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Generalized R-contraction by using triangular α-orbital admissible

Ferhan Şola Erduran[1]

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Abstract – *This study presents Ciric type generalization of R-contraction and generalized R-contraction by using an α-orbital admissible function in metric spaces using the definition of R-contraction introduced by Roldan-Lopez-de-Hierro and Shahzad [New fixed-point theorem under R-contractions, Fixed Point Theory and Applications, 98(2015): 18 pages, 2015] and prove some fixed-point theorems for this type contractions. Thanks to these theorems, we generalize some known results.*

Subject Classification (2020): 54H25, 47H10.

1. Introduction

This section provides some of basic notions. The concept of fixed point appeared in 1922 with the Banach contraction principle (BCP) [1]. So far, many studies [2-5] has been conducted on this concept applied in many areas, such as differential equations theory and economics. The most striking of the results obtained by generalizing BCP is the Meir-Keeler contraction (MKC) provided in [6]:

Let T be a self-mapping on a complete metric space (X, d). Given $\varepsilon > 0$, there exist $\delta > 0$ such that

$$\varepsilon \leq d(x,y) < \varepsilon + \delta \text{ implies that } d(Tx,Ty) < \varepsilon$$

After that, many authors studied extensions of MKC. In [7], the authors presented the notion of simulation function (SF), an auxiliary function for improving BCP, and generalized MKC:

A simulation function ξ is a mapping from $[0,\infty) \times [0,\infty)$ to $\mathbb{R}$ such that

ξ_1) $\xi(0,0) = 0$

ξ_2) $\xi(t,s) < s - t$, for all $s, t \in \mathbb{N}$

ξ_3) If $\{t_n\}, \{s_n\}$ are sequences in $(0,\infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$ then $\limsup_{n\to\infty} \xi(t_n, s_n) < 0$

Afterwards, [8] modified the condition ξ_3 of SF to expand the family of SFs:

ξ_3) If $\{t_n\}, \{s_n\}$ are sequences in $(0,\infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$ and $t_n < s_n$, for all $n \in \mathbb{N}$, then $\limsup_{n\to\infty} \xi(t_n, s_n) < 0$.

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In [7], the researchers then put forward the Z-contraction mapping as follows:

Let (X, d) be a metric space and T be a self-mapping on X. If there exists $\xi \in Z$ (Z is the family of SFs), for all $x, y \in X$ with $x \neq y$,

$$\xi\big(d(Tx, Ty), d(x, y)\big) \geq 0$$

then T is Z-contraction concerning ξ. So, they generalized the Banach fixed point theorem in metric space using the auxiliary function ξ. Furthermore, the concept of manageable function (MF) provided by [2] to work multivalued contraction mappings is as follows:

A function $\eta: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is manageable if

η_1) $\eta(t, s) < s - t$, for all $s, t > 0$

η_2) For a bounded sequence $\{t_n\} \subset (0, \infty)$, a non-increasing sequence $\{s_n\} \subset (0, \infty)$, η provides

$$\limsup_{n\to\infty} \frac{t_n + \eta(t_n, s_n)}{s_n} < 1.$$

Besides, [7] defined $\widehat{Man(R)}$-contraction for single-valued mapping as follows:

Let (X, d) be a metric space and T be self-mapping on X. If there exists $\eta \in \widehat{Man(R)}$ such that

$$\eta(d(Tx, T^2x), d(x, Tx)) \geq 0$$

for all $x \in X$, then T is $\widehat{Man(R)}$-contraction.

Recently, [9] have introduced R-function for considering a true extension of MKC as follows:

Let $A \subset \mathbb{R}$, $A \neq \emptyset$, and $\varrho: A \times A \to \mathbb{R}$ be a function. Then, ϱ is called an R-function:

(ϱ_1) If a sequence $\{a_n\} \subset (0, \infty) \cap A$ and $\varrho(a_{n+1}, a_n) > 0$ for all $n \in \mathbb{N}$, then $\{a_n\} \to 0$.

(ϱ_2) If two sequence $\{a_n\}, \{b_n\} \subset (0, \infty) \cap A$ converges to $L \geq 0$ such that $L < a_n$ and $\varrho(a_n, b_n) > 0$ for all $n \in \mathbb{N}$, then $L = 0$.

(ϱ_3) If $\{a_n\}, \{b_n\} \subset (0, \infty) \cap A$ are two sequences such that $\{b_n\} \to 0$ and $\varrho(a_n, b_n) > 0$ for all $n \in \mathbb{N}$, then $\{a_n\} \to 0$.

Let R_A denote the family of all R-functions, (X, d) be a metric space, and T be a mapping on X. T is R-contraction concerning ϱ if there exist $\varrho \in R_A$ such that $\operatorname{ran}(d) \subset A$ and

$$\varrho\big(d(Tx, Ty), d(x, y)\big) > 0$$

for all $x, y \in X$ with $x \neq y$

$$\operatorname{ran}(d) = \{d(x, y): x, y \in X\} \subset [0, \infty)$$

[9] also gave R-contraction concerning ϱ and showed a relationship between the class of some known functions and R-function and between some known contractions and R-contraction relating to ϱ as follows:

i. A SF is an R-function and verifies (ϱ_3),

ii. Any MF is an R-function and confirms (ϱ_3),

iii. A Geraghty function (GF) $\phi: [0, \infty) \to [0,1)$ holds if $\{t_n\} \subset [0, \infty)$ and $\{\phi(t_n)\} \to 1$, then $\{t_n\} \to 0$ [10]

If $\phi: [0, \infty) \to [0,1)$ is a GF, then $\varrho'_{\phi}: [0, \infty) \times [0, \infty) \to \mathbb{R}$, defined with

$$\varrho'_{\phi}(t, s) = \phi(s)s - t$$

for all $t, s \in [0, \infty)$, is an R-function on $[0, \infty)$ satisfying condition (ϱ_3), *iv.* Any MKC is R-contraction in respect of ϱ,

v. A Geraghty contraction (GC) is a self-mapping T on X such that for every $x, y \in X$ and ϕ is a GF $d(Tx,Ty) \leq \phi\big(d(x,y)\big)d(x,y)$ [10].

Every GC is R-contraction in respect of ϱ.

In [9], it is claimed that if $\varrho(t,s) \leq s - t$ for all $t, s \in A \cap (0,\infty)$, then (ϱ_3) is held.

[11] presented the concept of weakly Picard operator as follows:

Let (X,d) be a metric space and T be a self-mapping on X. Given a point $x_0 \in X$, the Picard sequence $\{x_n\}$ of T started with x_0 is given by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. T defined as a weakly Picard operator if, for all $x_0 \in X$, the Picard sequence of T converges to a fixed point of T. Also, T is a Picard operator if it is a weakly Picard operator, and T has a unique fixed point.

2. Main Result

This section proves the Ciric type generalization of R-contraction concerning ϱ, and presents a generalization of known results and illustrates them.

Definition 2.1. Let (X,d) be a metric space T be a self-mapping on X and $\varrho \in R_A$. T is generalized R-contraction in respect of ϱ the following case satisfying $\operatorname{ran}(d) \subset A$ and

$$\varrho\big(d(Tx,Ty), M(x,y)\big) > 0\#(2.1)$$

for all $x, y \in X$ and $x \neq y$, where

$$\mathrm{M(x,y)} = \max\left\{\mathrm{d(x,y)}, \mathrm{d(x,Tx)}, \mathrm{d(y,Ty)}, \frac{1}{2}[\mathrm{d(x,Ty)} + \mathrm{d(y,Tx)}]\right\}.$$

Theorem 2.2. Let (X,d) be a complete metric space and T be generalized R-contraction on X in respect of ϱ. Suppose that one of the followings hold.

i. T is continuous,
ii. ϱ satisfies the condition (ϱ_3),
iii. $\varrho(t,s) \leq s - t$ for all $t, s \in A \cap (0,\infty)$.

Then T is a Picard operator, and T has a unique fixed point.

Proof.

Let we take any $x_0 \in X$ and $\{x_n\}$ is a Picard sequence of T started with x_0. If there exists some $n_0 \in \mathbb{N}$, $x_{n_0+1} = Tx_{n_0} = x_{n_0}$ then x_{n_0} is a fixed point of T. Assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Since T is generalized R-contraction in respect of ϱ,

$$\varrho\big(d(Tx_{n-1},Tx_n), M(x_{n-1},x_n)\big) > 0\#(2.2)$$

where

$$\begin{aligned} \mathrm{M}(x_{n-1},x_n) &= \max\left\{\mathrm{d}(x_{n-1},x_n), \mathrm{d}(x_{n-1},\mathrm{T}x_{n-1}), \mathrm{d}(x_n,\mathrm{T}x_n), \frac{\mathrm{d}(x_{n-1},\mathrm{T}x_n) + \mathrm{d}(x_n,\mathrm{T}x_{n-1})}{2}\right\} \\ &= \max\left\{\mathrm{d}(x_{n-1},x_n), \mathrm{d}(x_{n-1},x_n), \mathrm{d}(x_n,x_{n+1}), \frac{\mathrm{d}(x_{n-1},x_{n+1}) + \mathrm{d}(x_n,x_n)}{2}\right\} \\ &= \max\{\mathrm{a_{n-1}}, \mathrm{a_n}\}. \end{aligned}$$

From (2.2), we get

$$\varrho(a_n, \max\{a_{n-1}, a_n\}) > 0 \#(2.3)$$

If $a_{n-1} \leq a_n$ for some $n \in \mathbb{N}$, then from (2.3)

$$\varrho(a_n, a_n) > 0$$

which is a contradiction. Therefore, $a_{n-1} > a_n$ for all $n \in \mathbb{N}$ and $\varrho(a_n, a_{n-1}) > 0$.

From (ϱ_1), we have $\{a_n = d(x_n, x_{n+1})\} \to 0$.

Now, we show the sequence $\{x_n\}$ is Cauchy. Assume $\{x_n\}$ is not a Cauchy sequence. There exist $\varepsilon > 0$, for all $k \geq n_1$, there exist $m(k) > n(k) > k$ and $d(x_{n(k)}, x_{m(k)}) \geq \varepsilon$. Let $m(k)$ be the smallest number and satisfies the conditions above. Then $d\left(x_{n(k)-1}, x_{m(k)}\right) < \varepsilon$. Hence,

$$\varepsilon \leq d\left(x_{n(k)}, x_{m(k)}\right) \leq d\left(x_{n(k)}, x_{m(k)-1}\right) + d\left(x_{m(k)-1}, x_{m(k)}\right) < \varepsilon + d\left(x_{m(k)-1}, x_{m(k)}\right).$$

As $k \to \infty$, $\lim_{k\to\infty} d\left(x_{n(k)}, x_{m(k)}\right) = \varepsilon$. Since

$$\left|d\left(x_{n(k)}, x_{m(k)-1}\right) - d\left(x_{n(k)}, x_{m(k)}\right)\right| \leq d\left(x_{m(k)-1}, x_{m(k)}\right)$$

we get $\lim_{k\to\infty} d\left(x_{n(k)}, x_{m(k)-1}\right) = \varepsilon$. Similarly, we obtain

$$\lim_{k\to\infty} d\left(x_{n(k)-1}, x_{m(k)}\right) = \varepsilon = \lim_{k\to\infty} d\left(x_{n(k)-1}, x_{m(k)-1}\right).$$

Let $L = \varepsilon > 0$, $\left\{t_k = d\left(x_{n(k)}, x_{m(k)}\right)\right\} \to L$, $\left\{s_k = d\left(x_{n(k)-1}, x_{m(k)-1}\right)\right\} \to L$ and

$$d\left(x_{n(k)-1}, x_{m(k)-1}\right) \leq M\left(x_{n(k)-1}, x_{m(k)-1}\right) = \max\left\{\begin{matrix} d\left(x_{n(k)-1}, x_{m(k)-1}\right), d\left(x_{n(k)-1}, Tx_{n(k)-1}\right), d\left(x_{m(k)-1}, Tx_{m(k)-1}\right), \\ \frac{1}{2}\left[d\left(x_{n(k)-1}, Tx_{m(k)-1}\right) + d\left(x_{m(k)-1}, Tx_{n(k)-1}\right)\right] \end{matrix}\right\}$$

Taking a limit $k \to \infty$, we have $\lim_{k\to\infty} M\left(x_{n(k)-1}, x_{m(k)-1}\right) = L$. Since $L = \varepsilon < d\left(x_{n(k)}, x_{m(k)}\right) = t_k$ and

$$\varrho\left(d\left(x_{n(k)}, x_{m(k)}\right), M\left(x_{n(k)-1}, x_{m(k)-1}\right)\right) > 0$$

for all $k \in \mathbb{N}$, then (ϱ_2) guarantees $L = \varepsilon = 0$. Consequently, $\{x_n\}$ is Cauchy. Since the metric space (X, d) is complete, there exist $z \in X$ such that $x_n \to z$. Let show that z fixed point.

Case 1: Suppose T is a continuous function. So $\{Tx_n = x_{n+1}\} \to Tz$, and $Tz = z$.

Case 2: In propositional logic, $p \Rightarrow q \equiv q' \Rightarrow p'$. Now we look at the proof of a fixed point of T concerning this point of view. Assume $d(z, Tz) > 0$.

$$a_n = d(Tx_n, Tz) = d(x_{n+1}, Tz) \text{ and so } \lim_{n\to\infty} a_n = d(z, Tz) > 0 \text{ and}$$

$$b_n = M(x_n, z) = \max\left\{d(x_n, z), d(x_n, Tx_n), d(z, Tz), \frac{1}{2}[d(z, Tx_n) + d(x_n, Tz)]\right\}$$

Let $n \to \infty$, we get $\lim_{n\to\infty} b_n = M(x_n, z) = d(z, Tz) > 0$, but

$$\varrho\left(d(Tx_n, Tz), M(x_n, z)\right) > 0.$$

It contradicts to (ϱ_3). Consequently, $d(z, Tz) = 0$.

Case 3: Assume $\varrho(t, s) < s - t$ for all $t, s \in A \cap (0, \infty)$. Proposition 1.2 means that Case 2 is applicable. z is a fixed point, so T is a weakly Picard operator.

Let $z \neq y$ and $z, y \in X$ be two fixed points. In this case, $a_n = d(z, y) > 0$ for all $n \in \mathbb{N}$.

$$\varrho(a_{n+1}, a_n) = \varrho(d(z, y), d(z, y)) = \varrho(d(Tz, Ty), M(z, y)) > 0$$

Applying (ϱ_1), $\{a_n\} \to 0$, which is a contradiction.

Example 2.3. Let $X = [0,1]$ and $d: X \times X \to \mathbb{R}$ be a usual metric. Let $T: X \to X$ as $Tx = \frac{x}{x+1}$ for all $x \in X$. We define $\varrho: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $\varrho(t,s) = \frac{s}{s+1} - t$. From Theorem 2.2, $x = 0$ is a fixed point of T.

We have the following corollaries by using Theorem 2.2. In this case, we generalized Corollary 28-33 in [9] by using similar $M(x,y)$.

Corollary 2.4. Any continuous generalized R-contraction has a unique fixed point.

Corollary 2.5. Any generalized Z-contraction has a unique fixed point.

Corollary 2.6. Every generalized $\widehat{\mathrm{Man(R)}}$-contraction has a unique fixed point.

Corollary 2.7. Let (X,d) be a complete metric space and $T: X \to X$. Assume that there exist $\varphi, \psi: [0,\infty) \to [0,\infty)$ such that

$$\psi(d(Tx,Ty)) \leq \psi\big(M(x,y)\big) - \varphi\big(M(x,y)\big)$$

for all $x, y \in X$. If φ is lower semi-continuous, ψ is nondecreasing, ψ continuous from right and $\varphi^{-1}(\{0\}) = \{0\}$, then Thas a unique fixed point.

Proof.

It is obvious Theorem 2.2 and Theorem 22 in [9].

Corollary 2.5. Every generalized GC has a unique fixed point.

Proof.

It is obvious Theorem 2.2 and Corollary 26 in [9].

Corollary 2.6. Every generalized MKC has a unique fixed point.

Proof.

It is obvious from Theorem 2.2 and Theorem 25 in [9].

