 5}{40}\right) \circ hours * men = \left(\frac{8 \bullet x \bullet 9}{60}\right) \circ hours * men$$

from which it results:

$$\frac{6 \bullet 8 \bullet 5}{40} = \frac{8 \bullet x \bullet 9}{60}$$

Therefore, we obtain the value of the unknown number:

$$x = \frac{6 \bullet 8 \bullet 5 \bullet 60}{40 \bullet 8 \bullet 9} = 5$$

Recovering the dimensional element of the unknown, the answer to the question is that 5 days of work are needed.

Application of the method to a more complex problem

Let's suppose the following problem: In a factory, 8 men and 12 women work. The men work 9 hours a day, and the women work 7 hours a day. Together, they produce 360 units of a product in 6 days. The factory decides to hire 4 more men and 6 more women, and also adjust the work schedule so that both men and women work 8 hours a day, but they just work 5 *days*.

To solve this problem, we must first identify the different magnitudes presented: men, women, hours per day, days, and units of product manufactured. It is necessary to distinguish between men and women because their working hours are different. Additionally, according to the problem statement, the contribution of men and women to the work may vary, meaning that the geometric segment defining these two magnitudes may also be different.

On the other hand, it is observed that the magnitudes *hours per day* and *days* share the same dimensional nature, which is *time*. Likewise, the relationship between *working time* and the number of *men* or *women* working determines the *total hours of work* based on gender, making them homogeneous. The total working time of men and women is then added together, and this total work results in the production of 360 units.

Symbolically, it would be represented as follows:

$$8\ \textit{men} * 9\ \textit{hours/day} * 6\ \textit{days} + 12\ \textit{women} * 7\ \textit{hours/day} * 6\ \textit{days} = 360\ \textit{units}$$

The question follows the same symbolic structure in its formulation, with the number of units produced as the unknown variable:

$$12\ \textit{men} * 8\ \textit{hours/day} * 5\ \textit{days} + 18\ \textit{women} * 8\ \textit{hours/day} * 5\ \textit{days} = \mathrm{X}\ \textit{units}$$

Dividing both equations term by term, we obtain the following expression, easily reaching the result:

$$\frac{8\, men * 9\, hours/day * 6\, days \oplus 12\, women * 7\, hours/day * 6\, days}{12\, men * 8\, hours/day * 5\, days \oplus 18\, women * 8\, hours/day * 5\, days} = \frac{360\, units}{\mathrm{X}\, units}$$

$$\frac{8 \times 9 \times 6 + 12 \times 7 \times 6}{12 \times 8 \times 5 + 18 \times 8 \times 5} = \frac{360}{X}$$

X≈461,54 *units*

Proposition of the new method based on the *First Algebra of Magnitudes*.

With all the above reasoning, the method based on the *First Algebra of Magnitudes* is proposed in the following terms. The *First Algebra of Magnitudes* proposes not eliminating magnitudes in physical operations by the algebra of magnitudes as a solution of the problem of the arithmetization of physics. Thus, every measurement consists of two elements: a numerical one and a dimensional one. For example, "5 meters long" is composed of the numerical element "5" and the dimensional "meter long", and algebraically they are multiplied. If more "meters long" are added, the abstract number 5 will increase, being a homogeneous proportion, as the dimensional element is the same: "meter long". If measurements of a different nature are added, that is, with a different dimensional element such as "2 meters wide", we will have a heterogeneous proportion, which is the geometric multiplication of the two measurements "5 meters long" and "2 meters wide", resulting a third measurement: "surface".

As with arithmetic operations, magnitudes can be operated on (although with different symbols as it is geometric algebra and not just arithmetic, symbolism that we are not going to introduce here). This is geometric algebra since it treats dimensional elements as segments of a certain length. Thus, "meter long" is a segment of a certain length. If more segments of this same dimension are added, they result in a longer segment determined by the abstract number, in our example "5" segments, which has been called homogeneous proportion. If segments of another dimension are added, such as "meter wide", they will have equal or different length, but they can no longer be added after the "meters long" because they are of a different nature, and they must be multiplied together, geometrically resulting in an area that represents the third resulting dimensional element, in this case the "surface". This is the heterogeneous proportion.

This method applied to problems with magnitudes consists of obtaining an equality, how much work is needed (a combination of measurements of equal or different dimensions) to obtain a result, that is:

Amount of work = Work result

Let's see it with an example: if 5 men working 6 hours a day have dug a 40-meter trench in 8 days, how many days will it take 9 men working 8 hours a day to dig a 60-meter trench?

For the given assumption, how much work is needed? 5 men, 6 hours a day and 8 days are needed. What is the result of the work? A 40-meter trench. Thus:

5 *men* × 6 *hours a day* × 8 *days* = 40 *meters*

To solve the unknown number, the question can be posed similarly, knowing that the work needed is: 9 men and 8 hours a day for x days (the dimensional elements are the same). The result of the work is a 60-meter trench:

9 *men* × 8 *hours a day* × *x days* = 60 *meters*

From here, it is sufficient to move everything to one side of the equation to isolate the variable (remember that "40 meters" and "60 meters" are algebraic multiplications). Thus, we obtain:

$$\frac{5\,men \times 6\,hours\ a\ day \times 8\,days}{40\,meters} = 1$$

Likewise:

$$\frac{9\,men \times 8\,hours\ a\ day \times X\,days}{60\,meters} = 1$$

Now, since both expressions equal 1, they are equal to each other. If we separate the numbers from the magnitudes, we get for the first case:

$$\frac{5 \times 6 \times 8}{40} \times \frac{men \times hours\ a\ day \times days}{meters} = 1$$

And for the second case:

$$\frac{9 \times 8 \times X}{60} \times \frac{men \times hours\ a\ day \times days}{meters} = 1$$

We can observe that we have the same magnitudes in both expressions, so all the magnitudes cancel out, leaving only the numerical part:

$$\frac{5 \times 6 \times 8}{40} = \frac{9 \times 8 \times X}{60}$$

Operating:

$$6 = 1{,}2 \times X$$

Solving for the unknown X, we get the result in a mere arithmetic equation:

$$6/1{,}2 = X = 5$$

Method

Research Model

The objective of the questionnaire is to measure the degree of effectiveness of each of the didactic methods of the compound rule of three.

