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WEAKLY $\mathcal{I}_{g\delta}$-CLOSED SETS

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Abstract – In this paper, the notion of weakly $\mathcal{I}_{g\delta}$-closed sets in ideal topological spaces is introduced and studied. The relationships of weakly $\mathcal{I}_{g\delta}$-closed sets and various properties of weakly $\mathcal{I}_{g\delta}$-closed sets are investigated.

Keywords – *generalized class of τ^*, weakly $\mathcal{I}_{g\delta}$-closed set, ideal topological space, generalized closed set, $\mathcal{I}_{g\delta}$-closed set, pre$^*_{\mathcal{I}}$-closed set, pre$^*_{\mathcal{I}}$-open set.*

1 Introduction and Preliminaries

In this paper, (X, τ) represents topological space on which no separation axioms are assumed unless explicitly stated. The closure and the interior of a subset G of a space X will be denoted by cl(G) and int(G), respectively.

In 1937, Stone [16] introduced and studied the notion of regular open sets in topological spaces. A subset G of X is said to be regular open [16] if int(cl(G))=G. The complement of regular open set is regular closed. In 1968, Veličko [19] introduced the notion of δ-open sets, which are stronger than open sets in order to investigate the characterization of H-closed spaces. A point x $\in$ X is called a δ-cluster point of G if G $\cap$ U $\neq \emptyset$ for every regular open set U containing x. The set of all δ-cluster points of G is called the δ-closure of G and is denoted by cl_δ(G). If cl_δ(G)=G, then G is called δ-closed. The complement of a δ-closed set is δ-open. In 1968, Zaitsav [20] introduced and studied the notion of π-open sets. A finite union of regular open sets is said to be π-open [20]. The complement of a π-open set is π-closed.

In 1999, Dontchev et al. studied the notion of generalized closed sets in ideal topological spaces called $\mathcal{I}_g$-closed sets [2]. In 2008, Navaneethakrishnan and Paulraj Joseph have studied some characterizations of normal spaces via $\mathcal{I}_g$-open sets [10].

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In 2013, Ekici and Ozen [6] introduced a generalized class of τ^*. Ravi et. al [14, 15] introduced another generalized classes of τ^* called weakly $\mathcal{I}_g$-closed sets and weakly $\mathcal{I}_{\pi g}$-closed sets respectively.

The main aim of this paper is to study the notion of weakly $\mathcal{I}_{g\delta}$-closed sets in ideal topological spaces. Moreover, this generalized class of τ^* generalize $\mathcal{I}_{g\delta}$-open sets and weakly $\mathcal{I}_{g\delta}$-open sets. The relationships of weakly $\mathcal{I}_{g\delta}$-closed sets and various properties of weakly $\mathcal{I}_{g\delta}$-closed sets are discussed.

Definition 1.1. A subset G of a topological space (X, τ) is said to be

1. g-closed [9] if cl(G) $\subseteq$ H whenever G $\subseteq$ H and H is open in X;
2. g-open [9] if X\G is g-closed;
3. weakly g-closed [17] if cl(int(G)) $\subseteq$ H whenever G $\subseteq$ H and H is open in X.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies

1. A $\in \mathcal{I}$ and B $\subseteq$ A imply B $\in \mathcal{I}$ and
2. A $\in \mathcal{I}$ and B $\in \mathcal{I}$ imply A$\cup$B $\in \mathcal{I}$ [8].

Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X if $\mathcal{P}$(X) is the set of all subsets of X, a set operator $(\bullet)^\star : \mathcal{P}(X) \to \mathcal{P}(X)$, called a local function [8] of A with respect to τ and $\mathcal{I}$, is defined as follows: for A $\subseteq$ X, $A^\star(\mathcal{I}, \tau) = \{x \in X \mid U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator $cl^\star(\bullet)$ for a topology $\tau^\star(\mathcal{I}, \tau)$, called the $\star$-topology and finer than τ, is defined by $cl^\star(A) = A \cup A^\star(\mathcal{I}, \tau)$ [18]. We will simply write $A^\star$ for $A^\star(\mathcal{I}, \tau)$ and $\tau^\star$ for $\tau^\star(\mathcal{I}, \tau)$. If $\mathcal{I}$ is an ideal on X, then (X, τ, $\mathcal{I}$) is called an ideal topological space. On the other hand, (A, τ_A, $\mathcal{I}_A$) where τ_A is the relative topology on A and $\mathcal{I}_A = \{A \cap J : J \in \mathcal{I}\}$ is an ideal topological space for an ideal topological space (X, τ, $\mathcal{I}$) and A $\subseteq$ X [7]. For a subset A $\subseteq$ X, cl*(A) and int*(A) will, respectively, denote the closure and the interior of A in (X, τ^*).

Definition 1.2. A subset G of an ideal topological space (X, τ, $\mathcal{I}$) is said to be

1. $\mathcal{I}_g$-closed [2] if G* $\subseteq$ H whenever G $\subseteq$ H and H is open in (X, τ, $\mathcal{I}$).
2. $\mathcal{I}_{rg}$-closed [11] if G* $\subseteq$ H whenever G $\subseteq$ H and H is regular open in (X, τ, $\mathcal{I}$).
3. $\mathcal{I}_{\pi g}$-closed [13] if G* $\subseteq$ H whenever G $\subseteq$ H and H is π-open in (X, τ, $\mathcal{I}$).
4. pre*$_{\mathcal{I}}$-open [5] if G $\subseteq$ int*(cl(G)).
5. pre*$_{\mathcal{I}}$-closed [5] if X\G is pre*$_{\mathcal{I}}$-open.
6. $\mathcal{I}$-R closed [1] if G = cl*(int(G)).
7. $*$-closed [7] if G = cl*(G) or G* $\subseteq$ G.

Remark 1.3. [6] In any ideal topological space, every $\mathcal{I}$-R closed set is $*$-closed but not conversely.

Definition 1.4. Let (X, τ, $\mathcal{I}$) be an ideal topological space. A subset G of X is said to be

1. a weakly $\mathcal{I}_g$-closed set [14] if (int(G))* $\subseteq$ H whenever G $\subseteq$ H and H is an open set in X.

2. a weakly $\mathcal{I}_{\pi g}$-closed set [15] if (int(G))* $\subseteq$ H whenever G $\subseteq$ H and H is a π-open set in X.

3. a weakly $\mathcal{I}_{rg}$-closed set [6] if (int(G))* $\subseteq$ H whenever G $\subseteq$ H and H is a regular open set in X.

Remark 1.5. [3] The following holds in any topological space:
regular open set $\Rightarrow$ π-open set $\Rightarrow$ δ-open set $\Rightarrow$ open set.

These implications are not reversible.

2 Properties of Weakly $\mathcal{I}_{g\delta}$-closed Sets

Definition 2.1. A subset G of an ideal topological space (X, τ, $\mathcal{I}$) is said to be

1. $\mathcal{I}_{g\delta}$-closed if G* $\subseteq$ H whenever G $\subseteq$ H and H is δ-open in (X, τ, $\mathcal{I}$).

2. weakly $\mathcal{I}_{g\delta}$-closed if (int(G))* $\subseteq$ H whenever G $\subseteq$ H and H is δ-open in (X, τ, $\mathcal{I}$).

Theorem 2.2. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. The following properties are equivalent:

1. G is a weakly $\mathcal{I}_{g\delta}$-closed set,

2. cl*(int(G)) $\subseteq$ H whenever G $\subseteq$ H and H is a δ-open set in X.

Proof. (1) $\Rightarrow$ (2) : Let G be a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$). Suppose that G $\subseteq$ H and H is a δ-open set in X. We have (int(G))* $\subseteq$ H. Since int(G) $\subseteq$ G $\subseteq$ H, then (int(G))* $\cup$ int(G) $\subseteq$ H. This implies that cl*(int(G)) $\subseteq$ H.

(2) $\Rightarrow$ (1) : Let cl*(int(G)) $\subseteq$ H whenever G $\subseteq$ H and H is a δ-open in X. Since (int(G))* $\cup$ int(G) $\subseteq$ H, then (int(G))* $\subseteq$ H whenever G $\subseteq$ H and H is a δ-open set in X. Therefore G is a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$).

Theorem 2.3. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is δ-open and weakly $\mathcal{I}_{g\delta}$-closed, then G is $*$-closed.

Proof. Let G be a δ-open and weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$). Since G is δ-open and weakly $\mathcal{I}_{g\delta}$-closed, cl*(G) = cl*(int(G)) $\subseteq$ G. Thus, G is a $*$-closed set in (X, τ, $\mathcal{I}$).

Theorem 2.4. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_{g\delta}$-closed set, then (int(G))* \G contains no any nonempty δ-closed set.

Proof. Let G be a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$). Suppose that H is a δ-closed set such that H $\subseteq$ (int(G))*\G. Since G is a weakly $\mathcal{I}_{g\delta}$-closed set, X\H is δ-open and G $\subseteq$ X\H, then (int(G))* $\subseteq$ X\H. We have H $\subseteq$ X\(int(G))*. Hence, H $\subseteq$ (Int(G))* $\cap$ (X\(int(G))*) = $\emptyset$. Thus, (int(G))*\G contains no any nonempty δ-closed set.

Theorem 2.5. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_{g\delta}$-closed set, then cl*(int(G))\G contains no any nonempty δ-closed set.

Proof. Suppose that H is a δ-closed set such that H $\subseteq$ cl*(int(G))\G. By Theorem 2.4, it follows from the fact that cl*(int(G))\G = ((int(G))* $\cup$ int(G))\G.

Theorem 2.6. Let (X, τ, $\mathcal{I}$) be an ideal topological space. The following properties are equivalent:

1. G is pre*$_{\mathcal{I}}$-closed for each weakly $\mathcal{I}_{g\delta}$-closed set G in (X, τ, $\mathcal{I}$),

2. Each singleton {x} of X is a δ-closed set or {x} is pre*$_{\mathcal{I}}$-open.

Proof. (1) $\Rightarrow$ (2) : Let G be pre*$_{\mathcal{I}}$-closed for each weakly $\mathcal{I}_{g\delta}$-closed set G in (X, τ, $\mathcal{I}$) and x $\in$ X. We have cl*(int(G)) $\subseteq$ G for each weakly $\mathcal{I}_{g\delta}$-closed set G in (X, τ, $\mathcal{I}$). Assume that {x} is not a δ-closed set. It follows that X is the only δ-open set containing X\{x}. Then, X\{x} is a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$). Thus, cl*(int(X\{x})) $\subseteq$ X\{x} and hence {x} $\subseteq$ int*(cl({x})). Consequently, {x} is pre*$_{\mathcal{I}}$-open.

(2) $\Rightarrow$ (1) : Let G be a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$). Let x $\in$ cl*(int(G)).

Suppose that {x} is pre*$_{\mathcal{I}}$-open. We have {x} $\subseteq$ int*(cl({x})). Since x $\in$ cl*(int(G)), then int*(cl({x})) $\cap$ int(G) $\neq \emptyset$. It follows that cl({x}) $\cap$ int(G) $\neq \emptyset$. We have cl({x} $\cap$ int(G)) $\neq \emptyset$ and then {x} $\cap$ int(G) $\neq \emptyset$. Hence, x $\in$ int(G). Thus, we have x $\in$ G.

Suppose that {x} is a δ-closed set. By Theorem 2.5, cl*(int(G))\G does not contain {x}. Since x $\in$ cl*(int(G)), then we have x $\in$ G. Consequently, we have x $\in$ G.

Thus, cl*(int(G)) $\subseteq$ G and hence G is pre*$_{\mathcal{I}}$-closed.

Theorem 2.7. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If cl*(int(G))\G contains no any nonempty $*$-closed set, then G is a weakly $\mathcal{I}_{g\delta}$-closed set.

Proof. Suppose that cl*(int(G))\G contains no any nonempty $*$-closed set in (X, τ, $\mathcal{I}$). Let G $\subseteq$ H and H be a δ-open set. Assume that cl*(int(G)) is not contained in H. It follows that cl*(int(G))$\cap$(X\H) is a nonempty $*$-closed subset of cl*(int(G))\G. This is a contradiction. Hence G is a weakly $\mathcal{I}_{g\delta}$-closed set.

Theorem 2.8. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_{g\delta}$-closed set, then int(G) = H\K where H is $\mathcal{I}$-R closed and K contains no any nonempty δ-closed set.

Proof. Let G be a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$). Take K = (int(G))*\G. Then, by Theorem 2.4, K contains no any nonempty δ-closed set. Take H = cl*(int(G)). Then H = cl*(int(H)). Moreover, we have H\K = ((int(G))* $\cup$ int(G))\((int(G))*\G) = ((int(G))* $\cup$ int(G)) $\cap$ (X\(int(G))* $\cup$ G) = int(G).

Theorem 2.9. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. Assume that G is a weakly $\mathcal{I}_{g\delta}$-closed set. The following properties are equivalent:

1. G is pre*$_{\mathcal{I}}$-closed,

2. cl*(int(G))\G is a δ-closed set,

3. (int(G))* \G is a δ-closed set.

Proof. (1) ⇒ (2) : Let G be pre$^*_{\mathcal{I}}$-closed. We have cl*(int(G)) ⊆ G. Then, cl*(int(G))\G = ∅. Thus, cl*(int(G))\G is a δ-closed set.

(2) ⇒ (1) : Let cl*(int(G))\G be a δ-closed set. Since G is a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$), then by Theorem 2.5, cl*(int(G))\G = ∅. Hence, we have cl*(int(G)) ⊆ G. Thus, G is pre$^*_{\mathcal{I}}$-closed.

(2) ⇔ (3) : It follows easily from that cl*(int(G))\G = (int(G))*\G.

Theorem 2.10. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G ⊆ X be a weakly $\mathcal{I}_{g\delta}$-closed set. Then G ∪ (X\(int(G))*) is a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$).

Proof. Let G be a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$). Suppose that H is a δ-open set such that G ∪ (X\(int(G))*) ⊆ H. We have X\H ⊆ X\(G ∪ (X\(int(G))*)) = (X\G) ∩ (int(G))* = (int(G))*\G. Since X\H is a δ-closed set and G is a weakly $\mathcal{I}_{g\delta}$-closed set, it follows from Theorem 2.4 that X\H = ∅. Hence, X = H. Thus, X is the only δ-open set containing G ∪ (X\(int(G))*). Consequently, G ∪ (X\(int(G))*) is a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$).

Corollary 2.11. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G ⊆ X be a weakly $\mathcal{I}_{g\delta}$-closed set. Then (int(G))*\G is a weakly $\mathcal{I}_{g\delta}$-open set in (X, τ, $\mathcal{I}$).

Proof. Since X\((int(G))*\G) = G ∪ (X\(int(G))*), it follows from Theorem 2.10 that (int(G))*\G is a weakly $\mathcal{I}_{g\delta}$-open set in (X, τ, $\mathcal{I}$).

Theorem 2.12. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G ⊆ X. The following properties are equivalent:

1. G is a ∗-closed and δ-open set,
2. G is $\mathcal{I}$-R closed and δ-open set,
3. G is a weakly $\mathcal{I}_{g\delta}$-closed and δ-open set.

Proof. (1) ⇒ (2) ⇒ (3) : Obvious.

(3) ⇒ (1) : Since G is δ-open and weakly $\mathcal{I}_{g\delta}$-closed, cl*(int(G)) ⊆ G and so G = cl*(int(G)). Then G is $\mathcal{I}$-R closed and hence it is ∗-closed.

Proposition 2.13. Every pre$^*_{\mathcal{I}}$-closed set is weakly $\mathcal{I}_{g\delta}$-closed but not conversely.

Proof. Let H ⊆ G and G be a δ-open set in X. Since H is pre$^*_{\mathcal{I}}$-closed, cl*(int(H)) ⊆ H ⊆ G. Hence H is weakly $\mathcal{I}_{g\delta}$-closed set.

Example 2.14. Let (X, τ, $\mathcal{I}$) be an ideal topological space such that X = {a, b, c}, τ = {∅, X, {c}, {a, c}} and $\mathcal{I}$ = {∅, {c}}. Then {a, c} is weakly $\mathcal{I}_{g\delta}$-closed set but not pre$^*_{\mathcal{I}}$-closed.

3 Further Properties

Theorem 3.1. Let (X, τ, $\mathcal{I}$) be an ideal topological space. The following properties are equivalent:

1. Each subset of (X, τ, $\mathcal{I}$) is a weakly $\mathcal{I}_{g\delta}$-closed set,
2. G is pre$^*_{\mathcal{I}}$-closed for each δ-open set G in X.

Proof. (1) ⇒ (2) : Suppose that each subset of (X, τ, $\mathcal{I}$) is a weakly $\mathcal{I}_{g\delta}$-closed set. Let G be a δ-open set in X. Since G is weakly $\mathcal{I}_{g\delta}$-closed, then we have cl*(int(G)) ⊆ G. Thus, G is pre*$_{\mathcal{I}}$-closed.

(2) ⇒ (1) : Let G be a subset of (X, τ, $\mathcal{I}$) and H be a δ-open set such that G ⊆ H. By (2), we have cl*(int(G)) ⊆ cl*(int(H)) ⊆ H. Thus, G is a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$).

Theorem 3.2. Let (X, τ, $\mathcal{I}$) be an ideal topological space. If G is a weakly $\mathcal{I}_{g\delta}$-closed set and G ⊆ H ⊆ cl*(int(G)), then H is a weakly $\mathcal{I}_{g\delta}$-closed set.

Proof. Let H ⊆ K and K be a δ-open set in X. Since G ⊆ K and G is a weakly $\mathcal{I}_{g\delta}$-closed set, then cl*(int(G)) ⊆ K. Since H ⊆ cl*(int(G)), then cl*(int(H)) ⊆ cl*(int(G)) ⊆ K. Thus, cl*(int(H)) ⊆ K and hence, H is a weakly $\mathcal{I}_{g\delta}$-closed set.

Corollary 3.3. Let (X, τ, $\mathcal{I}$) be an ideal topological space. If G is a weakly $\mathcal{I}_{g\delta}$-closed and open set, then cl*(G) is a weakly $\mathcal{I}_{g\delta}$-closed set.

Proof. Let G be a weakly $\mathcal{I}_{g\delta}$-closed and open set in (X, τ, $\mathcal{I}$). We have G ⊆ cl*(G) ⊆ cl*(G) = cl*(int(G)). Hence, by Theorem 3.2, cl*(G) is a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$).

Theorem 3.4. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G ⊆ X. If G is a nowhere dense set, then G is a weakly $\mathcal{I}_{g\delta}$-closed set.

Proof. Let G be a nowhere dense set in X. Since int(G) ⊆ int(cl(G)), then int(G) = ∅. Hence, cl*(int(G)) = ∅. Thus, G is a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$).

Remark 3.5. The reverse of Theorem 3.4 is not true in general as shown in the following example.

Example 3.6. In Example 2.14, {a, c} is a weakly $\mathcal{I}_{g\delta}$-closed set but not a nowhere dense set.

Remark 3.7. The intersection of two weakly $\mathcal{I}_{g\delta}$-closed sets in an ideal topological space need not be a weakly $\mathcal{I}_{g\delta}$-closed set.

Example 3.8. Let (X, τ, $\mathcal{I}$) be an ideal topological space such that X = {a, b, c, d}, τ = {∅, X, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}} and $\mathcal{I}$ = {∅, {d}}. Then A = {a, b, d} and B = {a, b, c} are weakly $\mathcal{I}_{g\delta}$-closed sets but their intersection {a, b} is not a weakly $\mathcal{I}_{g\delta}$-closed set.

Theorem 3.9. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G ⊆ X. Then G is a weakly $\mathcal{I}_{g\delta}$-open set if and only if H ⊆ int*(cl(G)) whenever H ⊆ G and H is a δ-closed set.

Proof. Let H be a δ-closed set in X and H ⊆ G. It follows that X\H is a δ-open set and X\G ⊆ X\H. Since X\G is a weakly $\mathcal{I}_{g\delta}$-closed set, then cl*(int(X\G)) ⊆ X\H. We have X\int*(cl(G)) ⊆ X\H. Thus, H ⊆ int*(cl(G)).

Conversely, let K be a δ-open set in X and X\G ⊆ K. Since X\K is a δ-closed set such that X\K ⊆ G, then X\K ⊆ int*(cl(G)). We have X\int*(cl(G)) = cl*(int(X\G)) ⊆ K. Thus, X\G is a weakly $\mathcal{I}_{g\delta}$-closed set. Hence, G is a weakly $\mathcal{I}_{g\delta}$-open set in (X, τ, $\mathcal{I}$).

Theorem 3.10. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G ⊆ X. If G is a weakly $\mathcal{I}_{g\delta}$-closed set, then cl*(int(G))\G is a weakly $\mathcal{I}_{g\delta}$-open set in (X, τ, $\mathcal{I}$).

Proof. Let G be a weakly $\mathcal{I}_{g\delta}$-closed set in (X, τ, $\mathcal{I}$). Suppose that H is a δ-closed set such that H $\subseteq$ cl*(int(G))\G. Since G is a weakly $\mathcal{I}_{g\delta}$-closed set, it follows from Theorem 2.5 that H = $\emptyset$. Thus, we have H $\subseteq$ int*(cl(cl*(int(G))\G)). It follows from Theorem 3.9 that cl*(int(G))\G is a weakly $\mathcal{I}_{g\delta}$-open set in (X, τ, $\mathcal{I}$).

Theorem 3.11. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_{g\delta}$-open set, then H = X whenever H is a δ-open set and int*(cl(G)) $\cup$ (X\G) $\subseteq$ H.

Proof. Let H be a δ-open set in X and int*(cl(G)) $\cup$ (X\G) $\subseteq$ H. We have X\H $\subseteq$ (X\int*(cl(G))) $\cap$ G = cl*(int(X\G))\(X\G). Since X\H is a δ-closed set and X\G is a weakly $\mathcal{I}_{g\delta}$-closed set, it follows from Theorem 2.5 that X\H = $\emptyset$. Thus, we have H = X.

Theorem 3.12. Let (X, τ, $\mathcal{I}$) be an ideal topological space. If G is a weakly $\mathcal{I}_{g\delta}$-open set and int*(cl(G)) $\subseteq$ H $\subseteq$ G, then H is a weakly $\mathcal{I}_{g\delta}$-open set.

Proof. Let G be a weakly $\mathcal{I}_{g\delta}$-open set and int*(cl(G)) $\subseteq$ H $\subseteq$ G. Since int*(cl(G)) $\subseteq$ H $\subseteq$ G, then int*(cl(G)) = int*(cl(H)). Let K be a δ-closed set and K $\subseteq$ H. We have K $\subseteq$ G. Since G is a weakly $\mathcal{I}_{g\delta}$-open set, it follows from Theorem 3.9 that K $\subseteq$ int*(cl(G)) = int*(cl(H)). Hence, by Theorem 3.9, H is a weakly $\mathcal{I}_{g\delta}$-open set in (X, τ, $\mathcal{I}$).

Corollary 3.13. Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_{g\delta}$-open and closed set, then int*(G) is a weakly $\mathcal{I}_{g\delta}$-open set.

Proof. Let G be a weakly $\mathcal{I}_{g\delta}$-open and closed set in (X, τ, $\mathcal{I}$). Then int*(cl(G)) = int*(G) $\subseteq$ int*(G) $\subseteq$ G. Thus, by Theorem 3.12, int*(G) is a weakly $\mathcal{I}_{g\delta}$-open set in (X, τ, $\mathcal{I}$).

Definition 3.14. A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is called $Q_{\mathcal{I}}$-set if A = M$\cup$N where M is δ-closed and N is pre*$_{\mathcal{I}}$-open.

Remark 3.15. Every pre*$_{\mathcal{I}}$-open (resp. δ-closed) set is $Q_{\mathcal{I}}$-set but not conversely.

Example 3.16. Let (X, τ, $\mathcal{I}$) be an ideal topological space such that X = {a, b, c, d}, τ = {$\emptyset$, X, {a}, {b}, {a, b}, {b, c}, {a, b, c}} and $\mathcal{I}$ = {$\emptyset$, {d}}. Then {b, d} is a $Q_{\mathcal{I}}$-set but it is neither pre*$_{\mathcal{I}}$-open nor δ-closed.

Theorem 3.17. For a subset H of (X, τ, $\mathcal{I}$), the following are equivalent.

1. H is pre*$_{\mathcal{I}}$-open.

2. H is a $Q_{\mathcal{I}}$-set and weakly $\mathcal{I}_{g\delta}$-open.

Proof. (1) $\Rightarrow$ (2): By Remark 3.15, H is a $Q_{\mathcal{I}}$-set. By Proposition 2.13, H is weakly $\mathcal{I}_{g\delta}$-open.

(2) $\Rightarrow$ (1): Let H be a $Q_{\mathcal{I}}$-set and weakly $\mathcal{I}_{g\delta}$-open. Then there exist a δ-closed set M and a pre*$_{\mathcal{I}}$-open set N such that H =M$\cup$N. Since M $\subseteq$ H and H is weakly $\mathcal{I}_{g\delta}$-open, by Theorem 3.9, M $\subseteq$ int*(cl(H)). Also, we have N $\subseteq$ int*(cl(N)). Since N $\subseteq$ H, N $\subseteq$ int*(cl(N)) $\subseteq$ int*(cl(H)). Then H = M$\cup$N $\subseteq$ int*(cl(H)). So H is pre*$_{\mathcal{I}}$-open.

The following example shows that the concepts of weakly $\mathcal{I}_{g\delta}$-open set and $Q_{\mathcal{I}}$-set are independent.

Example 3.18. In Example 3.16, {c} is weakly $\mathcal{I}_{g\delta}$-open set but not $Q_{\mathcal{I}}$-set. Also {d} is $Q_{\mathcal{I}}$-set but not weakly $\mathcal{I}_{g\delta}$-open set.

Remark 3.19. The following diagram holds for any ideal topological space:

$$\begin{array}{ccc} \mathcal{I}_{g\delta}\text{-closed set} & \longrightarrow & \text{weakly } \mathcal{I}_{g\delta}\text{-closed set} \\ \downarrow & & \downarrow \\ \mathcal{I}_{rg}\text{-closed set} & \longrightarrow & \text{weakly } \mathcal{I}_{rg}\text{-closed set} \end{array}$$

None of the implications is reversible as shown in the following examples and in [6].

Example 3.20. Let (X, τ, $\mathcal{I}$) be an ideal topological space such that X = {a, b, c, d}, τ = {∅, X, {a}, {b}, {a, b}, {b, c}, {a, b, c}} and $\mathcal{I}$ = {∅}. Then {a, b} is $\mathcal{I}_{rg}$-closed set but not $\mathcal{I}_{g\delta}$-closed.

Example 3.21. In Example 3.20, {a, b} is weakly $\mathcal{I}_{rg}$-closed set but not weakly $\mathcal{I}_{g\delta}$-closed.

Example 3.22. In Example 3.20, {c} is weakly $\mathcal{I}_{g\delta}$-closed set but not $\mathcal{I}_{g\delta}$-closed.

4 gδ-pre*$_{\mathcal{I}}$-normal Spaces

Definition 4.1. An ideal topological space (X, τ, $\mathcal{I}$) is said to be gδ-pre*$_{\mathcal{I}}$-normal if for every pair of disjoint δ-closed subsets A, B of X, there exist disjoint pre*$_{\mathcal{I}}$-open sets U, V of X such that A ⊆ U and B ⊆ V.

Theorem 4.2. The following properties are equivalent for a space (X, τ, $\mathcal{I}$).

1. X is gδ-pre*$_{\mathcal{I}}$-normal;
2. for any disjoint δ-closed sets A and B, there exist disjoint weakly $\mathcal{I}_{g\delta}$-open sets U, V of X such that A ⊆ U and B ⊆ V;
3. for any δ-closed set A and any δ-open set B containing A, there exists a weakly $\mathcal{I}_{g\delta}$-open set U such that A ⊆ U ⊆ cl*(int(U)) ⊆ B.

Proof. (1) ⇒ (2): The proof is obvious.

(2) ⇒ (3): Let A be any δ-closed set of X and B any δ-open set of X such that A ⊆ B. Then A and X\B are disjoint δ-closed sets of X. By (2), there exist disjoint weakly $\mathcal{I}_{g\delta}$-open sets U, V of X such that A ⊆ U and X\B ⊆ V. Since V is weakly $\mathcal{I}_{g\delta}$-open set, by Theorem 3.9, X\B ⊆ int*(cl(V)) and U∩int*(cl(V)) = ∅. Therefore we obtain cl*(int(U)) ⊆ cl*(int(X\V)) and hence A ⊆ U ⊆ cl*(int(U)) ⊆ B.

(3) ⇒ (1): Let A and B be any disjoint δ-closed sets of X. Then A ⊆ X\B and X\B is δ-open and hence there exists a weakly $\mathcal{I}_{g\delta}$-open set G of X such that A ⊆ G ⊆ cl*(int(G)) ⊆ X\B. Put U = int*(cl(G)) and V = X\cl*(int(G)). Then U and V are disjoint pre*$_{\mathcal{I}}$-open sets of X such that A ⊆ U and B ⊆ V. Therefore X is gδ-pre*$_{\mathcal{I}}$-normal.

Definition 4.3. A function f : (X, τ, $\mathcal{I}$) → (Y, σ) is said to be weakly $\mathcal{I}_{g\delta}$-continuous if f^{-1}(V) is weakly $\mathcal{I}_{g\delta}$-closed in X for every closed set V of Y.

Definition 4.4. A function f : (X, τ, $\mathcal{I}$) → (Y, σ, $\mathcal{J}$) is called weakly $\mathcal{I}_{g\delta}$-irresolute if f^{-1}(V) is weakly $\mathcal{I}_{g\delta}$-closed in X for every weakly $\mathcal{J}_{g\delta}$-closed of Y.

Definition 4.5. A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be δ-closed [4, 12] if f(V) is δ-closed in Y for every δ-closed set V of X.

Definition 4.6. A topological space (X, τ) is said to be δ-normal if for every pair of disjoint δ-closed subsets A, B of X, there exist disjoint open sets U, V of X such that A $\subseteq$ U and B $\subseteq$ V.

Theorem 4.7. Let f : X $\rightarrow$ Y be a weakly $\mathcal{I}_{g\delta}$-continuous δ-closed injection. If Y is δ-normal, then X is gδ-pre$^*{}_{\mathcal{I}}$-normal.

Proof. Let A and B be disjoint δ-closed sets of X. Since f is δ-closed injection, f(A) and f(B) are disjoint δ-closed sets of Y. By the δ-normality of Y, there exist disjoint open sets U and V in Y such that f(A) $\subseteq$ U and f(B) $\subseteq$ V. Since f is weakly $\mathcal{I}_{g\delta}$-continuous, then f^{-1}(U) and f^{-1}(V) are weakly $\mathcal{I}_{g\delta}$-open sets of X such that A $\subseteq$ f^{-1}(U) and B $\subseteq$ f^{-1}(V). Therefore X is gδ-pre$^*{}_{\mathcal{I}}$-normal by Theorem 4.2.

Theorem 4.8. Let f : X $\rightarrow$ Y be a weakly $\mathcal{I}_{g\delta}$-irresolute δ-closed injection. If Y is gδ-pre$^*{}_{\mathcal{I}}$-normal, then X is gδ-pre$^*{}_{\mathcal{I}}$-normal.

Proof. Let A and B be disjoint δ-closed sets of X. Since f is δ-closed injection, f(A) and f(B) are disjoint δ-closed sets of Y. Since Y is gδ-pre$^*{}_{\mathcal{I}}$-normal, by Theorem 4.2, there exist disjoint weakly $\mathcal{J}_{g\delta}$-open sets U and V in Y such that f(A) $\subseteq$ U and f(B) $\subseteq$ V. Since f is weakly $\mathcal{I}_{g\delta}$-irresolute, then f^{-1}(U) and f^{-1}(V) are disjoint weakly $\mathcal{I}_{g\delta}$-open sets of X such that A $\subseteq$ f^{-1}(U) and B $\subseteq$ f^{-1}(V). Therefore X is gδ-pre$^*{}_{\mathcal{I}}$-normal.

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INVESTIGATION OF USAGE IN DYEING OF TEXTILE OF POMEGRANATE (*PUNICA GRANATUM*) JUICE

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Abstract - In this study, the dyeing properties of pomegranate (*Punica granutum*) juice were investigated. Its juice was obtained by extraction (cool press) to obtain the dyebath. Aluminium sulphate $Al_2(SO_4)_3$, Iron (II) sulphate ($FeSO_4$), Copper (II) sulphate ($CuSO_4$) salts and (NH_3 + calcium oxalate + urea) solution 3% g/v) were used as mordants for mordanting of wool, viscose and linen fabrics. All fabrics were dyed at different pH values (4 and 7) using together mordanting, pre-mordanting and last mordanting methods). Consequently, 11 wool, 11 viscose and 11 linen fabric samples were dyed at two different pH degree (4 and 7). Color codes, fastness measurements and dyeing conditions were determined.

***Keywords* -** *Pomegranate, mordant, dyes, wool, viscose , linen*

1. Introduction

Punica granatum belong to *Lythraceae* family which has slightly sour and some slightly sweet [1].

100 mL of *Punica granatum* juice meets 16% need of human in daily. This juice is rich respect to antioxidant called as pro-anthocyanidine [2]. Antioxidants have an most important in the pharmacological studies. *S*kin of *Punica granatum* fruit has tanen (between 30% - 28%) and is used in leather industry. In addition, fruit skin has also been using in dyeing of fabric, leather and making the ink [3]. In the skin of *Punica granatum* has tannic acid, parinaric acid, palmitic acid, stearic acid, oleic acid and linoleic acid [4-5].

Figure 1 is shown the chemical structure of tannin (tannic acid).

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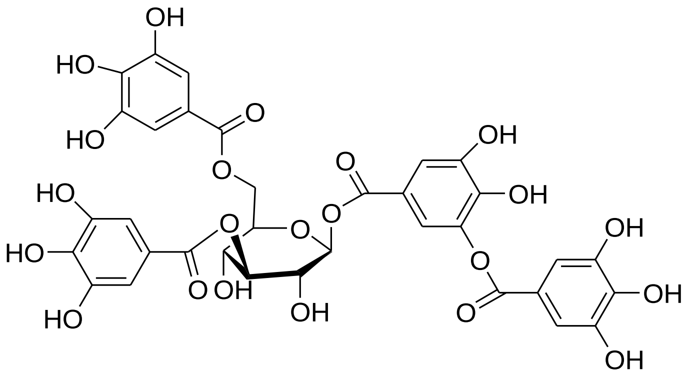


Figure 1. Molecule structure of tannic acid (Ferrell, Thorington and Richard, 2006).

Furthermore, the fruit of *Punica granatum* has been using to extend the life of containers.[6]. Its juise has B and K vitamine that is used in diet product [7]. In addition, its seeds are source of diet fiber [8]. Because of the high antioxidant values of skin, it is used the main source of pro-antociyanidine and kersetol either pharmaceuticals or other areas studies [9].

According to the literature surveys there is no enough research in dyeing of fabrics that using the fruit juice of *Punica granatum*. That is why, we aimed to investigate the dyeing properties or capacity of *Punica granatum* juice in dyeing of wool, linen and viscose fabric using some dyeing methods and mordant that described in experimental section.