3. Admissible Functions

[12] gave α-admissible concept as follows: let $T: X \to X$, $\alpha: X \times X \to \mathbb{R}$. T is said to be α-admissible if $\alpha(x,y) \geq 1$ implies $\alpha(Tx,Ty) \geq 1$. Then, [3] added the condition; $\alpha(x,z) \geq 1$, $\alpha(z,y) \geq 1$ imply $\alpha(x,y) \geq 1$, nearby the α-admissible condition and so they introduced triangular α-admissible notion. We understand from these definitions, triangular α-admissible implies α-admissible, but the converse is not valid. In 2014, Popescu [4] introduced α-orbital and triangular α-orbital admissible notions as follows:

Definition 3.1. [4] Let $T: X \to X$, $\alpha: X \times X \to \mathbb{R}$. T is said to be α-orbital admissible if $\alpha(x,Tx) \geq 1$ implies $\alpha(Tx,T^2x) \geq 1$.

Definition 3.2. [4] Let $T: X \to X$, $\alpha: X \times X \to \mathbb{R}$. T is said to be triangular α-orbital admissible if $T$is α-orbital admissible, $\alpha(x,y) \geq 1$, if $\alpha(y,Ty) \geq 1$ implies $\alpha(x,Ty) \geq 1$.

Every α-admissible mapping is an α-orbital admissible and every triangular α-admissible mapping is a triangular α-orbital admissible mapping. So that a triangular α-orbital admissible mapping is a very wide function class in the literature.

Lemma 3.3. [9] Let $T: X \to X$ be a triangular α-orbital admissible mapping. Assume that there exist $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$. Define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$. Then we have $\alpha(x_n, x_m) \geq 1$ for all $n, m \in \mathbb{N}$ with $n < m$.

Definition 3.4. Let (X, d) be a metric space, $T: X \to X$. T is an α-admissible R-contraction in respect of ϱ if there exist $\varrho: A \times A \to \mathbb{R}$ such that for all $x, y \in X$, $\alpha(x,y)d(Tx,Ty) \in A$, $\text{ran}(d) \subset A$, $\alpha: X \times X \to [0, \infty)$,

$$\varrho(\alpha(x,y)d(Tx,Ty), d(x,y)) > 0$$

for all $x, y \in X$ with $x \neq y$. If $\alpha(x,y) = 1$, then T is a R-contraction.

Theorem 3.5. Let (X, d) be a complete metric space, $\alpha: X \times X \to \mathbb{R}$, $T: X \to X$. If

T is an α-admissible R-contraction type mapping in respect of ϱ,

T is a triangular α-orbital admissible mapping, there exist $x_0 \in X$ and $\alpha(x_0, Tx_0) \geq 1$,

$\varrho(t,s) < s - t$ for all $t, s \in A \cup (0,1)$,

T is a continuous function.

Then, T is a Picard operator and has a fixed point in X.

Proof.

Let $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$ and let $\{x_n\}$ be a Picard sequence of T started with x_0 such that $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. If there exist $n_0 \in \mathbb{N}$, $x_{n_0+1} = x_{n_0}$, then x_{n_0} is a fixed point of T. In this case, suppose that $x_{n+1} \neq x_n$ or all $n \in \mathbb{N}$. Because of (*ii*) and (*iii*), we obtain

$$\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq 1 \Rightarrow \alpha(Tx_0, Tx_1) \geq 1$$

similarly,

$$\alpha(x_1, x_2) = \alpha(x_1, Tx_1) \geq 1 \Rightarrow \alpha(Tx_1, Tx_2) \geq 1$$

continuing this process, we derive $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$. T is an α-admissible R-contraction, then

$$0 < \varrho\big(\alpha(x_n, x_{n-1})d(Tx_n, Tx_{n-1}), d(x_n, x_{n-1})\big) < d(x_n, x_{n-1}) - \alpha(x_n, x_{n-1})d(x_{n+1}, x_n)$$

as a result, we get for all $n \in \mathbb{N}$

$$d(x_{n+1}, x_n) < \alpha(x_n, x_{n-1})d(x_{n+1}, x_n) < d(x_n, x_{n-1}) \#(3.1)$$

Hence, the sequence $\{x_n\}$ is decreasing, bounded from below. Consequently, there exists $L \geq 0$ such that $\lim_{n\to\infty} d(x_n, x_{n-1}) = L$. From equation (3.1), we get

$$\lim_{n\to\infty} \alpha(x_n, x_{n-1})d(x_{n+1}, x_n) = L.$$

Let $s_n = \alpha(x_n, x_{n-1})d(x_{n+1}, x_n)$, $t_n = d(x_n, x_{n-1})$ and we can easily see that $L < s_n$ for $n \in \mathbb{N}$. In this case, from the (ϱ_2) property, we have $L = 0$.

The sequence $\{x_n\}$ is Cauchy in X. Assume the sequence $\{x_n\}$ is not Cauchy. There exist $\varepsilon > 0$, for all $k \geq n_1$, there exist $m(k) > n(k) > k$ and $d(x_{n(k)}, x_{m(k)}) \geq \varepsilon$. Let $m(k)$ be the smallest and satisfies the above conditions. So $d\big(x_{n(k)-1}, x_{m(k)}\big) < \varepsilon$. Then

$$\varepsilon \leq d\big(x_{n(k)}, x_{m(k)}\big) \leq d\big(x_{n(k)}, x_{m(k)-1}\big) + d\big(x_{m(k)-1}, x_{m(k)}\big) < \varepsilon + d\big(x_{m(k)-1}, x_{m(k)}\big)$$

As $k \to \infty$, we get $\lim_{k\to\infty} d\big(x_{n(k)}, x_{m(k)}\big) = \varepsilon$. Since

$$\big|d\big(x_{n(k)}, x_{m(k)-1}\big) - d\big(x_{n(k)}, x_{m(k)}\big)\big| \leq d\big(x_{m(k)-1}, x_{m(k)}\big),$$

we get $\lim_{k\to\infty} d(x_{n(k)}, x_{m(k)-1}) = \varepsilon$. Similarly, we obtain

$$\lim_{k\to\infty} d(x_{n(k)-1}, x_{m(k)}) = \varepsilon = \lim_{k\to\infty} d(x_{n(k)-1}, x_{m(k)-1}).$$

By Lemma 4.3, we have $\alpha(x_{n(k)-1}, x_{m(k)-1}) \geq 1$. Thus, we deduce that

$$0 < \varrho\left(\alpha(x_{n(k)-1}, x_{m(k)-1}) d(Tx_{n(k)-1}, Tx_{m(k)-1}), d(x_{n(k)-1}, x_{m(k)-1})\right)$$
$$< d(x_{n(k)-1}, x_{m(k)-1}) - \alpha(x_{n(k)-1}, x_{m(k)-1}) d(Tx_{n(k)-1}, Tx_{m(k)-1})$$

for all $k \geq n_1$. Consequently,

$$0 < d(x_{n(k)}, x_{m(k)}) < \alpha(x_{n(k)-1}, x_{m(k)-1}) d(Tx_{n(k)-1}, Tx_{m(k)-1}) < d(x_{n(k)-1}, x_{m(k)-1})$$

for all $k \geq n_1$. Let $k \to \infty$, we have

$$\lim_{k\to\infty} \alpha(x_{n(k)-1}, x_{m(k)-1}) d(Tx_{n(k)-1}, Tx_{m(k)-1}) = \varepsilon.$$

Let $a_k = \alpha(x_{n(k)-1}, x_{m(k)-1}) d(Tx_{n(k)-1}, Tx_{m(k)-1})$ and $b_k = d(x_{n(k)}, x_{m(k)})$. We show that $\varepsilon < a_k$ for all $k \geq n_1$. In this case, from the (ϱ_2) property, we have $\varepsilon = 0$, which is a contradiction. Hence, the sequence $\{x_n\}$ is Cauchy. From (X, d) is complete, there exist $z \in X$, $\{x_n\} \to z$.

Assume the condition (v) satisfied. In this case, $\{x_{n+1} = Tx_n\} \to Tz$, and so $Tz = z$. Therefore, T is a weakly Picard operator.

Theorem 3.6. Let (X, d) be complete, $\alpha: X \times X \to \mathbb{R}$ and $T: X \to X$. Assume the followings are satisfied:

T is a α- admissible R-contraction type mapping concerning ϱ;

T is a triangular α- orbital admissible mappings,

There exist $x_0 \in X$ and $\alpha(\alpha, Tx_0) \geq 1$;

$\varrho(t, s) < s - t$ for all $t, s \in A \cup (0,1)$;

if $\{x_n\} \in X$, $\alpha(x_n, x_{n+1}) \geq 1$ for all n, $x_n \to x$ as $n \to \infty$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ and $\alpha(x_{n_k}, x) \geq 1$ for all $k \in \mathbb{N}$.

So, T is a Picard operator and has a fixed point in X.

Proof.

From the proof of the above theorem, the sequence $\{x_n\}$, $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$, converges to $z \in X$. By the condition (v), there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ and $\alpha(x_{n_k}, x) \geq 1$ for all $k \in \mathbb{N}$. Applying (i) for all k, we get

$$0 < \varrho(\alpha(x_{n_k}, z) d(Tx_{n_k-1}, Tz), d(x_{n_k}, z)) = \varrho(\alpha(x_{n_k}, z), d(x_{n_k}, Tz), d(x_{n_k}, z))$$
$$< d(x_{n_k}, z) - \alpha(x_{n_k}, z) d(x_{n_k}, Tz)$$

which is equivalent to

$$d(x_{n_k}, Tz) = d(Tx_{n_k-1}, Tz) \leq \alpha(x_{n_k}, z) d(x_{n_k}, Tz) < d(x_{n_k}, z).$$

Let $k \to \infty$, we have $d(z, Tz) = 0$, i.e., $z = Tz$.

From the uniqueness of fixed point of α-admissible R-contraction type mapping,

(H) For all $x \neq y$, there exists $v \in X$ and $\alpha(x, v) \geq 1$, $\alpha(y, v) \geq 1$, $\alpha(v, Tv) \geq 1$.

Replacing (*iii*) with (*H*) in the hypothesis of Theorem 3.5 and Theorem 3.6, we get the uniqueness of the fixed point of T. Assume z, t are two fixed points of T and $z \neq t$. From the condition (*H*), there exists $v \in X$ and

$$\alpha(z, v) \geq 1, \alpha(t, v) \geq 1, \alpha(v, Tv) \geq 1.$$

Because T is triangular α-orbital admissible, we obtain $\alpha(z, T^n v) \geq 1$ and $\alpha(t, T^n v) \geq 1$ for all $n \in \mathbb{N}$, we get

$$0 < \varrho\big(\alpha(z, T^n v)d(Tz, T^{n+1}v), d(z, T^n v)\big)$$
$$< d(z, T^n v) - \alpha(z, T^n v)d(Tz, T^{n+1}v)$$

and so

$$d(z, T^n v) = d(Tz, T^n v) \leq \alpha(z, T^n v)d(Tz, T^{n+1}v) < d(z, T^n v)$$

By the Theorem 3.5, we know that the sequence $\{T^n v\}$ converges to a fixed point t of T. As $n \to \infty$,

$s_n = (z, T^n v)d(Tz, T^{n+1}v) \to d(z, t)$ and $t_n = d(z, T^n v) \to d(z, t)$

From (ϱ_2), we $d(z, t) = 0$, which is a contradiction. Therefore, $z = t$.

Now, we can give some corollaries by using Theorem 3.5 and Theorem 3.6.

Corollary 3.7. Every α-admissible Z-contraction has a unique fixed point.

Corollary 3.8. Every α-admissible $\widehat{\mathrm{Man}(R)}$-contraction has a unique fixed point.

We prove the following corollary by using Theorem 3.5 and Theorem 2.2.

Corollary 3.9. Every α-admissible Z-contraction has a unique fixed point.

Corollary 3.10. Every α-MKC has a unique fixed point.

Conflicts of Interest

The author declares no conflict of interest.

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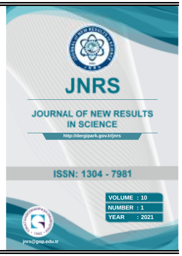

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A Mathematical modelling of success in education

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Abstract. In this study, to determine the value of success within any time interval or specified time intervals, a mathematical model has been developed by taking into account "Discipline Factors", "Teacher Factors", "Negative Factors" and "Student Factors" as factors affecting the success of the student, class or school, positively or negatively. These factors, determining the value of success, which are the parameters of the mathematical model, was defined as real-valued linear functions, considering internal and external influences and any interventions and with the help of these functions, situations in which success is increasing, decreasing, or stable in a time interval examined by this model.

Subject Classification (2020): 00A05, 97M10.

1. Introduction

Education and training are directly related to all segments of society because they are processes in which some outcomes, such as learning, knowing, getting habits, gaining behaviour, social, cultural and scientific development, getting a professional and growing up in a special field. Therefore, everyone involved in the training process is willing to see positive effects for the educated ones. Obviously, at the end of each training process, obtaining the predicted achievements provided that some criteria are provided, means to be successful. However, being successful or achieving the desired success is not an easy task. There are multiple factors that directly or indirectly affect the process of being successful and the education process. Particularly, Ministry, for the Elementary and Secondary School affiliated with Ministry of National Education, Directorate of National Education, school administration, environment, curriculum, teachers, disciplinary rules, school regulations and practices, student's family, economic reasons and many factors as an institution, person and situation that are uncontrolled in-school or out-of-school interventions concerning the school and the student are included in the education process. These or similar factors are also pointed for Universities and students of the Council of Higher Education. Because schools belonging to both institutions have similar directions in social and administrative terms and have similar academic, social, and cultural ideals. The goal of every school and every student is the success ultimately, that is, to be educated in both academic and social and cultural fields.

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Making specific classifications, factors affecting the success, will often and often find their place within these classifications. To ensure this, in this study, by determining the “set of disciplinary factors”, “set of teacher performance factors”, “set of negative factors”, the state of functions helping the model will be proposed according to the real provisions of the values in these sets and their averages. Therefore, the multiplicity of elements of these clusters in terms of values means considering the vast majority of education factors that are essential and which have the possibility of intervention.

In this study, “disciplinary factors”, “teacher performance factors”, and “negative factors” that may cause the success graph to increase, remain stable or decrease by concerning the school and the student primarily are considered. Functions are created for the values considered, and it is stated how to calculate the minimum, maximum and average real values for a certain time or period with the help of these functions. After calculating these real values, an exponential function has been considered as a criterion for evaluating success because it is thought that more explicit results of the success value graph can be seen with the help of an exponential function.

Mathematics has its alphabet and therefore a language, which is also an internationally used language of science. Mathematics takes its place in the school programs of all countries at a significant concentration and is taught and taught under various names.

Some of the reasons why mathematics is indispensable in human life can be listed as follows:

1) The fact that everyone can accept the rules of mathematics and that the claims, in theory, can be proved,

2) There is always mathematics in the person himself, his speech and his movements,

3) With the help of the univariate or multivariate functions and the solution methods of initial value problems, which are the main topics of mathematics, there is the ability to build and use mathematical models in many different fields such as water sciences, climate and atmosphere, energy sciences, food and agricultural sciences, biology, health and medical sciences, human social relations, radioactive decay [1].

4) With the help of mathematical topics in which differential equations, statistics, probability, analysis, numeric analysis are used predominantly, there is not only the ability to analyse movements but also the establishment of mathematical models in both natural sciences and earth sciences and technological fields such as medicine, forensic medicine, earthquake analysis, economics, gene movement, signals, all engineering, architecture, physics, chemistry, astronomy and robotic technology.

5) The password, formula, or correlation of almost any mystery found in the universe can be translated into mathematics.

It is possible to determine the success of a student, a class, or a school in a process by using the fundamental equation of change based on the analysis and formulation of mathematics. Therefore, models can be formed to determine the real value of success at a time point or the course of success in a time interval by taking into consideration the factors that affect students’ attitudes, behaviours, work, motivation and learning, as well as the teacher’s performance factors and discipline rules and rules.

A general definition of mathematical modelling is expressed in the most general sense as the process of trying to express mathematical or non-mathematical events, phenomena, relationships between events and to reveal mathematical patterns within these events and phenomena [2].