To achieve the objective, the questionnaire was structured as follows:

- Firstly, the following problem was presented: if 5 men working 6 hours a day have dug a 40-meter trench in 8 days, how many days will it take for 9 men working 8 hours a day to dig a 60-meter trench?

The objective of this first question is to determine if they remember how to solve this type of problem.

- The second part of the questionnaire aims to measure the didactic effectiveness of each of the methods in this research. To this end, the steps followed were:
- First, an explanation of the reduction to unity method with an example.
- Second, presentation of a problem to be solved by the student using the reduction to unity method.
- Third, an explanation of the proportion method with an example.
- Fourth, presentation of a problem to be solved by the student using the proportion method
- Fifth, an explanation of the practical method with an example.
- Sixth, presentation of a problem to be solved by the student using the practical method.
- Seventh, an explanation of the method based on the *First Algebra of Magnitudes* with an example.
- Eighth, presentation of a problem to be solved by the student using the method of the First Algebra of Magnitudes.

Each didactic method was explained in writng according to the bibliography studied in this research in the same terms presented, having given a time of ten minutes for the study of each one. The method of the *First Algebra of Magnitudes* was conducted with the same degree of demonstration and extension as the three traditional methods in the terms of section III of this article.

The problem explained as an example in each didactic method in steps one, three, five and seven was always the same: if 5 men working 6 hours a day have dug a 40-meter trench in 8 days, how many days will it take for 9 men working 8 hours a day to dig a 60-meter trench?

The problem presented to be solved according to each method in steps two, four, six and eight was always the same to maintain the level of difficulty: if 60 printers working 40 days at 8 hours a day print 1,000 books, how many days will it take 30 printers working 6 hours a day to print 750 books?

Since the objective is to measure the didactic efficiency of each method, they were not allowed to take notes that they could consult during the subsequent problem-solving, and the written explanation was removed.

Additionally, they were never told whether the result they obtained in the problems they had to solve was correct or not, to avoid conditioning them for subsequent solutions.

A final problem was presented, different from all the previous ones, so that they could solve it by the method they chose. The problem in questionwas the following: leaving 9 taps open for 8 hours at 20 degrees results in an expense of 48 euros. At what temperature should the water be if 15 taps are left open for 5 hours to produce an expense of 60 euros?

Participants

The subjects who participated in the study were 97 students of the Business Administration and Management Degree in the 1st, 2nd and 3rd years. The age of the students was 18-21 years old.

Results and Discussion

The first problem presented was solved correctly by 26 students (26.80% of the sample) out of the 97 who participated. Of these, 19 solved it by intuition, 1 by the reduction to unity method, 3 by the proportion method and 3 by the practical method.

The results obtained regarding point 2 of the previous section can be seen in Table 1. To present these results more simply, Illustration 1 is provided showing whether it was correctly solved or, on the contrary, it was incorrectly solved or not solved at all.

Table 1. Results of the solution to the presented problems according to each didactic method (point 2 of section VI.1):

Method	No reasoning	%	Incorrectly reasoning	%	Correctly reasoning	%
Reduction to unity	39	40,63	53	55,21	4	4,17
Proportions	33	34,02	57	58,76	7	7,22
Practical	21	21,65	44	45,36	32	32,99
First Algebra of Magnitudes	8	8,25	12	12,37	77	79,38

Note: The results obtained in solving the proposed problem are compared according to each method. It has been specified based on whether the solution was not formulated, leaving the answer blank, whether the solution was set up using the requested method, and whether the approach was correct.

It can be observed that the best result was obtained with the *First Algebra of Magnitudes* method, with which 79.38% have reasoned it correctly, while with the practical method this figure rises to 32.99%. The methods of reduction to unity and proportion had a very low success rate, remaining at 4.17% and 7.22% respectively.

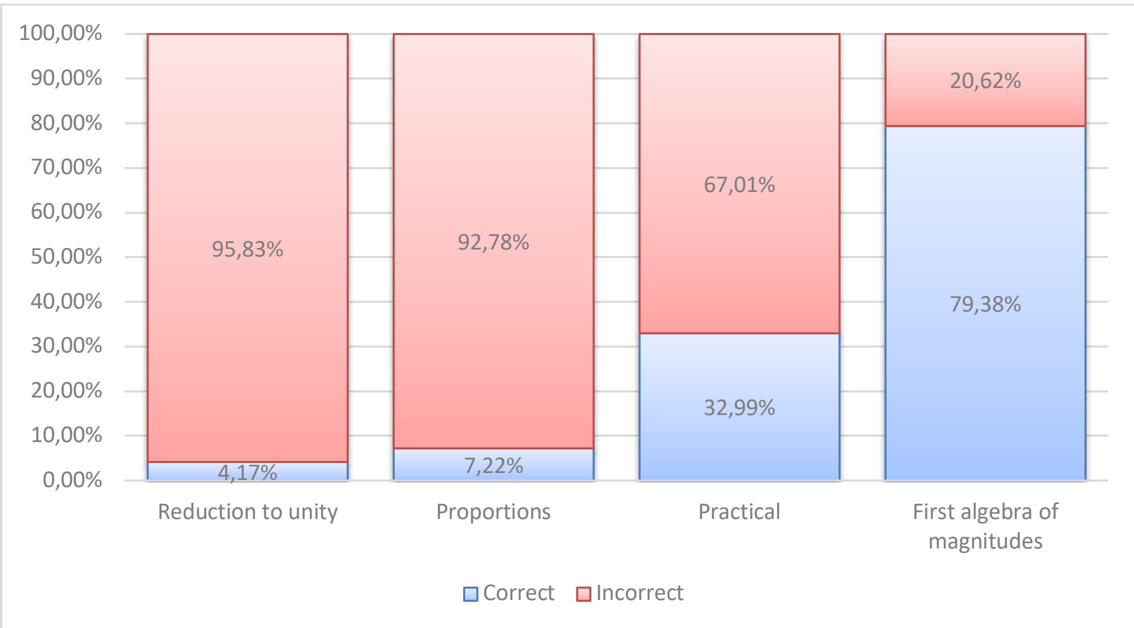


Figure 1. Results of the solutions to the presented problems according to each didactic method (point 2 of section IV.1)

Note: This illustration shows graphically the same results ot table 1 comparing if the problem was correctly solved according to each method.