2. Experimental

2.1. Reagents and equipments

All chemicals and mordants ($FeSO_4.7H_2O$, $AlK(SO_4)_2.12H_2O$ and $CuSO_4.5H_2O$) used in this work, were purchased from Merck. Distilled water was used for all steps. *Punica granatum* juice was obtained mechanically (cool press). Color codes were determined by using Pantone Color Guide. The wash-, crock- (wet, dry) and light fastness of all dyed samples were carried out according to ISO 105-C06 and to CIS, respectively, and fastness levels were determined by Atlas Weather-ometer, a Launder-ometer and a 255 model crock-meter, respectively [10].

2.2. Fabrics

Wool and cotton and fabrics were chosen as the fabric types to be studied. The characteristics of the fabrics are shown in Table 1.

Table 1 - Characteristics of the used fabrics

Fibre type	**Mass per unit area (g/m^2)**	**Surface type**	**Fabric density**
Wool	180	weaved	Weft:28, warp: 30
Linen	150	knitted	Course:18, Wale:13
Viscose	140	knitted	Course:15, Wale:12

2.3. Natural dye extraction and mordanting

The juice of *Punica granatum* fruit used as a natural dye source in the present study; these were supplied from Tokat bazaar (Turkey). The raw materials was pressed and diluted with distilled water before using. (the rate of natural dye source to distilled water was 1:1) The colored solutions were filtered and used in the dyeing process.

The metal salts iron sulfate , copper sulfate and aluminum sulfate were used as mordants; the dyeing procedure of the textile fabrics is pre-mordanting (T1), together mordanting (T2), and after-mordanting (T3). The experimental plan is listed in Table 2.

Table 2 - Experimental plan

Treatment type (T)	Mordant	Dyeing pH	Wool	Viscose	Linen
Pre-mordanting (T1)	Iron sulphate	4-7	+	+	+
	Copper "	4-7	+	+	+
	Aluminium "	4-7	+	+	+
Together-mordanting (T2)	Iron sulphate	4-7	+	+	+
	Copper "	4-7	+	+	+
	Aluminium "	4-7	+	+	+
Last-mordanting (T3)	Iron sulphate	4-7	+	+	+
	Copper "	4-7	+	+	+
	Aluminium "	4-7	+	+	+

3. Dyeing

Three dyeing methods including pre-mordanting (T1), together-mordanting (T2) and last-mordanting (T3) were applied to the wool, linen and viscose fabrics.

In the T1 procedure, fabric was initially dipped into 0.1 M mordant solution (100 ml) and then resulting solution was heated for 1 h at 90ºC. Then it was cooled and rinsed with double distilled water and then poured into the dye-bath solution (100 ml). Then further dyeing was carried out at 90ºC for 1 h. Finally, the material after dyeing was removed, washed with double distilled water and finally dried at room temperature [11].

In the T2 procedure, both mordant (in solid form that is equal to 0.1 M mordant solution) and the dye residue was transferred in a conical flask and the sample was poured into the mixture. Then the mixture was heated at 90ºC until 1 h. Then it was cooled and washed with distilled water, squeezed and finally it was dried [11].

In the T3 method, the non-colored material (1 g) was firstly given treatment with the dye solution for 1 h at 90ºC. Then sample was cooled, washed twice with distilled water and poured into 0.1 M mordant solution (100 ml). It was heated for 1 h at 90ºC and then, After dyeing, the washing of the dyed fabrics were carried out in cold, boiled, boiled with non-ionic detergents and cold rinsing[11].

3.1. Dyeing mechanism of the fabrics

Metal complex formation has been an outstanding property of textile dyeing from ancient times, since it was known that the technical performance, including fastness such as washing and light, of many natural dyes could be improved by treatment with definite metal ions, a method known as mordanting [12].

Al (III) and Fe (II) ions have a coordination number of six and they are able to make complexes in the octahedral configuration. So, in the proposed mechanisms which are given in Figure 2 the unoccupied sites of the metal ions may be occupied with H_2O molecules, oxochrome groups of the dyestuff or free amino and carboxyl groups of wool fabric [12]. Proposed mechanisms for dyeing of wool and cotton fiber with the extract of apple leaves are given in Figure2 and Figure 3.

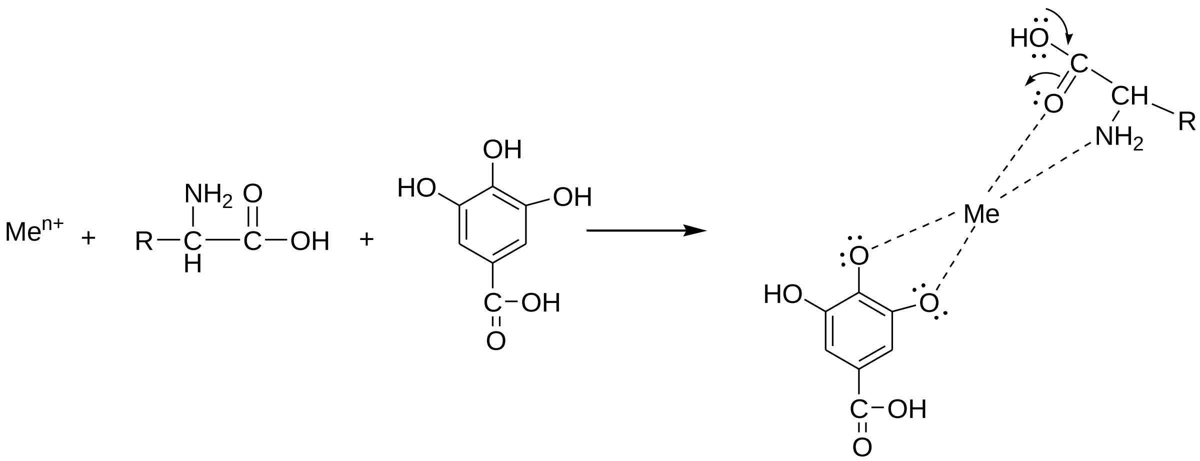


Figure 2. Proposed dyeing mechanism of wool (together mordanting.)

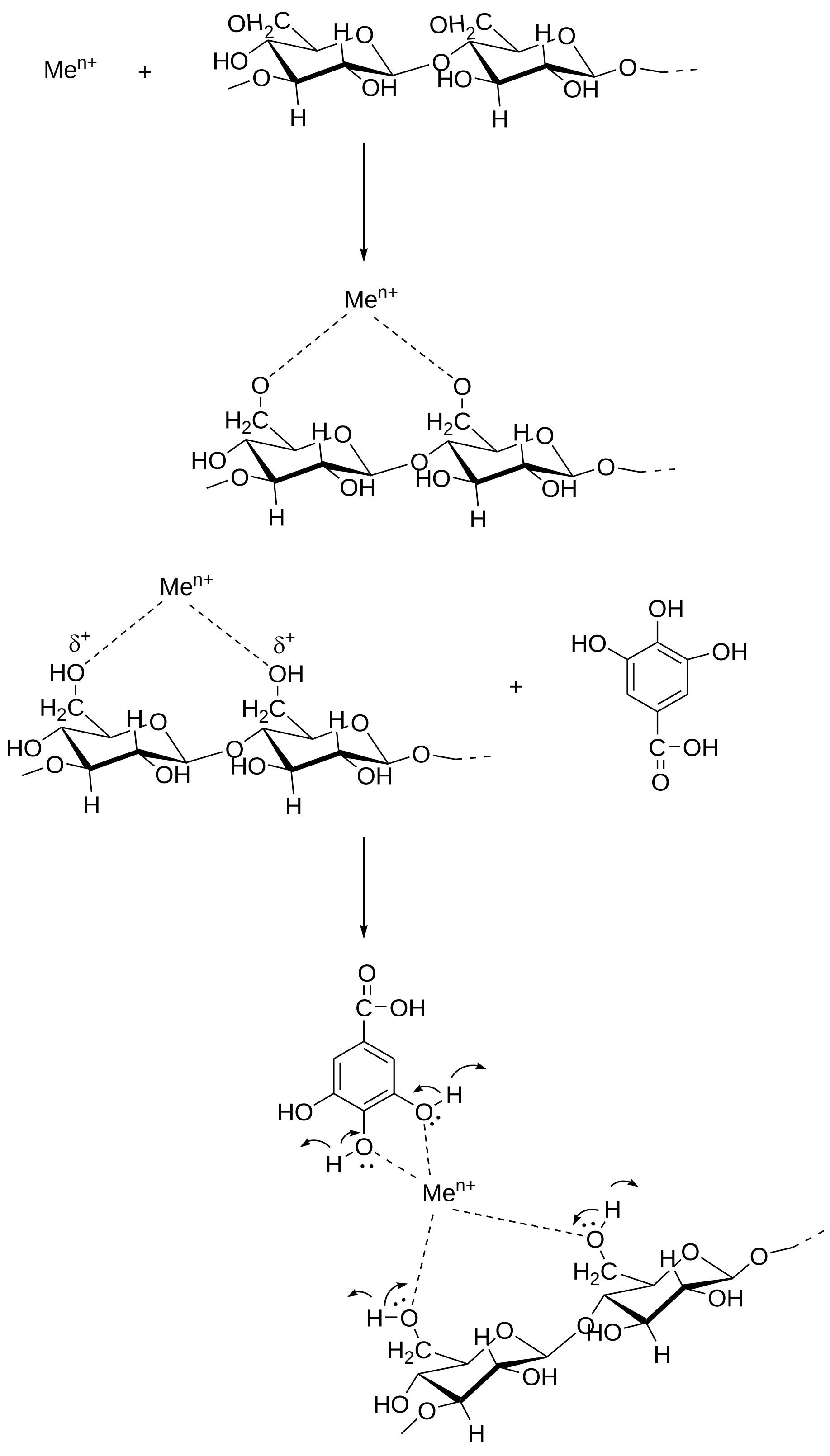


Figure 3. Proposed dyeing mechanism of linen (together mordanting.)

3.2 .Fastness results for viscose fabrics

Pre-mordantig, together mordanting and last – mordanting fastness values are given in table 1, table 2 and table 3, respectively.

Table 1. Fastness results of viscose fabrics with $FeSO_4$ mordant.

$FeSO_4$	pH	Rubbing			Light	Color Code
		Washing	Wet	Dry		
T1	4	4/5	3	4/5	5	7806-Y13R
	7	4/5	2	4/5	6	57005-Y20R
T2	4	3/4	4	4/5	6	6005-Y10R
	7	3/4	4	4/5	4	0505-Y0RS
T3	4	4/5	4	4/5	3	6005-Y10R
	7	4/5	4	4/5	2	6005-Y10R
Unmordant	4	1/2	3/4	4/5	5	0621-Y
	7	4/5	2	4/5	5	0621-Y
Urea+NH_3+Oxalate	4	1/2	3/4	4/5	4	0631-Y03R
	7	1/2	4/5	4/5	6	0505-Y05R

Table 2. Fastness results of viscose fabrics with $AlK(SO_4)_2$ mordant

$AlK(SO_4)_2$	pH	Rubbing			Light	Color Code
		Washing	Wet	Dry		
T1	4	4/5	4/5	4/5	1	S-1020-Y
	7	3/4	3/4	3/4	6	S-1030-Y
T2	4	4/5	4/5	4/5	6	S-1030-Y
	7	4/5	4/5	3/4	6	S-1020-Y
T3	4	4/5	4/5	4/5	2	1008-Y
	7	4/5	4/5	4/5	4	1008-Y

Table 3. Fastness results of viscose fabrics with $CuSO_4$ mordant

$CuSO_4$	pH	Rubbing			Light	Color Code
		Washing	Wet	Dry		
T1	4	4/5	3	4/5	6	S-2030-Y
	7	3/4	3	4/5	5	2894-Y32R
T2	4	4/5	4	4/5	6	6005-Y30S
	7	3/4	4	4/5	6	1952-Y30S
T3	4	4/5	4	4/5	6	3121-Y29R
	7	4/5	3/4	4/5	6	2013-Y32R

3.3. Fastness results for linen fabrics

Pre-mordanting, together mordanting and last – mordanting fastness values are given in table 4, table 5 and table 6, respectively.

Table 4. Fastness results for linen with $FeSO_4$ mordant

$FeSO_4$	pH	Rubbing			Light	Color Code
		Washing	Wet	Dry		
T1	4	4/5	3	4/5	5	7806-Y13R
	7	4/5	2	4/5	6	57005-Y20R
T2	4	3/4	4	4/5	6	6005-Y10R
	7	3/4	4	4/5	5	0505-Y0RS
T3	4	4/5	4	4/5	3	5005-Y10R
	7	4/5	4	4/5	6	6005-Y10R
Unmordant	4	1/2	3/4	4/5	5	0631-Y03R
	7	1/2	3	4/5	5	0539-G99Y
Urea+NH_3+Oxalate	4	1/2	3/4	4/5	4	0621-Y
	7	1/2	4/5	4/5	6	0611-G95Y

Table 5. Fastness results for linen with $AlK(SO_4)_2$ mordant

$AlK(SO_4)_2$	pH	Rubbing			Light	Color Code
		Washing	Wet	Dry		
T1	4	4/5	4/5	4/5	1	S-1020-Y
	7	3/4	3/4	4/5	5	S-1030-Y
T2	4	4/5	4/5	4/5	6	S-1030-Y
	7	4/5	3/4	4/5	6	S-1020-Y
T3	4	4/5	4/5	4/5	2	1008-Y
	7	4/5	4/5	4/5	4	1008-Y

Table 6. Fastness results for linen with $CuSO_4$ Mordant

$CuSO_4$	pH	Rubbing			Light	Color Code
		Washing	Wet	Dry		
T1	4	4/5	3	4/5	6	S-2030-Y
	7	3/4	3	4/5	6	2894-Y32R
T2	4	4/5	5	4/5	6	6005-Y10R
	7	3/4	4/5	4/5	6	1952-Y30S
T3	4	4/5	4	4/5	6	2013-Y32R
	7	4/5	4/5	4/5	6	2013-Y32R

3.4. Fastness results for wool fabrics

Pre-mordanting, together mordanting and last – mordanting fastness values are given in table 7, table 8 and table 9, respectively.

Table 7. Fastness results of wool fabrics with $FeSO_4$ mordant.

$FeSO_4$	pH	Rubbing			Light	Color Code
		Washing	Wet	Dry		
T1	4	1/2	1/2	1/2	1	S-5020
	7	1/2	1/2	1/2	4	7511-Y99R
T2	4	1/2	1/2	1/2	1	6128R
	7	1/2	1/2	1/2	1	4837-Y98R
T3	4	1/2	1/2	1/2	2	S-2030-Y90R
	7	1/2	1/2	1/2	3	S-2030-Y90R
Unmordant	4	1/2	1/2	1/2	5	1719-Y90R
	7	1/2	1/2	1/2	4	1719-Y90R
Urea+NH_3+Oxalate	4	1/2	1/2	1/2	5	1719-Y90R
	7	1/2	1/2	1/2	5	1719-Y90R

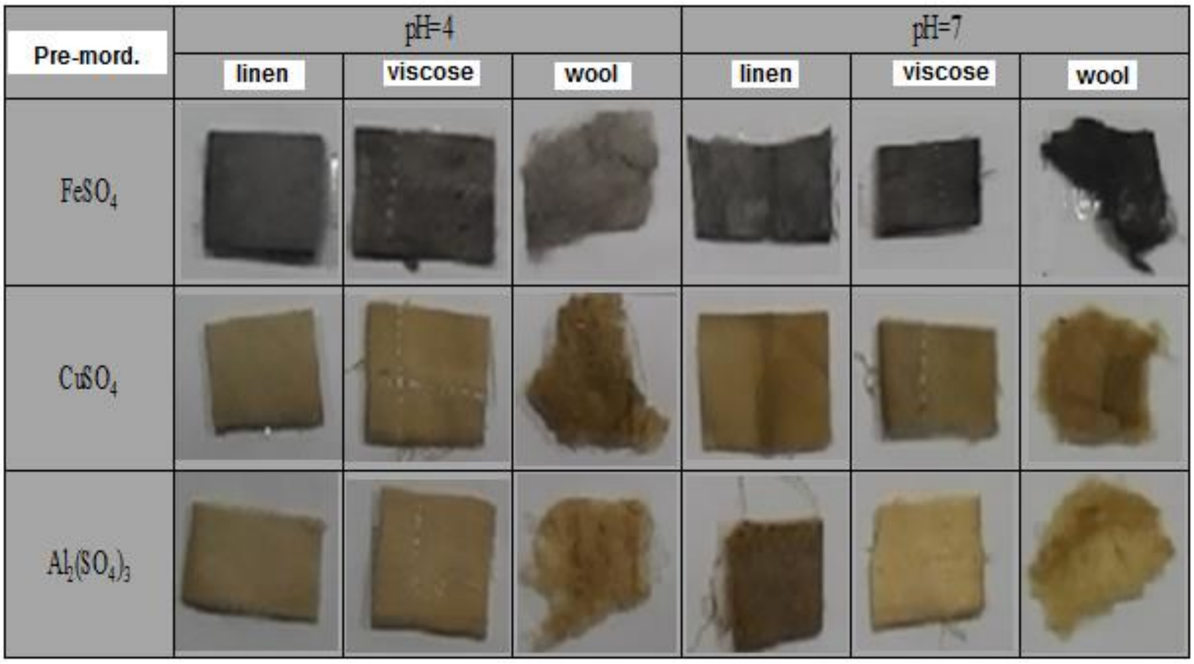
Table 8. Fastness results of wool fabrics with $AlK(SO_4)_2$ mordant

$AlK(SO_4)_2$	pH	Rubbing			Light	Color Code
		Washing	Wet	Dry		
T1	4	1/2	1/2	1/2	1	4837-Y98R
	7	1/2	1/2	1/2	5	7311-Y99R
T2	4	1/2	1/2	1/2	3	S-3040-Y20R
	7	1/2	1/2	1/2	6	S-3040-Y20R
T3	4	1/2	1/2	1/2	3	S-3010-Y20R
	7	1/2	1/2	1/2	4	S-3010-Y20R

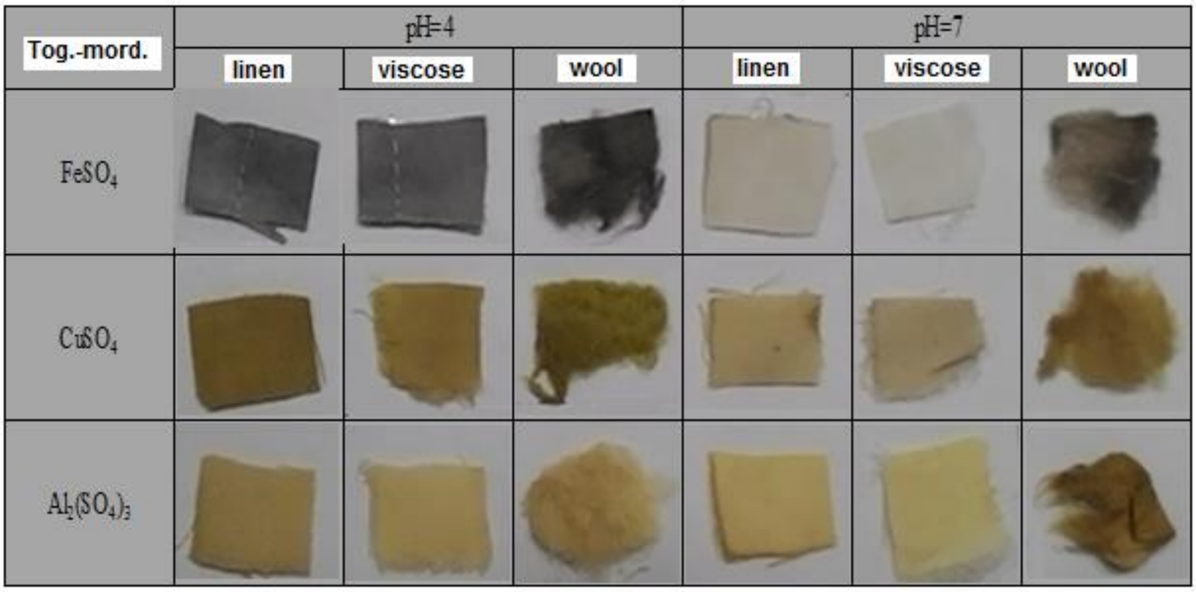
Table 9. Fastness results of wool fabrics with $CuSO_4$ mordant

$CuSO_4$	pH	Rubbing			Light	Color Code
		Washing	Wet	Dry		
T1	4	1/2	1/2	1/2	1	S-3040-Y0R
	7	1/2	1/2	1/2	5	S-3040-Y0R
T2	4	1/2	1/2	1/2	3	S-3446-Y19R
	7	1/2	1/2	1/2	6	S-3446-Y19R
T3	4	1/2	1/2	1/2	3	1619-Y34R
	7	1/2	1/2	1/2	4	1619-Y34R

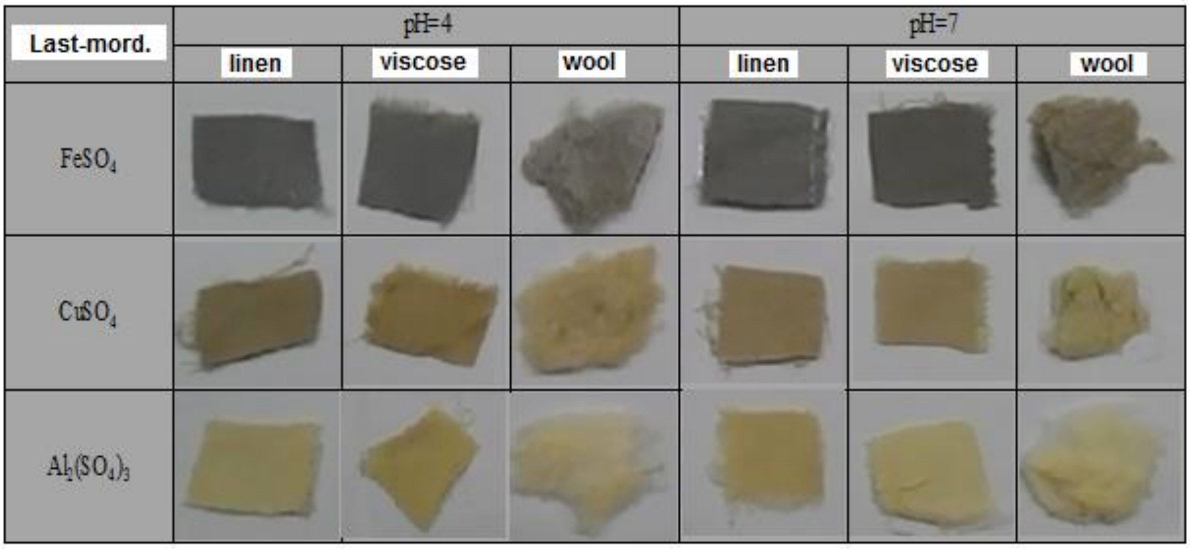
Dyed fabrics in this research are given in Picture 1, 2 and Picture 3, (urea+ammonia+oxalate) and unmordanting Picture 4, respectively.



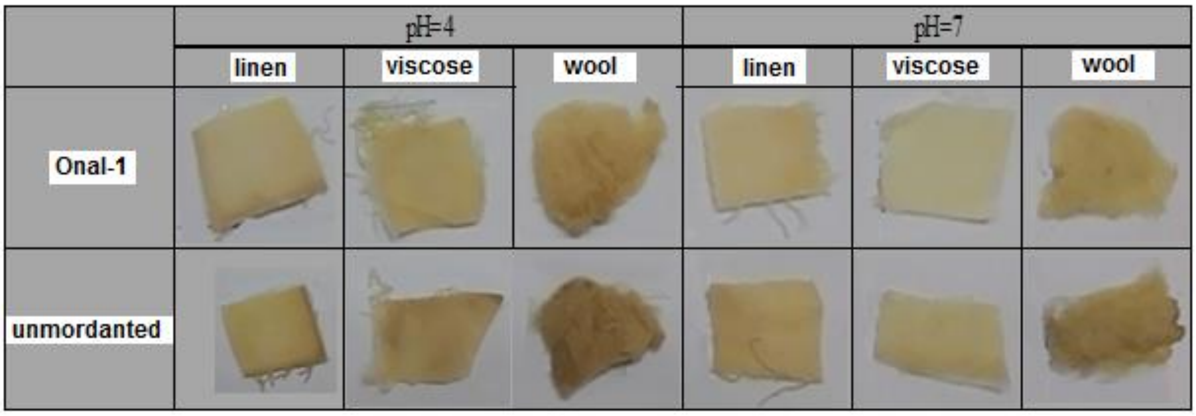
Picture 1. Dyed samples (linen, viscose, wool) with pre- mordanting method



Picture 2. Dyed samples (linen, viscose, wool) with tog.- mordanting method



Picture 3. Dyed samples (linen, viscon, wool) with last mordanting method



Picture 4. Dyed samples with Önal-1mordant and unmordanting method (wool,linen,viscose)

Dyed samples obtained using the $FeSO_4$ for each three methods are darkener from each other mordant ($AlK(SO_4)_2$ and $CuSO_4$). In addition, the lowest fastness values are obtained for wool. High fastness values have been obtained pH 4, in generally. Pre- mordanting method is very proper for each three samples (wool, linen, viscose) in dyeing.

The occurred dyeing using ($Urea+NH_3+CaC_2O_4$) solution has higher fastness values at pH 4 and pH 7, and darker colors were obtained than other mordants. We say that this solution has great importance each of pH values (pH 4 and pH 7). In here, NH_3 opens micelles of fabric. Urea is increases the solubility of dyestuff, and oxalate is makes stable of the complex molecule formed between dye, mordant and fabric [13-14].

According to the results, light fastness values is highest for $CuSO_4$, in generally. However, there is no considerable difference for light fastness at pH 4 and 7.

Good light fastness results were obtained at pH 7. However, there is no any important difference for each of pH values (4 and 7).

4. Conclusions

In this study, the pomegranate juice was used for dyeing wool, viscose and linen fabrics. Natural dye solution was extracted and applied to the selected fabrics using pre, together, and last mordanting techniques. The dyeing results of the study showed that pomegranate juice can be used as a natural dyestuff source in dyeing of wool, linen and viscose fabrics with suitable mordants.

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BIPOLAR (T,S)-FUZZY MEDIAL IDEAL OF BCI-ALGEBAS

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Abstract - In this paper, the concept bipolar (T,S)- fuzzy medial-ideals are introduced and several properties are investigated .Also, the relations between bipolar (T,S)- fuzzy medial-ideals and bipolar (T,S)- fuzzy BCI-ideals are given .The image and the pre-image of bipolar (T,S)- fuzzy medial-ideals under homomorphism of BCI-algebras are defined and how the image and the pre-image of bipolar (T,S)- fuzzy medial-ideals under homomorphism of BCI-algebras become bipolar (T,S)- fuzzy medial-ideals are studied. Moreover, the Cartesian product of bipolar (T,S)- fuzzy medial-ideals in Cartesian product BCI-algebras is established.

Keywords – Medial BCI-algebra, fuzzy medial-ideals, bipolar (T,S)-fuzzy medial-ideals, the pre-image of bipolar (T,S)- fuzzy medial-ideals in BCI-algebras, Cartesian product of bipolar (T,S)-fuzzy medial-ideals.

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1. Introduction

In 1966 Iami and Iseki [5,6,7] introduced the notion of BCK-algebras Iseki [5,7] introduced the notion of a BCI-algebra which is a generalization of BCK-algebra. Since then numerous mathematical papers have been written investigating the algebraic properties of the BCK / BCI-algebras and their relationship with other structures including lattices and Boolean algebras. There is a great deal of literature which has been produced on the theory of BCK/BCI-algebras, in particular, emphasis seems to have been put on the ideals theory of BCK/BCI-algebras . In 1956, Zadeh [17] introduced the notion of fuzzy sets. At present this concept has been applied to many mathematical branches. There are several kinds of fuzzy sets extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets etc. In [1,2,8,9,10,11,13] they introduced an extension of fuzzy sets named bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0,1] to [-1,1]. On

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the other hand, triangular norm is a powerful tool in the theory research and application development of fuzzy sets [4,12]. Li [12] generalized the operators "$\wedge$" and " $\vee$ " to T-norm and S-norm and defined the intuitionistic fuzzy groups of (T-S) - norms. as a generalization of the notion of fuzzy set. In 1991, Xi [16] applied the concept of fuzzy sets to BCI, BCK, MV-algebras. In [14] J.Meng and Y.B.Jun studied medial BCI-algebras. Mostafa et al. [15] introduced the notion of medial ideals in BCI-algebras, they stated the fuzzification of medial ideals and investigated its properties. Now, in this note we use the notion of Bipolar valued fuzzy set to establish the notion of bipolar valued (T,S) - fuzzy medial ideals of BCI-algebras; then we obtained some related properties, which have been mentioned in the abstract .

2. Preliminaries

Now we review some definitions and properties that will be useful in our results.

Definition 2.1 [5,7] An algebraic system $(X,*,0)$ of type (2, 0) is called a BCI-algebra if it satisfying the following conditions:

(BCI-1) $((x*y)*(x*z))*(z*y)=0,$

(BCI-2) $(x*(x*y))*y=0,$

(BCI-3) $x*x=0,$

(BCI-4) $x*y=0$ and $y*x=0$ imply $x=y$.

For all x, y and $z \in X$. In a BCI-algebra *X*, we can define a partial ordering"$\leq$" by $x \leq y$ if and only if $x*y=0$.

In what follows, *X* will denote a BCI-algebra unless otherwise specified.

Definition 2.2 [14] A BCI-algebra $(X,*,0)$ of type (2, 0) is called a medial BCI-algebra if it satisfying the following condition: $(x*y)*(z*u)=(x*z)*(y*u)$, for all x, y, z and $u \in X$.

Lemma 2.3 [14] An algebra (X,$*$, 0) of type (2, 0) is a medial BCI-algebra if and only if it satisfies the following conditions:

(i) $x*(y*z)=z*(y*x)$

(ii) $x*0=x$

(iii) $x*x=0$

Lemma 2.4 [14] In a medial BCI-algebra X, the following holds:

$$x*(x*y)=y, \text{ for all } x, y \in X .$$

Lemma 2.5 Let *X* be a medial BCI-algebra, then $0*(y*x)=x*y$, for all $x, y \in X$.

Proof . Clear.

Definition 2.6 A non empty subset *S* of a medial BCI-algebra *X* is said to be medial sub-algebra of *X*, if $x*y \in S$, for all $x, y \in S$.

Definition 2.7 [5,7] A non-empty subset I of a BCI-algebra X is said to be a BCI-ideal of X if it satisfies:

(I_1) $0 \in I$,

(I_2) $x * y \in I$ and $y \in I$ implies $x \in I$ for all $x, y \in X$.

Definition 2.8 [15] A non empty subset M of a medial BCI-algebra X is said to be a medial ideal of X if it satisfies:

(M_1) $0 \in M$,

(M_2) $z * (y * x) \in M$ and $y * z \in M$ imply $x \in M$ for all x, y and $z \in X$.

Definition 2.9 [15] Let μ be a fuzzy set on a BCI-algebra X, then μ is called a fuzzy BCI-subalgebra of X if

(FS_1) $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

Definition 2.10 [15] Let X be a BCI-algebra. a fuzzy set μ in X is called a fuzzy BCI-ideal of X if it satisfies:

(FI_1) $\mu(0) \geq \mu(x)$,

(FI_2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$, for all x, y and $z \in X$.

Definition 2.11 [15] Let X be a medial BCI-algebra. A fuzzy set μ in X is called a fuzzy medial ideal of X if it satisfies:

(FM_1) $\mu(0) \geq \mu(x)$,

(FM_2) $\mu(x) \geq \min\{\mu(z * (y * x)), \mu(y * z)\}$, for all x, y and $z \in X$.

Definition 2.12 [12] A triangular norm (t-norm) is a function $T : [0,1] \times [0,1] \to [0,1]$ that satisfies following conditions:

(T_1) boundary condition :T(x, 1) = x,

(T_2)) commutativity condition: T(x, y) = T(y, x),

(T_3) associativity condition :T(x, T(y, z)) = T(T(x, y), z),

(T_4) monotonicity : T(x, y) ≤ T(x, z) ,whenever y ≤ z for all x, y, z ∈[0, 1].

A simple example of such defined *t*-norm is a function $T(\alpha, \beta) = \min\{\alpha, \beta\}$.

In the general case $T(\alpha, \beta) \leq \min\{\alpha, \beta\}$ and $T(\alpha, 0) = 0$ for all $\alpha, \beta \in [0, 1]$.

Definition 2.13 [4] Let X be a BCI-algebra. A fuzzy subset μ in X is called a fuzzy sub-algebra of X with respect to a t-norm T (briefly, a T-fuzzy sub-algbra of X) if $\mu(x) \geq T\{\mu(x * y), \mu(y)\}$, for all $x, y \in X$.

Definition 2.14 [12] A triangular conorm (t-conorm S) is a mapping $S : [0,1] \times [0,1] \to [0,1]$ that satisfies following conditions:

(S1) S(x, 0) = x,

(S2) S(x, y) = S(y, x),

(S3) S(x, S(y, z)) = S(S(x, y), z),

(S4) S(x, y) ≤ S(x, z) , whenever y ≤ z for all x, y, z ∈[0, 1].

A simple example of such definition s-norm S is a function S(x, y) = max{x, y}.

Every S- conorm S has a useful property: max{α, β}≤ S (α, β) for all α, β ∈ [0, 1].

Definition 2.15 [4] Let X be a BCI-algebra. a fuzzy set μ in X is called T- fuzzy BCI- ideal of X if it satisfies:

(TI$_1$) $\mu(0) \geq \mu(x)$,

(TI$_2$) $\mu(x) \geq T\{\mu(x*y), \mu(y)\}$, for all x, y and $z \in X$.

Definition 2.16 [4] Let X be a BCI-algebra. a fuzzy set λ in X is called S- fuzzy BCI- ideal of X if it satisfies:

(SI$_1$) $\lambda(0) \leq \lambda(x)$,

(SI$_2$) $\lambda(x) \leq S\{\lambda(x*y), \lambda(y)\}$, for all x, y and $z \in X$.

Definition 2.17 Let X be a BCI-algebra. A fuzzy set μ in X is called T- fuzzy medial -ideal of X if it satisfies:

(FM$_1$) $\mu(0) \geq \mu(x)$,

(FM$_2$) $\mu(x) \geq T\{\mu(z*(y*x)), \mu(y*z)\}$, for all x, y and $z \in X$.

Definition 2.18 Let X be a BCI-algebra. A fuzzy set λ in X is called S- fuzzy medial -ideal of X if it satisfies:

(FS$_1$) $\lambda(0) \leq \lambda(x)$,

(FS$_2$) $\lambda(x) \leq S\{\lambda(z*(y*x)), \lambda(y*z)\}$, for all x, y and $z \in X$.