In the literature, there are mathematical models developed by Malthus [3] and Verhulst [4] on population growth, as an example of mathematical models in this sense. As an example of Malthus and Verhults models, the study of Pingle [5] on Malthus’s population principle using dynamic models, Brilhante, Gomes and Pestana [6] on the extension of the classical Verhults logistics model, Şekerci and

Petrovskii [7] on oxygen and plankton dynamics, Bender [8] on the details of mathematical modelling, and Junior [9] on the population growth of Sergipe can be given as source works.

On the other hand, many studies on the factors that affect student success [10] and some studies on the discipline factor and environmental conditions in education [11,12] have been conducted.

In this study, “discipline factors”, “teacher factors”, and “negative factors” were taken into consideration as the factors affecting the success of the student, class or school, and it was also stated that the personal contribution of the student could be taken into consideration. In this context, a mathematical model of success was tried to be proposed. First, an exponential success relation was obtained with the help of an initial value problem, and the coefficient of the time variable in the correlation was kept dependent on the mentioned factors. Since these factors may vary with time, they are functions according to time-independent variables. Therefore, since the graph of these functions will be formed according to the real values taken by these factors in any time interval, the graphs will be constant, increasing or decreasing according to the time variable so that it is a curve as convex or concave or a line in a plane.

For example, it is found a point for each factor in the set of disciplinary factors for any time interval, where the arithmetic means of the real discipline values predicted and obtained accordingly. Likewise, a point is obtained at any time. Depending on the disciplinary values obtained from the first point to the second point, it may be reached along a line or curve. If the process between $t_1$and t_2 can be observed, and it is determined that the arrival from the first point to the second point is along a curve, and if the function of this curve can be determined, the disciplinary values between these two points can be studied with the help of the determined function. However, if the disciplinary values are not observed in the process between t_1 and t_2 or if they are determined that the arrival from the first point to the second point is along a line or almost a line, the disciplinary values between these two points are studied with the help of linear function.

In this research, it has been accepted that obtaining the values of the factors affecting the success both in points and between these points is with the help of linear functions and non-linearity case is explained in the methods and applications sections.

2. Method

In establishing the criterion, the basic principle of change, differential equations, the solution of the initial value problem, mathematical models provided by [3,4] were considered.

2.1. Establishment of Student Success Model in Education

Preliminary Definitions

t: Independent time variable

$S(t)$: Success value obtained at any time t

Δt : Time interval in variation

D : Discipline factors

The cases such as positive or negative behaviours and behaviours in personal attitudes, behaviours and habits of the student, compliance or not compliance with the rules and rules of environment, everything’s being free or too extreme rules for the student, there being the necessary sanctions or not, the student’s wellbeing managed or not wellbeing managed [11].

T : Teacher performance factors

The teacher is the class administrator, trainer, instructor and leader, both socially and academically. The teacher is the person who knows and applies the rules and principles of teaching, duties and responsibilities. The ability of the teacher to carry and maintain the teaching mission is called teacher performance [10].

N : Negative factors

The negative intervention of the family to education, economic conditions or of future anxiety that negatively affect the student's work, causing the student not to be motivated or worried about his goals, external conditions such as whether the environment is suitable for education or not, whether the environment is clean and decent or not, passing a course without learning, and not failing because of discontinuity [10].

In this study, functions defined as $D = f(t)$, $T = g(t)$ and $N = h(t)$ for disciplinary factors, teacher performance factors and negative factors, respectively and given their detail in the following sections are real-valued functions, and in terms of the integrity study, all three are considered as linear functions. However, any one or all of these functions may not be linear. An explanation of this situation is given in "Applications (B)".

Taking these definitions into consideration as Malthusian population modelling, let k be the variation of the success, Δt be the small-time increment of the independent time variable t, the change of success function $S(t)$ is

$$S(t + \Delta t) - S(t) = kS(t)\Delta t \tag{1}$$

Then,

$$\frac{S(t + \Delta t) - S(t)}{\Delta t} = kS(t)$$

And if we take limit for$\Delta t \to 0$, we obtain the differential equation as follows:

$$\frac{dS(t)}{dt} = kS(t)$$

At the first instance, that is $t = t_0$, let the available success value be S_0, we can write the following initial value problem with this equation:

$$\frac{dS(t)}{dt} = kS(t), S(t_0) = S_0$$

When solved this problem, we obtain the below formula (relation)

$$S(t) = e^{k(t-t_0)}S_0 \tag{2}$$

If we take the first time when success value is determined as $t_0 = 0$, the formula (2) will be

$$S(t) = e^{kt}S_0 \tag{3}$$

Here, the value of k, at time $t = t_j$, will be determined by $D = f(t_j)$, $T = g(t_j)$ and $N = h(t_j)$

it will be $k = k_j$ for the time value in question.

Since the increase in the value of $D = f(t)$ and $T = g(t)$ will affect the success in a positive sense and the increase in the value of $N = h(t)$ will affect the success in a negative sense, the coefficientk_j at $t = t_j$ time will be determined as

$$k_j = \frac{f(t_j)g(t_j)}{h(t_j)} \in \mathbb{R}$$

Therefore, the value of success at time $t = t_j$, will be obtained as

$$S(t_j) = e^{k_j(t_j - t_0)} S_0 \tag{4}$$

according to the equality (2) and as

$$S(t_j) = e^{k_j t_j} S_0 \tag{5}$$

2.2. Aim

This study aims to create a mathematical model to examine the success of the student, class or school over a certain period of $[t_0, t_v]$ time. Informing this model, firstly, the factors that are certain to affect the success positively or negatively were determined. To determine the status of success, the course of success within the time interval considered will be examined with the help of this model.

First of all, the task is to determine the coefficients k_j in time values $t_j \in [t_0, t_v]$ utilizing linear functions of "disciplinary factors", "teacher factors", and "negative factors".

The values $S(t_j)$ can then be determined using (4) or (5), and comments can be made on the success of the time interval.

Let us now give detailed descriptions, explanations and assumptions of the unknowns of D, T and N by considering the formulas (2) and (3).

3. Results

3.1. The Case $t \in [0, t_v]$

1) Disciplinary factors D.

Disciplinary factors in an educational environment are determined by the help of the function $D = f(t)$ defined as

$$f: [0, t_v] \to \mathbb{R},\ a_1, b_1 \in \mathbb{R}.$$

Here,

a) For $t = 0$, the first moment of disciplinary value determination,$f(t) = b_1$. If $f(t) = 0$intime $t = 0$, that is, if disciplinary values are not available, the disciplinary function is in the form of as

$$f(t) = a_1 t$$

b) If$f(t) > 0$ in the time interval $[0, t_v]$, the arithmetic means of the values obtained during this time interval is as following

$$\overline{f(t)} = \frac{1}{n}\sum_{j=1}^{n} f(t_j) = \alpha$$

where the mean value of discipline function$f(t)$ is $\alpha \in \mathbb{R}$.

c) If an increasing discipline function at a time $t = t_1$continues to increase after reaching the necessary and sufficient legal and humanitarian disciplinary values at a time $t = t_1$, discipline values are considered negative and

$$f(t) = -a_1 t - b_1$$

is considered as a disciplinary function after this point. Where $a, b > 0$, since there will be excessive discipline after this point. The maximum point where the disciplinary value of transition from normal to extreme discipline will be expressed as $t = t_j = t_{max}$ in $t = t_j \in [0, t_v]$. According to this,

$$\max(f(t_{\max})) = a_1 t_{\max} + b_1 = c_1$$

where $c_1 \in \mathbb{R}$.

Here, excessive discipline is when the student is anxious, intimidated and subjected to violence beyond the normal rules of discipline. It is thought that in the case of excessive discipline, success is negatively affected. In this case, disciplinary function $f(t)$ is

$$f(t) = \begin{cases} a_1 t + b_1, 0 \leq t \leq t_{max} \\ -a_1 t - b_1, tv_{max} \end{cases}$$

where $a_1, b_1 > 0$.

d) In time interval $[0, t_v]$, if $f(t) > 0$and the value of disciplinary function$f(t)$ is constant, then

$$f(t) = b_1$$

where $b_1 > 0$.

e) If the disciplinary values for the time interval $[0, t_v]$ do not exist, then

$$f(t) = 0$$

shall be taken.

f) As long as the disciplinary function $f(t)$ in the time interval $[0, t_v]$ is under the t -axis until $f(t) = 0$, that is if the disciplinary situation in the environment is absolutely undesirable, then since the disciplinary function is in the following type

$$f(t) = -a_1 t,$$

$$f(t) = -a_1 t - b_1,$$

$$f(t) = -b_1$$

where $a_1, b_1 > 0$, in this case, $f(t) < 0$ and we consider

$$f(t) = f\left(\frac{t_1}{2}\right)$$

2) The function$T = g(t)$, expressing the teacher's performance factors, is defined as follows.

$$g: [0, t_v] \to (0, m], g(t) = a_2 t + b_2, a_2, b_2, m \in \mathbb{R}$$

a) The average contribution value of teacher performance over the time interval $[0, t_v]$ will be the arithmetic means of the values obtained during this period, that is,

$$\overline{g(t)} = \frac{1}{n} \sum_{j=1}^{n} f(t_j) = \beta$$

where $\beta \in (0, m]$.

b) It is thought that teacher contribution will be positive in good faith as it should be.

c) Anyway, if $g(t) \leq 0$, success will decrease compared to the first situation by formula.

d) The reason why the right endpoint is not included in the interval $(0, m]$ is that each teacher can have a maximum performance value. According to this, the maximum expression of teacher value is

$$\max\big(g(t_{\max})\big) = a_2 t_{\max} + b_2 = m$$

where the maximum point is $t = t_j = t_{max}$, for $t = t_j \in [0, t_v]$.

Note: If $g(t) < 0$and$f(t) < 0$, then it does not mean that $\frac{f(t)g(t)}{h(t)} > 0$.

3) The function $N = h(t)$, which expresses the state of negative factors, is defined as

$$h: [0, t_v] \to [1, n), h(t) = a_3 t + b_3, \;\; a_3, b_3, n \in \mathbb{R}$$

a) The average contribution value of negative factors to student achievement in the time interval $[0, t_v]$ that is, the arithmetic means of the values obtained during this time interval will be

$$\overline{h(t)} = \frac{1}{n}\sum_{j=1}^{n} f(t_j) = \delta$$

where $\delta \in [1, n)$.

b) The reason why the lower bound of the interval $[1, n)$ is $\delta = 1$ that in the event that the negative value is only the minimum value $\delta = 1$, the success will not be adversely affected. Moreover, the case that $\delta > 1$ is the state of$A$ negatively affecting success.

c) In the case of $\delta < 1$, it will be revealed as if negative situations would positively contribute to success.

d) The reason why the upper bound of the interval $[1, n)$ is not included is due to the fact that there may be an upper limit of negative conditions but not the maximum value.

3.2. The Case $t \in [t_0, t_v]$ ($t_0 < t_v$ and $t_0 \neq 0$):

1) For the discipline value D, linear discipline function $D = f(t)$ is as following:

$$f: [t_0, t_v] \to \mathbb{R}, f(t) = a_1 t + b_1, a_1, b_1 \in \mathbb{R}.$$

Here,

a) If $f(t) = 0$ in $t = t_0$ time, that is, there are no disciplinary values, then the disciplinary function is as follows:

$$f(t) = a_1 t + b_1 = 0$$

b) The average mean of the disciplinary function $D = f(t)$ in the time interval $[t_0, t_v]$ is again the arithmetic mean of the values obtained during this period, that is,

$$\overline{f(t)} = \frac{1}{n}\sum_{j=1}^{n} f(t_j) = \alpha$$

where $\alpha \in \mathbb{R}$.

c) If an increasing $f(t) > 0$, the disciplinary function is still increasing after the moment $t = t_2$ in time interval $[t_0, t_v]$, that is, if there is excessive discipline as stated in item 1.c regarding the case

$t \in [0, t_v]$, we consider the

$$f(t) = -a_1 t - b_1$$

as a disciplinary function where $a_1, b_1 > 0$.Here too, the maximum point $t = t_j$, which is a transition from normal disciplinary values to extreme disciplinary values, will be expressed as $t = t_j = t_{max}$. According to this,

$$\max(f(t_{\max})) = a_1 t_{\max} + b_1 = c_1$$

where $c_1 \in \mathbb{R}$.

In this case, the disciplinary function is

$$f(t) = \begin{cases} a_1 t + b_1, t_0 \leq t \leq t_{max} \\ -a_1 t - b_1, t1_{max} \end{cases}$$

where $a_1, b_1 > 0$.

d) In time interval $[t_0, t_v]$, if$f(t) > 0$ and the value of the disciplinary function$f(t)$ is constant, then

$$f(t) = b_1$$

where $b > 0$.

e) If there is no disciplinary provision in the time interval $[t_0, t_v]$, then again

$$f(t) = 0$$

is considered.

f) As long as the disciplinary function $f(t)$ is below the t -axis until $f(t) = 0$, in the time interval $[t_0, t_v]$ that is, if the disciplinary situation in the environment is absolutely undesirable, then since the disciplinary function will be as

$$f(t) = -a_1 t,$$

$$f(t) = -a_1 t - b_1,$$

$$f(t) = -b_1$$

where $a_1, b_1 > 0$, then$f(t) < 0$ and the value of discipline is obtained as

$$f(t) = f\left(\frac{t_0 + t_1}{2}\right)$$

2) The function $T = g(t)$ expressing the value of teacher's performance in the time interval $[t_0, t_v]$ is defined as follows:

$$g: [t_0, t_v] \to (0, m], g(t) = a_2 t + b_2, a_2, b_2, m \in \mathbb{R},$$

where a and $b$are positive real numbers.

a) The average contribution value of teacher performance is again the arithmetic mean of the values obtained during this time interval $[t_0, t_v]$, that is,

$$\overline{g(t)} = \frac{1}{n} \sum_{j=1}^{n} f(t_j) = \beta$$

where $\beta \in (0, m]$.

b) Maximum expression of teacher value is

$$\max(g(t_{\max})) = a_2 t_{\max} + b_2 = m$$

where $m \in \mathbb{R}$ and the maximum point is $t = t_j = t_{max}$ for $t = t_j \in [t_0, t_v]$.

The function $N = h(t)$ expressing the state of negative factors in the time interval $[t_0, t_v]$ is defined as

$$h: [t_0, t_v] \to [1, n), h(t) = a_3 t + b_3, a_3, b_3, n \in \mathbb{R},$$

where a_3 and b_3 are positive real numbers.

The average contribution value of negative factors to the success of the student is the arithmetic mean of the values obtained during this period, and it is as follows:

$$\overline{h(t)} = \frac{1}{n}\sum_{j=1}^{n} f(t_j) = \delta$$

where $\delta \in [1, n)$. Other features of the function $h(t)$ are expressed in item 3 of case $t \in [0, t_v]$.

4. Conclusion and Suggestions

4.1. Some Evaluations of Success Formula

1) If $f(t_1) = \alpha_1 = 0$, since $S(t) = S_0$, it means no success has been achieved regardless $t_1 \in [t_0, t_v]$, $\delta_1 \in [1, n)$ and of teacher value $\beta_1 \in (0, m]$ is.

2) If $f(t) < 0$ for $t_1 \in [t_0, t_v]$, negative case $\delta_1 \in [1, n)$ and the value of teacher$\beta_1 \in (0, m]$, for example, let$f(t_1) = \alpha_1 = -2$. In this case, since

$$S(t_1) = e^{-\frac{2\beta_1}{\delta_1}(t_1 - t_0)} S_0 = \frac{S_0}{e^{\frac{2\beta_1}{\delta_1}(t_1 - t_0)}}$$

our success is even behind the point we started in time. Again, let $t_1 > t_{max}$ where the other conditions are the same. In this case, since$f(t_1) = at_1 + b = \alpha_2 < 0$,

$$S(t_1) = e^{\frac{\alpha_2\beta_1}{\delta_1}(t_1 - t_0)} S_0 = \frac{S_0}{e^{\frac{\alpha_2\beta_1}{\delta_1}(t_1 - t_0)}}$$

is obtained, and again a negative result is obtained.

3) In the case of $f(t) > 0$, decreasing the contrary intervention value $\delta_1 \in [1, n)$, increasing the teacher value $\beta_1 \in (0, m]$, then the success of the student will increase.

4) We're where we started on behalf of success, since $S(t) = S_0$, where $f(t) > 0$ and $n \to \infty$.