With all this, a notable improvement in the understanding of this type of problem can be observed with the method of the *First Algebra of Magnitudes* compared to the others, considering that this method is completely new to the students since they had never encountered it before completing this questionnaire, which is not the case with at least one of the other three methods.

It is also observed that the practical method is easier to understand compared to the other two (reduction to unity and proportions), although the success rate in solving problems remains low.

Note that the number of students in the reduction to unity method is 96 (one less than the rest of the methods). This is because, once they began solving the problem according to this method, it was observed that one student had copied the explanation previously made having solved the problem with that material, so it has been removed from the results. Finally, Table 2 shows the results obtained regarding point 3 of section IV. 1, that is, the resolution of a new problem by the method freely chosen by each student. Additionally, Illustration 2 shows the graphical representation of these results.

Table 2. *Results of the last problem proposed according to the method chosen by each student (point 3 of section 3.2.).*

Method	Correct	%	Incorrect	%
Reduction to unity	0	0	3	3,09
Proportions	0	0	3	3,09
Practical	3	3,09	6	6,19
First Algebra of Magnitudes	72	74,23	1	1,03

Note: This table shows the results obtained in solving the last problem of the questionary. The students freely chose the method they preferred to solve it. The absolute and relative distribution of correct or incorrect solutions for each method is indicated.

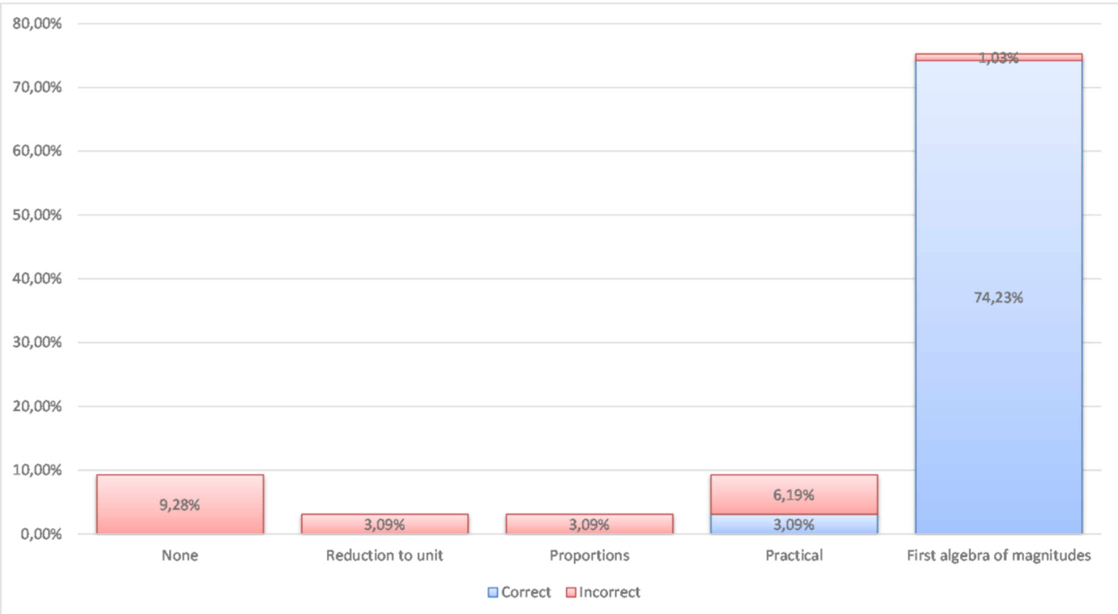


Figure 2. Results of the last problem proposed according to the method chosen by each student (point 3 of section 3.2.)

Note: Graphical representation of the results obtained in solving the last problem of the questionary, based on the method chosen by each student, indicating the relative distribution of correct and incorrect solutions for each method in relation to the total number of students.

It can be seen that 9.28% of the students did not choose any method, and were unable to reason the solution correctly. In most of these cases, there is an attempt to solve the problem by intuition. 75.26% (73 students) preferred the method of the *First Algebra of Magnitudes*, with only one student erring in the reasoning. 9 students preferred the practical method, with 66.67% of them erring, a proportion very similar to that in Illustration 1 for this same method. Finally, the 3 students who chose the proportion method reasoned it incorrectly. The same occurs with the 3 students who chose the reduction to unity method.

Conclusions

The primary conclusion is that the rule of three, as a didactic method, becomes obsolete due to its lack of foundation, relying solely on intuition, and is replaced by a true proportionality of magnitudes through the geometric algebra of dyads that are traditionally known as concrete numbers.As has been seen, the traditional rule of three relies on two theories that are displaced by the demonstrations of homogeneous and heterogeneous dyads based on the geometry of the algebra of magnitudes.

The theory of direct proportionality of magnitudes —“it is said that two magnitudes are directly proportional when, by multiplying one of them by a number, the other is multiplied by the same number, and by dividing one of them by a number, the other is divided by the same number”— loses rigor with the arithmetization of physics and is replaced by the property of dyadic addition or “proportion with homogeneous dyads” which can be summarized as: for two ratios between homogeneous or additive dyads to be proportional they must have the same abstract multiplier or quotient.
The theory of inverse proportionality of magnitudes —“it is said that two magnitudes are inversely proportional when, multiplying one of them by a number, the other is divided by the same number, and by dividing one of them by a number, the other is multiplied by the same number”— suffers from the same problem as the previous one, being replaced by the property of “proportion with heterogeneous dyads”, that is, “given the combination between two heterogeneous dyads that result in a third combined dyad, when the first dyad is multiplied or divided by an abstract number, to maintain this equality with the combined dyad being invariant, the second dyad will be divided or multiplied respectively by that same abstract number”.