Example 2.19 Let X = {0, 1, 2, 3, 4, 5} be a set with a binary operation $*$ defined by the following table:

*	0	1	2	3	4	5
0	0	0	0	0	4	4
1	1	0	1	0	4	4
2	2	2	0	0	4	4
3	3	2	1	0	4	4
4	4	4	4	4	0	0
5	5	4	5	4	1	0

we can prove that $(X,*,0)$ is a BCI-algebra. Let $T_m : [0,1] \times [0,1] \rightarrow [0,1]$ be a function defined by T_m (α, β) = max {α + β − 1, 0} for all α, β ∈ [0, 1] is a t-norm . By routine calculations, we known that a fuzzy set μ in X defined by

$$\mu(1) = 0.3 \text{ and } \mu(0) = \mu(2) = \mu(3) = \mu(4) = \mu(5) = 0.9$$

is a T_m-fuzzy medial-ideal , since

$$\mu_A(x) \geq T\{\mu_A(z*(y*x), \mu_A(y*z)\}.$$

and $S_m : [0,1] \times [0,1] \rightarrow [0,1]$ be a function defined by $S_m(\alpha, \beta) = \min\{1-(\alpha+\beta), 1\}$. Then By routine calculations, we known that a fuzzy set λ in X defined by

λ (5) = 0.8 and λ (0) = λ (1) = λ (2) = λ (3) = λ (4) = 0.3

is a S_m-fuzzy medial-ideal ,because

$$\lambda_A(x) \le S\{\lambda_A(z*(y*x), \lambda_A(y*z)\}$$, for all x, y and $z \in X$.

3. Bipolar (T,S)-fuzzy Medial Ideal

Now, we present some preliminaries on the theory of bipolar-valued fuzzy set.

Definition 3.1 [9] A bipolar valued fuzzy subset B in a nonempty set X is an object having the form $\mathrm{B} = \{(x, \mu^N(x), \mu^P(x)) \mid x \in X\}$ where $\mu^N : X \to [-1,0]$ and $\mu^P : X \to [0,1]$ are mappings. The positive membership degree $\mu^P(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set $\mathrm{B} = \{(x, \mu^N(x), \mu^P(x)) \mid x \in X\}$, and the negative membership degree $\mu^N(x)$ denotes the satisfaction degree of x to some implicit counter-property of a bipolar-valued fuzzy set $\mathrm{B} = \{(x, \mu^N(x), \mu^P(x)) \mid x \in X\}$. For simplicity, we shall use the symbol $\mathrm{B} = (x, \mu^N, \mu^P)$ for bipolar fuzzy set $\mathrm{B} = \{(x, \mu^N(x), \mu^P(x)) \mid x \in X\}$, and use the notion of bipolar fuzzy sets instead of the notion of bipolar-valued fuzzy sets.

Definition 3.2 [9] Let $\mathrm{B} = (x, \mu^N, \mu^P)$ be a bipolar fuzzy set and $(s,t) \in [-1,0] \times [0,1]$.
The set $\mathrm{B}_s^N = \{x \in X : \mu^N(x) \le s\}$ and $\mathrm{B}_t^P = \{x \in X : \mu^P(x) \ge t\}$ which are called the negative s-cut and the positive t-cut of $\mathrm{B} = (x, \mu^N, \mu^P)$, respectively.

Definition3.3 [9] A bipolar fuzzy set $\mathrm{B} = (x, \mu^N, \mu^P)$ in a BCI-algebra (X,$*$, 0) is called a bipolar (T,S)- fuzzy BCI-subalgebra of X if it satisfies the following condition:
For all $x, y \in X$, $\mu^N(x*y) \le S\{\mu^N(x), \mu^N(y)\}$, $\mu^P(x*y) \ge T\{\mu^P(x), \mu^P(y)\}$.

Lemma 3.4 If $\mathrm{B} = (x, \mu^N, \mu^P)$ is a bipolar (T,S)- fuzzy BCI-subalgebra of X , then

$$\mu^N(0) \le \mu^N(x) \text{ and } \mu^P(0) \ge \mu^P(x)$$

Proof: Put x= y in Definition3.3and use (BCI-3) . The proof is complete.

Defintion 3.5 A bipolar fuzzy set $\mathrm{B} = (x, \mu^N, \mu^P)$ in X is called a bipolar (T,S)- fuzzy BCI-ideal of X if it satisfies the following condition: for all $x, y \in X$
(b_1) $\mu^N(0) \le \mu^P(x)$ and $\mu^N(0) \ge \mu^P(x)$,
(b_2) $\mu^N(x) \le S\{\mu^N(x*y), \mu^N(y)\}$, $\mu^P(x) \ge T\{\mu^P(x*y), \mu^P(x)\}$.

Defintion 3.6 A bipolar fuzzy set $\mathrm{B} = (x, \mu^N, \mu^P)$ in X is called a bipolar (T,S)- fuzzy medial -ideal of X if it satisfies the following condition: for all $x, y, z \in X$
(b_1) $\mu^N(0) \le \mu^P(x)$ and $\mu^N(0) \ge \mu^P(x)$,

(B$_2$) $\mu^N(x) \le S\{\mu^N(z*(y*x)), \mu^N(y*z)\}$, $\mu^P(x) \ge T\{\mu^P(z*(y*x)), \mu^P(y*z)\}$.

Example 3.7 Let X = {0, 1, 2, 3} be a set with a binary operation $*$ define by the following table:

*	0	1	2	3
0	0	0	0	0
1	1	0	2	0
2	2	0	0	0
3	3	2	1	0

	0	1	2	3
μ^N	**-0.7**	**-0.7**	**- 0.6**	**- 0.4**
μ^P	**0.6**	**0.5**	**0.3**	**0.3**

Let $T:[0,1]\times[0,1]\to[0,1]$ be a function defined by T (α, β) = max {α + β − 1, 0} for all α, β ∈ [0, 1] and $S:[-1,0]\times[-1,0]\to[-1,0]$ be a function defined by $S(\tilde{\alpha},\tilde{\beta}) = \min\{1+(\tilde{\alpha}+\tilde{\beta}),0\}$, $\forall\ \tilde{\alpha},\tilde{\beta}\in[-1,0]$ .By routine calculations, we know that $\mathrm{B}=(x,\mu^N,\mu^P)$ is bipolar (T,S)- fuzzy medial-ideal of X .

Proposition 3.8 Every a bipolar fuzzy(T,S)- medial-ideal of X is a bipolar fuzzy(T,S)-BCI-ideal of X .

Proof: clear.

Proposition 3.9 If $\mathrm{B}=(x,\mu^N,\mu^P)$ is a bipolar (T,S)- fuzzy medial-ideal of X and $x\le y$, then $\mu^N(x)\le\mu^N(y)$ and $\mu^P(x)\ge\mu^P(y)$.

Proof: If $x\le y$, then $x*y=0$, since $\mathrm{B}=(x,\mu^N,\mu^P)$ is a bipolar fuzzy(T,S)- medial-ideal of X , we get

$$\mu^N(x)\le S\{\mu^N(0*(y*x)),\mu^N(y)\} \overset{by\ \ Lemma 2.5}{=} S\{\mu^N(x*y),\mu^N(y)\}$$
$$= S\{\mu^N(0),\mu^N(y)\} = \mu^N(y).$$

And

$$\mu^P(x)\ge T\{\mu^P(0*(y*x)),\mu^P(y)\} \overset{by\ \ Lemma 2.5}{=} T\{\mu^P(x*y),\mu^P(y)\}$$
$$= T\{\mu^P(0),\mu^P(y)\} = \mu^N(y).$$

Theorem 3.10 Every bipolar fuzzy (T,S)- medial-ideal of X is a bipolar (T,S)- fuzzy BCI-sub-algebra of X .

Proof . Let $B=(x,\mu^N,\mu^P)$ be bipolar (T,S)- fuzzy medial-ideal of X . Since $x*y\le x$, for all $x,y\in X$, then $\mu^N(x*y)\le\mu^N(x)$, $\mu^P(x*y)\ge\mu^P(x)$ .Put z = 0 in (b_1), (B_2), we have

$\mu^N(0)\le\mu^P(x)$ and $\mu^N(0)\ge\mu^P(x)$,

$$\mu^N(x*y)\le\mu^N(x)\le S\{\mu^N(0*(y*x),\mu^N(y*0)\}=S\{\mu^N(0*(y*x),\mu^N(y*0)\}$$
$$S\{\mu^N(x*y),\mu^N(y)\}\le S\{\mu^N(x),\mu^N(y)\}\ ,\ \textit{and}$$

$$\mu^P(x*y)\ge\mu^P(x)\ge T\{\mu^P(0*(y*x),\mu^P(y*0)\}=T\{\mu^P(0*(y*x),\mu^P(y*0)\}$$
$$T\{\mu^P(x*y),\mu^P(y)\}\ge T\{\mu^P(x),\mu^P(y)\}.$$

Then $B=(x,\mu^N,\mu^P)$ is bipolar (T,S)- fuzzy sub-algebra of *X*.

Proposition 3.11 Let $B=(x,\mu^N,\mu^P)$ be a bipolar (T,S)- fuzzy medial-ideal of X . If the inequality $x*y\le z$ holds in X, then

$\mu^N(x)\le S\{\mu^N(y),\mu^N(z)\}$ and $\mu^P(x)\ge T\{\mu^P(y),\mu^P(z)\}$ for all $x,y,z\in X$.

Proof . Let $x,y,z\in X$ be such that $x*y\le z$. Thus, put $z=0$ in (Defintion3.6), (using lemma2.5 and lemma 3.9), we get,

$$\mu^N(x)\le S\{\mu^N(0*(y*x),\mu^N(y*0)\}=S\{\mu^N(x*y),\mu^N(y)\}\le\overbrace{S\{\mu^N(z),\mu^N(y)\}}^{\text{since }\mu^N(x*y)\le\mu^N(z)}.$$

Similarly we have,

$$\mu^P(x)\ge T\{\mu^P(0*(y*x),\mu^P(y*0)\}=T\{\mu^P(x*y),\mu^P(y)\}\ge\overbrace{T\{\mu^P(z),\mu^P(y)\}}^{\text{since }\mu^P(x*y)\ge\mu^P(z)}.$$

Theorem 3.12 Let $B=(x,\mu^N,\mu^P)$ be a bipolar (T,S)- fuzzy sub-algebra of X .such that $\mu^P(x)\ge T\{\mu^P(y),\mu^P(z)\}$, $\mu^N(x)\le S\{\mu^N(y),\mu^N(z)\}$, satisfying the inequality $x*y\le z$ for all $x,y,z\in X$. Then $B=(x,\mu^N,\mu^P)$ is a bipolar (T,S)- fuzzy medial ideal of *X*.

Proof. Let $B=(x,\mu^N,\mu^P)$ be a bipolar fuzzy sub-algebra of *X*. Recall that $\mu^N(0)\le\mu^P(x)$ and $\mu^N(0)\ge\mu^P(x)$, for all $x\in X$. Since, for all $x,y,z\in X$,we have $x*(z*(y*x))=(y*x)*(z*x)\le y*z$, it follows from the hypothesis that $\mu^N(x)\le S\{\mu^N(z*(y*x),\mu^N(y*z)\}$, $\mu^P(x)\ge T\{\mu^P(z*(y*x),\mu^P(y*z)\}$.

Hence $B=(x,\mu^N,\mu^P)$ is a bipolar (T,S)- fuzzy medial ideal of *X*.

Definition 3.13 [9] Let $B=(x,\mu^N,\mu^P)$ be a bipolar fuzzy set and $(s,t)\in[-1,0]\times[0,1]$.
The set $B_s^N=\{x\in X:\mu^N(x)\le s\}$ and $B_t^P=\{x\in X:\mu^P(x)\ge t\}$ which are called the negative s-cut and the positive t-cut of $B=(x,\mu^N,\mu^P)$, respectively.

Example 3.14 Let $X = \{0,1,2,3\}$ be a set with a binary operation $*$ define by the following table:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

We can prove that $(X,*,0)$ is a BCI-algebra. We Define

X	0	1	2	3
$\mu^N(x)$	**-0.7**	**-0.6**	**-0.3**	**-0.4**
$\mu^P(x)$	**0.7**	**0.5**	**0.3**	**0.2**

Then

$\mathrm{B}_{-0.3}^N = \{x \in X : \mu^N(x) \le -03\} = \{0,1,2\}$, $\mathrm{B}_{0.5}^P = \{x \in X : \mu^P(x) \ge 0.5\} = \{0,1\}$

Theorem 3.14 An BFS $\mathrm{B} = (x, \mu^N, \mu^P)$ is a bipolar (T,S)- fuzzy medial ideal of *X* if and only if for all $s \in [-1,0], t \in [0,1]$, the set $B_s^{\ N}$ and $B_t^{\ P}$ are either empty or medial ideals of *X*.

Proof. Let $\mathrm{B} = (x, \mu^N, \mu^P)$ be bipolar (T,S)- fuzzy medial ideal of *X* and $B_t^{\ P} \ne \Phi \ne B_s^{\ N}$. Since $\mu^P(0) \ge t$ and $\mu^N(0) \le s$, let $x, y, z \in X$ be such that $z*(y*x) \in B_t^{\ P}$ and $y*z \in B_t^{\ P}$, then $\mu^P(z*(y*x)) \ge t$ and $\mu^P(y*z) \ge t$, it follows that $\mu^P(x) \ge T\{\mu^P(x*(y*z)), \mu^P(y*z)\} \ge t$, we get $x \in B_t^{\ P}$. Hence $B_t^{\ P}$ is a medial ideal of *X*.

Now let $x, y, z \in X$ be such that $z*(y*x) \in B_s^{\ N}$ and $y*z \in B_s^{\ N}$, then $\mu^N(z*(y*x)) \le s$ and $\mu^N(y*z) \le s$ which imply that $\mu^N(x) \le S\{\mu^N(z*(y*x)), \mu^N(y*z)\} \le s$. Thus $x \in B_s^{\ N}$ and therefore $B_s^{\ N}$ is a medial ideal of *X*.

Conversely, assume that for each $s \in [-1,0], t \in [0,1]$, the sets $B_t^{\ P}$ and $B_s^{\ N}$ are either empty or medial ideal of *X*. For any $x \in X$, let $\mu^P(x) = t$ and $\mu^N(x) = s$. Then $x \in B_t^{\ P} \cap B_s^{\ N}$ and so $B_t^{\ P} \ne \Phi \ne B_s^{\ N}$. Since $B_t^{\ P}$ and $B_s^{\ N}$ are medial ideals of *X*, therefore $0 \in B_t^{\ P} \cap B_s^{\ N}$. Hence $\mu^P(0) \ge t = \mu^P(x)$ and $\mu^N(0) \le s = \mu^N(x)$ for all $x \in X$.

If there exist $x', y', z' \in X$ be such that $\mu^P(x') < T\{\mu^P(z' * (y' * x')), \mu^P(y' * z')\}$. Then by taking $t_0 := \frac{1}{2}\{\mu^P(x') + T\{\mu^P(z' * (y' * x'), \mu^P(y' * z')\}\}$, we get

$$\mu^P(x') < t_0 < T\{\mu^P(z' * (y' * x')), \mu^P(y' * z')\}$$

and hence $x' \notin B_{t_0}{}^P$, $z' * (y' * x') \in B_{t_0}{}^P$ and $y' * z' \in B_{t_0}{}^P$, i.e. $B_{t_0}{}^P$ is not a medial ideal of *X*, which make a contradiction. Finally assume that there exist $a, b, c \in X$ such that $\mu^N(a) > S\{\mu^N(c * (b * a)), \mu^N(b * c)\}$.

Then by taking $s_0 := \frac{1}{2}\{\mu^N(a) + S\{\mu^N(c * (b * a), \mu^N(b * c)\}\}$, we get

$$S\{\mu^N(c * (b * a)), \mu^N(b * c)\} < s_0 < \mu^N(a)$$

Therefore, $(c * (b * a)) \in B_{s_0}{}^N$ and $b * c \in B_{s_0}{}^N$, but $a \notin B_{s_0}{}^N$, which make a contradiction. This completes the proof.

4. Image (Pre-image) Bipolar (T,S)-fuzzy Medial Ideal

Definition 4.1 Let $(X, *, 0)$ and $(Y, *', 0')$ be BCI-algebras. A mapping $f : X \to Y$ is said to be a homomorphism if $f(x * y) = f(x) *' f(y)$ for all $x, y \in X$. Note that if $f : X \to Y$ is a homomorphism of BCI-algebras, then $f(0) = 0'$.

Let $f : X \to Y$ be a homomorphism of BCI-algebras for any bipolar fuzzy set $\mathrm{B} = (x, \mu^N, \mu^P)$ in *Y*, we define new bipolar fuzzy set $B_f = (\mu_f^N, \mu_f^P)$ in *X* by $\mu_f^N(x) := \mu^N(f(x))$, and $\mu_f^P(x) := \mu^P(f(x))$ for all $x \in X$.

Theorem 4.2 Let $f : X \to Y$ be a homomorphism of BCI-algebras. If $\mathrm{B} = (x, \mu^N, \mu^P)$ is bipolar (T,S)- fuzzy medial ideal of *Y*, then $B_f = (\mu_f^N, \mu_f^P)$ is bipolar (T,S)- fuzzy medial ideal of *X*.

Proof. $\mu_f^N(x) := \mu^N(f(x)) \geq \mu^N(0) = \mu^N(f(0)) = \mu_f^N(0)$, and

$\mu_f^P(x) := \mu^P(f(x)) \leq \mu^P(0) = \mu^P(f(0)) = \mu_f^P(0)$, for all $x, y \in X$.

And

$$\begin{aligned}
\mu_f^P(x) &:= \mu^P(f(x)) \geq T\{\mu^P(f(z) * (f(y) * f(x))), \mu^P(f(y) * f(z))\} \\
&= T\{\mu^P(f(z) * f(y * x)), \mu^P(f(y * z))\} = T\{\mu^P(f(z * (y * x)), \mu^P(f(y * z))\} \\
&= T\{\mu_f^P(z * (y * x)), \mu_f^P(y * z)\},
\end{aligned}$$

and

$$\mu_f^N(x) := \mu^N(f(x)) \le S\{\mu^N(f(z)*(f(y)*f(x)), \mu^N(f(y*z))\}$$
$$= S\{\mu^N(f(z)*f(y*x)), \mu^N(f(y*z))\} = S\{\mu^N(f(z*(y*x)), \mu^N(f(y*z))\}$$
$$= S\{\mu_f^N(z*(y*x)), \mu_f^N(y*z)\}.$$

Hence $B_f = (\mu_f^N, \mu_f^P)$ is bipolar (T,S)- fuzzy medial ideal of *X*.

Theorem 4.3 Let $f: X \to Y$ be an epimorphism of BCI-algebras .If $B_f = (\mu_f^N, \mu_f^P)$ is bipolar (T,S)- fuzzy medial ideal of *X*, then $\mathrm{B} = (x, \mu^N, \mu^P)$ is bipolar (T,S)- fuzzy medial ideal of *Y*.

Proof. For any $a \in Y$, there exists $x \in X$ such that $f(x) = a$. Then

$$\mu^P(a) = \mu^P(f(x)) = \mu_f{}^P(x) \le \mu_A^f(0) = \mu^P(f(0)) = \mu^P(0),$$
$$\mu^N(a) = \mu^N(f(x)) = \mu_f{}^N(x) \ge \mu_f{}^N(0) = \mu^N(f(0)) = \mu^N(0).$$

Let $a, b, c \in Y$. Then $f(x) = a, f(y) = b, f(z) = c$, for some $x, y, z \in X$. It follows that

$$\mu^P(a) = \mu^P(f(x)) = \mu_f^P(x) \ge T\{\mu_f^P(z*(y*x)), \mu_f^P(y*z)\}$$
$$= T\{\mu^P(f(z*(y*x)), \mu^P(f(y*z))\}$$
$$= T\{\mu^P(f(z)*f(y*x)), \mu^P(f(y)*f(z))\} = T\{\mu^P(f(z)*(f(y)*f(x))), \mu^P(f(y)*f(z))\}$$
$$= T\{\mu^P(c*(b*a)), \mu^P(b*c)\},$$

$$\mu^N(a) = \mu^N(f(x)) = \mu_f^N(x) \le S\{\mu_f^N(z*(y*x)), \mu_f^N(y*z)\}$$
$$= S\{\mu^N(f(z*(y*x)), \mu^N(f(y*z))\}$$
$$= S\{\mu^N(f(z)*f(y*x)), \mu^N(f(y)*f(z))\} = S\{\mu^N(f(z)*(f(y)*f(x))), \mu^N(f(y)*f(z))\}$$
$$= S\{\mu^N(c*(b*a)), \mu^N(b*c)\}.$$

This completes the proof.

5. Product of Bipolar(T,S)-fuzzy Medial Ideals

Definition 5.1 Let μ and λ be two fuzzy sets in the set *X*. the product $\lambda \times \mu : X \times X \to [0,1]$ is defined by $(\lambda \times \mu)(x, y) = \min\{\lambda(x), \mu(y)\}$, for all $x, y \in X$.

Definition 5.2 Let $A = (X, \mu_A^N, \mu_A^P)$ and $B = (X, \lambda_B^N, \lambda_B^P)$ are two bipolar (T,S)- fuzzy set of *X*, the Cartesian product $A \times B = (X \times X, \mu_A^N \times \lambda_B^N, \mu_A^P \times \lambda_B^P)$ is defined by $\mu_A^P \times \lambda_B^P(x, y) = T\{\mu_A^P(x), \lambda_B^P(y)\}$ and

$\mu_A^N \times \lambda_B^N(x,y) = S\{\mu_A^N(x), \lambda_B^N(y)\}$, where $\mu_A^P \times \lambda_B^P : X \times X \to [0,1]$, $\mu_A^N \times \lambda_B^N : X \times X \to [-1,0]$ for all $x, y \in X$.

Remark 5.3 Let X and Y be medial BCI-algebras, we define* on $X \times Y$ by:
For every $(x,y),(u,v) \in X \times Y$, $(x,y)*(u,v) = (x*u, y*v)$. Clearly $(X \times Y; *, (0,0))$ is BCI-algebra.

Proposition 5.4 Let $A = (X, \mu_A^N, \mu_A^P)$ and $B = (X, \lambda_B^N, \lambda_B^P)$ are two bipolar (T,S)- fuzzy medial ideal of X, then $A \times B$ is bipolar (T,S)- fuzzy medial ideal of $X \times X$.

Proof. $\mu_A^P \times \lambda_B^P(0,0) = T\{\mu_A^P(0), \lambda_B^P(0)\} \geq T\{\mu_A^P(x), \lambda_B^P(y)\}\} = \mu_A^P \times \lambda_B^P(x,y)$, for all $x, y \in X$. And $\mu_A^N \times \lambda_B^N(0,0) = S\{\mu_A^N(0), \lambda_B^N(0)\} \leq S\{\mu_A^N(x), \lambda_B^N(y)\}\} = \mu_A^N \times \lambda_B^N(x,y)$, for all $x, y \in X$.

Now let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, then

$$
\begin{aligned}
& T\{(\mu_A^P \times \lambda_B^P)((z_1,z_2)*((y_1,y_2)*(x_1,x_2))), (\mu_A^P \times \lambda_B^P)((y_1,y_2)*(x_1,x_2))\} \\
&= T\{(\mu_A^P \times \lambda_B^P)((z_1,z_2)*(y_1*x_1, y_2*x_2)), (\mu_A^P \times \lambda_B^P)(y_1*x_1, y_2*x_2)\} \\
&= T\{(\mu_A^P \times \lambda_B^P)(z_1*(y_1*x_1), z_2*(y_2*x_2)), (\mu_A^P \times \lambda_B^P)(y_1*x_1, y_2*x_2)\} \\
&= T\{T\{\mu_A^P(z_1*(y_1*x_1)), \lambda_B^P(z_2*(y_2*x_2))\}, T\{\mu_A^P(y_1*x_1), \lambda_B^P(y_2*x_2)\}\} \\
&= T\{T\{\mu_A^P(z_1*(y_1*x_1)), \mu_A^P(y_1*x_1)\}, T\{\lambda_B^P(z_2*(y_2*x_2)), \lambda_B^P(y_2*x_2)\} \\
&= T\{\ T\ \{\ \mu_A^P(z_1*(y_1*x_1)), \mu_A^P(y_1*x_1)\}, T\ \ \{\ \lambda_B^P(z_2*(y_2*x_2)), \lambda_B^P(y_2*x_2)\} \\
&\leq T\{\mu_A^P(x_1), \lambda_B^P(x_2) = (\mu_A^P \times \lambda_B^P)(x_1,x_2) .
\end{aligned}
$$

and

$$
\begin{aligned}
& S\{(\mu_A^N \times \lambda_B^N)((z_1,z_2)*((y_1,y_2)*(x_1,x_2))), (\mu_A^N \times \lambda_B^N)((y_1,y_2)*(x_1,x_2))\} \\
&= S\{(\mu_A^N \times \lambda_B^N)((z_1,z_2)*(y_1*x_1, y_2*x_2)), (\mu_A^N \times \lambda_B^N)(y_1*x_1, y_2*x_2)\} \\
&= S\{(\mu_A^N \times \lambda_B^N)(z_1*(y_1*x_1), z_2*(y_2*x_2)), (\mu_A^N \times \lambda_B^N)(y_1*x_1, y_2*x_2)\} \\
&= S\{\max\{\mu_A^N(z_1*(y_1*x_1)), \lambda_B^N(z_2*(y_2*x_2))\}, S\{\mu_A^N(y_1*x_1), \lambda_B^N(y_2*x_2)\}\} \\
&= S\{S\{\mu_A^N(z_1*(y_1*x_1)), \mu_A^N(y_1*x_1)\}, S\{\lambda_B^N(z_2*(y_2*x_2)), \lambda_B^N(y_2*x_2)\} \\
&= S\ \{\ S\ \{\ \ \mu_A^N(z_1*(y_1*x_1)), \mu_A^N(y_1*x_1)\}, S\ \{\ \ \lambda_B^N(z_2*(y_2*x_2)), \lambda_B^N(y_2*x_2)\} \\
&\geq S\{\mu_A^N(x_1), \lambda_B^N(x_2)\} = \ (\mu_A^N \times \lambda_B^N)(x_1,x_2)
\end{aligned}
$$

.This completes the proof.

Example 5.5 Let X = {0, 1, 2, 3} be a set with a binary operation $*$ as example 3.14

Then $X \times X = \begin{Bmatrix} (0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3), \\ (2,0),(2,1),(2,2),(2,3),(3,0),(3,1),(3,2),(3,3) \end{Bmatrix}$

we define operation * on $X \times X$ by:for every

$(x,y),(u,v) \in X \times X$, $(x,y)*(u,v) = (x*u, y*v)$.

By routine calculations $(X \times X;*,(0,0))$ is BCI-algebra.

Let $T_m:[0,1]\times[0,1]\to[0,1]$ be a functions ,defined by $T_m(\alpha, \beta) = \max\{\alpha + \beta - 1, 0\}$ and $S_m:[-1,0]\times[-1,0]\to[-1,0]$ defined by $S_m(\alpha,\beta)=\min\{(\tilde{\alpha}+\tilde{\beta})-1,0\}$ for all $\tilde{\alpha},\tilde{\beta}\in[-1,0]$ Let $A\times B=(X\times X,\mu^P{}_A\times\lambda^P{}_B,\ \mu^N{}_A\times\lambda^N{}_{\mathbf{B}})$ bipolar fuzzy medial ideals of X *define* by

$$(\mu_A{}^P\times\lambda_B{}^P)(x,y)=\begin{cases}0.6\,if & (x\,,\ y)\in\{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1),(3,0),(3,1)\}\\ 0.3 & if\quad (x\,,\ y)\in\{,(2,2),(2,3),,(3,2),(3,3)\}\end{cases}$$

$$(\mu_A{}^N\times\lambda_B{}^N)(x,y)=\begin{cases}-0.7\quad if & (x\,,\ y)\in\{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1),(3,0),(3,1)\}\\ -0.3 & if\quad (x\,,\ y)\in\{,(2,2),(2,3),,(3,2),(3,3)\}\end{cases}$$

By routine calculations, we can prove that $A\times B=(X\times X,\mu^P{}_A\times\lambda^P{}_B,\ \mu^N{}_A\times\lambda^N{}_{\mathbf{B}})$, is bipolar (T_m,S_m)-fuzzy medial ideals of $X\times X$.

Definition 5.6 Let $A=(X,\mu_A^N,\mu_A^P)$ and $B=(X,\lambda_B^N,\lambda_B^P)$ are two bipolar fuzzy medial ideals of BCI-algebra *X*. for $s,t\in[0,1]$ the set

$$U(\mu_A^P\times\lambda_B^P,t):=\{(x,y)\in X\times X\mid(\mu_A\times\mu_B)(x,y)\geq t\}$$

is called upper t-level of $(\mu_A^P\times\lambda_B^P)(x,y)$ and the set

$$L(\mu_A^N\times\lambda_B^N,s):=\{(x,y)\in X\times X\mid(\lambda_A\times\lambda_B)(x,y)\leq s\}$$

is called lower s-level of $(\mu_A^P\times\lambda_B^P)(x,y)$.

Theorem 5.7 An bipolar fuzzy set $A=(X,\mu_A^N,\mu_A^P)$ and $B=(X,\lambda_B^N,\lambda_B^P)$ are bipolar(T,S)-fuzzy medial ideals of BCI-algebra *X* if and only if the non-empty set upper *t*-level cut $U(\mu_A^P\times\lambda_B^P,t)$ and the non-empty lower *s*-level cut $L(\mu_A^N\times\lambda_B^N,s)$ are medial ideals of $X\times X$ for any $s,t\in[0,1]$.

Proof. Let $A=(X,\mu_A^N,\mu_A^P)$ and $B=(X,\lambda_B^N,\lambda_B^P)$ are two bipolar (T,S)- fuzzy medial ideals of BCI-algebra *X*, therefore for any $(x,y)\in X\times X$,

$(\mu_A^P\times\lambda_B^P)(0,0)=T\{\mu_A^P(0),\lambda_B^P(0)\}\geq T\{\mu_A^P(x),\lambda_B^P(y)\}=(\mu_A^P\times\lambda_B^P)(x,y)$ and for $t\in[0,1]$, if $(\mu_A^P\times\lambda_B^P)(x_1,x_2)\geq t$, therefore $(x_1,x_2)\in U(\mu_A^P\times\lambda_B^P,t)$.

Let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$ be such that

$((z_1,z_2)*((y_1,y_2)*(x_1,x_2)))\in U(\mu_A^P\times\lambda_B^P,t)$, and $(y_1,y_2)*(z_1,z_2)\in U(\mu_A^P\times\lambda_B^P,t)$.

Now

$(\mu_A^P \times \lambda_B^P)(x_1, x_2) \geq$

$$T\{(\mu_A^P \times \lambda_B^P)((z_1, z_2) * ((y_1, y_2) * (x_1, x_2))), (\mu_A^P \times \lambda_B^P)((y_1, y_2) * (z_1, z_2))\}$$
$$= T\{(\mu_A^P \times \lambda_B^P)((z_1, z_2) * (y_1 * x_1, y_2 * x_2)), (\mu_A^P \times \lambda_B^P)(y_1 * z_1, y_2 * z_2)\}$$
$$= T\{(\mu_A^P \times \lambda_B^P)(z_1 * (y_1 * x_1), z_2 * (y_2 * x_2)), (\mu_A^P \times \lambda_B^P)(y_1 * z_1, y_2 * z_2)\}$$

$\geq T\{t, t\} = t$, Therefore $(x_1, x_2) \in U((\mu_A^P \times \lambda_B^P)(x, y), t)$ is a medial ideal of $X \times X$. Similar to above $L((\mu_A^N \times \lambda_B^N)(x, y), s)$ is a medial ideal of $X \times X$. This completes the proof .

6. Conclusion

we have studied the bipolar (T,S)- fuzzy of medial-ideal in BCI-algebras. Also we discussed few results of bipolar fuzzy of medial-ideal in BCI-algebras under homomorphism, the image and the pre- image of bipolar (T,S)- fuzzy of medial-ideal under homomorphism of BCI-algebras are defined. How the image and the pre-image of bipolar (T,S)- fuzzy of medial-ideal under homomorphism of BCI-algebras become bipolar fuzzy of medial-ideal are studied. Moreover, the product of bipolar (T,S)- fuzzy of medial-ideal to product bipolar (T,S)- fuzzy of medial-ideal is established. Furthermore. The main purpose of our future work is to investigate the foldedness of other types of fuzzy ideals with special properties such as a intuitionistic bipolar (interval value) fuzzy n-fold of medial-ideal in BCI-algebras.

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Conflicts of Interest

State any potential conflicts of interest here or “The authors declare no conflict of interest”.

Algorithm for BC I-algebras

Input (X :set, $*$:binary operation)

Output (“ X is a BCI -algebra or not”)

Begin

If $X = \phi$ then go to (1.);

End If

If $0 \notin X$ then go to (1.);

End If

Stop: =false;

$i := 1$;

While $i \leq |X|$ and not (Stop) do

If $x_i * x_i \neq 0$ then

Stop: = true;

End If

$j := 1$

While $j \leq |X|$ and not (Stop) do

If $(x_i * (x_i * y_j)) * y_j \neq 0$, then

Stop: = true;

End If

End If

$k := 1$

While $k \leq |X|$ and not (Stop) do

If $((x_i * y_j) * (x_i * z_k)) * (z_k * y_i) \neq 0$, then

Stop: = true;

End If

End While

End While

End While

If Stop then

(1.) Output (" X is not a BCI-algebra")

Else

Output (" X is a BCI -algebra")

End If.

Algorithm for fuzzy subsets

Input (X : BCI-algebra, $\mu : X \to [0,1]$);

Output (" A is a fuzzy subset of X or not")

Begin

Stop: =false;

$i := 1$;

While $i \leq |X|$ and not (Stop) do

If ($\mu(x_i) < 0$) or ($\mu(x_i) > 1$) then

Stop: = true;

End If

End If While

If Stop then

Output (" μ is a fuzzy subset of X ")

Else

Output (" μ is not a fuzzy subset of X ")

End If

End.

Algorithm for medial -ideals

Input (X : BCI-algebra, I : subset of X);

Output (" I is an medial -ideals of X or not");

Begin

If $I = \phi$ then go to (1.);

End If

If $0 \notin I$ then go to (1.);

End If

Stop: =false;

$i := 1$;

While $i \leq |X|$ and not (Stop) do

$j := 1$

While $j \leq |X|$ and not (Stop) do

$k := 1$

While $k \leq |X|$ and not (Stop) do

If $z_k * (y_j * x_i) \in I$ and $y_j * z_k \in I$ then

If $x_i \notin I$ then

Stop: = true;

End If

End If

End While

End While

End While

If Stop then

Output (“ I is is an medial -ideals of X ”)

Else

(1.) Output (“ I is not is an medial -ideals of X ”)

End If

End .

Algorithm for Bipolar medial ideal of X

Input (X :BCI-algebra, $*$:binary operation, μ^N *and* μ^P fuzzy subsets of X);

Output (“$\mathrm{B} = (x, \mu^N, \mu^P)$ is bipolar fuzzy medial ideal of X or not”)

Begin

Stop: =false;

$i := 1$;

While $i \leq |X|$ and not (Stop) do

If $\mu^N(0) > \mu^P(x)$ and $\mu^N(0) < \mu^P(x)$ then

Stop: = true;

End If

$j := 1$

While $j \le |X|$ and not (Stop) do

$k := 1$

While $k \le |X|$ and not (Stop) do

If $\mu^N(x) > \max\{\mu^N(x*y), \mu^N(y)\}$, $\mu^P(x) < \min\{\mu^P(x*y), \mu^P(x)\}$ then

Stop: = true;

End If

End While

End While

End While

If Stop then

Output (“ $B = (x, \mu^N, \mu^P)$ is not bipolar fuzzy medial ideal of X ”)

Else

Output(“ $B = (x, \mu^N, \mu^P)$ is bipolar fuzzy medial ideal of X ”)

End If.

End.