5) According to formula (2), to obtain an average achievement in education, it must be reached

$$\overline{S(t)} = e^{\frac{(ort f(t))(ort(g(t)))}{ort(h(t))}(t - t_0)} S_0 = e^{\frac{\alpha\beta}{\delta}(t - t_0)} S_0$$

6) According to formula (2), for maximum success in education,

$$\max(S(t)) = e^{\frac{(\max f(t))(\max g(t))}{\min h(t)}(t - t_0)} S_0 = e^{\frac{c_1 m}{1}(t - t_0)} S_0 = e^{c_1 m(t - t_0)} S_0$$

must be reached.

7) Let $t_1, t_2 \in [t_0, t_v]$, $(t_2 < t_3)$, disciplinary value $f(t_j) = \alpha_j$, teacher value $g(t_j) = \beta_j$, contrary intervention value $h(t_j) = \delta_j$ for $(j = 1,2)$.

a) To continue the success of $B(t)$, it must be as following:

$$\left.\frac{\alpha_j\beta_j}{\delta_j}\right|_{t=t_{j+1}} > \left.\frac{\alpha_j\beta_j}{\delta_j}\right|_{t=t_j}$$

b) In the case that $\frac{\alpha_j\beta_j}{\delta_j} = 1$ for $t_j \in [t_0, t_v]$, then $S(t_j) = e^{t_j - t_0}S_0$. In this case, since the values of D, T and N do not affect success and S_0 has only a time-dependent coefficient, success will be considered as

$$S(t) = e^{t-t_0}S_0 = \text{constant}$$

c) In the case that $\left.\frac{\alpha_j\beta_j}{\delta_j}\right|_{t=t_j} > 1$ for $t_j \in [t_0, t_v]$, since $\alpha_j\beta_j > \delta_j$ must be, if the value δ_j increase, the value of β_j will have to grow steadily to achieve inequality because the value of α_j can increase to a maximum of c. In fact, to increase the success, while $\alpha_j\beta_j > 1$, $\delta_j \to 1$ is the desired situation.

Now, here is a comparative example. Let $[0, t_v]$ be any time interval. In $t_j = 2 \in [0, t_v]$ time, assume that the disciplinary value is the number $f(2) = \alpha_1 = 3$ as regards disciplinary function, where $\alpha_1 \in \mathbb{R}$, the negative case value is the number $h(2) = \delta_1 = 2$ as regards the negative factors function and teacher value is the number $g(2) = \beta_1 = 10$ as regards the teacher factor function, where $\beta_1 \in (0, m]$. In this case,

$$S(t) = e^{\frac{3.10}{2}2}S_0 = e^{30}S_0$$

is obtained. Now, let us just take $h(2) = 5$ and not change the others. In this case,

$$S(t) = e^{\frac{3.10}{5}2}S_0 = e^{12}S_0$$

is obtained. As seen, the increasing value of $h(t)$ decreases the value of success. Under the same conditions, if α_1 is to be fixed and $\delta_1 > 30$, $\beta_1 > 10$ should be to ensure inequality since $\alpha_1\beta_1 = 30$, which means that the teacher should exceed his current performance.

4.2. The Case of Consideration of Student Contribution

Considering that the student personally can contribute to the student achievement formula (2) and (3), a function that expresses the student's contribution value in terms of personal and school interest and interest can be considered. Since social issues related to students' attitude, behaviour and habit movements are included in the discipline values, students' contribution to academic achievements such as studying, doing homework, questioning, research, interest, and attention to the course can only be considered.

Here if the academic contribution function to the success of a student's class or school is shown as $R = \varphi(t)$, then it is defined as follows:

$$\varphi: [t_0, t_v] \to (0, r], \varphi(t) = a_4 t + b_4,\ a_4, b_4 r \in \mathbb{R}.$$

Since the student's academic contribution is considered to be positive, the value set of the function is taken as a positive interval. Here,

a) The fact that zero is not included in the value set is that the student is at least in the classroom is considered a positive value.

b) The inclusion of the right endpoint in the value set is that the student can make a maximum contribution to success.

c) Since $k = \frac{DT}{N}$, when considering the student's academic contribution, as this coefficient is $k = \frac{RDT}{N}$, the success formula in the time interval$[0, t_v]$ is as following:

$$S(t) = e^{\frac{RDT}{N}t}S_0$$

Moreover, in the time interval $[0, t_v]$, the mean value of the function $R(t)$ is the arithmetic mean of the values obtained during this time interval and shall be

$$\overline{\varphi(t)} = \frac{1}{n}\sum_{j=1}^{n} f(t_j) = \gamma$$

If the maximum point of the function $R(t)$ is $t = t_j = t_{max}$, then

$$\max(\varphi(t_{\max})) = a_4 t_{\max} + b_4 = r$$

d) The success formula in the time interval $[t_0, t_v]$ will be

$$S(t) = e^{\frac{RDT}{N}(t-t_0)} S_0.$$

On the other hand, in the time interval $[t_0, t_v]$, the mean value of the $R(t)$ function will again be the arithmetic mean of the values obtained during this time interval, i.e.,

$$\overline{\varphi(t)} = \frac{1}{n}\sum_{j=1}^{n} f(t_j) = \gamma$$

Therefore, the average value of success is as follows:

$$\overline{S(t)} = e^{\frac{\overline{\varphi(t) f(t) g(t)}}{\overline{h(t)}}(t-t_0)} S_0 = e^{\frac{\gamma\alpha\beta}{\delta}(t-t_0)} S_0.$$

The maximum value will be

$$\max(\varphi(t_{\max})) = a_4 t_{\max} + b_4 = r$$

where the point $t = t_j = t_{\max}$ is the maximum contribution value of the student is achieved academically. In this case, the maximum value of the success is obtained as:

$$\max(S(t)) = e^{\frac{(\max\varphi(t))(\max f(t))(\max g(t))}{\min h(t)}(t-t_0)} S_0$$
$$= e^{\frac{rc_1 m}{1}(t-t_0)} S_0$$
$$= e^{rc_1 m(t-t_0)} S_0$$

Here, for $\overline{S(t)}$ and $\max(S(t))$, $t_0 = 0$ in time interval $[0, t_v]$.

Explanation

1) The fact that $S(t) = S_0$ and especially $0 < S(t) < 1$ is definitely a failure situation. In such a situation, each value of success increases negatively, that is $k \leq 0$. Since the value of success $S(t)$would also represent a success above the status of failure can be graded in terms of success.

2) Since only the coefficient k determines the value of success $S(t)$ if it is desired the multiplier $(t - t_0)$ to be ineffective, the length of time intervals can be selected to be fixed; that is, we can take $t_{j+1} - t_j$ is constant.

5. Applications

A) The case where the functions $\boldsymbol{D = f(t), T = g(t)}$ and $\boldsymbol{N = h(t)}$ are linear:

Let D be the set consisting of disciplinary values D_1,D_2,D_3,D_4,D_5, i.e.

$$D = \{D_1, D_2, D_3, D_4, D_5\}$$

and the corresponding the real numbers of the disciplinary value D_1, D_2, D_3, D_4, D_5 is d_{j1}, d_{j2}, d_{j3}, d_{j4}, d_{j5} ($j = 1{,}2$), respectively in time interval $t = t_j$.

For $t = t_1$, we consider the number

$$d_{t_1} = \frac{d_{11} + d_{12} + d_{13} + d_{14} + d_{15}}{5}$$

and for $t = t_2$,

$$d_{t_2} = \frac{d_{21} + d_{22} + d_{23} + d_4 + d_{25}}{5}$$

as disciplinary value. According to this, since $f(t_1) = a_1 t_1 + b_1$ and $f(t_2) = a_1 t_2 + b_1$, we obtain the following system:

$$\begin{cases} a_1 t_1 + b_1 = d_{t_1} \\ a_1 t_2 + b_1 = d_{t_2} \end{cases}$$

From this system, the coefficients a and b are found as follows:

$$a_1 = \frac{d_{t_2} - d_{t_1}}{t_2 - t_1} \text{ and } b_1 = \frac{t_2 d_{t_1} - t_1 d_{t_2}}{t_2 - t_1}$$

So, the equation of a line passing through points (t_1, d_{t_1}) and (t_2, d_{t_2}) will be

$$f(t) = a_1 t + b_1 = \frac{d_{t_2} - d_{t_1}}{t_2 - t_1} t + \frac{t_2 d_{t_1} - t_1 d_{t_2}}{t_2 - t_1}$$

A disciplinary value between points $t = t_1$ and $t = t_2$ can also be estimated using this line equation. Here, disciplinary values in the set $D = \{D_1, D_2, D_3, D_4, D_5\}$ is reproducible or reducible.

Same way, after determining the $T = \{T_1, T_2, T_3, T_4, T_5\}$ for the teacher value T, and

$N = \{N_1, N_2, N_3, N_4, N_5\}$ for the negative factors A, real values for these values can be given, and linear functions $g(t)$ and $h(t)$ can be written.

As a result, after obtaining the coefficient

$$k_j = \frac{f(t_j) g(t_j)}{h(t_j)}$$

and by determining the real value of $f(t_j)$,$g(t_j)$ and $h(t_j)$, the success value $S(t_j)$ is found in any time $t = t_j$. If desired, considering the student's academic contribution value, the success value $S(t_j)$ will be determined with the help of the coefficient

$$k_j = \frac{\varphi(t_j) f(t_j) g(t_j)}{h(t_j)}$$

B) The case where the functions$\boldsymbol{D = f(t)}$, $\boldsymbol{T = g(t)}$ and $\boldsymbol{N = h(t)}$ are not linear:

Any or all of these functions are nonlinear. For example, assume that the disciplinary function is quadratic as following:

$$D = f(t) = at^2 + bt + c$$

Moreover, let disciplinary values be d_{t_1}, $d_{t_2}$and d_{t_3}, respectively for $t = t_1$, $t = t_2$and$t = t_3$, where $t_1 < t_2 < t_3$. According to this, we obtain the following system:

$$\begin{cases} at_1^2 + bt_1 + c = d_{t_1} \\ at_2^2 + bt_2 + c = d_{t_2} \\ at_3^2 + bt_3 + c = d_{t_3} \end{cases}$$

The coefficients a, $b$and c belonging to disciplinary functions are determined as following from the system:

$$a = \frac{d_{t_1}t_2 + d_{t_3}t_1 + d_{t_2}t_3 - d_{t_3}t_2 - d_{t_1}t_3 - d_{t_2}t_1}{t_1^2t_2 + t_3^2t_1 + t_2^2t_3 - t_3^2t_2 - t_1^2t_3 - t_2^2t_1}$$

$$b = \frac{d_{t_2}t_1^2 + d_{t_1}t_3^2 + d_{t_3}t_2^2 - d_{t_2}t_3^2 - d_{t_3}t_1^2 - d_{t_1}t_2^2}{t_1^2t_2 + t_3^2t_1 + t_2^2t_3 - t_3^2t_2 - t_1^2t_3 - t_2^2t_1}$$

$$c = \frac{d_{t_3}t_1^2t_2 + d_{t_2}t_3^2t_1 + d_{t_1}t_2^2t_3 - d_{t_1}t_3^2t_2 - d_{t_2}t_1^2t_3 - d_{t_3}t_2^2t_1}{t_1^2t_2 + t_3^2t_1 + t_2^2t_3 - t_3^2t_2 - t_1^2t_3 - t_2^2t_1}$$

The disciplinary value can now be determined at a time $t = t_j$ using the disciplinary function whose coefficients are known. Namely, if the functions $T = g(t)$ and $N = h(t)$ are quadratic, their coefficients can be determined as above.

Two of these functions can be quadratic, and the third is linear, or one can be quadratic, and the other two are linear. To determine the coefficients of the quadratic function, we use the points (t_1, d_{t_1}), (t_2, d_{t_2}) and (t_3, d_{t_3}) above. Since there will be time interval $[t_1, t_3]$ for a linear function, the coefficients of this function are calculated separately for time intervals $[t_1, t_2]$ and $[t_2, t_3]$ two functions are determined concerning these time intervals. That is to say, the coefficients of the first function will be obtained with the help of the points (t_1, d_{t_1}), (t_2, d_{t_2}) concerning the linear function and the coefficients of the second function will be obtained with the help of the points (t_2, d_{t_2}), (t_3, d_{t_3}) in time interval $[t_1, t_3]$. So, this linear function will be as following:

$$\begin{cases} a_1t + b_1, t \in [t_1, t_2] \\ a_2t + b_2, t \in [t_2, t_3] \end{cases}$$

For any time $t_j \in [t_1, t_3]$, the nonlinear function will be used directly and for the linear function, the function of the time interval in which the time value $t = t_j$ is to be considered.

6. Results and Discussion

In this study, some real results will be obtained with the help of the functions installed for the "discipline factors", "teacher performance factors", and "negative factors". Therefore, in the case of the value of the success desired to be learnt is the negative according to the results, the function or the function or the functions which cause the failure can be determined. The function or functions which adversely affect the success of the function are improved. For example, if the failure is due to more disciplinary values, the real value of decreasing the mean by decreasing the average is small; it will make to correct that value or that values by examining the earlier determined discipline values. After determining the disciplinary values for two points of a time interval, the linear line equation of the function can be determined with the help of their averages. By considering the line of this equation, it is possible to determine the disciplinary values for a time value between these time points. On the other hand, the model can be implemented with the help of the model and the functions depending on the model to achieve positive results for success because it is not intended to determine success only. Besides, it is possible to make suggestions based on the results obtained in terms of the elements of the sets of values.

Of course, comments can be made in the name of values derived from functions established in the name of "discipline factors", "teacher factors", and "negative factors". On the other hand, since there will be a linear function for the mentioned values, it will first determine the failing or featured value by reading the status of these functions by good reading. For this reason, the study aims to make an application in the name of success by meaning the clusters of the mentioned values, determining the status of the success for an instant or a time interval, or making a suggestion in cases or cases where the success is negative.

This study aims to establish a criterion for examining the success of the student, class, or school in a certain period. The way of success can be examined in the time taken into consideration with the help of this criterion. For this purpose, the first thing to do is to determine the coefficient in time values with the help of linear functions installed for the values, which will be defined as "discipline factors", "teacher factors", and "negative factors". Then, the success values can be determined depending on the mentioned values by using the relation obtained as a solution of an initial value problem and comments on success can be made in the aforesaid time interval.

Conflicts of Interest

The author declares no conflict of interest.

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Recent applications of microencapsulated phase change materials

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Keywords:
Microcapsule, Thermochromic system, Thermotropic system, Electrospinning method

Abstract — This study aims to inform about the up-to-date knowledge on the applications of microencapsulated phase change materials (MEPCMs). The most recent applications of MEPCMs are in the fields of thermotropic, thermochromic and electrospinning, which can be considered as smart systems. MEPCMs serve as protector materials for those unresistant against thermal environmental conditions. The common treatment of them is thermal storage. They serve other or hybrid properties after functionalization. Especially in solar energy systems, thermotropic materials are an important technology for overheating protection. Applications of reversible thermochromic microcapsules are extensively applied in many ways: sinks, coatings of smart materials, cement, textiles, luminescent thermosensors, and colour indicators. Nanofibers obtained by electrospinning method have many uses such as cosmetic applications, tissue engineering, filtration applications, agricultural applications, nanosensors, biomedical tools, protective clothing, reinforced composite making, controlled active substance release and will serve shortly responding thermal systems to environmental changes due to extended surface area and body structure due to fiber formation.

Subject Classification (2020):

1. Introduction

Storage of low or high heat energy temporarily for later use is defined as thermal energy storage. This feature acts as a transition between the energy need and the use of energy [1]. There are three types of heat storage methods: latent heat, sensible heat, and chemical reaction heat. Among the different thermal energy storage techniques, one of the most effective thermal energy storage methods is latent heat storage due to its high energy storage capacity. Latent heat is defined as the heat stored or emitted during phase change [2-6]. Substances that can absorb and store heat during the transition from one phase to another, i.e., during the phase change process, and on the contrary, can dissipate this stored heat in the case of phase change are known as phase changing substances (PCM) [3,4]. Phase changing substances transition from one phase to another within a certain temperature range [5]. Substances show four types of phase changes: solid-liquid, liquid-gas, solid-gas and solid-solid phase change [7-9]. PCM's temperatures are constant during melting and freezing/crystallization. With this feature, PCMs

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absorb more heat than other materials [10-12]. During the phase change, its temperature remains constant until the phase changing substance melts or solidifies [13-15].