Thus, the limitations of the intuitive principles on which traditional methods are based are overcome, and the problem of the arithmetization of physics is resolved, where the traditional solution relies on eliminating the magnitudes from operations, turning them into mere arithmetic, which, as has been justified in this article, is not the best idea. Instead, magnitudes with their respective numerical elements should be maintained, forming dyads.
Thus, the problems of the simple “rule of three” —with two magnitudes— and the compound “rule of three” —with more than two magnitudes— are reduced to a mere proportionality of magnitudes, whose only difficulty lies in the correct reasoning according to the relationship between the different dyads, which is determined by the nature of their dimensional elements.

If they are of the same nature, they will be homogeneous dyads consisting of addition, which is directly reflected in the numerical element of the dyad, being increased or decreased while maintaining the dimensional element. Conversely, if they are heterogeneous dyads, their respective dimensional elements or magnitudes are of a different nature and the operation between them is based on geometric multiplication and division, resulting in another dyad with a different dimensional element resulting from the combination of the previous ones.

In the problems of the so-called “compound rule of three”, a series of homogeneous and heterogeneous dyads are always presented, which combination results in another dyad, and two assumptions are provided where only the numerical elements vary, leaving one as the unknown to be calculated. Thus, the problems of the “compound rule of three” are reduced to a mere algebraic proportional reasoning of magnitudes.

Finally, a didactic method based on the *First Algebra of Magnitudes* has been proposed —without prejudice to the fact that the method can be improved by including even the real symbolism of the algebra of magnitudes— and its didactic effectiveness has been verified with a questionnaire which results are summarized below.
The method of the *First Algebra of Magnitudes* has greater ease of understanding than the other three traditional methods, reaching a 79.38% correct reasoning of the problem proposed. The students who correctly reasoned the problem according to the other methods were quite inferior (32.99% the practical method; 7.22% the proportion method; 4.17% the reduction to unity method).

It has also been concluded that students prefer the method based on the *First Algebra of Magnitudes* (75.26% compared to 9.28% of the practical method, 3.09% of the proportion method, 3.09% of the reduction to unity method and 9.28% for none).

This new method, based on the *First Algebra of Magnitudes*, is important as a logical reasoning approach for solving typical problems of the classic rule of three. It gives meaning to the chosen title for this topic, “direct and inverse proportion of magnitudes”, since, although this name is commonly used, there is no actual proportional calculation of magnitudes. Instead, it falls into the problem of the arithmetization of physics, which is simply resolved by removing the magnitudes from the operations.

Being the first topic in applied mathematics to physics, it is crucial to resolve the problem of arithmetization, preventing adolescents from developing an aversion to something so simple at an early age. Additionally, based on the *First Algebra of Magnitudes*, it is concluded that the possible existence of dismetric magnitudes could be considered—that is, the idea that the same magnitude may not always have the same measure depending on the surrounding circumstances. A meter in the vacuum of space may not be the same as a meter in a black hole.

This is precisely the case observed with the last proposed and reasoned problem in section 4.6. Although the problem was solved using isometric magnitudes—meaning that all men and women are considered equal—the reality is that each man is different from the others, just as each woman is, and no two people are exactly alike. Additionally, the same person is not always the same from one day to the next, so his contribution to work is not always the same.

This leads us to the concept of dismetric magnitudes, and these types of problems can be solved by taking this factor into account. However, operations involving such magnitudes become more complex, even requiring the use of tensors. In conclusion, to begin with, a review of the didactic methods of the classic rule of three is necessary, incorporating a true proportionality of magnitudes that corrects what has so far been referred to as direct and inverse proportionality.

Recomendations for Researchers

Based on this study, the following lines of research are suggested:

First: repeat the same statistical study to students who are learning the different methods of the compound rule of three for the first time, that is, to 12-year-old students. However, the explanation of the method based on the *First Algebra of Magnitudes* can be improved (even considering the possibility of teaching the method with the symbolism of the *First Algebra of Magnitudes* and all the reasoning presented in this research) as follows:

The *First Algebra of Magnitudes* proposes not eliminating magnitudes in physical operations by the algebra of magnitudes as a solution of the problem of the arithmetization of physics. Thus, every measurement consists of two elements: a numerical one and a dimensional one. For example, "5 meters long" is composed of the numerical element "5" and the dimensional "meter long", and algebraically they are multiplied. If more "meters long" are added, the abstract number 5 will increase, being a homogeneous proportion, as the dimensional element is the same: "meter long". If measurements of a different nature are added, that is, with a different dimensional element such as "2 meters wide", we will have a heterogeneous proportion, which is the geometric multiplication of the two measurements "5 meters long" and "2 meters wide", resulting a third measurement: "surface".

As with arithmetic operations, magnitudes can be operated on (although with different symbols as it is geometric algebra and not just arithmetic, symbolism that we are not going to introduce here). This is geometric algebra since it treats dimensional elements as segments of a certain length. Thus, "meter long" is a segment of a certain length. If more segments of this same dimension are added, they result in a longer segment determined by the abstract number, in our example "5" segments, which has been called homogeneous proportion. If segments of another dimension are added, such as "meter wide", they will have equal or different length, but they can no longer be added after the "meters long" because they are of a different nature, and they must be multiplied together, geometrically resulting in an area that represents the third resulting dimensional element, in this case the "surface". This is the heterogeneous proportion.

Here its is observed that dividing two homogeneous dyads results in an abstract number. This is because if we add 2 segments to the same dimension three times, for example, "meter", it results in:

$$2meters + 2meters + 2meters = 3 \times (2meters) = 6meters \to \frac{6meters}{2meters} = \frac{6}{2} = 3$$

This method applied to problems with magnitudes consists of obtaining an equality, how much work is needed (a combination of measurements of equal or different dimensions) to obtain a result, that is:

Amount of work = Work result

Let's see it with an example: if 5 men working 6 hours a day have dug a 40-meter trench in 8 days, how many days will it take 9 men working 8 hours a day to dig a 60-meter trench?