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INTUITIONISTIC FUZZY STRONGLY α-GENERALIZED SEMI CLOSED SETS

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Abstaract − In this paper, intuitionistic fuzzy strongly α-generalized semi closed sets and intuitionistic fuzzy strongly α-generalized open sets are introduced. Some of their properties are discussed with existing intuitionistic fuzzy generalized closed and intuitionistic fuzzy generalized open sets.

Keywords − *Fuzzy sets, Intuitionistic fuzzy sets, Intuitionistic fuzzy topological spaces, Intuitionistic fuzzy α-generalized semi closed sets.*

1 Introduction

Zadeh [12] introduced the concept of fuzzy sets and later Atanassov [1] generalized this idea to the new class of intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Levine(1970) introduced the notion of generalized closed (briefly g-closed) sets in topological spaces. The aim of this paper is to introduce and study stronger form of alpha generalized semi closed sets in intuitionistic fuzzy topological spaces for which we introduce the concepts of intuitionistic fuzzy strongly α-generalized semi closed sets and intuitionistic fuzzy strongly α-generalized semi open sets. Moreover, We study their properties.

2 Preliminary

Definition 2.1. [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

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$$A = \{\langle\ x,\ \mu_A(x),\ \nu_A(x)\ \rangle \mid x \in X\}$$

where the function $\mu_A(x)$:X $\rightarrow$ [0,1] denotes the degree of membership(namely $\mu_A(x)$) and the function $\nu_A(x)$:X $\rightarrow$ [0,1] denotes the degree of non-membership(namely $\nu_A(x)$) of each element x $\in$ X to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x $\in$ X.

IFS(X) denotes the set of all intuitionistic fuzzy sets in X.

Definition 2.2. [1] Let A and B be IFSs of the form
A=$\{\langle\ x,\ \mu_A(x),\ \nu_A(x)\ \rangle \mid x \in X\}$ and B=$\{\langle\ x,\ \mu_B(x),\ \nu_B(x)\ \rangle \mid x \in X\}$. Then

1. A $\subseteq$ B if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all x $\in$ X,
2. A = B if and only if A $\subseteq$ B and B $\subseteq$ A,
3. $A^c = \{\langle\ x,\ \nu_A(x),\ \mu_A(x)\ \rangle \mid x \in X\}$,
4. $A \cap B = \{\langle\ x,\ \mu_A(x) \wedge \mu_B(x),\ \nu_A(x) \vee \nu_B(x)\ \rangle \mid x \in X\}$,
5. $A \cup B = \{\langle\ x,\ \mu_A(x) \vee \mu_B(x),\ \nu_A(x) \wedge \nu_B(x)\ \rangle \mid x \in X\}$.

For sake of simplicity, we shall use the notation A=$\langle\ x,\ \mu_A,\ \nu_A\ \rangle$ instead of
$A = \{\langle\ x,\ \mu_A(x),\ \nu_A(x)\ \rangle \mid x \in X\}$.

Definition 2.3. [1] The intuitionistic fuzzy sets $0_\sim = \{\langle\ x,\ 0,\ 1\ \rangle : x \in X\}$ and $1_\sim = \{\langle\ x,\ 1,\ 0\ \rangle : x \in X\}$ are the empty set and the whole set of X respectively.

Definition 2.4. [2] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

1. $0_\sim,\ 1_\sim \in \tau$,
2. $G_1 \cap G_2 \in \tau$ for any $G_1,\ G_2 \in \tau$,
3. $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.
The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.5. [2] Let (X, τ) be an IFTS and A = $\langle\ x,\ \mu_A,\ \nu_A\ \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:

1. int(A) = $\cup\{G \mid G$ is an IFOS in X and $G \subseteq A\}$,
2. cl(A) = $\cap\{K \mid K$ is an IFCS in X and $A \subseteq K\}$.

Proposition 2.6. [2] For any IFSs A and B in (X, τ), we have

1. int(A) $\subseteq$ A,
2. A $\subseteq$ cl(A),

3. A $\subseteq$ B $\Rightarrow$ int(A) $\subseteq$ int(B) and cl(A) $\subseteq$ cl(B),

4. int(int(A)) = int(A),

5. cl(cl(A)) = cl(A),

6. cl(A$\cup$B) = cl(A)$\cup$cl(B),

7. int(A$\cap$B) = int(A)$\cap$int(B).

Proposition 2.7. [2] For any IFS A in (X, τ), we have

1. int($0_\sim$)= $0_\sim$ and cl($0_\sim$)= $0_\sim$,

2. int($1_\sim$)= $1_\sim$ and cl($1_\sim$)= $1_\sim$,

3. $(\text{int(A)})^c$ = cl(A^c),

4. $(\text{cl(A)})^c$ = int(A^c).

Proposition 2.8. [3] If A is an IFCS in X then cl(A) = A and if A is an IFOS in X then int(A) = A. The arbitrary union of IFCSs is an IFCS in X.

Definition 2.9. An IFS A = $\langle$ x, μ_A, ν_A $\rangle$ in an IFTS (X, τ) is said to be an

1. intuitionistic fuzzy regular closed set (IFRCS in short) if A = cl(int(A)) [3],

2. intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A))) $\subseteq$ A [5],

3. intuitionistic fuzzy semi closed set (IFSCS in short) if int(cl(A)) $\subseteq$ A [3],

4. intuitionistic fuzzy pre closed set (IFPCS in short) if cl(int(A)) $\subseteq$ A [3],

5. intuitionistic fuzzy γ-closed set (IFγCS in short) if cl(int(A))$\cap$int(cl(A)) $\subseteq$ A [4],

6. intuitionistic fuzzy semipreclosed set (IFSPCS in short) if there exists an IF-PCS B such that int(B) $\subseteq$ A $\subseteq$ B [7].

Definition 2.10. An IFS A = $\langle$ x, μ_A, ν_A $\rangle$ in an IFTS (X, τ) is said to be an

1. intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)) [3],

2. intuitionistic fuzzy α-open set (IFαOS in short) if A $\subseteq$ int(cl(int(A))) [5],

3. intuitionistic fuzzy semiopen set (IFSOS in short)if A $\subseteq$ cl(int(A)) [3],

4. intuitionistic fuzzy preopen set (IFPOS in short) if A $\subseteq$ int(cl(A)) [3],

5. intuitionistic fuzzy γ-open set (IFγOS in short) if A $\subseteq$ int(cl(A))$\cup$cl(int(A)) [4],

6. intuitionistic fuzzy semipreopen set (IFSPOS in short) if there exists an IFPOS B such that B $\subseteq$ A $\subseteq$ cl(B)[7].

Definition 2.11. Let A be an IFS of an IFTS (X, τ). Then

1. αcl(A) = $\cap$\{K | K is an IFαCS in X and A $\subseteq$ K\}[8],

2. αint(A) = ∪{K | K is an IFαOS in X and K ⊆ A}[8],
3. sint(A) = ∪{K | K is an IFSOS in X and K ⊆ A}[11],
4. scl(A) = ∩{K | K is an IFSCS in X and A ⊆ K}[11].

Result 2.12. Every IFαCS in (X, τ) is an IFαGSCS in (X, τ) but not conversely [6].

Definition 2.13. An IFS A of an IFTS (X, τ) is an

1. intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X [10],
2. intuitionistic fuzzy generalized semiclosed set (IFGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X [9].

The complements of the above mentioned intuitionistic fuzzy generalized closed sets are called their respective intuitionistic fuzzy generalized open sets.

Definition 2.14. [6] An IFS A in (X, τ) is said to be an intuitionistic fuzzy α-generalized semi-closed set (IFαGSCS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ). The complement A^c of an IFαGSCS A in an IFTS (X, τ) is called an intuitionistic fuzzy α generalized semi open set (IFαGSOS in short) in X.

Remark 2.15. [8] Let A be an IFS in (X, τ). Then

1. αcl(A) = A∪cl(int(cl(A))),
2. αint(A) = A∩int(cl(int(A))).

Definition 2.16. [10] Two IFSs are said to be q-coincident (A q B in short) if and only if there exists an element x ∈ X such that μ_A(x) $> \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$. For any two IFSs A and B of X, A $\bar{q}$ B if and only if A ⊆ B^c.

3 Intuitionistic Fuzzy Strongly α-generalized Semiclosed Sets

In this section we introduce intuitionistic fuzzy strongly α-generalized semi-closed sets and study some of its properties.

Definition 3.1. An IFS A in (X, τ) is said to be an intuitionistic fuzzy strongly α-generalized semi-closed set (IFsαGSCS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFGSOS in (X, τ) and the family of all IFsαGSCS of an IFTS (X, τ) is denoted by IFsαGSC(X).

Example 3.2. Let X={a, b}. Let τ={$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨ x, (0.7, 0.6), (0.3, 0.4)⟩. Then the IFS A = ⟨ x, (0.2, 0.3), (0.8, 0.7)⟩ is an IFsαGSCS in (X, τ).

Theorem 3.3. Every IFCS in (X, τ) is an IFsαGSCS but not conversely.

Proof. Assume that A is an IFCS in (X, τ). Let us consider an IFS A $\subseteq$ U where U is an IFGSOS in X. Since αcl(A) $\subseteq$ cl(A) and A is an IFCS in X, αcl(A) $\subseteq$ cl(A)= A $\subseteq$ U and U is IFGSOS. That is αcl(A) $\subseteq$ U. Therefore A is IFsαGSCS in X.

Example 3.4. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.8, 0.6), (0.2, 0.4)$\rangle$. Then the IFS A = $\langle$x, (0.1, 0.3), (0.9, 0.7)$\rangle$ is IFsαGSCS but not an IFCS in X.

Theorem 3.5. Every IFαCS in (X, τ) is an IFsαGSCS in (X, τ) but not conversely.

Proof. Let A be an IFαCS in X. Let us consider an IFS A $\subseteq$ U where U is an IFGSOS in (X, τ). Since A is an IFαCS, αcl(A) = A. Hence αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is IFGSOS. Therefore A is an IFsαGSCS in X.

Example 3.6. Let X = {a, b}. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where G_1 = $\langle$x, (0.4, 0.3), (0.5, 0.6)$\rangle$ and G_2 = $\langle$x, (0.3, 0.2), (0.6, 0.7)$\rangle$. Consider an IFS A = $\langle$ x, (0.8, 0.8), (0.1, 0.1)$\rangle$ which is IFsαGSCS but not IFαCS, since cl(int(cl(A))) = $1_\sim \nsubseteq$ A.

Theorem 3.7. Every IFRCS in (X, τ) is an IFsαGSCS in (X, τ) but not conversely.

Proof. Let A be an IFRCS in (X, τ). Since every IFRCS is an IFCS, A is an IFCS in X. Hence by Theorem 3.3, A is an IFsαGSCS in X.

Example 3.8. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.6, 0.6), (0.4, 0.4)$\rangle$. Consider an IFS A = $\langle$ x, (0.3, 0.2), (0.7, 0.8)$\rangle$ which is an IFsαGSCS but not IFRCS in X as cl(int(A))= $0_\sim \neq$ A.

Theorem 3.9. Every IFsαGSCS in (X, τ) is an IFαGSCS in (X, τ) but not conversely.

Proof. Assume that A is an IFsαGSCS in (X, τ). Let us consider IFS A $\subseteq$ U where U is an IFSOS in X. Since every IFSOS is an IFGSOS and by hypothesis αcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFGSOS in X. We have αcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSOS in X. Hence A is an IFαGSCS in X.

Example 3.10. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.8), (0.4, 0.2)$\rangle$. Then the IFS A = $\langle$x, (0.5, 0.3), (0.2, 0.3)$\rangle$ is an IFαGSCS but not an IFsαGSCS in X.

Remark 3.11. An IFP closedness is independent of IFsαGS closedness.

Example 3.12. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.8, 0.7), (0.2, 0.3)$\rangle$. Then the IFS A = $\langle$x, (0.4, 0.5), (0.5, 0.4)$\rangle$ is IFPCS but not IFsαGSCS.

Example 3.13. Let X = {a, b}. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.3, 0.6), (0.7, 0.4)$\rangle$. Then the IFS A = $\langle$x, (0.6, 0.3), (0.4, 0.7)$\rangle$ is IFsαGSCS but not an IFPCS.

Remark 3.14. An IFSP closedness is independent of IFsαGS closedness.

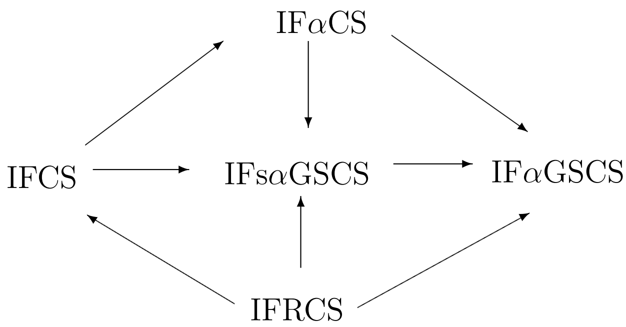
Example 3.15. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.6, 0.3), (0.2, 0.2)$\rangle$. Then the IFS A = $\langle$x, (0.7, 0.4), (0.1, 0.1)$\rangle$ is IFSPCS but not IFsαGSCS.

Example 3.16. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = ⟨x, (0.6, 0.3), (0.2, 0.2)⟩. Then the IFS A = ⟨x, (0.1, 0.1), (0.7, 0.5)⟩ is IFsαGSCS but not IFSPCS.

Remark 3.17. An IFγCS in (X, τ) need not be an IFsαGSCS.

Example 3.18. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = ⟨x, (0.3, 0.4), (0.6, 0.5)⟩. Then the IFS A = ⟨x, (0.4, 0.4), (0.5, 0.4)⟩ is IFγCS but not IFsαGSCS.

The relations between various types of intuitionistic fuzzy closed sets are given in the following diagram.



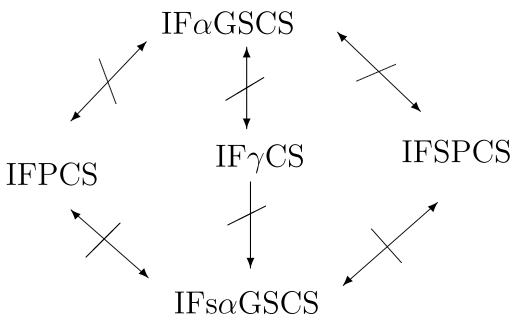
The reverse implications are not true in general.

Remark 3.19. The intersection of any two IFsαGSCS is not an IFsαGSCS in general as can be seen in the following Example.

Example 3.20. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = ⟨x, (0.2, 0.4),(0.8, 0.6)⟩. Then the IFS A = ⟨x, (0.3, 0.7), (0.6, 0.3)⟩ and B = ⟨x, (0.9, 0.4), (0.1, 0.5)⟩ are IFsαGSCS in X. Now A∩B = ⟨x, (0.3, 0.4), (0.6, 0.5)⟩ $\subseteq$ U = ⟨x, (0.4, 0.5), (0.3, 0.4)⟩ and U is IFGSOS in X. But αcl(A∩B) = $1_\sim \nsubseteq$ U. Therefore A∩B is not an IFsαGSCS in X.

Theorem 3.21. Let (X, τ) be an IFTS. Then for every A $\in$ IFsαGSC(X) and for every IFS B in X, A$\subseteq$ B $\subseteq$ αcl(A) implies B$\in$ IFsαGSC(X).

Proof. Let B$\subseteq$U where U is an IFGSOS in X. Since A $\subseteq$ B, A $\subseteq$ U. Since A is an IFsαGSCS in X, αcl(A) $\subseteq$ U. By hypothesis B $\subseteq$ αcl(A). This implies αcl(B) $\subseteq$ αcl(A) $\subseteq$ U. Therefore αcl(B) $\subseteq$ U. Hence B is an IFsαGSCS in X. The independent relations between various types of intuitionistic fuzzy closed sets are given in the following diagram.



In this diagram, A $\nleftrightarrow$ B denotes A and B are independent and A $\nrightarrow$ B denoted A need not be B.

Theorem 3.22. If A is an IFGSOS and an IFsαGSCS, then A is an IFαCS in X.

Proof. Let A be an IFGSOS in X. Since A $\subseteq$ A, by hypothesis αcl(A) $\subseteq$ A. But always A $\subseteq$ αcl(A). Therefore αcl(A) = A. Hence A is an IFαCS in X.

Theorem 3.23. Let (X, τ) be an IFTS. Then A is an IFsαGSCS if and only if A $\overline{q}$ F implies αcl(A) $\overline{q}$ F for every IFGSCS F of X.

Proof. Necessary Part: Let F be an IFGSCS and A$\overline{q}$F. Then A $\subseteq F'$ where F' is an IFGSOS in X. By assumption αcl(A) $\subseteq F'$. Hence αcl(A) $\overline{q}$ F.
Sufficient Part: Let F be IFGSCS in X such that A $\subseteq F'$. By hypothesis, A $\overline{q}$ F implies αcl(A) $\overline{q}$ F. This implies αcl(A) $\subseteq F'$ whenever A $\subseteq F'$ and F' is an IFGSOS in X. Hence A is an IFsαGSCS in X.

4 Intuitionistic Fuzzy Strongly α-generalized Semi-open Sets

In this section we introduce intuitionistic fuzzy strongly α-generalized semi-open sets and study some of its properties.

Definition 4.1. An IFS A is said to be intuitionistic fuzzy strongly α-generalized semi-open set (IFsαGSOS in short) in (X, τ) if the complement A^c is an IFsαGSCS in X. The family of all IFsαGSOS of an IFTS (X, τ) is denoted by IFsαGSO(X).

Theorem 4.2. For any IFTS (X, τ), every IFOS is an IFsαGSOS but not conversely.

Proof. Let A be an IFOS in X. Then A^c is an IFCS in X. By Theorem 3.3, A^c is an IFsαGSCS in X. Hence A is an IFsαGSOS in X.

Example 4.3. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = $\langle$x, (0.8, 0.6), (0.2, 0.4)$\rangle$. Consider the IFS A = $\langle$x, (0.9, 0.7), (0.1, 0.3)$\rangle$. Since A^c is an IFsαGSCS, A is an IFsαGSOS but not IFOS in X.

Theorem 4.4. In any IFTS (X, τ) every IFαOS is an IFsαGSOS but not conversely.

Proof. Let A be an IFαOS in X. Then A^c is an IFαCS in X. By Theorem 3.5, A^c is an IFsαGSCS in X. Hence A is an IFsαGSOS in X.

Example 4.5. Let X = {a, b}. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where G_1 = $\langle$x, (0.4, 0.3), (0.5, 0.6)$\rangle$ and G_2 = $\langle$x, (0.3, 0.2), (0.6, 0.7)$\rangle$. Then the IFS A = $\langle$x, (0.1, 0.1), (0.8, 0.8)$\rangle$ is an IFsαGSOS in X but not an IFαOS in X.

Theorem 4.6. In any IFTS (X, τ), every IFROS is an IFsαGSOS but not conversely.

Proof. Let A be an IFROS in X. Then A^c is an IFRCS in X. By Theorem 3.7, A^c is an IFsαGSCS in X. Hence A is an IFsαGSOS in X.

Example 4.7. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = ⟨x, (0.6, 0.6), (0.4, 0.4)⟩. Then the IFS A = ⟨x, (0.7, 0.8), (0.3, 0.2)⟩ is an IFsαGSOS in X but not an IFROS in X.

Theorem 4.8. In any IFTS (X, τ), every IFsαGSOS is an IFαGSOS but not conversely.

Proof. Let A be an IFsαGSOS in X. Then A^c is an IFsαGSCS in X. By Theorem 3.9, A^c is an IFαGSCS in X. Hence A is an IFαGSOS in X.

Example 4.9. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = ⟨x, (0.3, 0.8), (0.4, 0.2)⟩. Then the IFS A = ⟨x, (0.2, 0.3), (0.5, 0.3)⟩ is an IFαGSOS in X but not an IFsαGSOS in X.

Remark 4.10. The union of any two IFsαGSOS is not an IFsαGSOS in general.

Example 4.11. Let X = {a, b}. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where G = ⟨x, (0.2, 0.4),(0.8, 0.6)⟩. Then the IFS A = ⟨x, (0.6, 0.3), (0.3, 0.7)⟩ and B = ⟨x, (0.1, 0.5), (0.9, 0.4)⟩ are IFsαGSOS in X. Now A∪B = ⟨x, (0.6, 0.5), (0.3, 0.4) ⟩ is not an IFsαGSOS in X.

Theorem 4.12. An IFS A of an IFTS (X, τ) is an IFsαGSOS if and only if F ⊆ αint(A) whenever F is an IFGSCS in X and F ⊆ A.

Proof. Necessary Part: Let A be an IFsαGSOS in X. Let F be an IFGSCS in X and F ⊆ A. Then F' is an IFGSOS in X such that $A' \subseteq F'$. Since A' is an IFsαGSCS, we have $\alpha cl(A') \subseteq F'$. Hence $(\alpha int(A))' \subseteq F'$. Therefore F ⊆ αint(A).
Sufficient Part: Let A be an IFS in X and let F ⊆ αint(A) whenever F is an IFGSCS in X and F ⊆ A. Then $A' \subseteq F'$ and F' is an IFGSOS. By hypothesis, $(\alpha int(A))' \subseteq F'$, which implies $\alpha cl(A') \subseteq F'$. Therefore A' is an IFsαGSCS in X. Hence A is an IFsαGSOS in X.

Theorem 4.13. If A is an IFsαGSOS in (X, τ), then A is an IFGSOS in (X, τ).

Proof. Let A be an IFsαGSOS in X. This implies A is an IFαGSOS in X. Since every IFαGSOS is an IFGSOS, A is an IFGSOS in X.

5 Conclusion

In this paper we introduced the stronger form of intuitionistic fuzzy α- generalized semi closed set and some of its properties are discussed with existing intuitionistic fuzzy generalized closed sets. Also some comparable examples are given and some important notions of Intuitionistic fuzzy strongly α- generalized semi open sets are discussed.

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APPLICATIONS OF FUZZY GROUP DECISION MAKING PROBLEMS VIA BORDA SCORE METHOD

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Abstract - Inventory models in which the demand rate on the inventory level are based on the common real life observation that greater product availability tends to stimulate more sales. Theory of constraints (TOC) is a production planning philosophy that tries to improve the throughput of the system management of inventory levels. Due to the existing of inventory levels in a production system the demands of all products can not be fully met. So one of the most important decisions made in production systems is product mix problem. Although many algorithms have been developed in the fields using the concept of theory of constraints. This paper benefits from a variety of advantages. In order to consider the importance of all inventory levels, group decision making approach is applied and the optimal product mix is reached. In the algorithm presented in this paper, each inventory level is considered as a decision maker. The new algorithm benefits from the concept of fuzzy group decision making and optimizes the product mix problem in inventory environment where all parameters are fuzzy values.

Keywords **-** *Fuzzy group decision making, product mix optimization, multi-stage decision making, theory of constraints, inventory level cost and triangular fuzzy number.*

1. Introduction

Theory of constraints (TOC) which has been first introduced in the Goal [4] is a production planning philosophy that aims to improve the system through put by efficient use of

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inventory levels. In this paper product mix optimization is considered as a decision making problem. Regarding this analogy decision making criteria should be first defined [5]. Two important criteria are throughput and the later delivery cost. Later delivery cost is the most of mission one unit of each product. Assuming each inventory level as a decision maker [6], product mix optimization is a group decision making problem. In all previous researchers all parameters (such as processing time, demand etc) are assumed as crisp values. In this paper, a new algorithm is developed to optimize the product mix problem with all inputs are fuzzy values and Borda methods is used in group decision making process as ordinal techniques are preferred to cardinal ones [9].

2. Algorithms

The following notations are used in the new algorithm.

t_{ij} = processing time of product i on resource j.
D_i = Demand of product i.
Sp_i = selling price of product i.
Rm_i = Raw material cost of product i.
Ac_j =Available capacity of resource j.
Rc_j = Required capacity of resource j.
n = number of products.
m = number of inventory levels.

In this paper, all parameters are considered triangular fuzzy numbers and are shown as (x,y,z) where x<y<z, μ_y= 1 and $\mu_x = \mu_z = 0$. so let define t_{ij}, D_i, Sp_i, Rm_i, and Ac_j as follows.

$t_{ij} = (L_{ij}, M_{ij}, U_{ij})$,
$D_i = (L_i', M_i', U_i')$
$Sp_i = (A_i, B_i\ C_i)$
$Rm_i = (A_i', B_i', C_i')$,
$Ac = (\alpha_i, \beta_i, \chi_i)$

Step 1: Identify the system of inventory levels. As t_{ij} and D_i are positive fuzzy numbers, the required capacity of resource j is calculated as follows.

$$Rc_j = \sum_{i=1}^{n} t_{ij} D_i = \left(\sum_{i=1}^{n} L_{ij}\, L_{i'}\ ,\ \sum_{i=1}^{n} M_{ij}\, M_{i'}\ ,\ \sum_{i=1}^{n} U_{ij}\, U_{i'} \right) \tag{1}$$

For simplicity Rc_j is shown as (a_j, b_j, c_j). In order to determine whether j is an inventory level., Rc_j and Ac_j are compared using fuzzy ranking techniques. Due to the efficency of ranking of ranking methods based on lest and right scores, the method by chen is applied (chen and Hwang 1992). In this method, right and left scores of a fuzzy number refer to its intersection with the fuzzy max and the fuzzy min respectively. The fuzzy max and fuzzy min are defined as follows,

$$\mu_{max}(x) = \frac{x - x\,\mu_{min}}{x_{max} - x_{min}} \qquad \text{and} \qquad \mu_{min}(x) = \frac{x_{max} - x}{x_{max} - x_{min}} \tag{2}$$

where x_{max} is max (b_i , χ_i) and x_{min} (a_i , α_i). As Rc_i and Ac_i are triangular fuzzy numbers, they are convex, continous and normal. So their right scores may be determined by taking the intersection of their non-increasing part and $\mu_{max}(x)$. similarly their scores are determined by taking the intersection of their non-decreasing part and μ_{min} (x).As higher right score μ_R (x) and lower left score indicate large fuzzy number, the total scorec of Rc and Ac are defined as follows.

$$\mu_{Total}\,(Rc_j) = \frac{\mu_{Right}\,(Rc_j) + 1 - \mu_{Left}\,(Rc_j)}{2} \qquad (3)$$

$$\mu_{Total}\,(Ac_j) = \frac{\mu_{Right}\,(Ac_j) + 1 - \mu_{Left}\,(Ac_j)}{2} \qquad (4)$$

If μ_{Total} (Rc_j) > μ_{Total} (Ac_j), then j is an inventory level.

Step 2: Form decision matrices. Throughput (X_{ij}) is the first criterion considered in optimizing product mix. It is calculated as X_{ik}= Cm_i / t_{ij} …………….. (5) Where Cm_i is determined by the difference of the selling price and raw material cost of the product i.

Cm_i = (A_i-C_i', B_i-B_i', C_i-A_i') …………. (6) As Cm_i and t_{ij} are triangular fuzzy numbers, X_{ik} is calculated as follows.

$$X_{ik} = (A_i / U_{ij}, B_i / M_{ij}, C_i / L_{ij},) \qquad (7)$$

The other criteria is late delivery cost (Rashidi Komijan and Sadjadi 2005) . Although it is a crisp value in most cases, it is considered as LDC_i (p_i, q_i, r_i). Decision matrix of decision make K can be set as follows.

$$DM_k = \begin{matrix} Z_1 \\ .. \\ Z_i \\ .. \\ Z_k \end{matrix} \begin{pmatrix} X_{1k} & LDC_1 \\ .. & .. \\ X_{ik} & LDC_i \\ .. & .. \\ X_{nk} & LDC_n \end{pmatrix} \qquad (9)$$

Where r_{ijk} is the rank of alternative I assigned by decision maker K given the criterion j.

Step 4: For each criterion, set an agreed matrix that shows the ranks assigned to the alternatives by decision makers.

$$R_{j'} = \begin{array}{c} \\ Z_1 \\ \cdots \\ Z_i \\ \cdots \\ Z_n \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & \cdots & k & \cdots & m \end{array} \\ \begin{pmatrix} r_{1j1} & \cdots & r_{1jk} & \cdots & r_{1jm} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{ij1} & \cdots & r_{ijk} & \cdots & r_{ijm} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{nj1} & \cdots & r_{njk} & \cdots & r_{njm} \end{pmatrix} \end{array} \tag{10}$$

Step 5: Form Borda score matrices.

$$B_{j'} = \begin{array}{c} \\ Z_1 \\ \cdots \\ Z_i \\ \cdots \\ Z_n \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & \cdots & k & \cdots & m \end{array} \\ \begin{pmatrix} b_{1j1} & \cdots & b_{1jk} & \cdots & b_{1jm} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{ij1} & \cdots & b_{ijk} & \cdots & b_{ijm} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{nj1} & \cdots & b_{njk} & \cdots & b_{njm} \end{pmatrix} \end{array} \tag{11}$$

Where $b_{ijk} = n - r_{ijk}$

Step 6: Set score matrices by summing the values of each row.

$$SM_j = \begin{array}{c} Z_1 \\ .. \\ Z_i \\ .. \\ Z_n \end{array} \begin{pmatrix} S_{1j} \\ .. \\ S_{ij} \\ .. \\ S_{nj} \end{pmatrix} \tag{12}$$

Where $S_{ij} = \sum_{k=1}^{n} b_{ijk}$

Step 7: Set the agreement matrix. Firstly, values of the score matrices should be ranked. Then the agreement matrix is set by aggregating these ranks.

$$R_G = \begin{array}{c} Z_1 \\ .. \\ Z_i \\ .. \\ Z_n \end{array} \begin{pmatrix} G_{11} & G_{12} \\ .. & .. \\ G_{i1} & G_{i2} \\ .. & .. \\ G_{n1} & G_{n2} \end{pmatrix} \tag{13}$$

Where $G_{i,1}$ and $G_{i,2}$ are the agreement ranks of alternative 'i' given through put and late delivery cost respectively.

Step 8: Set the collective weighted agreement matrix. It is a nxn matrix in which rows and columns are alternatives and ranks respectively.

$$Q = \left[q_i \ell = \sum_{j=1}^{2} G_{i'} \ell_j W_j \right] \qquad (14)$$

Where w_i is the weight of criteria j and $G_{i'} \ell_j = 1$ if alternative i is assigned rank ℓ given criterion j. otherwise it is zero.

Step 9: Formulate a mathematical model. In order to obtain final ranks of alternatives, the classical assignment problem is considered. This is a zero–one model in which decision variable ($x_i\ell$) is one if rank ℓ is assigned to alternative i, otherwise it is zero.

$$\text{Max} \sum_{i=1}^{n} \sum_{\ell=1}^{n} q_i \ell \; x_i \ell$$

subject to

$$\sum_{i=1}^{n} x_i \ell = 1, \quad \ell = 1,2,3 \ldots n$$

$$\sum_{\ell=1}^{5} x_i \ell = 1, \; i = 1,2,3 \ldots\ldots\ldots n \qquad (15)$$

$x_i\ell$ is binary. Solving the above model represents the final ranking of alternatives.

3. Numerical Example

A company produces five products a,b,c,d,e. Demand, selling price, raw material cost and delivery cost of the products are triangular fuzzy numbers as shown in Table 1. Processing time and available capacity are shown in Table 2.

Table – 1 Demand, selling price, Raw material cost of each product and Late delivery cost.

Product	Demand	Selling price (dollar)	Raw material cost (dollar)	CM (SP-RMC)	Late delivery cost (dollar)
a	(4,6,7,9,)	(3,5,7,10,)	(5,7,10,12,)	(3,4,6,9,)	9
b	(2,3,5,9,)	(5,7,9,13,)	(4,6,9,12,)	(5,6,7,10,)	4
c	(7,9,10,12,)	(6,7,9,10,)	(7,9,10,13,)	(6,7,10,13,)	2
d	(4,5,7,9,)	(5,7,12,15,)	(7,9,13,15,)	(2,4,10,13,)	1
e	(5,7,10,14,)	(4,10,13,15,)	(3,7,9,13,)	(2,3,10,14,)	8

Table – 2 processing time of each product.

Station-1	Station - 2	Station-3	Station-4	Station-5
(3,7,10,13,1)	(4,7,10,14,1)	(5,15,20,30,)	(5,10,15,20,)	(10,15,20,)
(10,15,20,30)	(5,15,20,35)	(5,10,15,25)	(10,15,20,30)	(5,15,20,25)
(5,10,20,40)	(10,15,20,35)	(10,15,25,30)	(5,10,15,25)	(10,15,25,35)
(10,15,25,30)	(10,20,30,40)	(5,10,15,25)	(10,15,25,35)	(15,20,25,40)
(5,10,15,30)	(0,0,0,0)	(0,0,0,0)	(5,10,15,20)	(0,0,0,0)

The available capacity is (300,950,2000,2500), (250,1250,3500,4000), (150,650,1200,1800), (175,600,1000,1200) , (200,350,600,800).

Step 1: The required capacity of each station is calculated as follows.

$Rc_1 = \sum_{i=a}^{e} t_{i1} D_i$ = (275, 900,1900,2400)

$Rc_2 = \sum_{i=a}^{e} t_{i2} D_i$ = (225, 725,1800,2200)

$Rc_3 = \sum_{i=a}^{e} t_{i3} D_i$ = (200, 750,1325,1365)

$Rc_4 = \sum_{i=a}^{e} t_{i4} D_i$ = (275, 900,2000,2100)

$Rc_5 = \sum_{i=a}^{e} t_{i5} D_i$ = (275, 975,1825,2000)

Since $Rc_2 > Ac_2$, $Rc_3 > Ac_3$, $Rc_5 > Ac_5$ so stations 2,3 and 5 are inventory level but station 1 is not. It can be easily concluded whether station 4 is an inventory level. , Ac_4 and Rc_4 are compared using left and right.

$$\mu_{AC4} = \begin{cases} \frac{x-175}{425}, & 175 < x < 600 \\ \frac{1000-x}{400}, & 600 < x < 1000 \\ \frac{1200-x}{200} & 1000 < x < 1200 \end{cases} \qquad \mu_{max} = \frac{x-175}{1825}$$

$$\mu_{RC4} = \begin{cases} \frac{x-275}{625}, & 275 < x < 900 \\ \frac{2000-x}{1100}, & 900 < x < 2000 \\ \frac{2100-x}{100} & 2000 < x < 2100 \end{cases} \qquad \mu_{min} = \frac{2000-x}{1825}$$

This right score of fuzzy number's are the intersecting of their non-increasing parts and μ_{max} (x)

μ_{Right} (Ac_4) = 0.42 and μ_{Right} (Rc_4) = 0.53 similarly the left and total scores of Rc_4 and Ac_4 are calculated as

μ_{Left} (Ac_4) = 076 and μ_{Left} (Rc_1) = 0.68 , μ_{Total} (Ac_4) = 0.32 and μ_{Total} (Rc_4) = 0.41. so Rc_4 is greater then Ac_4 and state 4 is an inventory level.