The fact that they absorb a large amount of latent heat and emit this latent heat during melting and solidification processes without a significant temperature change causes phase changing substances to be in great demand as a heat storage source. The latent heat energy that occurs during the solid-liquid phase change is much higher than the energy (sensible heat) generated during the cooling/heating of the substance [4-6]. PCMs with different melting temperatures are used in various fields for thermal energy storage. For example, they are used in building materials, textile products, transportation and storage of temperature-sensitive materials (medical products, food, etc.), cooling of electronic devices, active and passive heating and cooling systems [16]. Classification of PCMs used for thermal energy storage is given in Figure 1.

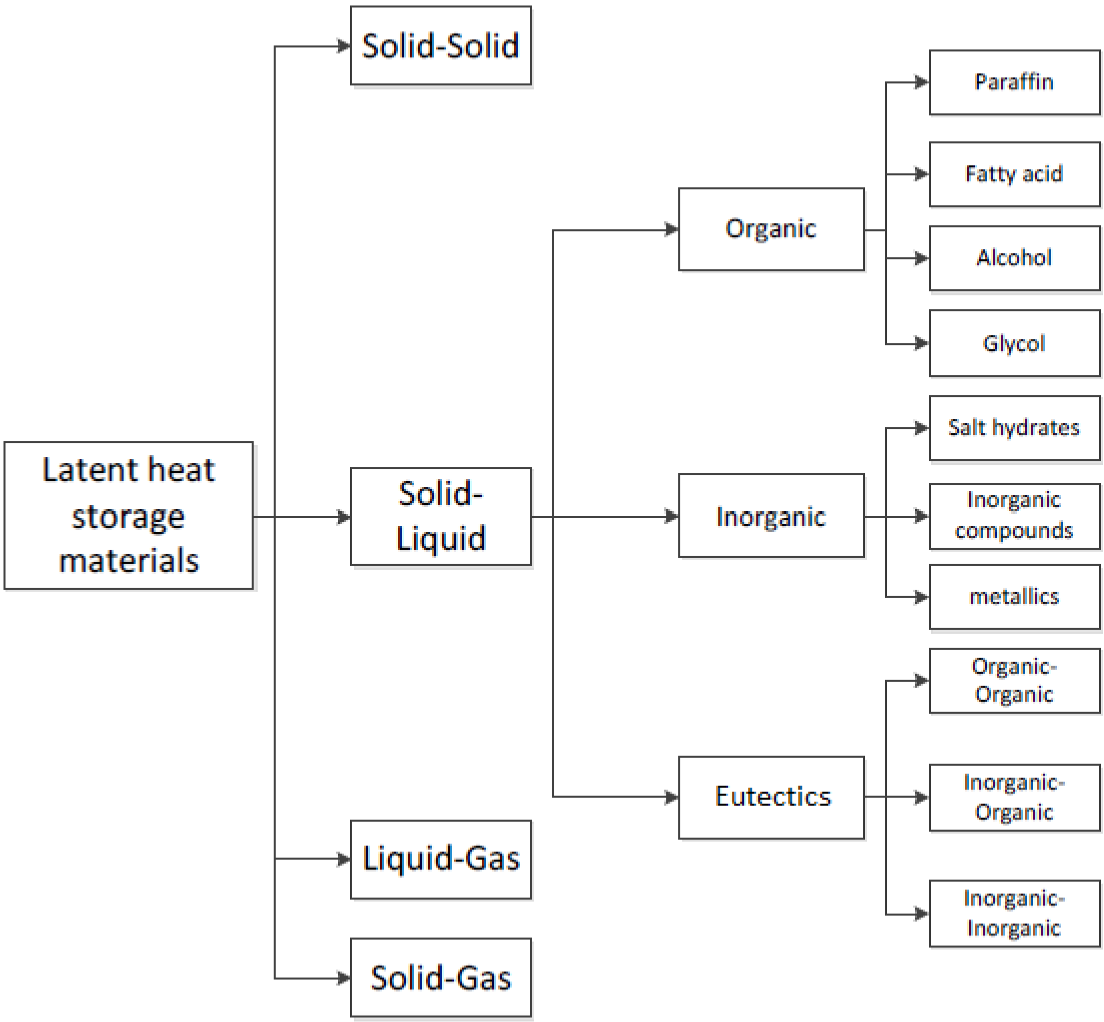

Figure 1. Classification of PCMs used for thermal energy storage [6]

Some basic parameters such as phase change temperature, phase change enthalpy, solid and liquid heat capacity and thermal conductivity should be considered while selecting PCM. Substances that have phase change in the range of 0-120 °C are candidates for use as PCM. These are grouped as organic, inorganic, and mixtures and also divided into subgroups within themselves [17]. Solid-liquid organic PCM species can leak into their environment during the heat storage process if applied directly to the material without encapsulation. Most organic PCMs are flammable, posing a serious potential fire hazard, in addition to having low thermal conductivity and poor thermal response. However, most inorganic PCMs are corrosive, which can cause irreversible damage to storage containers [10]. Such problems can be overcome using microencapsulated phase change materials (MEPCMs) or nanocapsulated phase change material (NEPCM) with different encapsulation technology. As shown in Figure 2, a typical structure of MEPCM or NEPCM consists of core and shell layers that can be classified as mononuclear, polynuclear, or matrix type [6]. The thermophysical properties of some paraffin are as in Table 1. Also, the thermophysical properties of some eutectic PCMs are given in Table 2.

Table1. Thermophysical properties of some paraffin [6]

Name	Chemical Formula	T_m (°C)	$H\ (kj/kg)$	$k\ (W/mK)$	$\rho\ (kg/m^3)$	$Cp\ (kj/kg)$
n-Dodecane	$C_{12}H_{26}$	-12	216	0.21 (s), 0.21 (l)	750	n.a.
n-Tridecane	$C_{13}H_{28}$	-6	n.a.	n.a.	756	n.a.
n-Tetradecane	$C_{14}H_{30}$	4.5-5.6	231	n.a.	771	n.a.
n-Pentadecane	$C_{15}H_{32}$	10	207	0.17	768	n.a.
n-Hexadecane	$C_{16}H_{34}$	18.2	238	0.21 (s)	774	n.a.
n-Heptadecane	$C_{17}H_{36}$	22	215	n.a.	778	n.a.
n-Octadecane	$C_{18}H_{38}$	28.2	245	0.35 (s), 0.149 (l)	814 (s), 775 (l)	2.14 (s), 2.66 (l)
n-Nonadecane	$C_{19}H_{40}$	31.9	222	0.21 (s)	912 (s), 769(l)	n.a.
n-Eicosane	$C_{20}H_{42}$	37	247	n.a.	n.a.	n.a.
n-Heneicosane	$C_{21}H_{44}$	41	215	n.a.	n.a.	n.a.
n-Docosane	$C_{22}H_{46}$	44	249	n.a.	n.a.	n.a.
n-Tricosane	$C_{23}H_{48}$	47	234	n.a.	n.a.	n.a.
n-Tetracosane	$C_{24}H_{50}$	51	255	n.a.	n.a.	n.a.
n-Pentacosane	$C_{25}H_{52}$	54	238	n.a.	n.a.	n.a.

Table 2. Thermophysical properties of some eutectic PCMs [6]

Name	Composition $(wt.\%)$	T_m (°C)	$H\ (kj/kg)$
Diethylene glycol	n.a.	-10	247
Tetradecane+octadecane	n.a.	-4.02	227.52
Water+polyacrylamide	n.a.	0	295
Tetradecane+docosane	n.a.	1.5-5.6	234.33
Tetradecane+hexadecane	91.67+8.33	1.7	156.2
Tetradecane+geneicosane	n.a.	3.54-5.56	200.28
Na_2SO_4+NaCl+KCl+H_2O	31+13+16+40	4	234
Tetrahidrofurano (THF)	n.a.	5	280
Pentadecane+heneicosane	n.a.	6.23-7.21	128.25
Pentadecane+docosane	n.a.	7.6-8.99	214.83
Pentadecane+octadecane	n.a.	8.5-9.0	271.93
Na_2SO_4+NaCl+KCl+H_2O	32+14+12+42	11	n.a.
$C_5H_5C_6H_5+(C_6H_5)_2O$	26.5+73.5	12	97.9
Triethylolethane+water+urea	38.5+31.5+30	13.4	160

(Cp: Specific heat (kJ / kg), H: Latent heat (kJ / kg), k: Thermal conductivity (W / m • K), T_m: Melting temperature (°C), ρ: Density (kg / m^3).

Microcapsules are nano, micro or macro-sized particles produced by placing an active ingredient in a polymeric wall structure as the core material. Phase-changing microcapsules are microcapsules containing PCM as the core material. By microencapsulation, the PCM in the liquid phase can be packed in micro size, and the permanence in the structure can be ensured [18]. Microcapsules can be single-core, multi-core and matrix-shaped structures. The compatibility of core-wall materials is also essential. Also, the morphology of microcapsules can be a smooth, symmetrical shape, depending on the chemicals used and the production methods, or they can be porous and rough. A schematic representation of the microcapsule morphology is given in Figure 2.

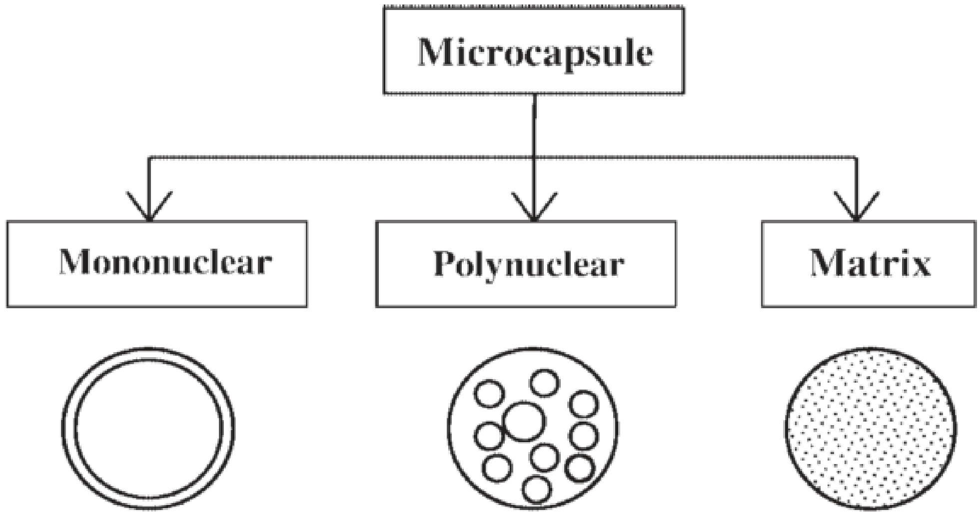

Figure 2. Structure of MEPCMs and NEPCMs[17]

The working principle of MEPCM is given in Figure 3.

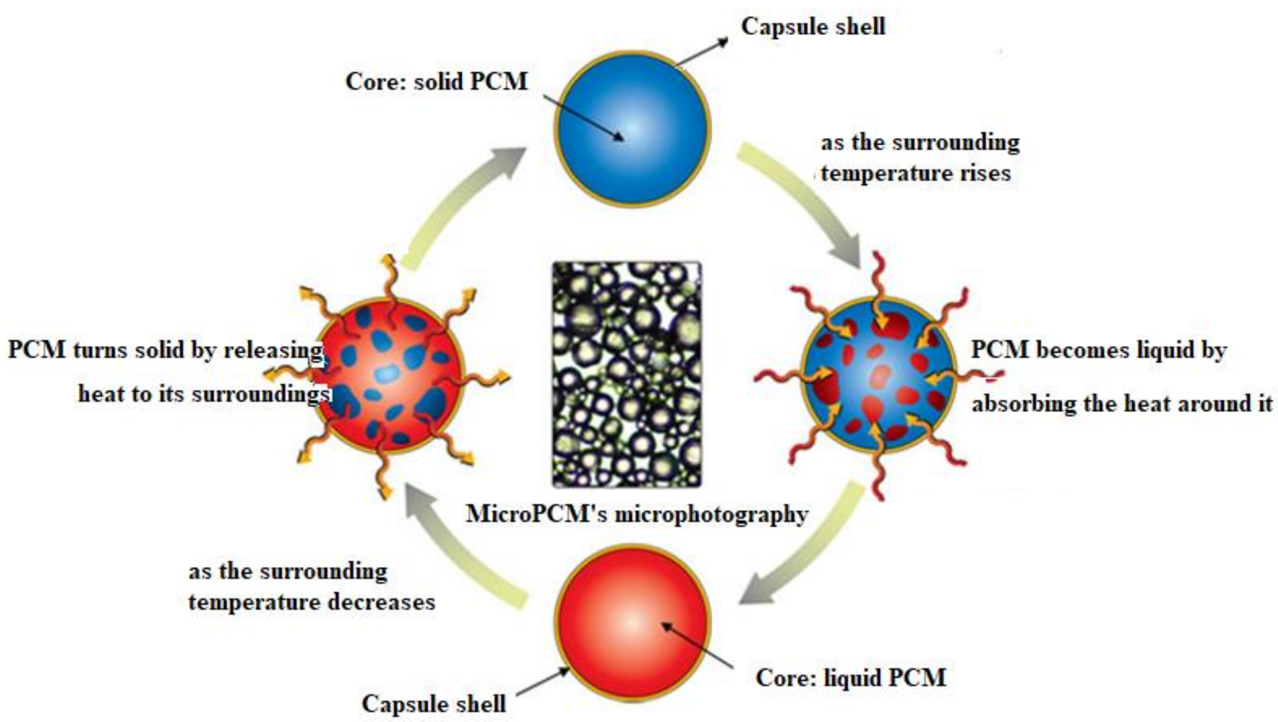

Figure 3. Working principle of a MEPCM in an energy storage system [19]

Commonly used microcapsule production techniques are as follows.

Chemical Process	**Mechanical Process**
Interface Polymerization	Spray Drying Method
Emulsion Polymerization	Cool-Drying Method
Micel Polymerization	Hot Melt Microencapsulation
Radical Chain Polymerization	Centrifuge Method
Polycondensation Polymerization	Rotational Suspension Separation
In-Situ Polymerization	Fluid Bed Method
Phase Separation Method (Coacervation) 1. Simple Coacervation 2. Complex Coacervation	
Supercritical Fluid Method	
Molecular Encapsulation Method	

With the advancement of technology, the demand for micro/nano-sized materials increases day by day. Microcapsules take their place in almost every aspect of our lives. Issues related to MEPCMs [20-23] with excellent latent thermal energy storage capacity [24- 28] have been extensively investigated in recent years. It has made significant progress in fundamental research MEPCMs, solar-thermal conversion systems [29], thermal energy storage, and the like. Also, significant progress has been made in some areas, such as encapsulated autonomous healing materials, drug delivery, photochromic materials and reversible thermochromic materials, thermotropic materials, and nanofiber production [30].