For the given assumption, how much work is needed? 5 men, 6 hours a day and 8 days are needed. What is the result of the work? A 40-meter trench. Thus:

$$5 \, men \times 6 \, hours \, a \, day \times 8 \, days = 40 \, meters$$

To solve the unknown number, the question can be posed similarly, knowing that the work needed is: 9 men and 8 hours a day for x days (the dimensional elements are the same). The result of the work is a 60-meter trench:

$$9 \, men \times 8 \, hours \, a \, day \times X \, days = 60 \, meters$$

From here, it is enough to divide both equalities term by term, obtaining the following homogeneous operation:

$$\frac{5 \, men \times 6 \, hours \, a \, day \times 8 \, days}{9 \, men \times 8 \, hours \, a \, day \times X \, days} = \frac{40 \, meters}{60 \, meters}$$

Since the magnitudes are divided by themselves, the operation becomes merely arithmetic:

$$\frac{5 \times 6 \times 8}{9 \times 8 \times X} = \frac{40}{60}$$

Solving for the unknown X, we get the value of X:

$$X = \frac{5 \times 6 \times 8 \times 60}{9 \times 8 \times 40} = 5$$

Second: apply the *First Algebra of Magnitudes* to different topics in Physics with the corresponding didactic method, also conducting a statistical study to compare the teaching effectiveness and student acceptance.

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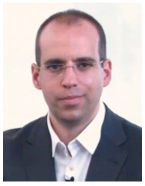

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Research Article

Self-Regulation Scale in Mathematics: short form development and validity-reliability study[1]

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Abstract

Self-regulation in mathematics is the organization of the student in line with the mathematics course outcomes. Students' ability to organize the mathematics learning process in a healthy way can increase their mathematics achievement. Knowing students' self-regulation towards mathematics will contribute to providing the necessary education for mathematics achievement. The purpose of this study is to develop a valid and reliable short-form scale to measure middle school students' self-regulation skills towards mathematics. In this context, existing self- regulation scales were examined and an item pool of 56 items was created by utilizing the items in the literature and the statements developed by the researchers. In line with expert opinions, a preliminary form of 25 items consisting of positive statements was prepared and this form was applied to 342 middle school students. As a result of exploratory factor analysis, the number of items was reduced to 14 and the scale had a four-factor structure: Effort, Integrated, Review and Getting Support. Confirmatory factor analysis confirmed this structure. The overall Cronbach's alpha reliability coefficient of the scale was calculated as .78, and the coefficients related to the sub- dimensions were found to be acceptable. The developed Short Mathematics Self-Regulation Scale (S-MSRS) is recommended as a tool that can be applied in a short time, has understandable expressions and can be used at different grade levels.

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Introduction

In recent years, the focus of studies on academic achievement has been on the concept of self- regulation in which students play an active role in their own learning process (Ainley & Patrick, 2006). Self-regulation is an active and effective process in which an individual is aware of his/her own learning, can control himself/herself, set learning goals, adjust his/her metacognitive capacity and behaviors (Bandura, 1986).

Self-regulation is self-organization. It also includes cognitive strategies such as planning and reviewing one's learning. Self-effort and self-evaluation are also important for self-regulation. It has gained importance by emphasizing the ability of learners to take responsibility for their own learning, to monitor what, when and how much they learn, and to apply

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appropriate strategies (Schuitema et al., 2012). People with well-developed self-regulation have more self- regulation and effort in the problems they will face in their courses or throughout their lives. These individuals determine a strategy to achieve their goals and use this strategy and then monitor their efforts and achievements, and as a result, they arrive at an evaluation. They organize their learning processes and strategies according to the evaluation results (Çiltaş, 2011).

Today, measuring and developing self-regulation in various courses is gaining importance. However, the self-regulation of students in mathematics is the most emphasized in research. Because mathematics courses have a special importance in terms of providing an understanding of the world we live in, having interesting methods and relationships and being in a closer relationship with the mental activities of the individual (Üredi & Üredi, 2005). Self-regulation involves students' active learning processes. Likewise, the relationship between mathematics and self-regulation is important since it is of great importance for students to perform active learning in mathematics lessons. Mathematical self-regulation is the organization of the student in line with the mathematics course objectives. The student's organization of the mathematics learning process leads to an increase in mathematics achievement. For this reason, self-regulation skills are important for mathematics lessons

Many studies have shown that students with self-regulation in mathematics are more successful in mathematics (Clearly & Chen, 2009; Lee et al., 2010, Usta, 2014). Clearly and Chen (2009), in their research examining the differences in self-regulation, motivation, and mathematics achievement in middle school according to grade level and mathematics context, showed that students' use of regulatory strategies during mathematics learning was the primary motivational predictor and emphasized the importance of identifying changing student motivation and self- regulation in the early middle school years and the potential role that context may have in these processes. Students who can control their own learning processes make all the necessary efforts by using metacognitive processes such as planning, organizing, and evaluating in the learning process, in other words, self-regulation skills (Çiltaş, 2011).

Studies conducted at the primary and secondary school level reveal that students' self-regulation skills towards mathematics course can show significant differences according to various variables (Aktan, 2012; Erdoğan & Şengül, 2014; İpek, 2019). Aktan (2012) examined the effect of 5th grade students' self-regulatory learning strategies on academic achievement and stated that especially female students scored higher in elaboration, organization, metacognitive self-regulation, time management and help-seeking strategies. Similarly, Erdoğan and Şengül (2014), in their study with 6th, 7th and 8th grade students, found that there were significant differences in students' self-regulation and metacognitive skills according to grade level and gender variables, and that these differences were in favor of female students. İpek (2019) examined the relationships between mathematics anxiety, self-efficacy beliefs, and self- regulation skills of middle school students; stated that as students' self-regulation scores increased, their self-regulation skills towards mathematics courses also increased. He also emphasized that there were significant differences in the sub-dimensions of "openness" and "seeking" according to grade level; especially 5th grade students had higher scores in these sub- dimensions.

Various scales have been developed to find out the level of students' self-regulation (Arslan & Gelişli, 2015; Çokçalışkan et al., 2015; Kadıoğlu et al., 2011; Pintrich et al., 1991). Arslan and Gelişli (2015) developed the Perceived Self-Regulation Scale. The study was conducted on 604 middle school students. The scale consists of 16 items and two factors (openness and seeking). The Self-Regulatory Learning Strategies Scale (MSLQ Motivated Strategies for Learning Questionnaire) developed by Pintrich et al. was adapted into Turkish by Aktan (2012). The scale consists of 40 items and 8 sub-dimensions. Kadıoğlu et al. (2011) developed the Self- Regulatory Learning Strategies Scale (SLSS) to evaluate the self-regulatory learning strategies of high school students. The scale consists of a total of 29 items and 8 sub-dimensions. Çokçalışkan et al. (2019) developed the Self-Regulated Learning Inventory (SLI) to assess the self-regulated learning skills of 4th grade primary school students. The inventory consists of 35 items and 3 sub-dimensions.