Step 2: Late delivery costs are assumed crisp values, However, the algorithm would be efficient, if they were fuzzy. Decision matrices are set as follows. Note that the first column of the following matrices are calculated by dividing C_{mi} into t_{ij}

$$DM_1 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 0.32 & 2 & 12 & 5 \\ 0.36 & 2.13 & 2.7 & 2.5 \\ 0.32 & 0 & 2 & 8 \\ 0.5 & 2.22 & 8 & 8 \\ 0.42 & 2.5 & 7 & 2.4 \end{pmatrix} \qquad DM_2 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 0.4 & 6 & 8 & 5 \\ 0.42 & 2.11 & 5 & 4.2 \\ 0.28 & 0.57 & 2 & 5.3 \\ 0.42 & 2 & 9 & 2.8 \\ - & - & - & - \end{pmatrix}$$

$$DM_3 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 0.43 & 1 & 13 & 12.3 \\ 0.43 & 2.01 & 5 & 5.4 \\ 0.1 & 0.34 & 2 & 3.9 \\ 7 & 3 & 6 & 6.8 \\ - & - & - & 3.7 \end{pmatrix} \qquad DM_4 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 0.78 & 4 & 3 & 2.3 \\ 0.35 & 1.76 & 4.6 & 7 \\ 0.54 & 2 & 4 & 4 \\ 0.43 & 1.89 & 2 & 5 \\ 0.43 & 1.7 & 7 & 8 \end{pmatrix}$$

$$DM_5 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 0.81 & 5 & 6 & 5 \\ 0.56 & 1.76 & 8 & 0 \\ 0.9 & 0.76 & 9 & 6 \\ 0.59 & 1.76 & 0 & 8 \\ - & - & - & - \end{pmatrix}$$

Step 3: Ranking alternatives given the first criterion is done by applying left and right scores method. Consider the column of DM_1. It is clear that the ranks assigned to c,d, and e are 5,4,3. so the ranks assigned to ‘a’ and ‘b’ are 1 and 2 respectively. The ordinal rank matrices for DM_1 are set as follows.

$$R_1 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 4 & 1 & 2 \\ 3 & 5 & 1 \\ 2 & 2 & 4 \\ 5 & 4 & 5 \\ 1 & 3 & 3 \end{pmatrix} \qquad R_2 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 3 & 4 & 3 \\ 5 & 5 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \\ 4 & 3 & 5 \end{pmatrix} \qquad R_3 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 5 & 4 & 3 \\ 4 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 5 \\ 3 & 5 & 2 \end{pmatrix}$$

Step 4: The agreed matrix given thought R_1 is set by

$$R_1^1 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 5 & 1 & 3 & 5 \\ 2 & 2 & 2 & 1 \\ 1 & 5 & 5 & 4 \\ 4 & 3 & 1 & 3 \\ 3 & 4 & 4 & 2 \end{pmatrix} \qquad R_2^1 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 5 & 2 & 3 & 3 \\ 4 & 4 & 1 & 4 \\ 1 & 5 & 5 & 1 \\ 2 & 3 & 2 & 5 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

Step 5: Agreed matrices are converted into Borda score matrices

$$B_1 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 3 & 4 & 4 & 4 \\ 4 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 2 & 3 \\ 2 & 3 & 3 & 2 \end{pmatrix} \qquad B_2 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 0 & 3 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 4 & 0 & 1 & 1 \\ 0 & 2 & 2 & 0 \\ 2 & 4 & 4 & 3 \end{pmatrix}$$

Step 6: score matrices are set by summing the values of each row.

$$Sm_1 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 15 \\ 5 \\ 2 \\ 6 \\ 10 \end{pmatrix} \qquad Sm_2 = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 10 \\ 4 \\ 6 \\ 4 \\ 13 \end{pmatrix}$$

Step 7: In order to get the agreement matrix the values of Sm_1 and Sm_2 are ranked and form the first and second columns of the agreement matrix respectively.

$$R_G = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \begin{pmatrix} 3 & 1.3 \\ 5 & 5 \\ 4 & 3 \\ 1 & 4 \\ 2 & 1 \end{pmatrix}$$

Step 8: Assume that the weight vector of criteria are (0.7,0.3). The collective weight agreement matrix is set as follows.

$$R_G = \begin{matrix} \\ a \\ b \\ c \\ d \\ e \end{matrix} \begin{matrix} R1 & R2 & R3 & R4 & R5 \\ \end{matrix} \begin{pmatrix} \mathbf{0.6} & 0.9 & 0.0 & 0.0 & 0.0 \\ 0.3 & 0.3 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.9 & \mathbf{0.4} \\ 0.0 & 0.0 & 0.0 & 0.3 & 0.6 \end{pmatrix}$$

or instance $a_{11} = 0.6$, $a_{35} = 0.4$ because rank 1 is assigned to alternative 'a' given the first criterion.

Step 9: The assignment model is formulated as follows.

$$\text{Max } z = 0.6\, Xa_1 + 0.9Xa_2 + 0.3Xb_1 + \ldots + 0.6Xe_5$$

Subject to $\sum_{i=a}^{e} x_i \ell = 1$, $\ell = 1,2,3,4,5$.

Subject to $\sum_{\ell=1}^{5} x_i \ell = 1$, i = a,b,c,d,e.

The optimal solution is $Xa_1 = Xb_2 = Xc_3 = Xd_4 = Xe_5 = 1$. It means that 'a' has the highest production priority while 'd' has the lowest one.

4. Conclusion

The improved algorithm benefits from the advantage of reaching optimal solution. In the previous researchers all inputs of the were considered as crisp values. The assumption is not in real cases. This paper considers product mix problem as a group decision making problem in which all inputs are fuzzy. In this paper, a new algorithm for optimizing product mix under fuzzy parameters is developed. For this method, ordering methods are used in order to make decision in a fuzzy group decision making environment.

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THE RESTRICT AND EXTEND OF SOFT SET

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Abstract – In 1999, Molodtsov [1] introduced the concept of soft sets. In 2003, Maji et al. [2] presented a detailed theoretical study of soft sets which includes soft subset of a soft set, equality of soft sets and operations on soft sets such as union and intersection. In 2009, Ali et al. [3] studied and discussed the basic properties of these operations and defined some new operations in soft set theory as restricted soft intersection, restricted soft union and extended soft intersection. In this paper, we have introduced new concepts of soft sets: restrict of soft set, extent of soft set and mutual soft sets and studied some relations between them and operations on soft sets

Keywords – *Soft Sets, Soft Subsets, Soft Equal, Restrict of Soft Sets, Extent of Soft Sets, Mutual Soft Sets.*

1 Introduction

Dealing with uncertainties is a major problem in many areas such as economics, engineering, environmental science, medical science and social sciences. These kinds of problems cannot be dealt with by classical methods, because classical methods have inherent difficulties. To overcome these kinds of difficulties, Molodtsov [1] proposed a completely new approach, which is called soft set theory, for modeling uncertainty. Then Maji et al. [2] introduced several operations on soft sets. The main purpose of this paper is to introduce new concepts in soft set theory (restrict, extend) and studied their relations with restricted soft intersection, restricted soft union, extended soft intersection and extended soft union. Also we have defined mutual soft sets and found an equivalent condition to exist a unique soft union of two soft sets over a common universe and we have generalized that of a non-empty family of soft sets over a common universe.

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2 Preliminaries

In this section, we give some basic definitions for soft sets. Throughout this paper, U denotes an initial universe set and E is a set of parameters, the power set of U is denoted by $P(U)$ and $A \neq \emptyset$ is a subset of E.

Definition 2.1. [1] A pair (F,A) is called a *soft set* over U, where F is a mapping given by $F: A \to P(U)$.

Definition 2.2. Let (F,A) and (G,B) be two soft sets over a common universe U. Then (G,B) is called a *soft subset* of (F,A), denote by $(G,B) \tilde{\subseteq} (F,A)$, if it satisfies the following:
1) $B \subseteq A$,
2) $G(x) \subseteq F(x)$ For all $x \in B$.

Definition 2.3. [2] Two soft sets (F,A) and (G,B) over a common universe U are called *soft equal*, denote by $(F,A) = (G,B)$, if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A). i.e. $G(x) = F(x)\ \ \forall x \in A = B$.

Definition 2.4. Let (F,A) be a soft set over U and $\emptyset \neq B \subseteq A$. The *restrict* of (F,A) on B is defined as the soft set (F_B, B) where F_B is restrict of F on B. i.e. $F_B(x) = F(x)\ \forall x \in B$. It is clear that $(F_B, B) \tilde{\subseteq} (F,A)$.

This definition is equivalent to definition of soft subset at [2].

Definition 2.5. Let (F,A) and (G,B) be two soft sets over a common universe U. The *extend* of (F,A) by (G,B) is defined as the soft set $(\bar{F}_G, A \cup B)$ where,

$$\bar{F}_G(x) = \begin{cases} F(x) \ ; & x \in A \\ G(x) \ ; & x \in B - A \end{cases}$$

It is clear that:

1) $(F,A) \tilde{\subseteq} (\bar{F}_G, A \cup B)$.
2) If $B \subseteq A$, then $\bar{F}_G(x) = F(x)\ \ \forall x \in A$. Thus $(\bar{F}_G, A) = (F,A)$.
3) $\left(\left(\overline{\bar{F}_G}\right)_G, A \cup B\right) = (\bar{F}_G, A \cup B)$.

Definition 2.6. [2] A soft set (F,A) over U is called a *null soft set*, denoted by (Φ, A), if $F(x) = \Phi(x) = \emptyset\ \ \forall x \in A$.

Definition 2.7. [2] A soft set (F,A) over U is called an *absolute soft set*, denoted by (Ω, A), if $F(x) = \Omega(x) = U\ \ \forall x \in A$.

Definition 2.8. [3] Let (F,A) be a soft set over U. The *soft complement* of (F,A) is defined as the soft set (F^c,A) where $F^c(x)=U-F(x)\ \forall x\in A$.

It is clear that $\left(F^{c^c},A\right)=(F,A)$.

Definition 2.9. [4] Let (F,A) be a soft set over U and $f:U\to U'$ be a mapping of sets. Then we can define a soft set $(f(F),A)$ over U' where $f(F):A\to P(U')$ is defined as $(f(F))(x)=f(F(x))$ for all $x\in A$.

Definition 2.10. Let (F,A) be a soft set over U' and $f:U\to U'$ be a mapping of sets. Then we can define a soft set $(f^{-1}(F),A)$ over U where $f^{-1}(F):A\to P(U)$ is defined as $\left(f^{-1}(F)\right)(x)=f^{-1}(F(x))$ for all $x\in A$.

Proposition 2.11. Let (F,A) be a soft set over U and $\varnothing\neq B\subseteq A$. Then:

$$\left((F^c)_B,B\right)=\left((F_B)^c,B\right)$$

Proof. Let x be an element of B. Then:

$$(F^c)_B(x)=F^c(x)=U-F(x)=U-F_B(x)=(F_B)^c(x)$$

Proposition 2.12. Let (F,A) and (G,B) be two soft sets over a common universe U. Then:

$$\left((\bar{F}_G)^c,A\cup B\right)=\left((\overline{F^c})_{G^c},A\cup B\right)$$

Proof. Let x be an element of $A\cup B$. Then:

$$\begin{aligned}(\bar{F}_G)^c(x)=U-(\bar{F}_G)(x)\ &=\begin{cases}U-F(x)\ ;\ x\in A\\ U-G(x)\ ;\ x\in B-A\end{cases}\\ &=\begin{cases}F^c(x)\ ;\ x\in A\\ G^c(x)\ ;\ x\in B-A\end{cases}\\ &=\left((\overline{F^c})_{G^c}\right)(x)\end{aligned}$$

Proposition 2.13. Let (F,A) be a soft set over U, and $f:U\to U'$ be a mapping of sets. If $\varnothing\neq B\subseteq A$, Then:

$$\left(f(F_B),B\right)=\left((f(F))_B,B\right)$$

Proof. Let x be an element of B. Then:

$$\left(f(F_B)\right)(x)=f\left(F_B(x)\right)=f\left(F(x)\right)=\left(f(F)\right)(x)=\left(f(F)\right)_B(x)$$

Proposition 2.14. Let (F,A) and (G,B) be two soft sets over a common universe U and $f:U\to U'$ be a mapping of sets. Then:

$$\left(f(\bar{F}_G),A\cup B\right)=\left(\left(\overline{f(F)}\right)_{f(G)},A\cup B\right)$$

Proof. Let x be an element of $A\cup B$. Then:

$$\left(f(\bar{F}_G)\right)(x)=f\left(\bar{F}_G(x)\right)=\begin{cases} f\left(F(x)\right) \; ; \; x\in A \\ f\left(G(x)\right) \; ; \; x\in B-A \end{cases}$$

$$=\begin{cases} \left(f(F)\right)(x) \; ; \; x\in A \\ \left(f(G)\right)(x) \; ; \; x\in B-A \end{cases}$$

$$=\left(\left(\overline{f(F)}\right)_{f(G)}\right)(x)$$

Proposition 2.15. Let (F,A) be a soft set over U', and $f:U\to U'$ be a mapping of sets. If $\varnothing\neq B\subseteq A$, Then:

$$\left(f^{-1}(F_B),B\right)=\left(\left(f^{-1}(F)\right)_B,B\right)$$

Proof. Let x be an element of B. Then:

$$\left(f^{-1}(F_B)\right)(x)=f^{-1}\left(F_B(x)\right)=f^{-1}\left(F(x)\right)=\left(f^{-1}(F)\right)(x)=\left(f^{-1}(F)\right)_B(x)$$

Proposition 2.16. Let (F,A) and (G,B) be two soft sets over a common universe U' and $f:U\to U'$ be a mapping of sets. Then:

$$\left(f^{-1}(\bar{F}_G),A\cup B\right)=\left(\left(\overline{f^{-1}(F)}\right)_{f^{-1}(G)},A\cup B\right)$$

Proof. Let x be an element of $A\cup B$. Then:

$$\left(f^{-1}(\bar{F}_G)\right)(x)=f^{-1}\left(\bar{F}_G(x)\right)=\begin{cases} f^{-1}\left(F(x)\right) \; ; \; x\in A \\ f^{-1}\left(G(x)\right) \; ; \; x\in B-A \end{cases}$$

$$=\begin{cases} \left(f^{-1}(F)\right)(x) \; ; \; x\in A \\ \left(f^{-1}(G)\right)(x) \; ; \; x\in B-A \end{cases}$$

$$=\left(\left(\overline{f^{-1}(F)}\right)_{f^{-1}(G)}\right)(x)$$

3 Relations Between Restrict and Extend of Soft Sets and Operations on Soft Sets

Definition 3.1. [3] Let (F,A) and (G,B) be two soft sets over a common universe U such that $A\cap B\neq\varnothing$. The *restricted soft intersection* of (F,A) and (G,B) is defined as the soft set $\left(F\,\tilde{\cap}_r\,G, A\cap B\right)$ where,

$$\left(F\,\tilde{\cap}_r\,G\right)(x)=F(x)\cap G(x)\ \ \forall x\in A\cap B$$

It is clear that:

$$\left(F\,\tilde{\cap}_r\,G, A\cap B\right)\tilde{\subseteq}\left(F,A\right)\,\&\,\left(F\,\tilde{\cap}_r\,G, A\cap B\right)\tilde{\subseteq}\left(G,B\right)\,\&\,\left(F\,\tilde{\cap}_r\,F^c, A\right)=(\Phi,A)$$

Definition 3.2. [3] The *extended soft intersection* of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set $\left(F\,\tilde{\cap}_e\,G, A\cup B\right)$ where,

$$\left(F\,\tilde{\cap}_e\,G\right)(x)=\begin{cases} F(x) & ; x\in A-B \\ G(x) & ; x\in B-A \\ F(x)\cap G(x) & ; x\in A\cap B\end{cases}$$

It is clear that:

1) If $A\cap B=\varnothing$, then $\left(F\,\tilde{\cap}_e\,G, A\cup B\right)=\left(\bar{F}_G, A\cup B\right)=\left(\bar{G}_F, A\cup B\right)$.
2) If $A=B$, then $\left(F\,\tilde{\cap}_e\,G, A\right)=\left(F\,\tilde{\cap}_r\,G, A\right)$.

Definition 3.3. [3] Let (F,A) and (G,B) be two soft sets over a common universe U such that $A\cap B\neq\varnothing$. The *restricted soft union* of (F,A) and (G,B) is defined as the soft set $\left(F\,\tilde{\cup}_r\,G, A\cap B\right)$ where,

$$\left(F\,\tilde{\cup}_r\,G\right)(x)=F(x)\cup G(x)\ \ \forall x\in A\cap B$$

Definition 3.4. [2] The *extended soft union* (it is called as union in [2]) of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set $\left(F\,\tilde{\cup}_e\,G, A\cup B\right)$ where,

$$\left(F\,\tilde{\cup}_e\,G\right)(x)=\begin{cases} F(x) & ; x\in A-B \\ G(x) & ; x\in B-A \\ F(x)\cup G(x) & ; x\in A\cap B\end{cases}$$

It is clear that:

1) $\left(F,A\right)\tilde{\subseteq}\left(F\,\tilde{\cup}_e\,G, A\cup B\right)\,\&\,\left(G,B\right)\tilde{\subseteq}\left(F\,\tilde{\cup}_e\,G, A\cup B\right)\,\&\,\left(F\,\tilde{\cup}_e\,F^c, A\right)=(\Omega,A)$.
2) If $A\cap B=\varnothing$, then $\left(F\,\tilde{\cup}_e\,G, A\cup B\right)=\left(F\,\tilde{\cap}_e\,G, A\cup B\right)=\left(\bar{F}_G, A\cup B\right)=\left(\bar{G}_F, A\cup B\right)$.

3) If $A = B$, then $\left(F \tilde{\bigcup}_e G, A\right) = \left(F \tilde{\bigcup}_r G, A\right)$.

Proposition 3.5. Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \bigcap B \neq \varnothing$. Then we have the following:

1) $\left(\left(F \tilde{\bigcap}_e G\right)_{A \bigcap B}, A \cap B\right) = \left(F \tilde{\bigcap}_r G, A \cap B\right)$.

2) $\left(\left(F \tilde{\bigcup}_e G\right)_{A \bigcap B}, A \cap B\right) = \left(F \tilde{\bigcup}_r G, A \cap B\right)$.

Proof. Proof is straightforward.

Proposition 3.6. Let (F, A) and (G, B) be two soft sets over a common universe U . Then we have the following:

1) $\left(\left(\overline{F^c}\right)_G \tilde{\bigcap}_r \overline{F}_{G^c}, A \bigcup B\right) = \left(\left(\overline{F^c}\right)_G \tilde{\bigcap}_e \overline{F}_{G^c}, A \bigcup B\right) = (\Phi, A \bigcup B)$.

2) $\left(\left(\overline{F^c}\right)_G \tilde{\bigcup}_r \overline{F}_{G^c}, A \bigcup B\right) = \left(\left(\overline{F^c}\right)_G \tilde{\bigcup}_e \overline{F}_{G^c}, A \bigcup B\right) = (\Omega, A \bigcup B)$.

Proof. 1) Let x be an element of $A \bigcup B$. Then:

$$\left(\left(\overline{F^c}\right)_G \tilde{\bigcap}_r \overline{F}_{G^c}\right)(x) = \left(\overline{F^c}\right)_G (x) \bigcap \left(\overline{F}_{G^c}\right)(x) = \begin{cases} F^c(x) \bigcap F(x) = \varnothing & ; x \in A \\ G(x) \bigcap G^c(x) = \varnothing & ; x \in B - A \end{cases}$$
$$= \Phi(x)$$

2) Let x be an element of $A \bigcup B$. Then:

$$\left(\left(\overline{F^c}\right)_G \tilde{\bigcup}_r \overline{F}_{G^c}\right)(x) = \left(\overline{F^c}\right)_G (x) \bigcup \left(\overline{F}_{G^c}\right)(x) = \begin{cases} F^c(x) \bigcup F(x) = U & ; x \in A \\ G(x) \bigcup G^c(x) = U & ; x \in B - A \end{cases}$$
$$= \Omega(x)$$

Theorem. 3.7. Let (F, A) and (G, B) be two soft sets over a common universe U . Then we have the following:

1) $\left(\left(F \tilde{\bigcap}_e G\right)_A, A\right) \tilde{\subseteq} (F, A)$ & $\left(\left(F \tilde{\bigcap}_e G\right)_B, B\right) \tilde{\subseteq} (G, B)$.

2) $\left(F \tilde{\bigcap}_e G, A \bigcup B\right)$ is the largest soft set which satisfies the two soft inclusions in (1).

Proof. 1) Let x be an element of A . Then:

$$\left.\begin{array}{ll} if \;\; x \in A - B & \Rightarrow \left(F \tilde{\bigcap}_e G\right)(x) = F(x) \\ if \;\; x \in A \bigcap B & \Rightarrow \left(F \tilde{\bigcap}_e G\right)(x) = F(x) \bigcap G(x) \subseteq F(x) \end{array}\right\} \Rightarrow$$
$$\Rightarrow \left(F \tilde{\bigcap}_e G\right)(x) \subseteq F(x) \quad \forall x \in A$$
$$\Rightarrow \left(F \tilde{\bigcap}_e G\right)_A (x) \subseteq F(x) \quad \forall x \in A$$

Thus:

$$\left(\left(F\,\tilde{\cap}_e\,G\right)_A, A\right) \tilde{\subseteq} (F, A)$$

Let x be an element of B . Then:

$$\left.\begin{array}{ll} if\;\; x \in B - A & \Rightarrow \left(F\,\tilde{\cap}_e\,G\right)(x) = G(x) \\ if\;\; x \in A \cap B & \Rightarrow \left(F\,\tilde{\cap}_e\,G\right)(x) = F(x) \cap G(x) \subseteq G(x) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left(F\,\tilde{\cap}_e\,G\right)(x) \subseteq G(x) \quad \forall x \in B$$

$$\Rightarrow \left(F\,\tilde{\cap}_e\,G\right)_B (x) \subseteq G(x) \quad \forall x \in B$$

Thus:

$$\left(\left(F\,\tilde{\cap}_e\,G\right)_B, B\right) \tilde{\subseteq} (G, B)$$

2) Let $(H, A \cup B)$ be a soft set such that $(H_A, A) \tilde{\subseteq} (F, A)$ & $(H_B, B) \tilde{\subseteq} (G, B)$ and we will prove that:

$$(H, A \cup B) \tilde{\subseteq} \left(F\,\tilde{\cap}_e\,G, A \cup B\right)$$

Let x be an element of $A \cup B$. Then:

$$\left.\begin{array}{ll} if\;\; x \in A - B & \Rightarrow \left(F\,\tilde{\cap}_e\,G\right)(x) = F(x) \supseteq H_A(x) = H(x) \\ if\;\; x \in B - A & \Rightarrow \left(F\,\tilde{\cap}_e\,G\right)(x) = G(x) \supseteq H_B(x) = H(x) \\ if\;\; x \in A \cap B & \Rightarrow \left(F\,\tilde{\cap}_e\,G\right)(x) = F(x) \cap G(x) \supseteq H_A(x) \cap H_B(x) = H(x) \end{array}\right\} \Rightarrow$$

$$\Rightarrow H(x) \subseteq \left(F\,\tilde{\cap}_e\,G\right)(x) \quad \forall x \in A \cup B$$

Thus:

$$(H, A \cup B) \tilde{\subseteq} \left(F\,\tilde{\cap}_e\,G, A \cup B\right)$$

Theorem. 3.8. Let (F, A) and (G, B) be two soft sets over a common universe U . Then we have the following:

1) $(F, A) \tilde{\subseteq} \left(\left(F\,\tilde{\cup}_e\,G\right)_A, A\right)$ & $(G, B) \tilde{\subseteq} \left(\left(F\,\tilde{\cup}_e\,G\right)_B, B\right)$.

2) $\left(F\,\tilde{\cup}_e\,G, A \cup B\right)$ is the smallest soft set which satisfies the two soft inclusions in (1).

Proof. 1) Let x be an element of A . Then:

$$\left.\begin{array}{ll} if\ \ x\in A-B & \Rightarrow\left(F\tilde{\bigcup}_e G\right)(x)=F(x) \\ if\ \ x\in A\bigcap B & \Rightarrow\left(F\tilde{\bigcup}_e G\right)(x)=F(x)\bigcup G(x)\supseteq F(x) \end{array}\right\}\Rightarrow$$

$$\Rightarrow F(x)\subseteq\left(F\tilde{\bigcup}_e G\right)(x)\quad\forall x\in A$$

$$\Rightarrow F(x)\subseteq\left(F\tilde{\bigcup}_e G\right)_A(x)\quad\forall x\in A$$

Thus:

$$(F,A)\tilde{\subseteq}\left(\left(F\tilde{\bigcup}_e G\right)_A,A\right)$$

Let x be an element of B . Then:

$$\left.\begin{array}{ll} if\ \ x\in B-A & \Rightarrow\left(F\tilde{\bigcup}_e G\right)(x)=G(x) \\ if\ \ x\in A\bigcap B & \Rightarrow\left(F\tilde{\bigcup}_e G\right)(x)=F(x)\bigcup G(x)\supseteq G(x) \end{array}\right\}\Rightarrow$$

$$\Rightarrow G(x)\subseteq\left(F\tilde{\bigcup}_e G\right)(x)\quad\forall x\in B$$

$$\Rightarrow G(x)\subseteq\left(F\tilde{\bigcup}_e G\right)_B(x)\quad\forall x\in B$$

Thus:

$$(G,B)\tilde{\subseteq}\left(\left(F\tilde{\bigcup}_e G\right)_B,B\right)$$

2) Let $(H,A\bigcup B)$ be a soft set such that $\left(F,A\right)\tilde{\subseteq}\left(H_A,A\right)\ \&\ \left(G,B\right)\tilde{\subseteq}\left(H_B,B\right)$ and we will prove that:

$$\left(F\tilde{\bigcup}_e G,A\cup B\right)\tilde{\subseteq}\left(H,A\cup B\right)$$

Let x be an element of $A\bigcup B$. Then:

$$\left.\begin{array}{ll} if\ \ x\in A-B & \Rightarrow\left(F\tilde{\bigcup}_e G\right)(x)=F(x)\subseteq H_A(x)=H(x) \\ if\ \ x\in B-A & \Rightarrow\left(F\tilde{\bigcup}_e G\right)(x)=G(x)\subseteq H_B(x)=H(x) \\ if\ \ x\in A\bigcap B & \Rightarrow\left(F\tilde{\bigcup}_e G\right)(x)=F(x)\bigcup G(x)\subseteq H_A(x)\bigcup H_B(x)=H(x) \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left(F\tilde{\bigcup}_e G\right)(x)\subseteq H(x)\ \forall x\in A\bigcup B$$

Thus:

$$\left(F\tilde{\bigcup}_e G,A\cup B\right)\tilde{\subseteq}\left(H,A\cup B\right)$$

Theorem. 3.9. Let (F,A) and (G,B) be two soft sets over a common universe U . Then there is an unique soft set $(H,A\bigcup B)$ over U such that:

$$\left(H_A,A\right)=(F,A)\ \&\ \left(H_B,B\right)=(G,B)$$

if and only if $F(x)=G(x)\ \forall x\in A\bigcap B$.

Proof. Suppose that $(H, A \cup B)$ is a soft set over U such that:

$$(H_A, A) = (F, A) \;\&\; (H_B, B) = (G, B)$$

Then:

$$\forall x \in A \cap B \;\Rightarrow F(x) = H_A(x) = H(x) \;\&\; G(x) = H_B(x) = H(x) \Rightarrow F(x) = G(x)$$

Conversely, suppose that $F(x) = G(x)\ \forall x \in A \cap B$.

Indeed the soft set $(H, A \cup B) = (F \tilde{\cap}_e G, A \cup B)$ where,

$$H(x) = (F \tilde{\cap}_e G)(x) = \begin{cases} F(x)\ ;\ x \in A \\ G(x)\ ;\ x \in B \end{cases}$$

And holds:

$$\forall x \in A \Rightarrow H_A(x) = H(x) = F(x) \Rightarrow (H_A, A) = (F, A)$$
$$\forall x \in B \Rightarrow H_B(x) = H(x) = G(x) \Rightarrow (H_B, B) = (G, B)$$

Now, let $(H', A \cup B)$ be a soft set such that $(H'_A, A) = (F, A) \;\&\; (H'_B, B) = (G, B)$ and we will prove that:

$$(H', A \cup B) = (H, A \cup B)$$

Let x be an element of $A \cup B$. Then:

$$\left.\begin{array}{l} \text{if}\quad x \in A \;\Rightarrow H'(x) = H'_A(x) = F(x) = H_A(x) = H(x) \\ \text{if}\quad x \in B \;\Rightarrow H'(x) = H'_B(x) = G(x) = H_B(x) = H(x) \end{array}\right\} \Rightarrow$$
$$\Rightarrow H'(x) = H(x)\ \forall x \in A \cup B$$

Thus:

$$(H', A \cup B) = (H, A \cup B)$$

Definition 3.10. Two soft sets (F, A) and (G, B) over a common universe U are called *mutual soft sets* if $F(x) = G(x)\ \forall x \in A \cap B$.

Proposition 3.11. Two soft sets (F, A) and (G, B) over a common universe U are *mutual soft sets* if and only if $(\bar{F}_G, A \cup B) = (\bar{G}_F, A \cup B)$.

Proof. Suppose that (F, A) and (G, B) are mutual soft sets. Then:

$$F(x) = G(x)\ \forall x \in A \cap B$$

Thus:

$$\bar{F}_G(x) = \begin{cases} F(x) \ ; x \in A \\ G(x) \ ; \ x \in B - A \end{cases}$$
$$= \begin{cases} F(x) & ; x \in A - B \\ F(x) = G(x) & ; x \in A \cap B \\ G(x) & ; x \in B - A \end{cases}$$
$$= \begin{cases} F(x) \ ; \ x \in A - B \\ G(x) \ ; \ x \in B \end{cases}$$
$$= \bar{G}_F(x)$$

Conversely, suppose that $\left(\bar{F}_G, A \cup B\right) = \left(\bar{G}_F, A \cup B\right)$. Then:

$$\forall x \in A \cap B \Rightarrow F(x) = \bar{F}_G(x) = \bar{G}_F(x) = G(x) \Rightarrow F(x) = G(x) \ \forall x \in A \cap B$$

Theorem. 3.12. Let $\left\{\left(F_i, A_i\right)\right\}_{i \in I}$ be a non-empty family of soft sets over a common universe U . Then there is an unique soft set $\left(H, \bigcup_{i \in I} A_i\right)$ over U such that:

$$\left(H_{A_\alpha}, A_\alpha\right) = \left(F_\alpha, A_\alpha\right) \ \forall \alpha \in I$$

if and only if $\left\{\left(F_i, A_i\right)\right\}_{i \in I}$ are pairwise mutual soft sets

Proof. Suppose that $\left(H, \bigcup_{i \in I} A_i\right)$ is a soft set over U such that:

$$\left(H_{A_\alpha}, A_\alpha\right) = \left(F_\alpha, A_\alpha\right) \ \forall \alpha \in I$$

Then:

$$\forall \alpha, \beta \in I, \forall x \in A_\alpha \cap A_\beta \Rightarrow F_\alpha(x) = H_{A_\alpha}(x) = H(x) \ \& \ F_\beta(x) = H_{A_\beta}(x) = H(x)$$
$$\Rightarrow F_\alpha(x) = F_\beta(x) \ \forall x \in A_\alpha \cap A_\beta \ , \ \forall \alpha, \beta \in I$$

Thus $\left\{\left(F_i, A_i\right)\right\}_{i \in I}$ are pairwise mutual soft sets.

Conversely, suppose that $\left\{\left(F_i, A_i\right)\right\}_{i \in I}$ are pairwise mutual soft sets. Then:

$$F_i(x) = F_j(x) \ \forall x \in A_i \cap A_j \ , \ \forall i, j \in I$$

We will define $\left(H, \bigcup_{i\in I} A_i\right)$ as: $\forall x \in \bigcup_{i\in I} A_i \Rightarrow \exists \alpha \in I; x \in A_\alpha$ we set $H(x) = F_\alpha(x)$.

H is well define because if $\beta \in I$ such that $x \in A_\beta$, then $x \in A_\alpha \cap A_\beta$, Thus:

$$H(x) = F_\alpha(x) = F_\beta(x).$$

H holds:

$$\forall \alpha \in I, \forall x \in A_\alpha \Rightarrow H_{A_\alpha}(x) = H(x) = F_\alpha(x) \Rightarrow \left(H_{A_\alpha}, A_\alpha\right) = \left(F_\alpha, A_\alpha\right) \ \forall \alpha \in I$$

Now, let $\left(H', \bigcup_{i\in I} A_i\right)$ be a soft set such that $\left(H'_{A_\alpha}, A_\alpha\right) = \left(F_\alpha, A_\alpha\right) \ \forall \alpha \in I$.

Then:

$$\forall x \in \bigcup_{i\in I} A_i \Rightarrow \exists \alpha \in I; x \in A_\alpha$$
$$\Rightarrow H'(x) = H'_{A_\alpha}(x) = F_\alpha(x) = H(x)$$

Thus:

$$H'(x) = H(x) \ \ \forall x \in \bigcup_{i\in I} A_i$$

Hence:

$$\left(H', \bigcup_{i\in I} A_i\right) = \left(H, \bigcup_{i\in I} A_i\right)$$

4 Conclusions

In this paper, we have introduced new concepts in soft set theory: restrict of soft set, extent of soft set and mutual soft sets. And we studied their relations with soft complement, restricted soft intersection, restricted soft union, extended soft intersection and extended soft union. To extend this work, one could extend study these concepts and relations of soft sets in other algebraic structures such as soft groups, soft rings, etc.

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ON SOFT EXPERT SETS

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Abstract – Alkhazaleh and Salleh defined the concept of soft expert sets [Advances in Decision Sciences, Article ID 757868, 2011]. In this paper, we make some modification to the soft expert sets. On the modified soft expert sets we then construct a decision making method which selects an elements from the alternatives. We finally give an example to shows this method can be successfully applied to some many uncertainty problems.

Keywords – *Soft sets, soft expert sets, soft operations, decision making.*

1 Introduction

The concept of soft sets was first introduced by Molodtsov [14]. Until now many versions of it have been developed and applied to a lot of areas from algebra to decision making problems. One of these versions is soft expert sets introduced by Alkhazaleh and Salleh [5]. They also propounded fuzzy soft expert sets [6] by using soft expert sets and fuzzy soft sets [12]. Afterwards, Hazaymeh et al. [9] improved generalized fuzzy soft expert sets. Then, Alhazaymeh and Hassan [1, 2] developed generalized vague soft expert (*gvse*) sets and gave an application of them in decision making. They also studied mapping on *gvse*-sets [3].

Although the concept of soft expert sets is important for the development of soft sets, it has some own difficulties arising from some definitions. This situation necessitates to arrange some parts of it. For example, although the idea based on the principle of time-dependent change of the experts' opinion is impressive, this scenario has not been modelled by using adequate parameterizations in [5]. So, we will ignore this idea for the time being. In addition to this case, we should emphasize that the soft expert sets have become consistent in itself. In other words, some arranges can be necessary when the other types of soft expert sets, as fuzzy parameterized soft expert sets [7] and fuzzy parameterized fuzzy soft expert sets [10], are taken into consideration.

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2 Soft Expert Sets

In this section, we recall some basic notions with some remarks and updates in soft expert sets [5]. Let U be a universe, E be a set of parameters, X be a set of experts (agents), $O = \{0, 1\}$ be a set of opinions, $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.1. A pair (F, A) is called a soft expert set over U, where F is a mapping given by

$$F : A \to P(U)$$

where $P(U)$ denotes the power set of U.

Example 2.2. Suppose that a company produces some new products and wants to obtain the opinion of some experts about these products. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of products, $E = \{e_1, e_2, e_3\}$ be a set of decision parameters where e_i ($i \in \{1, 2, 3\}$) denotes the parameters as *easy to use*, *quality* and *cheap*, respectively, and let $X = \{p, q, r\}$ be a set of experts.