2. Thermochromic systems

Applications of reversible thermochromic microcapsules are extensively applied in areas such as inks [31], coatings of smart materials [32], cement, textiles [33], luminescent thermosensors [34] and colour indicators [35]. Recently, luminescent dye or leuco dye-based thermochromic (TC) systems [36] have been chosen as core materials for fabricating the temperature-sensitive thermochromic microcapsule [30, 37]. Chromic materials can respond to external stimuli such as light, temperature, humidity, pH change, and electric and magnetic fields [38-41]. Thermochromism is defined as the reversible change in the component's colour with temperature [40, 41]. Reversible thermochromic are technically direct or indirect systems. Direct thermochromic systems show colour change with direct heat. Unlike direct systems, the colour change in indirect thermochromic systems depends on the change in the ambient temperature. Examples of direct thermochromic systems are stereoisomerism, liquid crystals and molecular rearrangements [42-45]. Indirect thermochromic systems themselves do not show chromism; they require a combination of a leuko dye, a developer and a solvent in certain proportions. Therefore, these systems can be named multicomponent systems [46]. Organic thermochromic materials have many advantages, such as regulating colour change according to temperature and regulating the variety of colour changes and therefore have become the main research focus in recent years [38-41]. Thermochromic materials have broadly important application areas such as security printing, plastic tape thermometers, food packaging, medical thermography, non-destructive testing of engineering products, electronic circuitry systems [45], the pharmaceutical industry and the limited range textile industry. In multicomponent systems, reversible colour change at a certain temperature from one colour to another or colourless to colour depends on a developer's interaction in an environment created by a colour generator (a leuco dye) and a co-solvent. The solvent creates a phase change medium for the three-component thermochromic system. The melting and crystallization points of the solvent determine the system's colour change temperatures [42,46]. Alcohols, hydrocarbons, esters, ethers, ketones, and fatty acids are examples of solvents for thermochromic composites [43-50]. The colour improver is a weak acid that acts as proton donors to create the coloured state of the leuco dye components. The most commonly used improvers are bisphenol-A, gallates and phenols. Leuco dyes are electron donors such as spirolactone, spiropyrans or fluoranes, the most common of which is the crystalline purple lactone (CVL) dye. In the three-component thermochromic system, leuco dye and developer interact with each other at low temperature. In that case, the lactone ring opens up and the system gives a strong colour in the solid phase. When the temperature rises, the system is colourless due to the closed lactone ringin the liquid phase [40, 41, 46-48, 51, 52]. Thermochromic systems show lower enthalpy since they consist of phase change agent, dye and dye auxiliary chemical. The dissolved dye and dye auxiliary chemical cause the phase change temperature to decrease due to the cryoscopic effect. This is a typical situation for thermochromic systems, and the decrease in enthalpy is a disadvantage. In contrast, the decrease in the phase change temperature is a significant advantage since the operating temperature can be adjusted. Figure 4 shows the formation process of thermochromic microencapsulated phase change materials (TC-MEPCM) prepared by the in-situ polymerization method. In Figure 5, a nanocapsule drawing with a thermochromic energy storage feature is given.

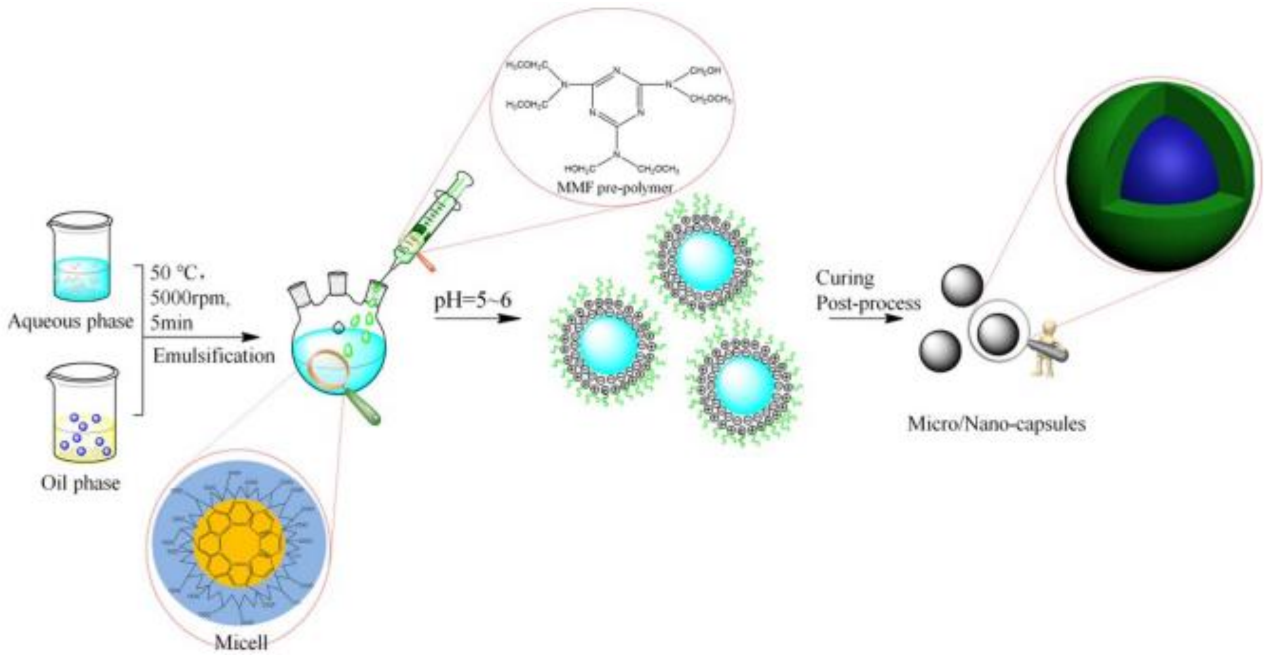

Figure 4. Formation process of TC-MEPCMs [30]

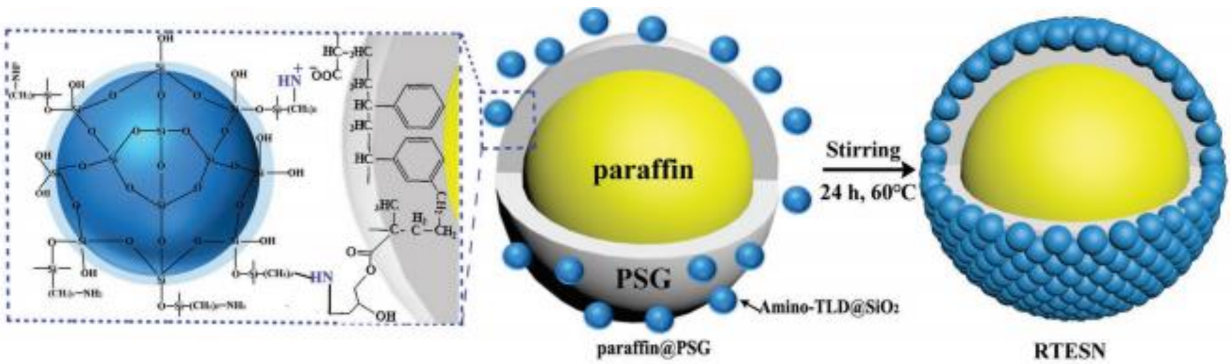

Figure 5. Raspberry-shaped nanocapsule drawing with thermochromic energy storage property [53]

3. Thermotropic systems

Chromogenic materials are perfectly suited for solar control glass applications [54-56]. Scientists and engineers have turned to develop new electrochromic, thermochromic and thermotropic materials for potential technical applications. Usually, conventional non-modifiable protective solar films combine reflection and absorption of infrared and UV rays, resulting in a noticeable temperature drop during the summer months [57]. In contrast, thermochromic and thermotropic systems can be compared in their impact on the energy efficiency of buildings. Here, thermotropic materials that act based on reflective effects appear to be more advantageous. Reflection partially prevents sunlight from entering the building and windows and facade elements and is, therefore, more advantageous compared to thermochromic systems. Thermochromic systems come to the fore in applications where a permanent appearance is needed from the inside out. Thermotropic systems exhibit light scattering feature depending on temperature change. If increased scatter is associated with significant backscatter due to temperature rise, the materials are suitable for an application in solar control. This area investigated thermotropic effects of phase separation, an isotropic and an anisotropic (liquid-crystalline) phase transition and areas between the case and the matrix of a refractive index caused by extremely different temperature dependence [58]. Fixed-area (TSFD) thermotropic systems change their light transmission from transparent to light scatter when they reversibly reach a certain threshold temperature. [59, 60]. Due to their autonomous temperature-triggered mode of operation, they are superior to actively operated shading devices which can be more prone to component malfunction [61]. Thus, TSFD can provide efficient overheating protection for buildings and solar thermal collectors [62, 63]. Thermotropic overheating protection is an important technology, especially for installing solar collectors made of cost-effective plastics [63, 64]. TSFD consists of at least two components: a matrix material and a thermotropic additive as the minor component finely dispersed in it. Both components exhibit similar refractive indices at temperatures below the threshold temperature. As a result, incoming solar radiation is not scattered, and the TSFD appears transparent [65]. Schematic sections of a thermotropic transparent outer insulation and coating (TEIF) system are given in Figure 6.

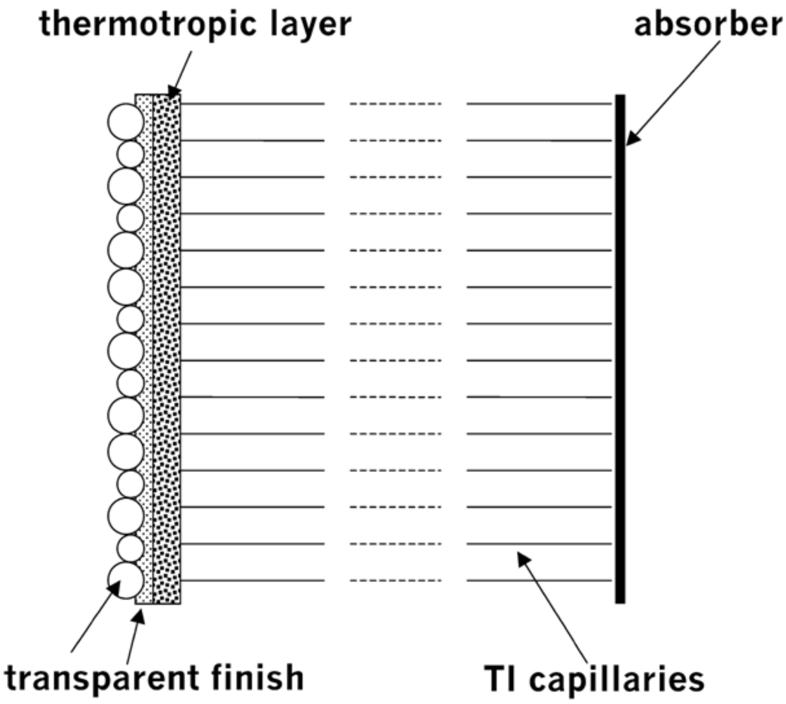

Figure 6. Schematic cross-sectional representation of a thermotropic transparent outer insulation and coating (TEIF) system [66]

4. Electrospinned MEPCMs

The electrospinning process [67-70] produces superfine fibers with diameters ranging from 10 mm to 10 nm by forcing a polymer melt or a solution through a spinneret with an electric field and then pulling the formed filaments as they solidify or coagulate. Due to the ease with which nanometer-sized fibers can be produced from a range of natural and synthetic polymers, they have received much attention in technology recently [71]. Nanofibers have a diameter of less than 1μm, and they are filamentous structures whose lengths are considerably higher than their diameters. Polymeric nanofibers can be produced by different techniques such as drawing, mould synthesis, phase separation, self-assembly and electrospinning [72]. Among these methods, the electrospinning method has many advantages such as high encapsulation efficiency, ease of application, high loading capacity, the ability to work with synthetic and natural polymers, obtaining fibers in sizes ranging from a few nanometers to micrometers, obtaining mechanically durable and flexible fibers, no need for purification because one type of fiber is obtained, two or three-dimensional fiber production, obtaining very long fibers from a few centimeters to meter dimensions and low-cost method.

In addition to these advantages, the disadvantage of the method can be considered the unstable jet formation, the fact that many parameters significantly affect the fiber formation and structure [73]. Nanofibers have a high molecular orientation. Due to their small size, they have fewer structural defects and thus have very well mechanical properties. Due to the small diameter, they have high surface/volume ratios or surface/ mass ratios, so they have high surface areas. The fact that nanofibers form structures with a large surface area increases their capacity to retain or emit functional groups, ions and a wide variety of nano-level particles [74]. Nanofibers obtained by the electrospinning method have many uses such as cosmetic applications, tissue engineering, filtration applications, agricultural applications, nanosensors, biomedical tools, protective clothing, reinforced composite making, controlled active substance release [75]. One of the alternative applications is to transform microcapsules into nanofibers with the electrospinning method to increase their applicability in different areas by expanding their surface area. This method, known as electrospinning, is a method developed to produce nanofibers from polymer solutions and is one of the easiest methods applied to form polymer-based nanofibers. This method makes it possible to create nanofibers with various morphological properties in a controlled manner with polymer solutions within the electric field generated by high voltage. The schematic representation of the electrospinning system is given in Figure 7.

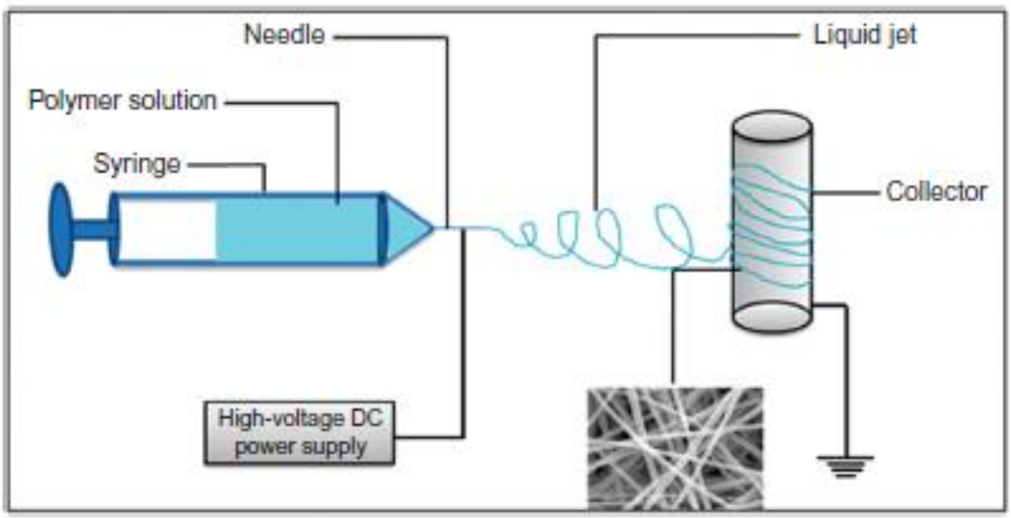

Figure 7. Schematic representation of the electrospinning system [75]

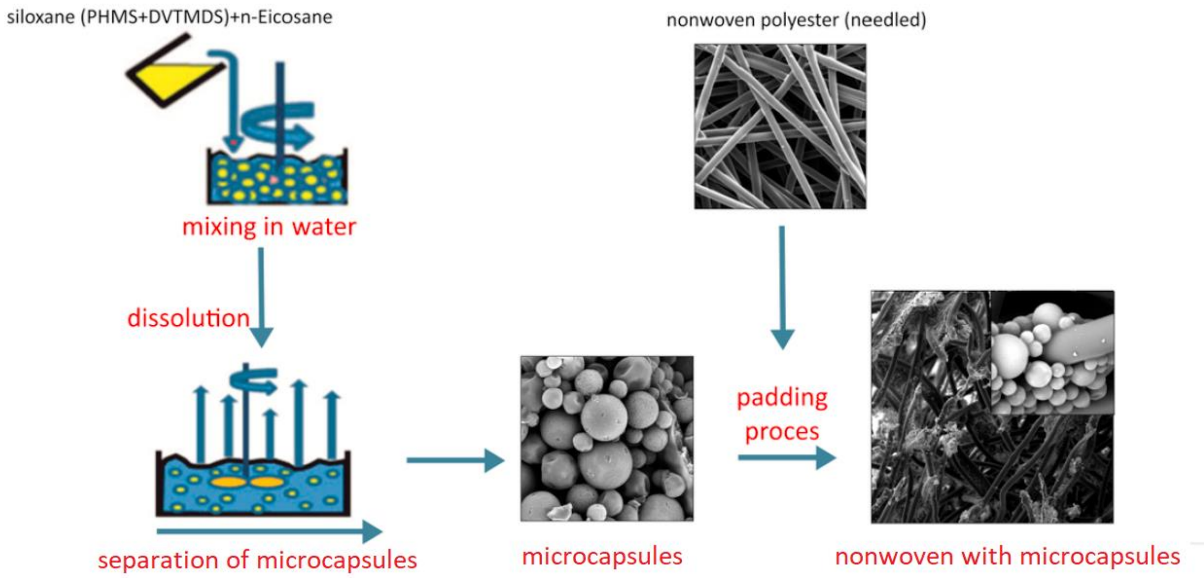

Figure 8. Schematic representation of the microencapsulated nanofiber production [76]

5. Conclusion

Microencapsulated phase change materials (MEPCMs) were first used in applications such as buildings and textiles with direct doping methods for heating and air conditioning. For this purpose, they are used for coating purposes in the interior of buildings by adding to plaster, as well as bonded to textile fiber cross-sections or surfaces. After a while, they have also been validated for different purposes, such as thermotropic systems, thermochromic and sensor materials, and electrospinning fabrics. Especially in solar energy systems, thermotropic materials are an important technology for overheating protection. Thermotropic systems are also used as a heating component in hybrid systems, benefiting from the greenhouse effect. Applications of reversible thermochromic microcapsules are extensively applied in many areas such as inks, coatings of smart materials, cement, textiles, luminescent thermosensors and colour indicators. Nanofibers obtained by the electrospinning method have many uses, such as cosmetic applications, tissue engineering, filtration applications, agricultural applications, nanosensors, biomedical devices, protective clothing, reinforced composite production, controlled active substance release. In the present study, it is aimed to summarize the innovations and applications of MEPCMs. It is thought that these materials will take place as an important group in the smart materials class in the next period. It is estimated that the usage areas will develop by adding new ones every day.