It is critical for students to take an active role in mathematics learning processes and to be able to plan and control their learning in order to increase learning retention and success. In this context, self-regulation skills include functions such as students' taking responsibility for their own learning, setting goals, using strategies, and evaluating their learning processes; and it stands out as an important variable that directly affects the quality of learning in areas that require cognitive effort such as mathematics. In the national and international literature, it is noteworthy that the number of scales developed to assess students' self-regulation levels in mathematics is limited and the existing scales are mostly either designed for a single level of education or have very general and lengthy statements. This situation may cause difficulties in the application process and may cause validity and reliability problems for students of different ages and developmental levels.

Based on this need, it was aimed to develop the Short Self-Regulation in Mathematics Scale (S-MSRS), which is shorter, simpler, clearer and adaptable to different educational levels. It is predicted that this scale to be developed will be a more practical and functional tool than the existing scales, especially in terms of the shortness of the application period, the use of age- appropriate language, and limiting the measurement scope to the mathematics-specific context. Thus, it will be possible to effectively monitor and support students' self-regulation skills in both educational research and teaching processes.

Method

Research Model

This study was conducted within the framework of descriptive survey design, which is one of the quantitative research methods. Descriptive survey design is a research approach that aims to define an existing situation or a phenomenon as it exists and to express the opinions, characteristics or behaviors of individuals with numerical data (Karasar, 2012). Within the scope of this study, it was aimed to develop a short and valid scale to measure middle school students' self-regulation skills towards mathematics course.

In the research process, basic stages such as creating an item pool, obtaining expert opinions, conducting a pilot study, and then conducting validity and reliability analyses were followed. In analyzing the psychometric properties of the scale, construct validity and internal consistency reliability were evaluated using appropriate statistical techniques.

Study Group

The first study group, which was formed to test the construct validity of the scale with exploratory factor analysis (EFA), consisted of 342 middle school students studying in a public middle school in Kocaeli city center in the fall semester of the 2022-2023 academic year. During the pre-analysis process, forms with missing or incorrect data were eliminated and the analyses were conducted on 332 students. The grade levels of the participants were 5th, 6th, 7th and 8th grades, and convenience sampling method was used in sample selection. The second study group, which was formed to test the validity of the factor structure obtained as a result of EFA, consisted of 254 middle school students studying in different public schools in Kocaeli in the same academic year

Research Process and Data Collection

Data for the first application form were collected in October 2022 and data for the second application form were collected in November 2022. The data collection tool used in the study was started with an item pool consisting of original items developed by examining existing self- regulation scales and statements adapted from scales with proven validity in the literature. A pilot form was prepared in line with expert opinions and applied to the students. During the data collection process, the purpose of the study was explained to the students, it was stated that participation was voluntary, and the answers would be kept confidential. The application took an average of 20-25 minutes.

In the scale development process, firstly, valid and reliable self-regulation scales in the literature were examined and items that could be adapted to mathematics course were determined. In this context, the Perceived Self-Regulation Scale developed by Arslan and Gelişli (2015) consists of 16 items, and as a result of the exploratory factor analysis, it was determined that the items were grouped into two factors: "openness" and "seeking". Cronbach Alpha internal consistency coefficient for the whole scale was reported as 0.90. In addition, the Motivational Strategies for Learning Questionnaire (MSLQ) developed by Pintrich et al. (1991) was adapted into Turkish by Aktan (2012) and applied to 5th grade students as the Self- Regulatory Learning Strategies Scale. This scale consists of 40 items and 8 sub-dimensions: elaboration, organization, metacognitive self-regulation, time management, effort regulation, seeking help, repetition, learning from peers. The overall Cronbach Alpha coefficient obtained as a result of confirmatory factor analysis was 0.95, indicating that the scale was highly reliable.

The items deemed appropriate from these two scales were revised to assess students' self- regulation levels in mathematics courses and combined with the original statements developed by the researchers to form an item pool of 56 items in total. This pool was submitted to expert opinion in order to support the content validity of the scale to be developed, and the first application form (pilot) was created with 25 items, all of which were positive. A five-point Likert-type rating (1- Strongly Disagree, 2-Disagree, 3-Somewhat Agree, 4-Agree, 5-Strongly Agree) was used for each item

Data analysis

SPSS 16.0 and SmartPLS 4 programs were used to analyze the data obtained from the scale items. Scale items were scored as 1-2-3-4-5 in the SPSS 16.0 package program. Thus, self- regulation scores in mathematics were obtained from each scale applied. A high score indicates that students' self-regulation in mathematics is high, while a low score indicates that their self- regulation in mathematics is negative. Before testing whether the scale was unidimensional or multidimensional, and if multidimensional, which items were grouped under which dimension, the item-total correlation was examined with the SPSS 16.0 package program and it was concluded that 14 items had correlation values that could remain in the scale. The 14-item scale was subjected to factor analysis. Factor analysis was used to prove the construct validity of the scale. In order to apply exploratory factor analysis, the data file must be suitable for analysis. This suitability was determined through Kaiser-Meyer-Olkin (KMO) test and Bartlett Sphericity. As a result of the exploratory factor analysis, 3 items with factor loading values less than 30 and the difference between factor loading values in two or more factors was less than10 were removed from the scale. After this revision, the resulting 22-item form was administered to a group of middle school students. In the preliminary analyses, a total of 8 items with low factor loadings or high loadings on more than one factor according to item-total correlations and exploratory factor analysis were removed from the scale. Thus, a four-factor structure (Effort, Integrated, Review, Support) consisting of a total of 14 items was obtained. This structure was tested with EFA and CFA and the construct validity of the scale was supported.