Assume that the company has distributed a questionnaire to three experts to make decisions on the products, and the results of this questionnaire are as in the following,

$$\begin{array}{lll}
F(e_1,p,1) = \{u_1,u_2,u_4\}, & F(e_1,q,1) = \{u_1,u_4\}, & F(e_1,r,1) = \{u_3,u_4\}, \\
F(e_2,p,1) = \{u_4\}, & F(e_2,q,1) = \{u_1,u_3\}, & F(e_2,r,1) = \{u_1,u_2,u_4\}, \\
F(e_3,p,1) = \{u_3,u_4\}, & F(e_3,q,1) = \{u_1,u_2\}, & F(e_3,r,1) = \{u_4\}, \\
F(e_1,p,0) = \{u_3\}, & F(e_1,q,0) = \{u_2,u_3\}, & F(e_1,r,0) = \{u_1,u_2\}, \\
F(e_2,p,0) = \{u_1,u_2,u_3\}, & F(e_2,q,0) = \{u_2,u_4\}, & F(e_2,r,0) = \{u_3\}, \\
F(e_3,p,0) = \{u_1,u_2\}, & F(e_3,q,0) = \{u_3,u_4\}, & F(e_3,r,0) = \{u_1,u_2,u_3\}
\end{array}$$

Then the soft expert set (F, Z) as in the following,

$$\begin{aligned}
(F,Z) = \; & \{((e_1,p,1),\{u_1,u_2,u_4\}),((e_1,q,1),\{u_1,u_4\}),((e_1,r,1),\{u_3,u_4\}), \\
& ((e_2,p,1),\{u_4\}),((e_2,q,1),\{u_1,u_3\}),((e_2,r,1),\{u_1,u_2,u_4\}), \\
& ((e_3,p,1),\{u_3,u_4\}),((e_3,q,1),\{u_1,u_2\}),((e_3,r,1),\{u_4\}), \\
& ((e_1,p,0),\{u_3\}),((e_1,q,0),\{u_2,u_3\}),((e_1,r,0),\{u_1,u_2\}), \\
& ((e_2,p,0),\{u_1,u_2,u_3\}),((e_2,q,0),\{u_2,u_4\}),((e_2,r,0),\{u_3\}), \\
& ((e_3,p,0),\{u_1,u_2\}),((e_3,q,0),\{u_3,u_4\}),((e_3,r,0),\{u_1,u_2,u_3\})\}
\end{aligned}$$

In this example, the expert p agrees that *easy to use* products are u_1, u_2 and u_4. The expert q agrees that the *easy to use* products are u_1 and u_4, and the expert r agrees that the *easy to use* products are u_3 and u_4. Notice also that all of them agree that product u_4 is *easy to use.*

Remark 2.3. In a soft set, for the parameter e_1, $F(e_1)$ and $G(e_1)$ can be different since the functions F and G may be different. However, in a soft expert set, for the parameter $(e_1, p, 1)$, $F(e_1, p, 1)$ and $G(e_1, p, 1)$ have to be the same since any variable causing changes, such as time, in the choices of expert p does not exist. In other words, for $t_1 \neq t_2$, $F(e_1, p, 1, t_1)$ and $G(e_1, p, 1, t_2)$ can be different.

From now on, since an expert p can not claim that a product either provides or does not provide the parameter in the same time, all of the examples given in [5] have been updates.

Definition 2.4. For two soft expert sets (F, A) and (G, B) over U, (F, A) is called a soft expert subset of (G, B), denoted by $(F, A)\tilde{\subseteq}(G, B)$, if $F(\alpha) \subseteq G(\alpha)$, for all $\alpha \in A$.

If $(F, A)\tilde{\subseteq}(G, B)$, then (G, B) is called a soft expert superset of (F, A).

Proposition 2.5. Let (F, A) and (G, B) be two soft expert sets over U. Then

$$(F, A)\tilde{\subseteq}(G, B) \Leftrightarrow (F, A) \subseteq (G, B) \Leftrightarrow A \subseteq B$$

Definition 2.6. Two soft expert sets (F, A) and (G, B) over U are said to be equal if $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\subseteq}(F, A)$.

Proposition 2.7. Let (F, A) and (G, B) be two soft expert sets over U. Then

$$(F, A) = (G, B) \Leftrightarrow A = B$$

Example 2.8. Let

$$\begin{aligned}(F, A) = \ & \{((e_1, p, 1), \{u_1, u_2, u_4\}), ((e_1, q, 1), \{u_1, u_4\}), ((e_2, q, 1), \{u_1, u_3\}), \\ & ((e_2, r, 1), \{u_1, u_2, u_4\}), ((e_3, p, 1), \{u_3, u_4\}), ((e_3, r, 1), \{u_4\}), \\ & ((e_1, r, 0), \{u_1, u_2\}), ((e_2, p, 0), \{u_1, u_2, u_3\}), ((e_3, q, 0), \{u_3, u_4\})\}\end{aligned}$$

and

$$\begin{aligned}(G, B) = \ & \{((e_1, p, 1), \{u_1, u_2, u_4\}), ((e_1, q, 1), \{u_1, u_4\}), ((e_2, q, 1), \{u_1, u_3\}), \\ & ((e_2, r, 1), \{u_1, u_2, u_4\}), ((e_1, r, 0), \{u_1, u_2\}), ((e_2, p, 0), \{u_1, u_2, u_3\})\}\end{aligned}$$

Therefore $(G, B)\tilde{\subseteq}(F, A)$. Clearly $B \subseteq A$.

Definition 2.9. An agree-soft expert set $(F, A)_1$ which is also a soft expert subset of (F, A) over U is defined as in the following,

$$(F, A)_1 = \{(\alpha, F(\alpha)) : \alpha \in A_1\}$$

where, $A_1 \subseteq Z_1$ such that $Z_1 := E \times X \times \{1\}$.

Example 2.10. Let's consider Example 2.2. Then

$$\begin{aligned}(F, Z)_1 = \ & \{((e_1, p, 1), \{u_1, u_2, u_4\}), ((e_1, q, 1), \{u_1, u_4\}), ((e_1, r, 1), \{u_3, u_4\}), \\ & ((e_2, p, 1), \{u_4\}), ((e_2, q, 1), \{u_1, u_3\}), ((e_2, r, 1), \{u_1, u_2, u_4\}), \\ & ((e_3, p, 1), \{u_3, u_4\}), ((e_3, q, 1), \{u_1, u_2\}), ((e_3, r, 1), \{u_4\})\}\end{aligned}$$

Definition 2.11. A disagree-soft expert set $(F,A)_0$ which is a soft expert subset of (F,A) over U is defined as in the following,

$$(F,A)_0 = \{(\alpha, F(\alpha)) : \alpha \in A_0\}$$

where, $A_0 \subseteq Z_0$ such that $Z_0 := E \times X \times \{0\}$.

Example 2.12. Let's consider Example 2.2. Then

$$\begin{aligned}(F,Z)_0 = \ & \{((e_1,p,0),\{u_3\}),((e_1,q,0),\{u_2,u_3\}),((e_1,r,0),\{u_1,u_2\}),\\ & ((e_2,p,0),\{u_1,u_2,u_3\}),((e_2,q,0),\{u_2,u_4\}),((e_2,r,0),\{u_3\}),\\ & ((e_3,p,0),\{u_1,u_2\}),((e_3,q,0),\{u_3,u_4\}),((e_3,r,0),\{u_1,u_2,u_3\})\}\end{aligned}$$

Remark 2.13. According to the definition of soft expert sets given in [5], it has been studied over a subset of the parameter set Z. However, from the definition of 'not α' and 'NOT Z', defined by $\neg\alpha = (\neg e_i, x_j, o_k)$ and $\rceil Z = \{\neg\alpha : \alpha \in Z\}$, respectively, $\rceil Z \nsubseteq Z$.

On the other hand, $\rceil A_1 \subseteq \rceil Z_1$. That is, $\rceil A_1 \subseteq \rceil E \times X \times \{1\}$. Since, $\rceil A_1 \neq A_0$, the propositions given in [5]

ii. $(F,A)_1^{\tilde{c}} = (F,A)_0$

iii. $(F,A)_0^{\tilde{c}} = (F,A)_1$

are not held according to the definition of equality of two soft expert sets in [5]. It can be overcome this kind of difficulties by accepting as $(\neg e_1, p, 1) = (e_1, p, 0)$. So, $\rceil Z_1 = Z_0$. In other words, the propositions

ii. $(F,Z)_1^{\tilde{c}} = (F,Z)_0$

iii. $(F,Z)_0^{\tilde{c}} = (F,Z)_1$

are held.

In the view of such information, the definition of *not set* and *soft expert complement* can be rewritten as in the following,

Definition 2.14. Let $\alpha = (e_i, x_j, o_k) \in Z$. Then *not* α and *NOT* Z are defined by $\neg\alpha = (e_i, x_j, 1 - o_k)$ and $\rceil Z = \{\neg\alpha : \alpha \in Z\}$, respectively. It can easily be seen that $\rceil Z = Z$ but $\rceil A \neq A$, for some $A \subseteq Z$.

Definition 2.15. The complement of a soft expert set (F,A) is denoted by $(F,A)^{\tilde{c}}$ and is defined by $(F,A)^{\tilde{c}} = (F^{\tilde{c}}, \rceil A)$ where $F^{\tilde{c}} : \rceil A \to P(U)$ is mapping given by $F^{\tilde{c}}(\neg\alpha) = U - F(\alpha)$, for all $\neg\alpha \in \rceil A$.

Proposition 2.16. Let (F,A) be a soft expert set over U. Then $((F,A)^{\tilde{c}})^{\tilde{c}} = (F,A)$.

Example 2.17. Let's consider Example 2.2. Then

$$\begin{aligned}(F,Z)^{\tilde{c}} = \; & \{((e_1,p,0),\{u_3\}),((e_1,q,0),\{u_2,u_3\}),((e_1,r,0),\{u_1,u_2\}),\\ & ((e_2,p,0),\{u_1,u_2,u_3\}),((e_2,q,0),\{u_2,u_4\}),((e_2,r,0),\{u_3\}),\\ & ((e_3,p,0),\{u_1,u_2\}),((e_3,q,0),\{u_3,u_4\}),((e_3,r,0),\{u_1,u_2,u_3\}),\\ & ((e_1,p,1),\{u_1,u_2,u_4\}),((e_1,q,1),\{u_1,u_4\}),((e_1,r,1),\{u_3,u_4\}),\\ & ((e_2,p,1),\{u_4\}),((e_2,q,1),\{u_1,u_3\}),((e_2,r,1),\{u_1,u_2,u_4\}),\\ & ((e_3,p,1),\{u_3,u_4\}),((e_3,q,1),\{u_1,u_2\}),((e_3,r,1),\{u_4\})\} = (F,Z)\end{aligned}$$

Definition 2.18. The union of two soft expert sets (F,A) and (G,B) over U, denoted by $(F,A)\widetilde{\cup}(G,B)$, is the soft expert set (H,C) where $C = A \cup B$, and for all $\alpha \in C$,

$$H(\alpha) = \begin{cases} F(\alpha), & \alpha \in A - B, \\ G(\alpha), & \alpha \in B - A, \\ F(\alpha) = G(\alpha), & \alpha \in A \cap B. \end{cases}$$

Proposition 2.19. Let (F,A) and (G,B) be two soft expert sets over U. Then

$$(F,A)\widetilde{\cup}(G,B) = (F,A) \cup (G,B)$$

Example 2.20. Let

$$\begin{aligned}(F,A) = \; & \{((e_1,p,1),\{u_1,u_2,u_4\}),((e_1,q,1),\{u_1,u_4\}),((e_2,q,1),\{u_1,u_3\}),\\ & ((e_2,r,1),\{u_1,u_2,u_4\}),((e_3,p,1),\{u_3,u_4\}),((e_3,r,1),\{u_4\}),\\ & ((e_1,r,0),\{u_1,u_2\}),((e_2,p,0),\{u_1,u_2,u_3\}),((e_3,q,0),\{u_3,u_4\})\}\end{aligned}$$

and

$$\begin{aligned}(G,B) = \; & \{((e_1,p,1),\{u_1,u_2,u_4\}),((e_1,q,1),\{u_1,u_4\}),((e_2,q,1),\{u_1,u_3\}),\\ & ((e_2,r,1),\{u_1,u_2,u_4\}),((e_3,q,1),\{u_1,u_2\}),((e_1,r,0),\{u_1,u_2\}),\\ & ((e_2,p,0),\{u_1,u_2,u_3\}),((e_3,p,0),\{u_1,u_2\})\end{aligned}$$

Then

$$\begin{aligned}(F,A)\widetilde{\cup}(G,B) = \; & \{((e_1,p,1),\{u_1,u_2,u_4\}),((e_1,q,1),\{u_1,u_4\}),((e_2,q,1),\{u_1,u_3\}),\\ & ((e_2,r,1),\{u_1,u_2,u_4\}),((e_3,p,1),\{u_3,u_4\}),((e_3,q,1),\{u_1,u_2\}),\\ & ((e_3,r,1),\{u_4\}),((e_1,r,0),\{u_1,u_2\}),((e_2,p,0),\{u_1,u_2,u_3\}),\\ & ((e_3,p,0),\{u_1,u_2\}),((e_3,q,0),\{u_3,u_4\})\}\end{aligned}$$

Proposition 2.21. Let (F,A), (G,B) and (H,C) be three soft expert sets over U. Then

i. $(F,A)\widetilde{\cup}(F,A) = (F,A)$

ii. $(F,A)\widetilde{\cup}(G,B) = (G,B)\widetilde{\cup}(F,A)$

iii. $(F,A)\widetilde{\cup}((G,B)\widetilde{\cup}(H,C)) = ((F,A)\widetilde{\cup}(G,B))\widetilde{\cup}(H,C)$

Remark 2.22. For all $\alpha \in A\cap B$, $F(\alpha) = G(\alpha)$. That is, $(F,A)\widetilde{\cup}(G,B)$=$(F,A)\widetilde{\cap}(G,B)$ in [5]. Therefore, for the intersection of two soft expert sets (H,C), the set C may consider as $A \cap B$.

Definition 2.23. The intersection of two soft expert sets (F,A) and (G,B) over U, denoted by $(F,A)\widetilde{\cap}(G,B)$ is the soft expert set (H,C) where $C = A \cap B$, for all $\alpha \in C$, and

$$H(\alpha) = \begin{cases} F(\alpha) = G(\alpha), & if\ C \neq \emptyset \\ \emptyset, & otherwise \end{cases}$$

Proposition 2.24. Let (F,A) and (G,B) be two soft expert sets over U. Then

$$(F,A)\widetilde{\cap}(G,B) = (F,A) \cap (G,B)$$

Example 2.25. Let's consider the Example 2.17. Then

$$\begin{aligned}(F,A)\widetilde{\cap}(G,B) = \ & \{((e_1,p,1),\{u_1,u_2,u_4\}),((e_1,q,1),\{u_1,u_4\}),((e_2,q,1),\{u_1,u_3\}),\\ & ((e_2,r,1),\{u_1,u_2,u_4\}),((e_1,r,0),\{u_1,u_2\}),((e_2,p,0),\{u_1,u_2,u_3\})\}\end{aligned}$$

Proposition 2.26. Let (F,A), (G,B) and (H,C) be three soft expert sets over U. Then

i. $(F,A)\widetilde{\cap}(F,A) = (F,A)$

ii. $(F,A)\widetilde{\cap}(G,B) = (G,B)\widetilde{\cap}(F,A)$

iii. $(F,A)\widetilde{\cap}((G,B)\widetilde{\cap}(H,C)) = ((F,A)\widetilde{\cap}(G,B))\widetilde{\cap}(H,C)$

Proposition 2.27. Let (F,A), (G,B) and (H,C) be three soft expert sets over U. Then

i. $(F,A)\widetilde{\cup}((G,B)\widetilde{\cap}(H,C)) = ((F,A)\widetilde{\cup}(G,B))\widetilde{\cap}((F,A)\widetilde{\cup}(H,C))$

ii. $(F,A)\widetilde{\cap}((G,B)\widetilde{\cup}(H,C)) = ((F,A)\widetilde{\cap}(G,B))\widetilde{\cup}((F,A)\widetilde{\cap}(H,C))$

Definition 2.28. Let (F,A) and (G,B) be two soft expert sets over U. Then (F,A) AND (G,B), denoted by $(F,A) \wedge (G,B)$, is defined by

$$(F,A) \wedge (G,B) = (H, A \times B)$$

where $H(\alpha,\beta) = F(\alpha) \cap G(\beta)$, for all $(\alpha,\beta) \in A \times B$.

Definition 2.29. Let (F,A) and (G,B) be two soft expert sets over U. Then (F,A) OR (G,B), denoted by $(F,A) \vee (G,B)$, is defined by

$$(F,A) \vee (G,B) = (O, A \times B)$$

where $O(\alpha,\beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha,\beta) \in A \times B$.

Example 2.30. Let

$$(F,A) = \{((e_2,q,1),\{u_1,u_3\}),((e_3,p,1),\{u_3,u_4\}),((e_3,r,1),\{u_4\}), ((e_1,r,0),\{u_1,u_2\}),((e_3,q,0),\{u_3,u_4\})\}$$

and

$$(G,B) = \{((e_1,q,1),\{u_1,u_4\}),((e_2,q,1),\{u_1,u_3\}),((e_2,r,1),\{u_1,u_2,u_4\})\}$$

Then

$$\begin{aligned}(F,A)\wedge(G,B) = \{&(((e_2,q,1),(e_1,q,1)),\{u_1\}),(((e_2,q,1),(e_2,q,1)),\{u_1,u_3\}),\\ &(((e_2,q,1),(e_2,r,1)),\{u_1\}),(((e_3,p,1),(e_1,q,1)),\{u_4\}),\\ &(((e_3,p,1),(e_2,q,1)),\{u_3\}),(((e_3,p,1),(e_2,r,1)),\{u_4\}),\\ &(((e_3,r,1),(e_1,q,1)),\{u_4\}),(((e_3,r,1),(e_2,q,1)),\emptyset),\\ &(((e_3,r,1),(e_2,r,1)),\{u_4\}),(((e_1,r,0),(e_1,q,1)),\{u_1\}),\\ &(((e_1,r,0),(e_2,q,1)),\{u_1\}),(((e_1,r,0),(e_2,r,1)),\{u_1,u_2\})\}\end{aligned}$$

and

$$\begin{aligned}(F,A)\vee(G,B) = \{&(((e_2,q,1),(e_1,q,1)),\{u_1,u_3,u_4\}),(((e_2,q,1),(e_2,q,1)),\{u_1,u_3\}),\\ &(((e_2,q,1),(e_2,r,1)),U),(((e_3,p,1),(e_1,q,1)),\{u_1,u_3,u_4\}),\\ &(((e_3,p,1),(e_2,q,1)),\{u_1,u_3,u_4\}),(((e_3,p,1),(e_2,r,1)),U),\\ &(((e_3,r,1),(e_1,q,1)),\{u_1,u_4\}),(((e_3,r,1),(e_2,q,1)),\{u_1,u_3,u_4\}),\\ &(((e_3,r,1),(e_2,r,1)),\{u_1,u_2,u_4\}),(((e_1,r,0),(e_1,q,1)),\{u_1,u_2,u_4\}),\\ &(((e_1,r,0),(e_2,q,1)),\{u_1,u_2,u_3\}),(((e_1,r,0),(e_2,r,1)),\{u_1,u_2,u_4\})\}\end{aligned}$$

Proposition 2.31. Let (F,A) and (G,B) be two soft expert sets over U. Then

i. $((F,A)\wedge(G,B))^{\tilde{c}} = (F,A)^{\tilde{c}} \vee (G,B)^{\tilde{c}}$

ii. $((F,A)\vee(G,B))^{\tilde{c}} = (F,A)^{\tilde{c}} \wedge (G,B)^{\tilde{c}}$

Proposition 2.32. Let (F,A), (G,B) and (H,C) be three soft expert sets over U. Then

i. $((F,A)\wedge((G,B)\wedge(H,C)) = ((F,A)\wedge(G,B))\wedge(H,C)$

ii. $((F,A)\vee((G,B)\vee(H,C)) = ((F,A)\vee(G,B))\vee(H,C)$

Remark 2.33. Since the domains of functions which lay on the right side of the equalities are different from the other side of them, the propositions

iii. $((F,A)\vee((G,B)\wedge(H,C)) = ((F,A)\vee(G,B))\wedge((F,A)\vee(H,C))$

iv. $((F,A)\wedge((G,B)\vee(H,C)) = ((F,A)\wedge(G,B))\vee((F,A)\wedge(H,C))$

are not held as it is also shown in [4] for the soft sets.

3 An Application of Soft Expert Sets

In this section, we show that the algorithm given in [5] has some unnecessary steps and that the results of this algorithm and Maji et al's algorithm [13] without reduction are equivalent. Afterwards, we suggest a new algorithm and give an application on decision making by using updated definitions and propositions as a result of remarks above.

Let's consider the algorithm in [5] as in the following,

Algorithm 1.

(1) Input the soft expert set (F, Z),

(2) Find an agree-soft expert set and a disagree-soft expert set,

(3) Find $c_j = \Sigma_i R_X(\alpha_i, u_j)$ for agree-soft expert set,

(4) Find $k_j = \Sigma_i R_X(\alpha_i, u_j)$ for disagree-soft expert set,

(5) Find $s_j = c_j - k_j$,

(6) Find m, for which $s_m = \max_j s_j$.

It is easy to show that, from the Definition 2.11,

$$k_j = |E \times X| - c_j$$

then

$$s_j = c_j - \{|E \times X| - c_j\} = 2c_j - |E \times X|$$

and

$$c_i \leq c_j \Leftrightarrow 2c_i \leq 2c_j \Leftrightarrow (2c_i - |E \times X|) \leq (2c_j - |E \times X|) \Leftrightarrow s_i \leq s_j$$

where, the symbol $|E \times X|$ is the cardinality of $E \times X$. That is, s_j and $\max_j\{s_j\}$ are redundant. So step 5, step 4 and the last part of step 2 are unnecessary. Hence, the algorithm has become Maji et al's algorithm, i.e.,

(1) Input the soft expert set (F, Z),

(2) Find the agree-soft expert set,

(3) Find $c_j = \Sigma_i R_X(\alpha_i, u_j)$ for the agree-soft expert set,

(4) Find m, for which $c_m = \max_j c_j$.

To illustrate, let's consider the application given in [5]. Assume that a company wants to fill a position. There are eight candidates who form the universe $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$. The hiring committee considers a set of parameters, $E = \{e_1, e_2, e_3, e_4, e_5\}$ where the parameters $e_i (i = 1, 2, 3, 4, 5)$ stand for *experience*, *computer knowledge*, *young age*, *elocution* and *friendly*, respectively. Let $X = \{p, q, r\}$ be a set of experts (committee members). Suppose that, after a serious discussion, the committee constructs the soft expert set (F, Z) as in the following,

$$
\begin{aligned}
(F,Z) = \; & \{((e_1,p,1),\{u_1,u_2,u_4,u_7,u_8\}),((e_1,q,1),\{u_1,u_4,u_5,u_8\}),\\
& ((e_1,r,1),\{u_1,u_3,u_4,u_6,u_7,u_8\}),((e_2,p,1),\{u_3,u_5,u_8\}),\\
& ((e_2,q,1),\{u_1,u_3,u_4,u_5,u_6,u_8\}),((e_2,r,1),\{u_1,u_2,u_4,u_7,u_8\}),\\
& ((e_3,p,1),\{u_3,u_4,u_5,u_7\}),((e_3,q,1),\{u_1,u_2,u_5,u_8\}),((e_3,r,1),\{u_1,u_7,u_8\}),\\
& ((e_4,p,1),\{u_1,u_7,u_8\}),((e_4,q,1),\{u_1,u_4,u_5,u_8\}),((e_4,r,1),\{u_1,u_6,u_7,u_8\}),\\
& ((e_5,p,1),\{u_1,u_2,u_3,u_5,u_8\}),((e_5,q,1),\{u_1,u_4,u_5,u_8\}),((e_1,p,0),\{u_3,u_5,u_6\}),\\
& ((e_1,q,0),\{u_2,u_3,u_6,u_7\}),((e_1,r,0),\{u_2,u_5\}),((e_3,p,0),\{u_1,u_2,u_6,u_8\}),\\
& ((e_5,r,1),\{u_1,u_3,u_5,u_7,u_8\}),((e_2,p,0),\{u_1,u_2,u_4,u_6,u_7\}),((e_2,q,0),\{u_2,u_7\}),\\
& ((e_3,q,0),\{u_3,u_4,u_6,u_7\}),((e_3,r,0),\{u_2,u_3,u_4,u_5,u_6\}),((e_2,r,0),\{u_3,u_5,u_6\}),\\
& ((e_4,p,0),\{u_2,u_3,u_4,u_5,u_6\}),((e_4,q,0),\{u_2,u_3,u_6,u_7\}),((e_4,r,0),\{u_2,u_3,u_4,u_5\}),\\
& ((e_5,p,0),\{u_4,u_6,u_7\}),((e_5,q,0),\{u_2,u_3,u_6,u_7\}),((e_5,r,0),\{u_2,u_4,u_6\})\}
\end{aligned}
$$

Then the table representation (or briefly table) of $(F,Z)_1$ as in Table 1.

Table 1. The table of agree-soft expert set

R_X	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
$(e_1,p,1)$	1	1	0	1	0	0	1	1
$(e_2,p,1)$	0	0	1	0	1	0	0	1
$(e_3,p,1)$	0	0	1	1	1	0	1	0
$(e_4,p,1)$	1	0	0	0	0	0	1	1
$(e_5,p,1)$	1	1	1	0	1	0	0	1
$(e_1,q,1)$	1	0	0	1	1	0	0	1
$(e_2,q,1)$	1	0	1	1	1	1	0	1
$(e_3,q,1)$	1	1	0	0	1	0	0	1
$(e_4,q,1)$	1	0	0	1	1	0	0	1
$(e_5,q,1)$	1	0	0	1	1	0	0	1
$(e_1,r,1)$	1	0	1	1	0	1	1	1
$(e_2,r,1)$	1	1	0	1	0	0	1	1
$(e_3,r,1)$	1	0	0	0	0	0	1	1
$(e_4,r,1)$	1	0	0	0	0	1	1	1
$(e_5,r,1)$	1	0	1	0	1	0	1	1
$c_j = \Sigma_i R_X(\alpha_i,u_j)$	$c_1=13$	$c_2=4$	$c_3=6$	$c_4=8$	$c_5=9$	$c_6=3$	$c_7=8$	$c_8=14$

Here, R_X is a relation on $Z \times U$, defined by $R_X(\alpha_i,u_j) = \chi_{F(\alpha_i)}(u_j)$ such that $R_X(\alpha_i,u_j)$ is the entries corresponding the ith row and jth column in table representation of R_X and

$$\chi_{F(\alpha_i)}(u_j) = \begin{cases} 1, & u_j \in F(\alpha_i) \\ 0, & otherwise \end{cases}$$

Hence, the committee can choose candidate 8 for the job since $\max_j c_j = c_8$.

Note that the order of c_j,

$$c_8 > c_1 > c_5 > c_4 = c_7 > c_3 > c_2 > c_6$$

obtained by Maji et al's algorithm without reduction, is the same as the order obtained by Alkhazaleh and Salleh's algoritm.

Let's give a new definition and an algorithm which is different from the others.

Definition 3.1. The soft expert set (F, A) is called p-part of (F, Z), denoted by $p(F, Z)$, such that $A = E \times \{p\} \times O$ for $p \in X$.

For example,

$$\begin{aligned} p(F,Z) = \; & \{((e_1,p,1),\{u_1,u_2,u_4\}),((e_2,p,1),\{u_4\}),((e_3,p,1),\{u_3,u_4\}),\\ & ((e_1,p,0),\{u_3\}),((e_2,p,0),\{u_1,u_2,u_3\}),((e_3,p,0),\{u_1,u_2\})\} \end{aligned}$$

is a part of (F, Z) given in Example 2.2.

Note that $p(F, Z)_1$ can be seen as a soft set over U and written simply as in the following,

$$p(F,Z)_1 = \{(e_1,\{u_1,u_2,u_4\}),(e_2,\{u_4\}),(e_3,\{u_3,u_4\})\}$$

Algorithm 2.

(1) Construct a soft expert set,

(2) Find the parts of agree-soft expert set,

(3) Find the consensus soft set by using s-intersection to all parts of agree-soft expert set.

(4) Find $c_j = \Sigma_i R_C(e_i, u_j)$ for consensus,

(5) Find $\{u_k : c_k = \max_j c_j\}$.

To illustrate, let's consider the application above. Then the table representation of all parts of agree-soft expert sets as in the following,

Table 2. The table of $p(F, Z)_1$

R_p	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
e_1	1	1	0	1	0	0	1	1
e_2	0	0	1	0	1	0	0	1
e_3	0	0	1	1	1	0	1	0
e_4	1	0	0	0	0	0	1	1
e_5	1	1	1	0	1	0	0	1

Table 3. The table of $q(F,Z)_1$

R_q	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
e_1	1	0	0	1	1	0	0	1
e_2	1	0	1	1	1	1	0	1
e_3	1	1	0	0	1	0	0	1
e_4	1	0	0	1	1	0	0	1
e_5	1	0	0	1	1	0	0	1

Table 4. The table of $r(F,Z)_1$

R_r	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
e_1	1	0	1	1	0	1	1	1
e_2	1	1	0	1	0	0	1	1
e_3	1	0	0	0	0	0	1	1
e_4	1	0	0	0	0	1	1	1
e_5	1	0	1	0	1	0	1	1

Here, R_p is a relation on $E \times U$, defined by $R_p(e_i, u_j) = \chi_{F(e_i)}(u_j)$ such that $R_p(e_i, u_j)$ is the entries corresponding the ith row and jth column in table representation of R_p and

$$\chi_{F(e_i)}(u_j) = \begin{cases} 1, & u_j \in F(e_i) \\ 0, & otherwise \end{cases}$$

Let's obtain the consensus soft set by soft intersection of all parts of the agree-soft expert set and show as in the following,

Table 5. The table of the consensus soft set

R_C	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
e_1	1	0	0	1	0	0	0	1
e_2	0	0	0	0	0	0	0	1
e_3	0	0	0	0	0	0	0	0
e_4	1	0	0	0	0	0	0	1
e_5	1	0	0	0	1	0	0	1
$c_j = \Sigma_i R_C(e_i, u_j)$	$c_1 = 3$	$c_2 = 0$	$c_3 = 0$	$c_4 = 1$	$c_5 = 1$	$c_6 = 0$	$c_7 = 0$	$c_8 = 4$

By Table 5, we have the following results;

$$c_8 > c_1 > c_4 = c_5 > c_2 = c_3 = c_6 = c_7$$

Since $\max_j c_j = c_8$, the committee can choose the candidate with number 8 for the job.

4 Conclusion

The concept of soft sets has idiosyncratic serious problems because of some of their definitions as the soft complement. Enginoğlu [8] overcame such problems by characteristic sets. Similarly, the concept of soft expert sets can provide dealing with the difficulty arising from the definition of soft complement in [11] by assuming $(\neg e_i, p_j, 1) = (e_i, p_j, 0)$. This is important for the development of soft sets, and it is worth doing the study on it when viewed from this aspect. People who want to study on this concept should not ignore this detail.

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AN EXTENDED GREY RELATIONAL ANALYSIS BASED INTERVAL NEUTROSOPHIC MULTI ATTRIBUTE DECISION MAKING FOR WEAVER SELECTION

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Abstract – The paper proposes a multi-attribute decision making method based on extended grey relation analysis under interval neutrosophic environment. The interval neutrosophic set is an important decision making apparatus that can handle imprecise, indeterminate, inconsistence information. The rating of the alternatives with respect to certain attribute considered by the expert is characterized by linguistic variables that can be represented by interval neutrosophic sets. In the selection process, the attributes are identified from the experts' opinion. The weight of each attribute is completely unknown and maximizing deviation method is employed in order to determine them. Then, an extended grey relational analysis technique is developed to find the ranking order of all alternatives. Finally, an illustrative numerical example for weaver selection in Khadi Institution is provided to show the effectiveness and applicability of the developed approach.

***Keywords* –** *Multi-attribute decision making, linguistic variable, interval neutrosophic set, grey relational analysis, weaver selection.*

1 Introduction

Zadeh [25] coined the term 'degree of membership' and defined the concept of fuzzy set in order to deal with uncertainty. Atanassov [1] incorporated the degree of non-membership in the concept of fuzzy set as an independent component and defined the concept of intuitionistic fuzzy set. Smarandache [13,14] grounded the term 'degree of indeterminacy as an independent component and defined the concept of neutrosophic set from the philosophical point of view to deal with incomplete, indeterminate and inconsistent

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information which exist in real world decision making problems. In neutrosophic set, truth membership, indeterminacy membership, falsity membership functions are independent and they are real standard or non-standard subsets of]0^-, 1^+[. However, NS is difficult to apply in practical decision making situation. To employ the concept of neutrosophic set in practical fields, Wang et al. [19] restricted the concept of neutrosophic set to single valued neutrosophic set (SVNS) since single value is an instance of set value. In SVNS, the truth membership, indeterminacy membership, falsity membership functions are subsets of [0, 1]. SVNS is identified as a useful tool for practical scientific and engineering applications. However, decision information may be provided with intervals rather than real numbers due to lack of knowledge of the decision maker. Therefore, Wang et al. [18] defined set-theoretic operators on interval neutrosophic set (INS) which is more flexible and practical than SVNS. INS is much easier to handle incomplete, indeterminate and inconsistent information.

Multi-attribute decision making (MADM) is one of the fastest developing areas during past few decades and it has been employed to solve different practical problems such as economic evaluation, planning and design, investment, transportation, marketing, operations research, management science, etc. The objective of MADM is to select the most desirable alternative from a set of alternatives with respect to multiple and often conflicting attributes. During last five years many methodologies [2-7, 10-12, 15-17, 20-24] have been proposed for MADM under neutrosophic environment. Ye [24] studied MADM method by using correlation coefficient of SVNSs. Ye and Zhang [23] proposed similarity measures between SVNSs based on maximum and minimum operators and developed a MADM method based on weighted similarity measures of SVNSs under single valued neutrosophic assessments. Liu and Wang [11] proposed a single valued neutrosophic normalized weighted Bonferroni mean operator for solving multi-attribute group decision making (MAGDM) problem. Broumi and Smarandache [6] proposed a neutrosophic trapezoid linguistic weighted arithmetic averaging aggregation operator and a neutrosophic trapezoid linguistic weighted geometric aggregation operator for MADM problems with single valued neutrosophic assessment. Biswas et al. [5] extended the concept of technique of order preference by similarity to ideal solution (TOPSIS) for solving MADM problems with SVNS information.

Chi and Liu [7] established an extended TOPSIS method for MADM problems where the attribute weights are unknown and attribute values are expressed in terms of INSs. Ye [21] discussed distance-based similarity measures for solving MADM problems with completely unknown weights for decision makers and attributes under single valued neutrosophic environment. Ye [22] proposed an interval neutrosophic linguistic weighted arithmetic average operator and an interval neutrosophic linguistic weighted geometric average operator and developed a method for solving MADM problems with interval neutrosophic linguistic information. Şahin and Karabacak [12] developed a simple inclusion measure for solving MADM problem under interval neutrosophioc environment.