Author Contributions

Ruhan Altun Anayurt: Conceptualization, investigation, writing original draft, review and editing.
Cemil Alkan: Investigation, writing original draft, review and editing, supervision.

Conflicts of Interest

The authors declare that they have no known competing commercial interests or personal relationships that could have influenced the work reported in this paper.

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n-complete crossed modules and wreath products of groups

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Abstract — In this paper we examine the n-completeness of a crossed module and we show that if $X = (W_1, W_2, \partial)$ is an n-complete crossed module, where $W_i = A_i wr B_i$ is the wreath product of groups A_i and B_i, then A_i is at most n-complete, for $i = 1, 2$. Moreover, we show that when $X = (W_1, W_2, \partial)$ is an n-complete crossed module, where A_i is nilpotent and B_i is nilpotent of class n, for $i = 1, 2$, then if A_i is an abelian group, then it is cyclic of order p_i. Also, if $W_i = C_p wr C_2$, where p is prime with $p > 3$, $i = 1, 2$, then $X = (W_1, W_2, \partial)$ is not an n-complete crossed module.

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1. Introduction

The notion of crossed module is investigated by Whitehead [1]. After him, many mathematicians applied crossed modules in many directions such as homology and cohomology of groups, algebraic structures, K-theory, and so on. Actor crossed module of algebroid is defined by Alp in [2]. Actions and automorphisms of crossed modules is studied by Norrie [3]. Tensor product modulo n of two crossed modules is introduced by Conduche and Rodriguez-Fernandez [4]. The concepts of q-commutator and q-center of a crossed module (where q is a non-negative integer) is studied by Doncel-Juarez and Crondjean-Valcarcel [5].

Let $X = (T, G, \partial)$ be a crossed module and $X = (T, G, \partial) = \gamma_1(X), \dots, \gamma_n(X), \dots$ be the lower central series of $X = (T, G, \partial)$. We define the series $K_1, \dots, K_n, \dots$ where K_n consists of the automorphisms of X which induce the identity on the quotient crossed module $\frac{X}{\gamma_{n+1}(X)}$. Now, in this paper, we present the definition of an n-complete crossed module which is an extension of the definition of a semi-complete crossed module.

2. n-commutator crossed submodule

It is well known that an action of the group G on the group T is a homomorphism $G \to Aut(T)$ or, a map $\mu : T \times G \to T$ such that

1. $\mu(t_1 t_2, x) = \mu(t_1, x)\mu(t_2, x)$,

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2. $\mu(t, x_1x_2) = \mu(\mu(t, x_1), x_2)$,

for all $t_1, t_2 \in T$ and $x, x_1, x_2 \in G$.

As usual, we will consider the notation $\mu(t, x) =^x t$ in continue. Indeed, a crossed module [6] is a 4-tuple $X = (T, G, \mu, \partial)$ or 3-tuple (T, G, ∂), where T and G are groups, μ is an action of T on G, and $\partial : G \to T$ is a homomorphism. The map ∂ is called the boundary, and it satisfies the following statements:

1. X Mod 1: $\partial(^t x) = t^{-1}\partial(x)t$ for all $x \in G$ and $t \in T$.

2. X Mod 2: $^{\partial(y)}x = y^{-1}xy$ for all $x, y \in G$.

If T and G are finite groups, then the crossed module is called finite.

Example 2.1. Let G be a group. We denote by RG the crossed module $(G, 1, \mu, \partial)$, where 1 is the trivial subgroup of G, and the action μ and the boundary map ∂ are trivial.

Example 2.2. Let G be a group. We denote by DG the crossed module (G, G, μ, id), where μ is the conjugation action, and $id : x \to x$ is the trivial map.

From the definition, we immediately conclude that $K = \text{Ker}\, \partial$ is a central subgroup of G, $I = \text{im}\, \partial$ is a normal subgroup of T, and obtain the following exact sequence $1 \to K \to G \to T \to C \to 1$, where $C = \frac{T}{I}$ is the cokernel of ∂. Specially, for a finite crossed module we have $|G||C| = |K||T|$ [7]. A morphism $\phi : X \to Y$ between two crossed modules $X = (T_1, G_1, \mu_X, \partial_X)$ and $y = (T_2, G_2, \mu_Y, \partial_Y)$ is a pair (ϕ_1, ϕ_2), where $\phi_1 : T_1 \to T_2$, $\phi_2 : G_1 \to G_2$ are group homomorphisms, and the following relations hold:

$$\partial_Y \circ \phi_2 = \phi_1 \circ \partial_X, \quad \mu_Y \circ (\phi_2 \times \phi_1) = \phi_2 \circ \mu_X.$$

This yields the commutativity of the following diagrams:

$$\begin{array}{ccc} G_1 & \xrightarrow{\partial_X} & T_1 \\ {\scriptstyle \phi_2}\downarrow & & \downarrow{\scriptstyle \phi_1} \\ G_2 & \xrightarrow[\partial_Y]{} & T_2 \end{array} \qquad \begin{array}{ccc} G_1 \times T_1 & \xrightarrow{\mu_X} & G_1 \\ {\scriptstyle \phi_2 \times \phi_1}\downarrow & & \downarrow{\scriptstyle \phi_2} \\ G_2 \times T_2 & \xrightarrow[\mu_Y]{} & G_2 \end{array}$$

Definition 2.3. Suppose that (T, G, ∂) is a crossed module and n is a non-negative integer. We define the notion of n-commutator crossed submodule of (T, G, ∂) as $\partial : D^n_G(T) \to G \neq_n G$, where $D^n_G(T)$ is the subgroup of T generated by the set

$$\left\{ {}^x a a^{-1} b^n \mid x \in G,\ a, b \in T \right\},$$

and in a general case, if N is a normal subgroup of G, then $G \neq_n G$ is the n-commutator subgroup of G and N, i.e., the subgroup generated by the

$$\left\{ [x, a] a'^n \mid x \in G, \quad a, a' \in N \right\}.$$

The n-commutator crossed submodule of (T, G, ∂) is a normal crossed submodule.

Example 2.4. The group G acts on N by conjugation if N is a normal subgroup. The triple (N, G, i) is a crossed module, where i is the inclusions. The n-commutator crossed submodule of (N, G) equals $(G \neq_n$

$N,\ G \neq_n G,\ i)$. This implies that for any group G, the triple (G, G, id) is a crossed module and $(G \neq_n G,\ G \neq_n G,\ id)$ is its n-commutator.

Let (T, G, ∂) be a crossed module with trivial center. According to [3], we can obtain a sequence of crossed modules as follows:

$$(T, G, \partial),\ \mathscr{A}(T, G, \partial), \mathscr{A}(\mathscr{A}(T, G, \partial)), \ldots$$

in which each term embeds in its successor. This sequence is called the actor tower of (T, G, ∂).

We say the crossed module (T, G, ∂) is complete if $Z(T, G, \partial) = 1$ and the canonical morphism $< \eta, \gamma >$: $(T, G, \partial) \to \mathscr{A}(T, G, \partial)$ is an isomorphism. Notice that the crossed module (T, G, ∂) is semi complete if $< \eta, \gamma >$ is an epimorphism. Consequently, a semi complete crossed module with trivial center is complete.

3. n-complete crossed modules

A crossed module (T, G, ∂) is said to be n-complete if n is the smallest positive integer such that K_n is sub-crossed module $I_{nn}(T, G, \partial)$, where $I_{nn}(T, G, \partial)$ is the crossed module of the inner automorphisms of (T, G, ∂).

Proposition 3.1. Let (T, G, ∂) is an n-complete crossed module. Then, T and G are at most n-complete and nilpotent of class at most n.

Example 3.2. If (G, G, i) is an n-complete crossed module, then G is n-complete and nilpotent of class n.

In Proposition 3.3 we give a relation between nilpotent groups and n-complete crossed modules.

Proposition 3.3. If (T, G, ∂) is a crossed module and groups T, G are nilpotent of class at most n, then (T, G, ∂) is an n-complete crossed module for some m with $m \leq n$.

Suppose that (R, K, ∂) is a normal crossed submodule of (T, G, ∂) and (S, H, ∂') is a crossed module such that $(T/R, G/K) \cong (S, H)$, then we call (T, G) an extension of (R, K) by (S, H). If there exists a surjective morphism $\psi = (\psi_1, \psi_2) : (X_1, X_2) \to (T, G)$, the trivially (X_1, X_2) is an extension of the crossed module $\ker\phi$ by (T, G). An extension $((X_1, X_2), \psi)$ by (T, G) is n-central extension if $\ker\psi = (\ker\psi_1, \ker\psi_2)$ is contained in $Z^n(X_1, X_2)$.

Let (M, G, μ) and (N, G, ν) be two crossed modules, and consider the pullback

$$\begin{array}{ccc} M \times_G N & \xrightarrow{\pi_2} & N \\ {\scriptstyle \pi_1}\downarrow & & \downarrow{\scriptstyle \nu} \\ M & \xrightarrow[\mu]{} & G \end{array}$$

Then, $M \times_G N = \{(a, b) \mid a \in M,\ b \in N,\ \mu(a) = \nu(b)\}$. If we write $\alpha = \mu\pi_1 = \nu\pi_2$, then for $c \in M \times_G N,\ a \in M,\ b \in N$, we get

$$^{\pi_1(c)}a = {}^{\alpha(c)}a = {}^{\pi_2(k)}a,\ {}^{\pi_1(c)}b = {}^{\alpha(c)}b = {}^{\pi_2(c)}b.$$

The tensor product $M \otimes^q N$ is defined as the group generated by the symbols $a \otimes b$ and $\{c\}$, $a \in M, b \in N, c \in M \times_G N$, with the following relations:

1. $a \otimes bb' = (a \otimes b)(^b a \otimes^b b')$.
2. $aa' \otimes b = (^a a' \otimes^a b)(a \otimes b)$.
3. $\{c\}(a \otimes b)\{c\}^{-1} = {}^{\alpha(c)^q} a \otimes^{\alpha(c)^q} b$.

4. $[\{c\},\{c'\}] = \pi_1(c)^q \otimes \pi_2(c')^q$.
5. $\{cc'\} = \{c\}\left(\prod_{i=1}^{q-1}(\pi_1(c)^{-1} \otimes ({}^{\alpha(c)^{1-q+i}}\pi_2(c'))^i)\right)\{c'\}$.
6. $\{(a^b a^{-1}, {}^a b b^{-1})\} = (a \otimes b)^q$.

Note that the structure of the tensor product mode q is bifunctorial. Under this conditions there exists an action of G on $M \otimes^q N$ defined as follows:

$$^x(a \otimes b) = {}^x a \otimes {}^x b, \ {}^x\{c\} = \{{}^x c\}$$

$a \in M$, $b \in N$, $c \in M \times_G N$, $x \in G$. The group M (resp. N) acts on $M \otimes^q N$ through the homomorphism μ (respectively ν) and if $a \in M$, $b \in N$, $c \in M \times_G N$, then

$$^a\{c\} = (a \otimes \pi_2 c^q)\{c\}, \ \ {}^b\{c\} = \{c\}(\pi_1 c^{-q} \otimes b).$$

Now let (T, G, ∂) and (G, G, id) be crossed modules. We can consider the tensor product $T \otimes^q G$, it was first defined by Brown. In this case $T \times_G G \cong T$, $\pi_1 = id_T$, $\pi_2 = \partial$. Similarly, we consider $G \otimes^q G$. Then, we have the following crossed modules:

$$\begin{array}{llll} (T \otimes^q G, T, \lambda), & \lambda(t \otimes g) = t^g t^{-1}, & \lambda(\{t\}) = t^q, & t \in T,\ g \in G; \\ (T \otimes^q G, T, \lambda'), & \lambda'(t \otimes g) = [\partial(t), g], & \lambda'(\{t\}) = \partial(t)^q, & t \in T,\ g \in G; \\ (G \otimes^q G, G, \xi), & \xi(g \otimes h) = [g, h], & \xi(\{g\}) = g^q, & g, h \in G. \end{array}$$

Theorem 3.4. If (T, G, ∂) is an n-complete crossed module, then $(T \otimes^n G, G \otimes^n G, (\lambda, \epsilon))$ is an n-complete extension by (T, G, ∂).

The restricted standard wreath product $W = AwrB$ of two groups A and B is the splitting extension of the direct power A^B by the group B, with B acting on A^B according to the rule, if $b \in B$ then $f^b(x) = f(xb^{-1})$ for all $f \in A^B$, $x \in B$. The base group A^B is characteristic in W, in all cases, except when A is of order 2, or is a dihedral group of order $4k+1$ and B is of order 2. In the following it is assumed that A^B is characteristic in W. The next theorem is of great importance for the sequel. But first we need the following results from [8].

Proposition 3.5. [8] If $\alpha \in Aut(A)$, we define $\alpha^* \in Aut(W)$ by $(bf)^{\alpha^*} = bf^{\alpha^*}$ for all $b \in B$, $f \in \mathscr{F}$, where $f^{\alpha^*}(x) = \big(f(x)\big)^\alpha$, for all $x \in B$, then the group A^* of all such automorphisms is isomorphic to $Aut(A)$.

Proposition 3.6. [8] If $\beta \in Aut(B)$, we define $\beta^* \in Aut(W)$ by $(bf)^{\beta^*} = b^\beta f^{\beta^*}$ for all $b \in B$, $f \in \mathscr{F}$, where $f^{\beta^*}(x) = f(x^{\beta^{-1}})$ for all $x \in B$, then the group B^* of all such automorphisms is isomorphic to $Aut(B)$.

Theorem 3.7. [8]

1. The automorphism group of the wreath product W of two groups A and B can be expressed as a product, $Aut(W) = KI_1B^*$, where
    - K is the subgroup of $Aut(W)$ consisting of those automorphisms which leave B element wise fixed.
    - I_1 is the subgroup of $Aut(W)$ consisting of those inner automorphisms corresponding to transformation by elements of the base group $\mathscr{F}$.

- B^* is defined as in Proposition 3.5.

2. The group K can be written as A^*H, where
   - A^* is defined as in Proposition 3.6.
   - H is the subgroup of $Aut(W)$ consisting of those automorphisms which leave both B and diagonal element wise fixed.

3. The subgroups A^*HI_1, HI_1B^*, HI_1, and I_1 are normal in $Aut(W)$ and $Aut(W)$ is splitting extension of A^*HI_1 by B^*. Furthermore, A^* intersects HB^* trivially.

In the following it is assumed that $W_1 = A_1 wr B_1$ and $W_2 = A_2 wr B_2$ are two standard wreath products of groups.

Theorem 3.8. If $X = (W_1, W_2, \partial)$ is an n-complete crossed module, then A_i is at most n-complete, for $i = 1,2$.

Proof.
If $(\alpha,\beta) \in K_n(X)$, then $\alpha \in K_n(A_1)$ and $f \in A_1^{B_1}$. Hence, $f^{\alpha^*}(x) = \big(f(x)\big)^{\alpha} = f(x)u_x$ for $x \in B_1$ and $u_x \in \gamma_{n+1}(A_1)$. If $g_1 \in A_1^{B_1}$, $g_1(x) = u_x$ for all $x \in B_1$, then $f^{\alpha^*}(x) = \big(fg_1(x)\big)$ for all $x \in B_1$. Therefor, $f^{\alpha^*} = fg_1$, where $g_1 \in \gamma_{n+1}(W_1)$. Since W_1 is n-complete, it follows that $K_n(W_1) \le I(W_1)$ and so $\alpha^* \in I(W_1)$. But according to [9], $\alpha^* \in I(W_1)$ if and only if $\alpha \in I(A_1)$. Hence, $K_n(A_1) \le I(A_1)$. The proof for $K_n(A_2) \le I(A_2)$ is similar.