Evidence regarding the reliability of the final scale formed as a result of the analysis was presented. The evidence for the reliability of the scale was presented by calculating the internal consistency coefficient. In addition, the relationship between the factors forming the scale based on the items under the factors was tested by correlation analysis method. As a result, exploratory factor analysis and confirmatory factor analysis were used for the validity study of the scale. For the reliability study, Cronbach Alpha was used. The lowest score that can be obtained from the scale is 14 and the highest score is 70. When interpreting the mean scores obtained from the scale: 1.00-1.80 is considered as "very low", 1.81-2.60 as "low", 2.61-3.40 as "medium", 3.41-4.20 as "high" and 4.21-5.00 as "very high".

Ethics

This study was conducted with the approval of Kocaeli University Social and Human Sciences Ethics Committee dated 20.09.2022 and numbered E-90104632-19.

Findings

Exploratory Factor Analysis (EFA)

In order to determine whether the scale items were divided into meaningful factors independent of each other, varimax-rotaied principal component analysis was used. At the end of this process and item-total correlation, 8 items were removed from the scale, leaving a total of 14 items related to the four factors identified in the scale.

Table 1. Findings related to factor load distributions of items as a result of factor analysis

Articles	Factor 1 (Effort)	Factor 2 (Integrated)	Factor 3 (Review)	Factor 4 (Receiving Support)
Item8	,757			
Item5	,750			
Item10	,748			
Item1	,598			
Itemm14	,592			
Item13		,702		
Item9		,680		
Item6		,632		
Itemm2		,578		
Item4			,741	
Itemm7			,719	
Itemm11			,516	
Item12				,878
Item3				,874
14 items	**5 items**	**4 items**	**3 items**	**2 items**

KMO: 0.792 BARTLETT:0.001

Table 1 shows the factor load distributions of the items in the scale as a result of the factor analysis. During the factor analysis, if the loading of an item in the scale was above 0.40, the item was counted in that factor. As seen in Table 1, the factor loadings for 14 items ranged between 0.516 and 0.878. Factor 1: 5 items explained 29.40%, Factor 2: 4 items explained 12.59%, Factor 3: 3 items explained 8.18%, Factor 4: 2 items explained 7.35%. The total variance explained in its final form is 57.52%. According to the factor analysis, 5 items were grouped under factor 1, 4 items under factor 2, 3 items under factor 3 and 2 items under factor. Accordingly, factor 1 is categorized under the title of "Effort", factor 2 under the title of "Integration", factor 3 under the title of "Review", and factor 4 under the title of "Getting Support". As a result, the developed scale consisted of a total of 14 self-regulation sentences, all of which were positive.

After the self-regulation scale in mathematics was finalized with 14 items and 4 factors, Cronbach alpha internal consistency coefficients for each dimension and the overall test were calculated for the reliability of the scale. Since this coefficient is calculated by taking into account all items in the scale, it is the coefficient that best reflects the overall reliability structure of the test compared to other coefficients (Özdamar, 2004). The alpha reliability of the final version of the scale was calculated as 0.78. Accordingly, it can be said that the reliability of the scale is high. The alpha reliabilities obtained from the sub-factors are 0.78 for "Effort" (item number 5), 0.64 for "Integrated" (item number 4), 0.58 for "Review" (item number 3), and 0.77 for "Getting Support" (item number 2). The calculated reliability coefficients reveal that the scale has an acceptable level of internal consistency for the overall scale and for each factor.

Confirmatory Factor Analysis (CFA)

Cronbach Alpha internal consistency coefficients were calculated for the reliability of the self- regulation scale in mathematics. Since this coefficient is calculated by considering all items in the scale, it is the coefficient that best reflects the overall reliability structure of the test compared to other coefficients (Özdamar, 2004). The alpha reliability of the final version of the scale was calculated as 0.78. The calculated reliability coefficients reveal that the scale has an acceptable level of internal consistency in general and for each factor.

Table 2. Validity and reliability values related to the K-PBSS

Latent Variable	Indicators	Loads	VIF	Composite Reliability (CR)	Cronbach Alpha	AVE
Effort	E1	0,791	1,691	0,853	0,784	0,538
	E2	0,694	1,395			
	E3	0,625	1,298			
	E4	0,769	1,611			
	E5	0,776	1,524			
Integrated	I1	0,677	1,226	0,787	0,640	0,482
	I2	0,723	1,253			
	I3	0,617	1,137			
	I4	0,753	1,309			
Review	R1	0,724	1,150	0,783	0,586	0,547
	R2	0,738	1,231			
	R3	0,756	1,195			
Getting Support	S1	0,657	1,660	0,828	0,773	0,715
	S2	0,999	1,660			

When the alpha values in Table 2 are analyzed, it is seen that all values except the review factor are above 0.60. The second value used for internal consistency in PLS analyses is the composite reliability (CR) coefficient (Hair et al. 2017; Gaskin, statwiki.com). When the results are analyzed, it is seen that the composite reliability coefficients are above 0.70. According to these results, it can be said that the variables considered are at the desired level in terms of internal consistency. When the AVE values are analyzed, it is seen that all values except the *integrated* factor are above the accepted values.(Hair et al. 2017), which is above the lower limit of 0.50. Some sources suggest that AVE values can be as low as 0.40 in scale development or adaptation studies (Fornell & Larcker, 1981). It is also stated that AVE > .50; CR > .70 and CR > AVE is a prerequisite for convergent validity; AVE values less than .50 will not be a problem if the CR value is greater than .70 (Fornell & Larcker, 1981). When VIF values are examined, it is desired that the coefficients should be below 5 in factor analysis (Hair et al., 2017). When the VIF values table is examined, it is seen that the coefficients are at an acceptable level.