Deng [8] originally developed grey relational analysis (GRA) in order to solve uncertainty problems under discrete data and incomplete information. In the field of neutrosophy, Biswas et al. [3] applied the concept of GRA to formulate an approach for solving MADM problem with SVNS information where the information about attribute weights are fully unknown to the DM. Biswas et al. [2] also studied neutrosophic MADM with unknown weight information using modified GRA.

The main objective of this paper is to extend the concept of GRA to develop a new approach for solving MADM problems under INS information. The attributes are obtained in terms of linguistic variables which can be transformed into INSs. Here, the weights of the attributes are completely unknown and maximizing deviation method [20] is applied in order to determine the unknown attribute weights. Then, virtual positive ideal solution (PIS) and negative ideal solution (NIS) [7] are identified by selecting the best values for each attribute from all alternatives. Finally, neutrosophic grey relational coefficient of each alternative is calculated in order to rank the alternatives.

The remaining part of the paper is constructed as follows: Section 2 presents preliminaries of neutrosophic set and also provides transformation rule between linguistic variables and INSs. Section 3 is devoted to develop an extended GRA method for solving MADM problems. In Section 4, an illustrative example is solved in order to demonstrate the effectiveness of the proposed approach. Finally, the last Section concludes the paper.

2 Preliminaries of Neutrosophic Sets

Neutrosophy [13] is a new branch of philosophy grounded by Smarandache. Neutrosophy is the origin of the concept of neutrosophic set.

2.1 Some Basic Definitions

Definition 2.1. [13] Let *X* be a space of objects with generic element in *X* represented by *x*. Then a NS is defined by

$$\mathrm{A} = \{x, \langle \mathrm{T_A}(x), \mathrm{I_A}(x), \mathrm{F_A}(x) \rangle \mid x \in \mathrm{X}\}$$

Where, $\mathrm{T_A}(x): X \to\ ]0^-, 1^+[$; $\mathrm{I_A}(x): X \to\ ]0^-, 1^+[$; $\mathrm{F_A}(x): X \to\ ]0^-, 1^+[$ are the truth-membership function, indeterminacy-membership function, and falsity-membership function, repectively. It is to be noted that $0^- \le \mathrm{T_A}(x) + \mathrm{I_A}(x) + \mathrm{F_A}(x) \le 3^+$.

Definition 2.2. [19] Let *X* be a universal space of objects with generic element in *X* represented by *x*. Then a SVNS $N \subset X$ is characterized by a truth-membership function $\mathrm{T_N}(x)$, an indeterminacy-membership function $\mathrm{I_N}(x)$, and a falsity-membership function $\mathrm{F_N}(x)$ with $\mathrm{T_N}(x)$, $\mathrm{I_N}(x)$, $\mathrm{F_N}(x) \in [0, 1]$ for each point $x \in X$. Here, it is to be noted that for a SVNS we have, $0 \le \sup \mathrm{T_N}(x) + \sup \mathrm{I_N}(x) + \sup \mathrm{F_N}(x) \le 3$.

Definition 2.3. [18] Consider *X* be a universal space of points with generic element in *X* represented by *x*. Then an INS is defined as follows

$$P = \{x, \langle \mathrm{T_P}(x), \mathrm{I_P}(x), \mathrm{F_P}(x) \rangle \mid x \in X\}$$

Here, $\mathrm{T_P}(x)$, $\mathrm{I_P}(x)$, $\mathrm{F_P}(x)$ are the truth-membership function, indeterminacy-membership function, and falsity-membership function respectively with $\mathrm{T_P}(x)$, $\mathrm{I_P}(x)$, $\mathrm{F_P}(x) \subseteq [0, 1]$ for each point $x \in X$ and $0 \le \sup(\mathrm{T_P}(x)) + \sup(\mathrm{I_P}(x)) + \sup(\mathrm{F_P}(x)) \le 3$. For convenience, we can write $x = ([\mathrm{T^L}, \mathrm{T^U}], [\mathrm{I^L}, \mathrm{I^U}], [\mathrm{F^L}, \mathrm{F^U}])$ and *x* is called interval neutrosophic value (INV).

2.2 The Operational Rules of INS

Consider a = ([T_1^L, T_1^U], [I_1^L, I_1^U],[F_1^L, F_1^U]) and b = ([T_2^L, T_2^U], [I_2^L, I_2^U],[F_2^L, F_2^U]) be two INVs, then the operational definitions [7] are presented as given as follows:

(1) The complement of a is $\bar{a}$ = ([F_1^L, F_1^U], [1- I_1^U, 1- I_1^L],[T_1^L, T_1^U])
(2) a+b = ([$T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U$], [$I_1^L I_2^L, I_1^U I_2^U$],[$F_1^L F_2^L, F_1^U F_2^U$])
(3) a.b = ([$T_1^L T_2^L, T_1^U T_2^U$], [$I_1^L + I_2^L - I_1^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U$],[$F_1^L + F_2^L - F_1^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U$])
(4) ma = ([1 - (1 - T_1^L)^m, 1 - (1 - T_1^U)^m], [(I_1^L)^m, (I_1^U)^m], [(F_1^L)^m, (F_1^U)^m]), m > 0
(5) a^m = ([(T_1^L)^m, (T_1^U)^m], 1 - (1 - I_1^L)^m, 1 - (1 - I_1^U)^m, [1 - (1 - F_1^L)^m, 1 - (1 - F_1^U)^m]), m > 0

Definition [7]. Let *a* = ([T_1^L, T_1^U], [I_1^L, I_1^U],[F_1^L, F_1^U]) and b = ([T_2^L, T_2^U], [I_2^L, I_2^U],[F_2^L, F_2^U]) be two INVs, then the Hamming distance between *a* and *b* is presented as follows.

$$r_H\,(a, b) = 1/6(|T_1^L - T_2^L| + |T_1^U - T_2^U| + |I_1^L - I_2^L| + |I_1^U - I_2^U| + |F_1^L - F_2^L| + |F_1^U - F_2^U|)$$

Definition [7]. Consider a = ([T_1^L, T_1^U], [I_1^L, I_1^U],[F_1^L, F_1^U]) and b = ([T_2^L, T_2^U], [I_2^L, I_2^U],[F_2^L, F_2^U]) be two INVs, then the Euclidean distance between a and b is defined as given below.

$$r_E(a, b) = \sqrt{1/6\left((T_1^L - T_2^L)^2 + (T_1^U - T_2^U)^2 + (I_1^L - I_2^L)^2 + (I_1^U - I_2^U)^2 + (F_1^L - F_2^L)^2 + (F_1^U - F_2^U)^2\right)}$$

Definition [7]. Let *A* = ([T_i^L, T_i^U], [I_i^L, I_i^U],[F_i^L, F_i^U]), (i = 1, 2, ..., m) and B = ([$\hat{T}_i^L, \hat{T}_i^U$], [$\hat{I}_i^L, \hat{I}_i^U$],[$\hat{F}_i^L, \hat{F}_i^U$]),(i = 1, 2, ..., m) be two INSs, then the Hamming distance between *A* and *B* is presented as follows.

$$r_H\,(A, B) = 1/6m \sum_{i=1}^{m} (|T_i^L - \hat{T}_i^L| + |T_i^U - \hat{T}_i^U| + |I_i^L - \hat{I}_i^L| + |I_i^U - \hat{I}_i^U| + |F_i^L - \hat{F}_i^L| + |F_i^U - \hat{F}_i^U|).$$

Definition [7]. Let *A* = ([T_i^L, T_i^U], [I_i^L, I_i^U],[F_i^L, F_i^U]), (i = 1, 2, ..., m) and *B* = ([$\hat{T}_i^L, \hat{T}_i^U$], [$\hat{I}_i^L, \hat{I}_i^U$],[$\hat{F}_i^L, \hat{F}_i^U$]),(i = 1, 2, ..., m) be two INSs, then the Euclidean distance between *A* and *B* is presented as given below.

$$r_E(A,B) = \sqrt{1/6m \sum_{i=1}^{m} \left((T_i^L - \hat{T}_i^L)^2 + (T_i^U - \hat{T}_i^U)^2 + (I_i^L - \hat{I}_i^L)^2 + (I_i^U - \hat{I}_i^U)^2 + (F_i^L - \hat{F}_i^L)^2 + (F_i^U - \hat{F}_i^U)^2\right)}$$

2.3 Transformation between Linguistic Variables and INS

A linguistic variable is a variable whose values are expressed in either words or sentences in a natural language. The rating of the alternatives with respect to certain qualitative attribute can be presented in terms of linguistic variable such as extreme good, very good, good, and medium good, etc. Linguistic variables can be transformed into INSs (see Table 1).

Table 1. Transformation between the linguistic variables and INVs

Linguistic variables	INVs
Extreme good (EG)	([0.95, 1], [0.05, 0. 1], [0, 0.1])
Very good (VG)	([0.75, 0.95], [0.1, 0.15], [0.1, 0.2])
Good (G)	([0.6, 0.75], [0.1, 0.2], [0.2, 0.25])
Medium Good (MG)	([0.5, 0.6], [0.2, 0.25], [0.25, 0.35])
Medium (M)	([0.4, 0.5], [0.2, 0.3], [0.35, 0.45])
Medium low (ML)	([0.3, 0.4], [0.15, 0.25], [0.45, 0.5])
Low (L)	([0.2, 0.3], [0.1, 0.2], [0.5, 0.65])
Very low (VL)	([0.05, 0.2], [0.1, 0.15], [0.65, 0.8])
Extreme low (EL)	([0, 0.05], [0.05, 0. 1], [0.8, 0.95])

3 An Extended GRA Method for Solving MADM Problems based on INS

Consider a MADM problem with p alternatives and q attributes. Let G = $\{g_1, g_2, \ldots, g_p\}$, $(p \geq 2)$ denotes the set of alternatives and H =$\{h_1, h_2, \ldots, h_q\}$, $(q \geq 2)$ represents the set of attributes. Also let W = $\{w_1, w_2, \ldots, w_q\}$ be the weighting vector of the attributes with $\sum_{j=1}^{q} w_j = 1$. w_j (> 0), (j = 1, 2, …, q) reflects the relative importance of the attributes and we assume that w_j, (j = 1, 2, …, q) is completely unknown in the decision making process. The attributes are obtained in linguistic variables, which can be expressed by INSs. In the following steps, we describe the extended GRA method under INSs for ranking the alternatives.

Step 1. Construction of decision matrix

Let the rating of alternative g_i, (i = 1, 2, …, p) with respect to the attribute h_j, (j = 1, 2, …, q) is obtained in terms linguistic variable that can be expressed in terms of INVs by using the Table 1. Then construct the decision matrix $C = [c_{ij}]_{p \times q}$ as follows:

$$C = [c_{ij}]_{p \times q} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1q} \\ c_{21} & c_{22} & \cdots & c_{2q} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ c_{p1} & c_{p2} & \cdots & c_{pq} \end{bmatrix} \tag{1}$$

where $c_{ij}=([T_{ij}^L, T_{ij}^U],[I_{ij}^L, I_{ij}^U],[F_{ij}^L, F_{ij}^U]),(i=1,2,...,p;j=1,2,...,q)$

Step 2. Standardization of decision matrix

To eliminate the influence of different physical dimensions to decision results, we standardize the decision matrix due to Chi and Liu [7]. Suppose the standardized decision matrix $S = [s_{ij}]_{p \times q}$ is presented as follows.

$$S = [s_{ij}]_{p\times q} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1q} \\ s_{21} & s_{22} & \cdots & s_{2q} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ s_{p1} & s_{p2} & \cdots & s_{pq} \end{bmatrix} \quad (2)$$

where $s_{ij} = ([\tilde{T}^L_{ij}, \tilde{T}^U_{ij}], [\tilde{I}^L_{ij}, \tilde{I}^U_{ij}], [\tilde{F}^L_{ij}, \tilde{F}^U_{ij}])$, $(i = 1, 2, ..., p; j = 1, 2, ..., q)$. Here, it is to be noted that

$c_{ij} = s_{ij}$, if j is benefit type of attribute

$c_{ij} = \bar{s}_{ij}$, if j is cost type of attribute,

where $\bar{s}_{ij}$ is the complement of s_{ij}.

Step 3. Determination of the attribute weights

The weights of the attributes are not always known to the DM in the decision making situation. Also, the weights are not equal in general. Since we assume that the weights of the attributes are completely unknown, we apply maximizing deviation method of Yang [20] in order to determine the unknown attribute weights. The method is based on the concept that if the attribute values of all alternatives for a specified attribute have a small deviations, then small weight is provided for this attribute. If the attribute values of all alternatives for a particular attribute have greater deviations, we can offer greater weight for this attribute. However, if the attribute values of all alternatives for a given attribute are equal then the weight of such attribute may be taken as 0.

The deviation values of alternatives G_i to all other alternatives with respect to attribute H_j can be described as $L_{ij}(w_j) = \sum_{t=1}^{p} r(c_{ij}, c_{tj})\, w_j$, then

$$L_j(w_j) = \sum_{i=1}^{p} L_{ij}(w_j) = \sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj}) w_j$$

denotes total deviations of all alternatives to the other alternatives for the attribute H_j.

$$L(w_j) = \sum_{j=1}^{q} L_j(w_j) = \sum_{j=1}^{q}\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj}) w_j$$

denotes the deviation of all attributes for all alternatives to the other alternatives. Then the optimization model is presented as follows:

$$\text{Maximize } L(w_j) = \sum_{j=1}^{q}\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj}) w_j$$

$$\text{Subject to } \sum_{j=1}^{q} w_j^2 = 1,\ w_j \geq 0,\ j = 1, 2, \ldots, q. \quad (3)$$

We can obtain attribute weight [20] as follows:

$$w_j = \frac{\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj})}{\sqrt{\sum_{j=1}^{q}\sum_{i=1}^{p}\sum_{t=1}^{p} r^2(c_{ij}, c_{tj})}}, \; j = 1, 2, \ldots, q. \tag{4}$$

Then, the normalized attribute weight based on the above model is obtained as given below.

$$w_j = \frac{\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj})}{\sum_{j=1}^{q}\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj})}, \; j = 1, 2, \ldots, q. \tag{5}$$

Step 4. Determination of the weighted decision matrix

The weighted decision matrix is constructed as follows:

$$Z = [z_{ij}]_{p\times q} = \begin{bmatrix} w_1 s_{11} & w_2 s_{12} & \cdots & w_q s_{1q} \\ w_1 s_{21} & w_2 s_{22} & \cdots & w_q s_{2q} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ w_1 s_{p1} & w_2 s_{p2} & \cdots & w_q s_{pq} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1q} \\ z_{21} & z_{22} & \cdots & z_{2q} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ z_{p1} & z_{p2} & \cdots & z_{pq} \end{bmatrix},$$

where $z_{ij} = ([\breve{T}_{ij}^{L}, \breve{T}_{ij}^{U}], [\breve{I}_{ij}^{L}, \breve{I}_{ij}^{U}], [\breve{F}_{ij}^{L}, \breve{F}_{ij}^{U}])$, $(i = 1, 2, ..., p;\ j = 1, 2, ..., q)$.

Step 5. Determination of interval PIS and NIS

Chi and Liu [7] defined the interval PIS (p_j^+) and interval NIS (n_j^+) for INS as given below.

$$p_j^+ = ([1, 1], [0, 0], [0, 0]), \; j = 1, 2, \ldots, q \tag{7}$$

$$n_j^- = ([0, 0], [1, 1], [1, 1]), \; j = 1, 2, \ldots, q \tag{8}$$

The virtual interval PIS and interval NIS can also be recognized by determining the best and worst values respectively for each attribute from all alternatives as shown below.

$$p_j^+ = ([\underset{i}{Max}\, \breve{T}_{ij}^{L}, \underset{i}{Max}\, \breve{T}_{ij}^{U}], [\underset{i}{Min}\, \breve{I}_{ij}^{L}, \underset{i}{Min}\, \breve{I}_{ij}^{U}], [\underset{i}{Min}\, \breve{F}_{ij}^{L}, \underset{i}{Min}\, \breve{F}_{ij}^{U}]) \tag{9}$$

$$n_j^+ = ([\underset{i}{Min}\, \breve{T}_{ij}^{L}, \underset{i}{Min}\, \breve{T}_{ij}^{U}], [\underset{i}{Max}\, \breve{I}_{ij}^{L}, \underset{i}{Max}\, \breve{I}_{ij}^{U}], [\underset{i}{Max}\, \breve{F}_{ij}^{L}, \underset{i}{Max}\, \breve{F}_{ij}^{U}]). \tag{10}$$

Step 6. Determination of neutrosophic grey relational coefficient of each alternative from PIS and NIS

The grey relational coefficient of each alternative from PIS is obtained from the following formula:

$$\zeta_{ij}^{+} = \frac{\underset{i}{Min}\underset{j}{Min}\Delta_{ij}^{+} + \sigma \underset{i}{Max}\underset{j}{Max}\Delta_{ij}^{+}}{\Delta_{ij}^{+} + \sigma \underset{i}{Max}\underset{j}{Max}\Delta_{ij}^{+}} \tag{11}$$

where $\Delta_{ij}^{+} = r\,(z_{ij},\ p_j^{+})$, (i = 1, 2, ..., p; j = 1, 2, ..., q).

Also, the grey relational coefficient of each alternative from NIS is obtained from the formula given below:

$$\zeta_{ij}^{-} = \frac{\underset{i}{Min}\underset{j}{Min}\Delta_{ij}^{-} + \sigma \underset{i}{Max}\underset{j}{Max}\Delta_{ij}^{-}}{\Delta_{ij}^{-} + \sigma \underset{i}{Max}\underset{j}{Max}\Delta_{ij}^{-}} \tag{12}$$

where $\Delta_{ij}^{-} = r\,(z_{ij},\ p_j^{-})$, (i = 1, 2, ..., p; j = 1, 2, ..., q).

Here, $\sigma \in [0, 1]$ denotes the the environmental coefficient and it is a free parameter. σ is used to adjust the difference of the relational coefficient. Generally, we set $\sigma = 0.5$ in the decision making circumstances.

Step 7. Determination of degree of neutrosophic grey relational coefficient

The degree of neutrosophic grey relational coefficient of each alternative from PIS and NIS are calculated respectively by using the follwing formula:

$$\zeta_i^{+} = \sum_{j=1}^{q} \zeta_{ij}^{+}\ ;\ \zeta_i^{-} = \sum_{j=1}^{q} \zeta_{ij}^{-}\ , \text{ for } (i = 1, 2, ..., p). \tag{13}$$

Step 8. Determination neutrosophic relative relational degree

The neutrosophic relative relational degree is obtained from the the following equation

$$R_i = \frac{\zeta_j^{+}}{\zeta_j^{+} + \zeta_j^{-}}\ , i = 1, 2, ..., p. \tag{14}$$

Step 9. Ranking the alternatives

Rank the alternatives g_i based on the relative relational degree. The highest value of R_i reflects the most important alternative.

4 An Illustrative Example

A Khadi Institution desires to recruit two most competent weavers g_1, g_2, g_3 from a list of three weavers. In order to identify the key attributes of weaver selection, we interviewed Khadi domain experts of Chak, a Gram Panchayet area of Murshidabad, West Bengal, India. After analyzing the data the seven most important attributes for weaver selection are identified as: skill (h_1), previous experience (h_2), honesty (h_3), physical fitness (h_4), locality of the weaver (h_5), personality (h_6), economic condition of the weaver (h_7) [9]. Here, the seven attributes are benefit type attributes and the weights of the attributes are completely unknown. The Khadi Institution then hire a Khadi expert in order to select the most suitable weaver based on the seven attributes. Generally, the Khadi expert is hired from the locality who knows the weavers strength and weakness very well. The Khadi expert provides linguistic variables to represent the rating of the weavers with respect to the above attributes as shown in the Table 2. Then our objective is to choose the most appropriate weaver based on the proposed approach. In the following steps, we present the proposed approach for weaver selection.

Step 1: We convert the linguistic decision matrix as shown in Table 2 into INVs decision matrix by using Table 1. The decision matrix is constructed as in Table 3.

Step 2: We use Euclidean distance defined in Eq. 5 to obtain r (c_{ij}, c_{tj}), i = t = 1, 2, …, p; j = 1, 2, …, q and we determine the weights of the attributes by using Eq. 7 as follows:

$w_1 = w_2 = 0.096$, $w_3 = w_4 = 0.176$, $w_5 = 0.096$, $w_6 = 0.151$, $w_7 = 0.207$ such that $\sum_{j=1}^{7} w_j = 1$, $w_j \geq 0$, j = 1, 2, …, 7.

Step 3: We determine z_{ij}, i = 1, 2, 3; j = 1, 2, …, 7 by using Eq. 6. The weighted decision matrix is provided in Table 4.

Step 4: The virtual interval PIS (p_j^+) and virtual interval NIS (n_j^+), j = 1, 2, …, 7 are identified as given below.

p_1^+ = ([0.125, 0.25], [0.802, 0.833], [0.802, 0.857]);
p_2^+ = ([0.125, 0.25], [0.802, 0.833], [0.802, 0.857]);
p_3^+ = ([0.216, 0.41], [0.667, 0.716], [0.667, 0.753]);
p_4^+ = ([0.216, 0.41], [0.667, 0.716], [0.667, 0.753]);
p_5^+ = ([0.125, 0.25], [0.802, 0.833], [0.802, 0.857]);
p_6^+ = ([0.129, 0.189], [0.706, 0.784], [0.784, 0.811]);
p_7^+ = ([0.173, 0.249], [0.621, 0.717], [0.717, 0.75]).

n_1^+ = ([0.084, 0.125], [0.802, 0.857], [0.857, 0.875]);
n_2^+ = ([0.084, 0.125], [0.802, 0.857], [0.857, 0.875]);
n_3^+ = ([0.115, 0.149], [0.753 0.783], [0. 783, 0.831]);
n_4^+ = ([0.115, 0.149], [0.753 0.783], [0. 783, 0.831]);
n_5^+ = ([0.084, 0.125], [0.802, 0.857], [0.857, 0.875]);
n_6^+ = ([0.074, 0.099], [0.784, 0.838], [0.853, 0.886]);
n_7^+ = ([0.071, 0.1], [0.717, 0.75], [0.848, 0.866]).

Step 5: The interval neutrosophic grey relational coefficient of each alternative from virtual PIS and NIS are calculated respectively by using the Eq.9 and Eq. 10 as follows:

$$\zeta_{ij}^{+} = \begin{bmatrix} 0.000 & 0.000 & 0.135 & 0.135 & 0.059 & 0.000 & 0.065 \\ 0.059 & 0.059 & 0.000 & 0.053 & 0.059 & 0.029 & 0.019 \\ 0.000 & 0.059 & 0.053 & 0.000 & 0.000 & 0.071 & 0.114 \end{bmatrix}$$

$$\zeta_{ij}^{-} = \begin{bmatrix} 1.000 & 1.000 & 0.333 & 0.333 & 0.534 & 1.000 & 0.509 \\ 0.534 & 0.534 & 1.000 & 0.560 & 0.534 & 0.699 & 0.780 \\ 1.000 & 0.534 & 0.560 & 1.000 & 1.000 & 0.487 & 0.372 \end{bmatrix}$$

Step 6: The degree or grade of interval neutrosophic grey relational coefficient of each alternative from PIS and NIS are obtained respectively by using the Eq. 11and Eq. 12 as given below.

$\zeta_1^{+} = 0.394,\ \zeta_2^{+} = 0.278,\ \zeta_3^{+} = 0.297;\ \zeta_1^{-} = 4.709,\ \zeta_2^{-} = 4.641,\ \zeta_3^{-} = 4.953$

Step 7: The interval neutrosophic relative relational degree is obtained as follows:

$$R_1 = = 0.077209,\ R_2 = = 0.056516,\ R_3 = = 0.056571$$

Finally, we rank the order of all alternatives according to the decending order of R_i as:

$$R_1 > R_3 > R_2$$

So, g_1, g_3 are the most suitable weavers for Khadi Institution.

5. Conclusion

In this paper, we have developed an alternative method for MADM problems with unknown weight information under interval neutrosophic environment. The attributes with respect to certain alternative are represented by linguistic variables rather than numerical values and the linguistic variables are expressed in terms of interval valued neutrosophic set. The unknown weights of the attributes are obtained by using maximizing deviation method. Then modified GRA method is proposed to solve the MADM problems. Finally, an illustrative numerical example for weaver selection in Khadi Institution is demonstrated to show the applicability of the proposed method. The authors hope that the proposed method can be effective for solving practical decision making problems such as pattern recognition, databases, medical diagnosis, decision making, etc.

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ON SOFT $b-I-$OPEN SETS WITH RESPECT TO SOFT IDEAL

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Abstaract — The aim of this work is to present and learning a novel set of soft I-open sets, namel soft b-open sets and acquire some of their features. Then debated the relations among soft semi-I-open sets, soft pre-I-open sets, soft $\beta-I$-open sets and soft $b-I$-open sets. We also researched the new notions of soft $b-I$-continuous functions and soft $b-I$-open (soft $b-I$-closed) functions.

Keywords — *Soft sets, soft $b-I-$open sets, soft $b-I-$closed sets, soft $b-I-$continuity, soft $b-I$-open functions, soft ideal.*

1 Introduction

Kuratowski [1] studied and introduced the concept of ideal topological spaces. The concept of I-open sets in topological spaces was presented by Jankovic and Hamlet [2], which formed via ideals. And in 1999, a Russian scientist Molodtsov [3] introduced the concept of soft sets. He excellently implemented the soft set theory. Later, Maji et al. [4,5] defined some operations on soft sets. On the other hand, Aktas and Cagman [6] compared soft sets with fuzzy sets and rough sets. Chang [7] studied the topological structures of set theories dealing with ambiguities first time. Then, Shabir and Naz [8] presented the concept of soft topological spaces that are described over an original universe with a fixed set of parameters. Additionally the soft separation axioms were presented for soft topological spaces by Shabir and Naz [8]. Zorlutuna et al.[9] presented the notion of soft continuity of functions and some of its features were studied. Then Aygunoglu and Aygun [10] continued to study continuous soft functions. Lately, Kharal and Ahmad [11] defined the concept of a function on soft grades and reviewed several features of images and reverse images of soft sets. Furthermore, these concepts were applied in medical by they Akdag and Ozkan [12] introduced the soft b-sets and soft b-continuous functions. Then the

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definition of soft ideal was gave by Kale and Guler [13] and they also presented the features of soft ideal topological space. Furthermore, the notion of soft I-regularity and soft I-normality were introduced by they. Later, Akdag and Erol [14] defined soft I-open sets and soft I-continuity of functions. They [15] also defined soft semi I-open sets and soft semi I-continuity of functions.

The aim of this work is to acquaint the notion of soft $b-I$-open sets, soft $b-I$-continuous functions, soft $b-I$-open functions and soft $b-I$-closed functions and to get some characterizations and fundamental features of this sets and functions. We debated the intercourses soft semi I-open sets, soft pre I-open sets, soft $\beta-I$-open sets and soft $b-I$-open sets. We also studied the relationships among soft $b-I$-continuous functions, soft semi-I-continuous functions, soft pre$-I$-continuous functions and soft $\beta-I$-continuous functions.

2 Preliminaries

In the valid part we will shortly recollection some fundamental descriptions and lemmas for soft sets.

Definition 1. [3] *Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A soft set F_A on the universe X is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) : e \in E, f_A(e) \in P(X)\}$, where $f_A : E \to P(X)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here, f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.*

Note that the set of all soft sets over X will be denoted by $S(X)$.

Definition 2. [4] *Let F_A and F_B be soft sets over a common universe X. Then F_A is said to be a soft subset of F_B if $f_A(e) \subset f_B(e)$, for all $e \in A$ and this relation is denoted by $F_A \widetilde{\subseteq} F_B$. Also, F_A is said to be a soft equal to F_B if $f_A(e) = f_B(e)$, for all $e \in A$ and this relation is denoted by $F_A = F_B$.*

Definition 3. [19] *The complement of a soft set F_A denoted by F_A^c is defined by $f_A^c : A \to P(X)$ is a mapping given by $f_A^c(e) = X - f_A(e)$, $\forall e \in A$. f_A^c is called the soft complement function of f_A. Clearly, $(f_A^c)^c$ is the same as f_A and $((F_A)^c)^c = F_A$.*

Definition 4. [4] *A soft set F_A over X is said to be a null soft set denoted by $\widetilde{\emptyset}$, if $\forall e \in A$, $f_A(e) = \emptyset$.*

Definition 5. [4] *A soft set F_A over X is said to be an absolute soft set denoted by $\widetilde{X}$, if $\forall e \in E$, $f_A(e) = X$.*

Clearly, $\widetilde{X}^c = \widetilde{\emptyset}$ and $\widetilde{\emptyset}^c = \widetilde{X}$.

Definition 6. [9] *The soft set F_A is called a soft point if there exists a $x \in X$ and $A \subset E$ such that $f_A(e) = \{x\}$, for all $e \in A$ and $f_A(e) = \emptyset$;, for all $e \in E - A$. A soft point is denoted by F_A^x .The soft point F_E^x is called absolute soft point. A soft point F_A^x is said to belong to a soft set G_B if $x \in g_B(e)$, for each $e \in A$, and symbolically denoted by $F_A^x \widetilde{\in} G_B$.*

Definition 7. [3] *The union of two soft sets of F_A and G_B over the common universe X is the soft set H_C, where $C = A \cup B$ and for all $e \in C$,*

$$h_C(e) = \begin{cases} f_A(e), & \text{if } e \in A - B, \\ g_B(e), & \text{if } e \in B - A, \\ f_A(e) \cup g_B(e), & \text{if } e \in A \cup B. \end{cases}$$

We write $F_A \tilde{\cup} G_B = H_C$.

Definition 8. [3] *The intersection of two soft sets F_A and G_B over the common universe X is the soft set H_C, where $C = A \cap B$ and for all $e \in C$, $h_C(e) = f_A(e) \cap g_B(e)$. This relationship is written as $F_A \tilde{\cap} G_B = H_C$.*

Definition 9. [20] *Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if,*
(a) $\tilde{\emptyset}$, $\tilde{X} \tilde{\in} \tau$
(b) the intersection of any two soft sets in τ belongs to τ
(c) the union of any number of soft sets in τ belongs to τ.

The triple (X, τ, E) is called a soft topological space over X. Every member of τ is called soft open in (X, τ, E). If complement of any soft set belongs to τ, then it is called soft closed set (X, τ, E).

Definition 10. [20, 9] *Let (X, τ, A) be a soft topological space over X and F_A be a soft set over X. The soft closure of F_A denoted by $cl(F_A)$ is the intersection of all closed soft super sets of F_A. The soft interior of F_A denoted by $int(F_A)$ is the union of all open soft subsets of F_A.*

Definition 11. [8] *A soft set F_A in a soft topological space (X, τ, A) is called a soft neighborhood (briefly: nbd) of the soft point $x_G \tilde{\in} \tilde{X}$ if there exists a soft open set H_A such that $x_G \tilde{\in} H_A \tilde{\subset} F_A$.*

Definition 12. [8] *Let F_A be a soft set over X and Y be a nonempty subset of X. Then the sub soft set of F_A over Y denoted by ${}^Y F_A$ is defined as ${}^Y F_A(e) = Y \cap f_A(e)$, for each $e \in A$. In other word ${}^Y F_A = \tilde{Y} \tilde{\cap} F_A$.*

Definition 13. [13] *A soft ideal I is a nonempty collection of soft sets over X if;*
(a) $F_A \tilde{\in} I$, $G_A \tilde{\subset} F_A$ implies $G_A \tilde{\in} I$.
(b) $F_A \tilde{\in} I$, $G_A \tilde{\in} I$ implies $F_A \tilde{\cup} G_A \tilde{\in} I$.
A soft topological space (X, τ, A) with a soft ideal I called soft ideal topological space and denoted by (X, τ, A, I).

Definition 14. [13] *Let F_A be a soft set in a soft ideal topological space (X, τ, A, I) and $(.)^*$ be a soft operator from $S(X)$ to $S(X)$. Then the soft local mapping of F_A defined by $F_A^*(I, \tau) = \left\{ F_A^x : X_A \tilde{\cap} F_A \tilde{\notin} I \text{ for every } X_A \tilde{\in} \nu(F_A^x) \right\}$ denoted by F_A^* simply.*

Lemma 1. [13] *Let (X, τ, A, I) be a soft ideal topological space and F_A, G_A be two soft sets. Then*
(a) $F_A \tilde{\subset} G_A$ implies $F_A^* \tilde{\subset} G_A^*$ and $(F_A \tilde{\cup} G_A)^* = F_A^* \tilde{\cup} G_A^*$.
(b) $F_A^* \tilde{\subset} cl(F_A)$ and $(F_A^*)^* \tilde{\subset} F_A^*$.
(c) If F_A is soft open $F_A \tilde{\cap} G_A \tilde{\in} I$ implies $F_A \tilde{\cap} G_A^* = \tilde{\emptyset}$
(d) F_A^* is soft closed.
(e) If F_A is soft closed then $F_A^* \tilde{\subset} F_A$.

Definition 15. [13] *Let* (X, τ, A, I) *be a soft ideal topological space. The soft set operator* cl^* *is called a soft*$^*-$*closure and is defined as* $cl^*(F_A) = F_A \widetilde{\cup} F_A^*$ *for a soft subset* F_A.

Proposition 1. [13] *Let* (X, τ, A, I) *be a soft ideal topological space and* F_A, G_A *be two soft sets. Then*

(a) $cl^*\left(\widetilde{\emptyset}\right) = \widetilde{\emptyset}$ *and* $cl^*\left(\widetilde{X}\right) = \widetilde{X}$.
(b) $F_A \widetilde{\subset} cl^*(F_A)$ *and* $cl^*(cl^*(F_A)) = cl^*(F_A)$.
(c) *If* $F_A \widetilde{\subset} G_A$ *then* $cl^*(F_A) \widetilde{\subset} cl^*(G_A)$.
(d) $cl^*(F_A) \widetilde{\cup} cl^*(G_A) = cl^*\left(F_A \widetilde{\cup} G_A\right)$.

Lemma 2. [13] *Let* (X, τ, A, I) *be a soft ideal topological space.*

(a) *If* $I = \left\{\widetilde{\emptyset}\right\}$, *then* $F_A^* = cl(F_A)$
(b) *If* $I = S(X)$, *then* $F_A^* = \widetilde{\emptyset}$.

Definition 16. [11] *Let* X_E *and* Y_K *be soft classes. Let* $u : X \to Y$ *and* $p : E \to K$ *be mappings. Then a mapping* $f : X_E \to Y_K$ *is defined as: for a soft set* F_A *in* X_E, $(f(F_A), B)$, $B = p(A) \subset K$ *is a soft set in* Y_K *given by* $f(F_A)(\beta) = u\left(\underset{\alpha \in p^{-1}(\beta) \cap A}{\cup f(\alpha)}\right)$ *for* $\beta \in K$. $(f(F_A), B)$ *is called a soft image of a soft set* F_A. *If* $B = K$, *then we shall write* $(f(F_A, K)$ *as* $f(F_A)$.

Definition 17. [11] *Let* $f : X_E \to Y_K$ *be a mapping from a soft class* X_E *to another soft class* Y_K, *and* G_C *a soft set in soft class* Y_K, *where* $C \subset K$. *Let* $u : X \to Y$ *and* $p : E \to K$ *be mappings. Then* $(f^{-1}(G_C), D)$, $D = p^{-1}(C)$ *is a soft set in the soft classes* X_E *defined as:* $f^{-1}(G_C)(\alpha) = u^{-1}(g(p(\alpha)))$ *for* $\alpha \in D \subset E$. $(f^{-1}(G_C), D)$ *is called a soft inverse image of* G_C. *Hereafter we shall write* $(f^{-1}(G_C), E)$ *as* $f^{-1}(G_C)$.

3 Soft $b - I$-Open Sets and Soft $b - I$-Closed Sets

Definition 18. *Let* (X, τ, A, I) *be a soft ideal topological space and a soft subset* F_A *in* X. *Then* F_A *is said;*

(a) [15] *soft semi-*$I-$*open set if* $F_A \widetilde{\subset} cl^*(int(F_A))$.
(b) *soft pre-*$I-$*open set if* $F_A \widetilde{\subset} int(cl^*(F_A))$.
(c) *soft* $\beta - I$*-open set if* $F_A \widetilde{\subset} cl(int(cl^*(F_A)))$.
(d) *soft* $b - I$*-open set if* $F_A \widetilde{\subset} cl^*(int(F_A)) \widetilde{\cup} int(cl^*(F_A))$.

By $SIO(X, \tau, A, I)$ (resp. $SSIO(X, \tau, A, I)$, $SPIO(X, \tau, A, I)$, $SbIO(X, \tau, A, I)$, $S\beta IO(X, \tau, A, I)$) we denote the family of all soft $I-$open (resp. soft semi-$I-$open, soft pre-$I-$open, soft $b - I-$open, soft $\beta - I-$open) sets of a soft topological space (X, τ, A, I).

Remark 1. *In following example indicatesed that every soft semi–*$I-$*open set is soft* $b - I-$*open set but the reverse is generally not true.*

Example 1. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}\right\}$ *and* $I = \left\{\widetilde{\emptyset}\right\}$.
Then $F_A = \{(e_1, \{h_1\})\}$ *is soft* $b - I-$*open set but is not soft semi–*I*-open set.*

Remark 2. *In following example shown that every soft pre−I-open set is soft $b-I$-open set but the inverse is usually not true.*

Example 2. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$ *and*
$\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_{A_1}, F_{A_2}, F_{A_3}\}$, *where*
$F_{A_1}, F_{A_2}, F_{A_3}$ *are soft sets over* X, *defined as follows:*
$F_{A_1} = \{(e_1, \{h_1\})\}$,
$F_{A_2} = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}$,
$F_{A_3} = \{(e_1, X), (e_2, \{h_2\})\}$,
Then τ *defines a soft topology on* X, *and thus* (X, τ, A, I) *is a soft ideal topological space, where* $I = \left\{\widetilde{\emptyset}\right\}$.
Then $F_A = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$;
is soft $b-I$*-open set but not soft pre-*I*-open set.*

Remark 3. *In following example shown that every soft* $b-I$*-open set is soft* $\beta-I$*-open set but the inverse is usually not true.*

Example 3. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\right\}$ *and* $I = \left\{\widetilde{\emptyset}\right\}$.
Then $F_A = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$ *is soft* $\beta-I-$*open set but is not soft* $b-I$*-open set.*

Remark 4. *In following example shows that every soft open set is soft* $b-I-$*open set but not usually reverse.*

Example 4. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}\right\}$ *and* $I = \left\{\widetilde{\emptyset}\right\}$.
Then $F_A = \{(e_1, \{h_2\})\}$ *is soft* $b-I-$*open set but is not soft-open set.*

Definition 19. [12] *A soft subset* F_A *of a soft topological space* (X, τ, A) *is said to be soft b-open set if* $F_A \widetilde{\subset} cl(int(F_A)) \widetilde{\cup} int(cl(F_A))$.

The collection of all soft $b-$open sets in (X, τ, A) is denoted $SbO(X)$.

Remark 5. *In following example shows that every soft* $b-I-$*open set is soft* $b-$*open set but not usually reverse.*

Example 5. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\right\}$ *and* $I = S(X)$.
Then $F_A = \{(e_1, X), (e_2, \{h_1\})\}$ *is soft* $b-I-$*open set but is not soft* $b-$*open set.*

Definition 20. *A soft subset* F_A *of a soft ideal topological space* (X, τ, A, I) *is said to be soft***-perfect if* $F_A = F_A^*$

Theorem 1. *For a soft subset* F_A *of a soft ideal topological space* (X, τ, A, I) *the following are true:*
(a) If $I = \{\widetilde{\emptyset}\}$ *and* F_A *is soft pre-open set, then* F_A *is soft* $b-I$*-open set.*
(b) If $I = S(X)$ *and* F_A *is soft* $b-I$*-open set, then* F_A *is soft-open set.*
(c) If F_A *is soft***-perfect and* F_A *is soft* $b-I$*-open set, then* F_A *is soft semi-*$I-$*open set.*

Proof. (a) Let F_A be a soft pre-open set. By Lemma 2, since $I = \{\widetilde{\emptyset}\}$, then $F_A^* = cl(F_A)$.
Therefore, $F_A \widetilde{\subset} int(cl(F_A)) = int(F_A^*) = int(cl^*(F_A))$
$\widetilde{\subset} int(cl^*(F_A)) \widetilde{\cup} cl^*(int(F_A))$. Hence F_A is soft $b-I$-open set.
(b) Let F_A be a soft $b-I$-open set. By Lemma 2, since
$I = S(X)$, then $F_A^* = \left\{\widetilde{\emptyset}\right\}$ and $cl^*(F_A) = F_A^* \widetilde{\cup} F_A = F_A$.
Thus $F_A \widetilde{\subset} cl^*(int(F_A)) \widetilde{\cup} int(cl^*(F_A))$
$= int(F_A) \widetilde{\cup} int(F_A) = int(F_A)$. Hence $F_A = int(F_A)$.
Therefore F_A is soft open set.
(c) Let F_A is soft*-perfect then $cl^*(F_A) = F_A \widetilde{\cup} F_A^* = F_A$.
Since F_A is soft $b-I$-open set then $F_A \widetilde{\subset} int(cl^*(F_A)) \widetilde{\cup} cl^*(int(F_A))$
$= int(F_A) \widetilde{\cup} int(F_A \widetilde{\cup} (intF_A)^*) = int(F_A) \widetilde{\cup} (intF_A)^* = cl^*(int(F_A))$.
Thus F_A is soft semi-I-open set. □

Proposition 2. *The union of two soft $b-I-$open sets in a soft ideal topological space (X, τ, A, I) is soft $b-I$-open set.*

Proof. Let F_A and G_A be two soft $b-I-$open sets. Then
$F_A \widetilde{\cup} G_A \widetilde{\subset} [cl^*(int(F_A)) \widetilde{\cup} int(cl^*(F_A))] \widetilde{\cup}$
$[cl^*(int(G_A)) \widetilde{\cup} int(cl^*(G_A)]$
$= [cl^*(int(F_A)) \widetilde{\cup} cl^*(int(G_A))] \widetilde{\cup}$
$[int(cl^*(F_A)) \widetilde{\cup} int(cl^*(G_A))]$
$\widetilde{\subset} cl^*[int(F_A) \widetilde{\cup} int(G_A)] \widetilde{\cup} int[cl^*(F_A) \widetilde{\cup} cl^*(G_A)]$
$\widetilde{\subset} cl^*(int[F_A \widetilde{\cup} (G_A)]) \widetilde{\cup} int(cl^*[F_A \widetilde{\cup} G_A])$.
Thus $F_A \widetilde{\cup} G_A$ is soft $b-I$-open set. □

Conclusion 1. *Let $\{(F_{A_i}) : i \in \Delta\}$ be a family of soft $b-I-$open sets. Then $\widetilde{\cup}_{i\in\Delta} (F_{A_i})$ is soft $b-I-$open set.*

Proof. Let $\{(F_{A_i})\}$ be a family of soft $b-I-$open sets. Then for each i,
$(F_{A_i}) \widetilde{\subset} cl^*(int(F_{A_i})) \widetilde{\cup} int(cl^*(F_{A_i}))$. Now
$\widetilde{\cup} (F_{A_i}) \widetilde{\subset} \widetilde{\cup} [cl^*(int((F_{A_i}))) \widetilde{\cup} cl^*(int(F_{A_i})]$
$\widetilde{\subset} [cl^*(int(\widetilde{\cup}(F, A)_\alpha))) \widetilde{\cup} int(cl^*(\widetilde{\cup}(F, A)_\alpha)))]$.
Therefore $\widetilde{\cup}_{i\in\Delta} (F_{A_i})$ is a soft $b-I-$open set. □

Remark 6. *The intersection of two soft $b-I-$open sets in a soft ideal topological space (X, τ, A, I) is not soft $b-I$-open in general as shown by the following example.*

Example 6. *Let $X = \{h_1, h_2\}$, $A = \{e_1, e_2, e_3\}$,*
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\})\}, \{(e_2, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\right\}$
and $I = \left\{\widetilde{\emptyset}\right\}$. Then $F_A = \{(e_1, \{h_1\}), (e_3, \{h_3\})\}$ and
$G_A = \{(e_2, \{h_2\}), (e_3, \{h_3\})\}$ are soft $b-I-$open sets
but $F_A \widetilde{\cap} G_A = \{(e_3, \{h_3\})\}$is not soft $b-I$-open set.

Theorem 2. *Let F_A and G_A be two soft subsets in a soft ideal topological space (X, τ, A, I). Then the following statements are hold:*

(a) If F_A is soft $b-I-$open set and G_A is soft open set then $F_A \widetilde{\cap} G_A$ is soft $b-I-$open set.

(b) If F_A is soft $b-I-$open set and G_A is soft $\alpha-I-$open set then $F_A \widetilde{\cap} G_A$ is soft $b-I-$open set.

Proof. (a) Let F_A is soft $b-I-$open set and G_A is soft open set, then

$F_A\widetilde{\cap}G_A\widetilde{\subset}[cl^*(int(F_A))\widetilde{\cup}int(cl^*(F_A))]\widetilde{\cap}G_A$
$= [cl^*(int(F_A))\widetilde{\cap}G_A]\widetilde{\cup}[int(cl^*(F_A))\widetilde{\cap}G_A]$
$= [(int(F_A)\widetilde{\cup}(int(F_A))^*)\widetilde{\cap}G_A]\widetilde{\cup}[int(F_A\widetilde{\cup}F_A^*)\widetilde{\cap}G_A]$
$= [(intF_A\widetilde{\cap}G_A)\widetilde{\cup}((int(F_A))^*\widetilde{\cap}G_A)]\widetilde{\cup}$
$[int(F_A\widetilde{\cup}G_A)^*)\widetilde{\cap}int(F_A)]$
$\widetilde{\subset}[(int(F_A)\widetilde{\cap}G_A)\widetilde{\cup}(int(F_A)\widetilde{\cap}G_A)^*]\widetilde{\cup}$
$int[(F_A\widetilde{\cup}F_A^*)\widetilde{\cap}G_A]$
$= [(int(F_A)\widetilde{\cap}int(G_A))\widetilde{\cup}(int(F_A)\widetilde{\cap}int(G_A))^*]\widetilde{\cup}$
$int[(F_A\widetilde{\cap}G_A)\widetilde{\cup}(F_A^*\widetilde{\cap}G_A)]$
$\widetilde{\subset}[int(F_A\widetilde{\cap}G_A)\widetilde{\cup}(int(F_A\widetilde{\cap}G_A))^*]\widetilde{\cup}$
$int[(F_A\widetilde{\cap}G_A)\widetilde{\cup}(F_A\widetilde{\cap}G_A)^*]$
$= cl^*(int(F_A\widetilde{\cap}G_A))\widetilde{\cup}int(cl^*(F_A\widetilde{\cap}G_A))$.

This shows that $F_A\widetilde{\cap}G_A$ soft $b-I-$open set.

(b) Straightforward. □

Definition 21. *Let F_A be a soft subset in a soft ideal topological space (X,τ,A,I). F_A is said to be soft $b-I$-closed set if F_A^c is soft $b-I$-open set.*

The collection of all soft $b-I-$closed sets subsets in (X,τ,A,I) will be denoted by $SbIC(X)$.

Theorem 3. *Let F_A be to a subset of a soft ideal topological space (X,τ,A,I). If F_A is soft $b-I$-closed set, then $cl^*(int(F_A))\widetilde{\cap}int(cl^*(F_A))\widetilde{\subset}F_A$.*

Proof. Since F_A is soft $b-I$-closed set, then $\widetilde{X}-F_A$ is soft $b-I$-open set in X. Thus,

$\widetilde{X}-F_A\widetilde{\subset}cl^*(int(\widetilde{X}-F_A))\widetilde{\cup}int(cl^*(\widetilde{X}-F_A))$
$\widetilde{\subset}cl(int(\widetilde{X}-F_A))\widetilde{\cup}int(cl(\widetilde{X}-F_A))$
$= (\widetilde{X}-(int(cl(F_A))))\widetilde{\cup}(\widetilde{X}-(cl(int(F_A))))$
$\widetilde{\subset}(\widetilde{X}-int(cl^*(F_A)))\widetilde{\cup}(\widetilde{X}-(cl^*(int(F_A))))$.

Hence we obtain $cl^*(int(F_A))\widetilde{\cap}int(cl^*(F_A))\widetilde{\subset}F_A$. □

Remark 7. *For soft subset F_A of a soft ideal topological space (X,τ,A,I) we have $\widetilde{X}-int(cl^*(F_A))\neq cl^*(int(\widetilde{X}-F_A))$ as seen in the following example.*

Example 7. *Let $X=\{h_1,h_2\}$, $A=\{e_1,e_2,e_3\}$,*

$\tau=\left\{\widetilde{\emptyset},\widetilde{X},\{(e_1,\{h_1\}),(e_2,\{h_2\})\},\{(e_1,\{h_2\}),(e_2,\{h_1\})\}\right\}$ *and* $I=S(X)$.

For $F_A=\{(e_1,\{h_1\}),(e_2,\{h_1\}),(e_3,X)\}$ *we have*

$\widetilde{X}-int(cl^*(F_A))=\{(e_1,\{h_2\}),(e_2,\{h_1\}),(e_3,X)\}$

but $cl^*(int(\widetilde{X}-F_A))=\widetilde{\emptyset}$.

Corollary 1. *Let F_A be a soft subset of a soft ideal topological space (X,τ,A,I) such that $\widetilde{X}-int(cl^*(F_A))=cl^*(int(\widetilde{X}-F_A))$. Then F_A is soft $b-I$-closed set if and only if $cl^*(int(F_A))\widetilde{\cup}int(cl^*(F_A)\widetilde{\subset}F_A$.*

Corollary 2. *In a soft ideal topological space (X,τ,A,I) the following statements are hold:*

(a) If F_A soft $b-I$-closed set and G_A soft open set, then $F_A\widetilde{\cup}G_A$ soft $b-I$-closed set.

(b) If F_A soft $b-I$-closed set and G_A soft $\alpha-I$-closed set, then $F_A\widetilde{\cup}G_A$ soft $b-I$-closed set.

Proof. It is obvious from Theorem 2. □

4 Soft $b - I-$Continuous Functions

Definition 22. *Let $f : (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ be a soft mapping. If $f^{-1}(G_B)$ is soft $b - I-$open set in (X, τ, E, I) for each soft open set G_B of (Y, σ, K), then f is called soft $b - I-$continuous function.*

Definition 23. *Let $f : (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ be a soft mapping.If $f^{-1}(G_B)$ is soft $\beta - I-$open set in (X, τ, E) for each soft open set (G_B) of (Y, σ, K), then f is called soft $\beta - I-$continuous function.*

Definition 24. [15] *A soft mapping $f : (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ is called soft semi$-I-$continuous if $f^{-1}(G_B)$ is soft semi$-I-$open set in (X, τ, E) for each soft open set G_B of (Y, σ, K).*

Definition 25. *Let $f : (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ be a soft mapping. If $f^{-1}(G_B)$ is soft pre$-I-$open set in (X, τ, E) for each soft open set G_B of (Y, σ, K), then f is called soft pre$-I-$continuous function.*

We can write the following results from the above descriptions.

Corollary 3. *Let $f : (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ be a soft function. Then:*
(a) If f is soft $b - I$-continuous, then f is soft $\beta - I-$continuous.
(b) If f is soft semi$-I$-continuous, then f is soft $b - I-$continuous.
(c) If f is soft pre$-I$-continuous, then f is soft $b - I-$continuous.

Not that the converses is not true in general. As the following examples shown.

Example 8. *Let $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,*
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\right\}$, $I = \left\{\widetilde{\emptyset}\right\}$ *and*
$F_A = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$. *Moreover, let $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$,*
$\sigma = \left\{\widetilde{\emptyset}, \widetilde{Y}, H_K\right\}$, *where $H_K = \{(k_1, \{y_1\}), (k_2, \{y_1\})\}$.*
Then $f : (X, \tau, A, I) \rightarrow (Y, \sigma, K)$ denoted by
$u(h_1) = y_1$, $u(h_2) = y_2$, $p(e_1) = k_1$, $p(e_2) = k_2$ *is soft $\beta - I-$continuous function but is not soft $b - I$-continuous function Because, for soft open set H_K in Y, $f^{-1}(H_K) = F_A$ is soft $\beta - I-$open set but is not soft $b - I$-open set.*

Example 9. *Let $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,*
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}\right\}$, $I = \left\{\widetilde{\emptyset}\right\}$
and $F_A = \{(e_1, \{h_1\})\}$. In addition to, let $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$, $\sigma = \left\{\widetilde{\emptyset}, \widetilde{Y}, H_K\right\}$,
where $H_K = \{(k_1, \{y_1\})\}$ .Then $f : (X, \tau, A, I) \rightarrow (Y, \sigma, K)$
denoted by $u(h_1) = y_1$, $u(h_2) = y_2$, $p(e_1) = k_1$, $p(e_2) = k_2$ is soft $b-I-$continuous function
but is not soft semi$-I$-continuous function. Because, for soft open set H_K in Y, $f^{-1}(H_K) = F_A$ is soft $b - I-$open set but is not soft semi$-I$-open set.

Example 10. *Let F_A be a soft set of a soft ideal topological space (X, τ, E, I) as in Example 2.*
Moreover, let $Y = \{y_1, y_2, y_3, y_4\}$, $K = \{k_1, k_2, k_3\}$, $\sigma = \left\{\widetilde{\emptyset}, \widetilde{Y}, H_K\right\}$, where
$H_K = \{(k_1, \{y_2, y_4\}), (k_2, \{y_1, y_3\}), (k_3, \{y_1, y_3, y_4\})\}$.

Then $f : (X, \tau, A, I) \to (Y, \sigma, K)$ *denoted by* $u(h_i) = y_i$, $p(e_j) = k_j$ *(for* $1 \leq i \leq 3$, $1 \leq j \leq 4$.*) is soft* $b - I-$*continuous function but is not soft semi*$-I$*-continuous function.*

Because, for soft open set H_K *in* Y, $f^{-1}(H_K) = F_A$ *is soft* $b - I-$*open set but is not soft semi*$-I$*-open set.*

Definition 26. [12] *Let* $f : (X, \tau, E, I) \to (Y, \sigma, K)$ *be a soft function. Then* f *is said to be soft b-continuous if* $f^{-1}(G_B)$ *is soft b-open set in* (X, τ, E) *for each soft open set* G_B *of* (Y, σ, K).

Remark 8. *It is clear that soft* $b - I$*-continuity implies soft b-continuity. But the converse is not true in general as shown by the following example:*

Example 11. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, \{(e_1, \{h_1\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\right\}$, $I = S(X)$ *and*
$F_A = \{(e_1, X), (e_2, \{h_1\})\}$. *Moreover, let* $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$, $\sigma = \left\{\widetilde{\emptyset}, \widetilde{Y}, H_K\right\}$,
where $H_K = \{(k_1, Y), (k_2, \{y_1\})\}$ *.Then* $f : (X, \tau, A, I) \to (Y, \sigma, K)$
denoted by $u(h_1) = y_1$, $u(h_2) = y_2$, $p(e_1) = k_1$, $p(e_2) = k_2$ *is soft* $b-I-$*continuous function*
but is not soft b-continuous function. Because, for soft open set H_K *in* Y, $f^{-1}(H_K) = F_A$
is soft $b - I-$*open set but not soft* $b-$*open set.*

Theorem 4. *Let* $f : (X, \tau, A, I) \longrightarrow (Y, \sigma, K)$ *be a soft function.* f *is soft* $b - I$*-continuous function if and only if then for each soft point* F_A^x *in* X *and each soft open set* V_K *in* Y *containing* $f(F_A^x)$ *there exists soft* $b - I$ *open set* G_A *containing* F_A^x *such that* $f(G_A)\widetilde{\subset}V_K$.

Proof. $\Rightarrow$: Let F_A^x be a soft point in X and V_K be soft open set in Y containing $f(F_A^x)$. Set $G_A = f^{-1}(V_K)$, then since f is soft $b - I$-continuous function, then G_A is soft $b - I$-open set containing F_A^x and $f(G_A)\widetilde{\subset}V_K$.

$\Leftarrow$: Let V_K be any soft open set in Y containing $f(F_A^x)$. Then by hypothesis there exists G_A soft $b - I$-open set such that $f(G_A)\widetilde{\subset}V_K$ and hence $G_A\widetilde{\subset}f^{-1}(V_K)$. Let $G_A = f^{-1}(V_K)$. Therefore $f^{-1}(V_K)$ is soft $b - I$-open set. This shows that f is soft $b - I$-continuous function. □

Theorem 5. *Let* $f : (X, \tau, E, I) \to (Y, \sigma, K)$ *be a soft function. If* f *is soft* $b - I$*-continuous function, then for each* $F_A^x \in X$ *the graph function* $g : X \to X \times Y$, *defined by* $g(F_A^x) = (F_A^x, f(F_A^x))$ *is soft* $b - I$*-continuous function.*

Proof. $\Rightarrow$: Let f is soft $b - I$-continuous function and $F_A^x \in X$ and $W_{A\times B}$ be any open set of $X \times\ Y$ containing $g(F_A^x)$. Then there exists a funtametal open set $U_A \times V_B$ such that $g(F_A^x) = (F_A^x, f(F_A^x))\widetilde{\in}U_A \times V_B\widetilde{\subset}W_{A\times B}$. In the cause of f is soft $b - I$-continuous function, there exists a soft $b - I$-open set U_{A_0} of X containing F_A^x such that $f(U_{A_0}) \subset V_B$. By Theorem 2, $U_{A_0} \cap U_A$ is soft $b - I-$open set in (X, τ) and $g(U_{A_0}\widetilde{\cap}U_A)\widetilde{\subset}U_A \times V_B\widetilde{\subset}W_{A\times B}$. Hence g is soft $b - I$-continuous function.

$\Leftarrow$: Let g is soft $b - I$-continuous function and $F_A^x \in X$ and G_B be any soft open set of Y containing $f(F_A^x)$. Then $\widetilde{X} \times V_B$ is soft open in $X \times Y$ and since g is soft $b - I$-continuity, we have a soft $b - I$-open set U_A in (X, τ) containing F_A^x such that $g(U_A) \subset X \times V_B$. Therefore, we obtain $f(U_A)\widetilde{\subset}V_B$. Hence f is soft $b - I$-continuous function. □

Definition 27. *Let $f : (X, \tau, E, I) \to (Y, \sigma, K)$ be a soft function. If $f^{-1}(G_B)$ is soft $b - I$-open set for every soft b-open set (G_B) of (Y, σ, K), then f is said to be soft $b - I$-irresolute function.*

Theorem 6. *Let $f : (X, \tau, E, I) \to (Y, \sigma, K)$ be a soft funciton. If $f : (X, \tau, E, I) \to (Y, \sigma, K)$ is soft $b-I$-irresolute, then for each soft point F_A^x in X and each soft b-open set V_K in Y containing F_A^x, there exists a soft $b - I$-open set U_A containing F_A^x such that $f(U_A) \widetilde{\subset} V_K$*

Proof. Let $F_A^x \widetilde{\in} X$ and V_K be any soft b-open set in Y containing $f(F_A^x)$.

By supposition, $f^{-1}(V_K)$ is soft $b - I$-open set in X.

Set $U_A = f^{-1}(V_K)$, then U_A is a soft $b - I$-open set in X containing F_A^x such that $f(U_A) \widetilde{\subset} V_K$. □

Theorem 7. *If is $f^{-1}(V_K) \widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$ for every soft b-open set V_K in Y, then $f^{-1}(H_K)$ is soft $b - I$-closed set in X for every soft b-closed set H_K in Y.*

Proof. Let H_K be any soft b-closed subset of Y and $V_K = \widetilde{Y} - H_K$.

Then V_K is soft b-open set in Y.

By hypothesis, $f^{-1}(V_K) \widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$.

Therefore $f^{-1}(H_K) = \widetilde{X} - f^{-1}(V_K)$ is soft $b - I$-closed set in X. □

Theorem 8. *If f is soft $b - I$-irresolute, then $f^{-1}(H_K)$ is soft $b - I$-closed set in X for every soft b-closed set H_K in Y.*

Proof. Let V_K be any soft b-open set in Y and $H_K = \widetilde{Y} - V_K$.

Then by hypothesis, $f^{-1}(H_K) = \widetilde{X} - f^{-1}(V_K)$ is soft $b - I$-closed in X.

This shows that $f^{-1}(V_K)$ is soft $b - I$-open set in X and f is soft $b - I$-irresolute function. □

Theorem 9. *For each soft point F_A^x in X and each soft b-open set V_K in Y containing F_A^x, if there exists a soft $b-I$-open set U_A containing F_A^x such that $f(U_A) \widetilde{\subset} V_K$, then for every soft b-open set V_K in Y, $f^{-1}(V_K) \widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$.*

Proof. Let V_K be any soft b-open set in Y and $F_A^x \widetilde{\in} f^{-1}(V_K)$.

By hypothesis, there exists a soft $b - I$-open set U_A of X containing F_A^x such that $f(U_A) \widetilde{\subset} V_K$.

Thus we attain $F_A^x \widetilde{\in} U_A \widetilde{\subset} cl^*(int(U_A)) \widetilde{\cup} int(cl^*(U_A))$
$\widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$ and hence
$F_A^x \widetilde{\in} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$.

Hence $f^{-1}(V_K) \widetilde{\subset} cl^*(int(f^{-1}(V_K))) \widetilde{\cup} int(cl^*(f^{-1}(V_K)))$. □

Theorem 10. *Let $f : (X, \tau, E, I) \to (Y, \sigma, K, J)$ and $g : (Y, \sigma, K, J) \to (Z, \eta, M)$ be two soft functions, where I and J are ideals on X and Y respectively. Then the following are hold:*

(a) if f is soft $b - I$-continuous and g is soft continuous then gof is soft continuous,

(b) if f^{-1} is soft $b - I$-irresolute and g is soft b-continuous then gof is soft $b - I$-continuous

Proof. (a) Let H_C be a soft open set of (Z, η, M). Since g is soft continuous then $g^{-1}(H_C)$ is soft open in (Y, σ, K, J). Since f is soft $b-I$-continuous then $f^{-1}(g^{-1}(H_C)) = (gof)^{-1}(H_C)$ is soft $b-I$-open set in (X, τ, E, I). Therefore we obtain gof is soft $b-I$-continuous.

(b) Let H_C be a soft open set of (Z, η, M). Since g is soft b-continuous then $g^{-1}(H_C)$ is soft b-open set in (Y, σ, K, J). Since f^{-1} is soft $b-I$-irresolute then $f^{-1}(g^{-1}(H_C)) = (gof)^{-1}(H_C)$ is soft $b-I$-open set in (X, τ, E, I).Therefore we obtain gof is soft $b-I$-continuous. □

Lemma 3. [13] *If (X, τ, E, I) is an soft ideal topological space and F_A is soft subset of X, we denote by $\tau|_{F_A}$ the soft relative topology on F_A and $I|_{F_A} = \{F_A \cap I | I \in I\}$ is obviously an ideal on F_A.*

Lemma 4. *Let (X, τ, E, I) be a soft ideal topological space and V_A, F_A subsets of X such that $V_A \subset F_A$. Then $B^*(\tau|_{F_A}, E, I|_{F_A}) = B^*(\tau, E, I) \cap F_A$.*

Proof. Obvious. □

Theorem 11. *In a soft ideal topological space (X, τ, A, I) if U_A is soft open and F_A is soft $b-I-$open set, then $U_A \widetilde{\cap} F_A$ is soft $b-I-$open in $(U_A, \tau|_{U_A}, I|_{U_A})$*

Proof. We have $int_{U_A} V_A = int(V_A) \widetilde{\cap} U_A$ for any soft subset V_A of U_A, since U_A is soft open. Hence, by using this real and Lemma 5, proof is completed. □

Theorem 12. *Let $f : (X, \tau, E, I) \to (Y, \sigma, K)$ be soft $b-I$-continuous function and U_A soft open set in X. Then the restriction $f|_{U_A} : (U_A, \tau|_{U_A}, E, I|_{U_A}) \to (Y, \sigma, K)$ is soft $b-I$-continuous.*

Proof. Let G_B be any soft open set of (Y, σ, K). Since f is soft $b-I$-continuous,
then $f^{-1}(G_B)$ is soft $b-I$-open set in X. For U_A soft open set,
by Theorem 8 $U_A \cap f^{-1}(G_B)$ is soft $b-I-$open set in $(U_A, \tau, E, I|_{U_A})$.
On the other hand, $(f|_{U_A})^{-1}(G_B) = U_A \cap f^{-1}(G_B)$ and
$(f|_{U_A})^{-1}(G_B)$ is soft $b-I$-open set in $(U_A, \tau|_{U_A}, E, I|_{U_A})$.
This shows that $f|_{U_A} : (U_A, \tau|_{U_A}, E, I|_{U_A}) \to (Y, \sigma, K)$ is soft $b-I$-continuous. □

5 Soft $b-I$-Open Functions and Soft $b-I$-Closed Functions

Definition 28. *A function $f : (X, \tau, E) \to (Y, \sigma, K, J)$ is said to be soft $b-I$-open (resp. soft $b-I$-closed) if the image of each soft open (resp. soft closed) set of X is soft $b-I$-open (resp. soft $b-I$-closed) set in (Y, σ, K, J).*

Definition 29. [12] *A function $f : (X, \tau, E) \to (Y, \sigma, K)$ is said to be soft b-open (resp. soft b-closed) if the image of each soft open (resp. soft closed) set of X is soft b-open (resp. soft b-closed) set in (Y, σ, K).*

We can give the following warning from the above two definitions

Remark 9. *(a) Every soft open function is soft $b-I$-open function.*
(b) Every soft $b-I$-open function is soft b-open function.

In the following examples as observed the converses are not true.

Example 12. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,
$\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, F_A\right\}$,*where* $F_A = \{(e_1, \{h_2\})\}$. *Also, let* $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$,
$\sigma = \left\{\widetilde{\emptyset}, \widetilde{Y}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}, \{(k_1, \{y_2\}), (k_2, \{y_1\})\}\right\}$ *and* $J = \left\{\widetilde{\emptyset}\right\}$
Then the soft function $f : (X, \tau, A, I) \to (Y, \sigma, K)$ *denoted by*
$u(h_1) = y_1$, $u(h_2) = y_2$, $p(e_1) = k_1$, $p(e_2) = k_2$ *is soft* $b - I-$*open set*
but is not soft open set. Because, for soft open set F_A *in* X,
$f(F_A) = H_K$ *is soft* $b - I-$*open set but is not soft -open set.*

Example 13. *Let* $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $\tau = \left\{\widetilde{\emptyset}, \widetilde{X}, F_A\right\}$,
where $F_A = \{(e_1, X), (e_2, \{h_1\})\}$. *Also, let* $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$,
$\sigma = \left\{\widetilde{\emptyset}, \widetilde{Y}, \{(k_1, \{y_1\})\}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}\right\}$ *and* $J = S(Y)$
Then the soft function $f : (X, \tau, A, I) \to (Y, \sigma, K)$ *denoted by*
$u(h_1) = y_1$, $u(h_2) = y_2$, $p(e_1) = k_1$, $p(e_2) = k_2$ *is soft* $b-$*open but*
is not soft $b - I$*-open set. Because, for soft open set* F_A *in* X,
$f(F_A) = H_K$ *is soft* $b-$*open set but is not soft* $b - I$*-open set.*

Theorem 13. *A function* $f : (X, \tau, E) \longrightarrow (Y, \sigma, J, K)$ *is a soft* $b - I$*-open if and only if for each* $F_A^x \widetilde{\in} X$ *and each soft open set* U_A *containing* F_A^x, *there exists a soft* $b - I$*-open set* W_K *containing* $f(F_A^x)$ *such that* $W_K \widetilde{\subset} f(U_A)$.

Proof. $\Rightarrow$:Let's face it $F_A^x \widetilde{\in} X$ and U_A be any soft open set containing F_A^x. Since f is soft $b - I$-open function, $f(U_A)$ is soft $b - I$-open set in Y. Set $W_K = f(U_A)$, then $f(F_A^x) \widetilde{\in} W_K$ and W_K is soft $b - I$-open set such that $W_K \widetilde{\subset} f(U_A)$.
$\Leftarrow$: Obvious. □

Theorem 14. *Let* $f : (X, \tau, E) \longrightarrow (Y, \sigma, J, K)$ *be a soft* $b - I$*-open function. If* W_K *is soft set in* Y *and* U_A *is soft closed set in* X *containing* $f^{-1}(W_K)$, *then there exists a soft* $b - I$*-closed set* H_K *in* Y *containing* W_K *such that* $f^{-1}(H_K) \widetilde{\subset} U_A$.

Proof. Let U_A be a soft closed set in X. Since $G_A = \widetilde{X} - U_A$ is soft open set in X. Since f is soft $b - I$-open function, $f(G_A)$ is soft $b - I$-open set in Y. Therefore $H_K = \widetilde{Y} - f(G_A)$ is soft $b - I$-closed set in Y and $f^{-1}(H_K) = f^{-1}(\widetilde{Y} - f(G_A)) = \widetilde{X} - f^{-1}(f(G_A)) \widetilde{\subset} \widetilde{X} - G_A = U_A$. □

Theorem 15. *The following phrases are equivalent for any bijective soft function* $f : (X, \tau, E, I) \longrightarrow (Y, \sigma, K, J)$*;*
(a) $f^{-1} : (Y, \sigma, J) \longrightarrow (X, \tau, I)$ *is soft* $b - I$*-continuous function,*
(b) f *is soft* $b - I$*-open function,*
(c) f *is soft* $b - I$*-closed function.*

Proof. Obvious. □

Theorem 16. *Let* $f : (X, \tau, E, I) \longrightarrow (Y, \sigma, K, J)$ *and* $g : (Y, \sigma, K, J) \longrightarrow (Z, \eta, L, K)$ *be two soft functions. The followings hold:*
(a) gof *is soft* $b-I$*-open function if* f *is soft open function and* g *is soft* $b-I$*-open function.*
(b) f *is soft* $b - I$*-open function if* gof *is soft open function and* g *is soft* $b - I$*-continuous function.*

Proof. This is obvious. □

6 Conclusion

Our purpose in this paper is to define upper and lower soft $b-I-$continuous functions and study their various properties. Moreover, we obtain some characterizations and several properties concerning such functions. We expect that results in this paper will be basis for further applications of soft mappings in soft sets theory and corresponding information systems.

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