Table 3. Discriminant validity coefficients of the subscales of the scale

Fornell-Larcker Criteria					**Heterotrait-Monotrait Ratio**				
Subscales of the Scale	**Effort**	**Getting Support**	**Integrated**	**Review**	**Subscales of the Scale**	**Effort**	**Getting Support**	**Integrated**	**Review**
Effort	**0.734**				**Effort**				
Getting Support	-0.024	**0.846**			**Getting Support**	0.064			
Integrated	0.465	0.133	**0.694**		**Integrated**	0.649	0.211		
Review	0.501	0.156	0.417	**0.739**	**Review**	0.725	0.211	0.682	

The discriminant validity findings in Table 3 were evaluated by considering Fornell-Larcker criteria and Heterotrait-Monotrait (HTMT) ratios. In the discriminant validity test proposed by Fornell and Larcker (1981), the root AVE (average variance explained) value of a construct should be greater than its correlations with other constructs. This criterion indicates that measures of similar constructions can be distinguished from each other. In this study, the Fornell-Larcker values ranged from 0.694 to 0.846 and were higher than the correlation values of each construct with other constructs. In addition, the HTMT ratios remained below .85, indicating that the discriminant validity was achieved in accordance with the criteria suggested by Henseler et al. (2015). These findings reveal that the measurement model has significant discrimination between constructions. In conclusion, it can be said that the four-factor structure of the Short Mathematics Self-Regulation Scale (B-MSRS) developed in line with the obtained analyses is valid and reliable and statistically supported in terms of both composite validity and discriminant validity. The scale showed structural consistency in samples of middle school students.

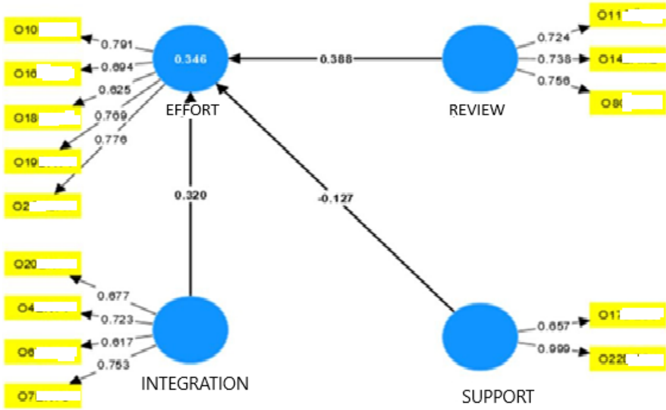

Figure 1. Diagram of SmartPLS4 Confirmatory Factor Analysis of the K-SCAS

When Figure 1 is examined, it is seen that the four-factor structure of the Short Mathematics Self-Regulation Scale (SMRS) was preserved as a result of the confirmatory factor analysis and each item had significant loading values under the factor to which it belonged. The path coefficients between the factors in the model and the observed variables ranged between 0.61 and 0.99, and these values are above the recommended minimum threshold value of 0.50 (Hair et al., 2017). The high correlation of all items with the relevant dimensions supports the construct validity of the scale. In addition, the presence of low-level cross-correlations between the factors in the diagram shows that the scale maintains its discriminant validity.

Conclusion

In this study, it was aimed to develop a valid and reliable short scale to measure middle school students' self-regulation skills towards mathematics course. For this purpose, the literature was reviewed, an item pool of 56 items was created, content validity was ensured with expert opinions, and a preliminary form of 25 items was applied. As a result of the exploratory factor analysis, the items with low loading values were eliminated and the scale was formed with 14 items and four factors: Effort, Integrated, Review, and Getting Support. The findings of confirmatory factor analysis showed that this structure was highly compatible with the data. The overall reliability of the scale and the Cronbach Alpha values of the sub-dimensions were at acceptable levels, indicating that the scale exhibited a consistent structure.

Although the overall validity and reliability results of the scale are sufficient, the “Getting Support” subscale contains only two items, which may limit the structural consistency of this factor. Therefore, it is recommended that this subscale be reconsidered and expanded in future studies based on qualitative data.

The composite reliability (CR) and average variance explained (AVE) values revealed that convergent validity was achieved. Moreover, the analyses using Fornell-Larcker criteria and HTMT ratios show that discriminant validity is also achieved. This suggests that the scale is not only valid and reliable but also structurally stable.

These findings are consistent with the theoretical context established between self-regulation and academic achievement (Bandura, 1986; Pintrich et al., 1991; Cleary & Chen, 2009). Self- regulation enables students to effectively use metacognitive processes such as planning, monitoring and evaluating learning processes. This is a factor that directly affects success, especially in an abstract and process-oriented field such as mathematics (Lee et al., 2010; Çiltaş, 2011). Therefore, the developed scale has the potential to be an important tool for educators and researchers. We recommended are for further studies and applicaitons;

- Age-related validity analyses can be conducted on different age groups (e.g., high school and university level).
- The scale can be evaluated in terms of cultural validity by testing it on students from various socioeconomic levels.
- With the digitalization of the scale, it can be used as an assessment tool for monitoring students' self-regulation levels in online teaching processes.

- Deeper analyses can be conducted with path analyses or structural equation modeling to reveal the effect of self-regulation skills on mathematics achievement.

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Appendix 1. Mathematics Self-Regulation Scale-Short Form

Mathematics Self-Regulation Scale-Short Form						
1 Absolutely Disagree, 2 Disagree, 3 Partially Agree, 4 Agree, 5 Strongly Agree						
		1	**2**	**3**	**4**	**5**
1	In math class, I strive to achieve my goals.					
2	I can use what I have learned in mathematics in daily life.					
3	I get help from my classmates in math lessons.					
4	I prepare a study plan for mathematics and stick to it.					
5	In math, I try to learn even the most difficult subjects.					
6	In mathematics, I try to find different ways to learn a topic.					
7	I organize my work in mathematics by looking at my notes.					
8	Even if I am not interested in mathematics, I manage to study until I finish the subject.					
9	I can use a topic I have already learned in mathematics in new subjects.					
10	I do not stop studying when subjects are difficult in mathematics					
11	In math class, I ask myself questions to make sure I understand the topic.					
12	In mathematics, when I do not understand a subject, I get help from my friends.					
13	I try to use what I have learned in mathematics in different courses.					
14	Even if I have difficulty in math lessons, I try to do it myself.					

Note: Items removed from the scale as a result of analysis
I don't get bored when I study math.
I get help from my teacher when I have difficulties in mathematics.
In math class, if something goes wrong, it doesn't demotivate me.
When studying mathematics, I can decide what I need to learn by thinking about the subject.
I use my time well when studying mathematics.
When I am studying math and I don't understand something, I go back and try to understand it.
If I don't understand something in math, I ask someone from my family.
I spend more time in math class.

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