Theorem 3.9. If $X = (W_1, W_2, \partial)$ is an n-complete crossed module, then B_i is nilpotent of class at most n, for $i = 1,2$.

Proof.
If $L(B_1)$ and $L(B_2)$ are the left regular representation of the groups B_1, B_2 respectively, then for each element $l_b \in L(B_1)$, $b \in B_1$, there corresponds an automorphism l_b^* of W_1 defined by $(cf)^{l_b^*} = cf^{l_b^*}$ for all $c \in B_1$, $f \in {A_1}^{B_1}$, where $f^{l_b^*}(x) = f(bx)$ for all $x \in B_1$.
If $f_1 \in {A_1}^{B_1}$ such that $f_1(1) = a$, $f_1(x) = 1$ for all $x \in B_1$, $x \ne 1$ and $b \in B_1$, $b \ne 1$, then $f_1^{l_b^*}(b^{-1}) = f_1(1) = a$ and $f_1^{l_b^*}(x) = f_1(bx) = 1$ for all $x \ne b^{-1}$.
Moreover, we obtain $f_1^{l_b^*} = f_1 g$, where $g(1) = a^{-1}$, $g(b^{-1}) = a$, $g(x) = 1$ for all $x \in B$, $x \ne 1, b^{-1}$. Also, by [10] for the element $g \in A_1^{B_1}$, $g = [b^{-1}, \varphi]$, where $\varphi \in A_1^{B_1}$ with $\varphi(1) = g(1)$ and $\varphi(x) = 1$ for all $x \ne 1$.
Now, if $X_i \in B_1$, we define the element $f_{x_i} \in A_1^{B_1}$ by $f_{x_i}(x_i) = a$ and $f_{x_i}(d) = 1$ for all $d \in B$, $d \ne x_i$, then $(f_{x_i})^{l_b^*} = f_{x_i} g^{x_i}$. If $b \in \gamma_n(B_1)$, then l_b^* belongs to the group $K_n(W_1) \le I(W_1)$. But $b \in Z(B_1)$ if and only if $l_b^* \in I(W_1)$. So, the group B_1 is nilpotent of class at most n, and similarly B_2 is nilpotent of class at most n.

Theorem 3.10. If $X = (W_1, W_2, \partial)$ is an n-complete crossed module, and B_i is nilpotent of class n, for $i = 1,2$, then A_i is directly indecomposable.

Proof.
Suppose that $A_i = U_i \times V_i$ is a non trivial direct decomposition of A_i for $i = 1,2$. If $f \in A_1^{B_1}$, then $f(x) = u_{1x}v_{1x}$ for all $x \in B_1$, where $u_{1x} \in U_1$ and $v_{1x} \in V_1$. If $g_f \in A_1^{B_1}$, $g_f(x) = u_{1x}$ for all $x \in B_1$ and $x \in \gamma_n(B_1) \le Z(B_1)$, $z \ne 1$, then $\eta : W_1 \to W_1$ by $(bf)^{\eta} = bf[g_f, z]$ is a map. Since $g_{fh} = g_f g_h$ and $g_f^y = g_{f^g}$ for all $f, h \in A_1^{B_1}$, $y \in B_1$, it follows that η is an outer automorphism of W_1 with $\eta \in K_n(W_1)$ and is a contradiction.

Theorem 3.11. If $X = (W_1, W_2, \partial)$ is an n-complete crossed module, where A_i is finite nilpotent and B_i is nilpotent of class n, then A_i is a p_i-group, (p_i is prime) for $i = 1,2$.

Proof.
By Theorem 3.10, the proof is straightforward.

Theorem 3.12. Let $X = (W_1, W_2, \partial)$ is an n-complete crossed module, where A_i is nilpotent and B_i is nilpotent of class n, for $i = 1,2$. If A_i is abelian group, then it is cyclic of order p_i.

Proof.
By Theorem 3.11, A_i is p_i-group. But A_i is abelian, and so A_i is cyclic of order p^r for some positive integer r. Now, we show that $r = 1$.
If r is not equal to 1, we choose an element $x \in \gamma_n(B_1)$, $x \neq 1$ and we define a mapping $\eta : W_1 \to W_1$ by $(bf)^\eta = bf[f,x]^p$, η is an automorphism of W_1 belonging to the group $K_n(W_1)$ by [9]. Since $r > 1$ and η is an outer automorphism, it follows that W_1 is not n-complete. Hence, $r = 1$, and A_2 is cyclic of order p_i, accordingly.

Corollary 3.13. If $X = (W_1, W_2, \partial)$ is an n-complete crossed module and A_i is finite nilpotent and B_i nilpotent of class n, for $i = 1,2$, then A_i is cyclic of prime order.

Now, we give examples of non n-complete crossed module. Let $W = AwrB$ be the restricted wreath product of A by B. The set $\sigma(f) = \{x \in B | f(x) \neq 1\}$ is the support of $f \in A^B$. Map $\pi : A^B \to \frac{A}{A'}$ given by

$$\pi(f) = \prod_{x \in \sigma(f)} f(x)A'$$

is well defined and obviously a homomorphism satisfying $\pi(f^b) = \pi(f)$ for all $b \in B$.

Proposition 3.14. [10] The derived subgroup W' of W is $W' = B'M$, where $M = Ker\pi$.

Theorem 3.15. If $W_i = C_p wr C_2$, where p is prime with $p > 3$, $i = 1,2$, then $X = (W_1, W_2, \partial)$ is not n-complete crossed module.

Proof.
If $W_1 = A_1 wr B_1$, then $W_1' = B_1' M_1$, where $M_1 = \{f | f \in A_1^{B_1}, \pi(f) \in A_1'\}$. But $B_1 = C_2$, $|M_1||A_1|^{|B_1|} = p^2$ and so $|M_1| = p$. W_1 is not nilpotent and thus $\gamma_n(W_1) = M_1$ for all $n \in Z^+$, $n \geq 2$. If $A_1 = C_p =< a >$, $B_1 = C_2 =< b >$, then $f_1 = (a^{p-1}, a^2)$, $f_2 = (a^2, a^{p-1})$, $g_1 = (a,1)$, $g_2 = (1,a)$, instill the mapping $g_1 \to f_1$, $g_2 \to f_2$ which can be extended to an automorphism γ of $A_1^{B_1}$, which commutes with the automorphism of $A_1^{B_1}$ induced by the element $b \in B_1$, since $A_1^{B_1} =< f_1, f_2 >=< g_1, g_2 >$ and $A_1^{B_1}$ is elementary abelian of rank 2 and $p \neq 3$. Thus, the automorphism γ can be extended to an automorphism of W_1, which fixes B_1 element wise [8]. On the other hand, we have

$$g_1^\gamma = (a,1)^\gamma = (a^{p-1}, a^2) = (a,1)(a^{p-2}, a^2),$$
$$g_2^\gamma = (1,a)^\gamma = (a^2, a^{p-1}) = (1,a)(a^2, a^{p-2}),$$

and $(a^{p-2}, a^2), (a^2, a^{p-2}) \in M_1 = \gamma_n(W_1)$, $n \geq 2$, so $\gamma \in K_n(W_1)$, $n \geq 2$ and γ is an outer automorphism. Hence $W_1 = C_p wr C_2$ is not n-complete. Therefore, $X = (W_1, W_2, \partial)$ is not n-complete crossed module.

Theorem 3.16. If $W_i = C_p wr B_i$, where p is prime with $p > 3$, $i = 1,2$, and B_i is nilpotent of class n with $k_i = |B_i| \geq 3$, $i = 1,2$, then $X = (W_1, W_2, \partial)$ is not n-complete crossed module.

Proof.
The group $A_i^{B_i}$ is an elementary abelian p-group, since A_i is $A_i = C_p =< a_i >$. The set $g_{x_i} \in A_i^{B_i}$ for all $x_i \in B_i = \{x_1, ..., x_{k_j}\}$ with $g_{x_i}(x_i) = a_i$, $g_{x_i}(x_j) = 1$, $x_j \neq x_i$ is a basis of $A_i^{B_i}$. Now, if we consider the mapping

$g_{x_i} \to f_{x_i} = g_{x_i}[b_1, g_{x_i}] = {g_{x_i}}^2({g_{x_i}}^{-1})^{b_1}$ for all $x_i \in B_i$, where $b_1 \in \gamma_n(B_i)$, then this mapping is extended to an automorphism $\overline{\gamma}$ of $A_i^{B_i}$, since the set f_{x_i}, $x_i \in B_i$ is a basis of $A_i^{B_i}$. On the other hand, since $C_p \cong Z_p$ and $p > 3$, it follows that the determinant of matrix

$$\begin{bmatrix} 2 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & -1 & 0 & 0 & \cdots & 0 \\ & & & & \ddots & & & \\ 0 & 0 & \cdots & -1 & 0 & 0 & \cdots & 2 \end{bmatrix}$$

is not zero in Z_p, where the element 2 is in the main diagonal and in each row and column we have once the element -1. But $\overline{\gamma}$ can be extended to an automorphism γ of the group W_i, which fixes B_i element wise, since the automorphism $\overline{\gamma}$ of $A_i^{B_i}$ commutes with the automorphisms of $A_i^{B_i}$ which are induced by the elements of the group B_i. The automorphism γ is an outer automorphism with $\gamma \in K_n(W_i)$. So, $X = (W_1, W_2, \partial)$ is not n-complete crossed module.

Proposition 3.17. [11] The wreath product $W = C_2 wr B$ is not n-complete, where B is finite abelian with $m = |B| \geq 4$ and m is an odd number.

Theorem 3.18. If $W_i = C_2 wr B_i$, where B_i is finite abelian with $m_i = |B_i| \geq 4$, $i = 1,2$, and m_i is an odd number, then $X = (W_1, W_2, \partial)$ is not n-complete crossed module.

Proof.

By Proposition 3.17, the proof is straightforward.

We have assumed up to this point that subgroup A^B is characteristic in $W = AwrB$. Now, we investigate the case of W in which A is a special dihedral group and B is of order 2. At this case A^B is not characteristic in W. We recall that D_m is $D_m =< a, b \mid a^m = 1, b^2 = 1, (ab)^2 = 1 >$.

Theorem 3.19. [11] The standard wreath product $W = D_n wr C_2$ is semi complete if and only if $n = 3$.

Theorem 3.20. Let $W = D_m wr C_2$, where $m = 2k+1$, $k \in N$, and C_2 is the cyclic group of order 2. Then, the crossed module $X = (W, W, i)$ is n-complete if and only if $m = 3$.

Proof.

In this case, we know that for the lower central series of the group D_m, is $\gamma_{k+1}(D_m) =< a^{2^k} >$, for all $k = 1, 2, \ldots$. Since m is an odd number, it follows that

$$\gamma_2(D_m) = \gamma_3(D_m) = \cdots = \gamma_k(D_m) = \gamma_{k+1}(D_m) = \cdots .$$

If the crossed module (W, W, i) is n-complete, then by Theorem 3.8, the group D_m is at most n-complete. This means that D_m is semi complete [12], and this is true if and only if $m = 3$.

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Research Article

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Comparison of the K factor in different areas on the slope

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Keywords:
USLE,
Soil Erosion,
K factor,
Tekneli,

Abstract — Erosion is among the leading factors threatening our sources of soil and water. The use of soil susceptibility indices to determine erosion and risk situations is vital, especially in erosion studies. The susceptibility of soil to erosion is the resistance of soil against decomposition due to some external forces. Carried out around Tokat-Tekneli village, the research aims to determine the erodibility factor (K) of the soil by its physical and chemical qualities in different slope segments of a field which is applied wheat-fallowing planting watch and has a convex slope in the study area. With this aim, three-times repeated deteriorated surface soil samples of 0-20 cm depth were taken from the peak, shoulder, ridge, face, and inch (finger) regions of the slope area, and they were analysed in the laboratory. With the formulation of the K factor, the soil erodibility value of each soil sample was calculated. The erodibility value of the region is between 0.07 and 0.12 t ha^{-1} Mj mm^{-1}, and it was determined that soils are classified in the soil class, which is sensitive to moderate erosion.

Subject Classification (2020):

1. Introduction

Today, soil erosion is considered one of the most critical factors leading to a decrease in agricultural land fertility [1]. Erosion, which causes environmental problems and land degradation, forms a coarse-textured soil structure by detaching fine particles from the soil [2]. It is stated that approximately 10 million hectares of agricultural land become undergo soil erosion every year [3,4]. In the agricultural lands where tillage practices are carried out for a long time, the crop yield decreases, and soil properties deteriorate as the fertile topsoil is eroded and moved [5]. Soil erosion in the world, especially in developing countries such as Turkey, leads to severe agricultural advancements and crop yield [6,7]. Therefore, the realistic estimation of soil loss in large areas is significant in conserving agricultural lands and increasing the yield [8].

Agricultural land is considered a rich source providing the necessary nutrients for the growth and development of plants [9]. This is because microbial activity, which allows the soil to be ventilated and the water to flow smoothly, is highly abundant in agricultural lands with high organic matter and humus content. This activity is deteriorated due to environmental and human factors, especially erosion. Soil

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erosion occurs due to the wearing away of the soil due to external forces such as wind, river or rainwater. The soil erodibility, which is expressed as abrasion, is defined as the resistance of the soil against decomposition and transportation [10]. Soils decomposed by precipitation or surface flow become convenient for erosion [11,12].

To raise the fertility of agricultural areas and ensure ecological balance, it is necessary to carry out agricultural research and develop state policies. A great many methods have been developed to estimate soil losses. USLE and its revised version, RUSLE, are the most widely used methods in the world today. USLE and RUSLE [13] are widely used in the whole world because they are easy to use, they require very little data and have a very reliable data set [14-16].

USLE [17] is one of the most advanced mathematical models used to predict the potential loss of soil likely to occur inland or basin due to surface erosion and gully erosion caused by precipitation [18]. It predicts soil losses by considering the rainfall erosivity factor, soil erodibility factor, the length and the slope factor of a hillside, the yield management factor, the soil conversion factor [19].

USLE is a model developed in line with the erosion data obtained from the uniform hillside lands divided into small parcels [20]. The K factor of USLE is widely used to determine and assess the soil losses occurring worldwide [17]. The K factor varies depending on the organic matter content of the soil, sand, very fine sand, silt and clay contents, soil structure, and permeability [21]. Also, erodibility is closely linked with soil texture, aggregate stability, shear stress, soil structure, infiltration capacity, soil depth, volume weight, soil organic matter content, and chemical composition [22]. Depending on the K value determined by these properties, it is ascertained how the soil affected by erosion can be improved or methods which can be taken to prevent soil losses [23].

In Tokat province, which generally shows arid and semi-arid climatic characteristics and where precipitation intensity-duration relationship and topographical condition are suitable for water erosion, the number of studies on the determination of soil loss is very limited [8]. Knowing the annual soil losses and tolerable soil erosion is essential in terms of taking effective erosion measures. This study aims to determine the value of the soil erosion occurring in different regions of a homogenously sloping hillside land where agricultural activities are carried out. The data obtained aims to guide the soil conservation studies to be carried out in the field in the future.

2.1. Material

The study was carried out in Tekneli village, 9 km away from Tokat province. Tekneli village is located between 36°30'13''E and 40°10'59''N and has an elevation of 1214m from the sea (Figure 1). In Tekneli village, where semi-arid climate conditions are present, the summers are hot and dry; winters are cold and rainy. The average annual rainfall is 492.1 mm, the average temperature is 8.1 °C, the highest snow depth is 86 cm, and the number of snow-covered days is 124. Considering the climate data of the Tokat region, the study area moisture regime is Ustic, and the soil temperature regime is Mesic. Tekneli village has shallow soil having A and C horizon and is formed on 10-12% sloping limestone. As the soil depth widens, the lime content increases and the amount of clay decreases. The dominant cations are Ca, and Mg and the pH vary between 7.70-7.86. The geological units of the basin are composed of magmatic, metamorphic and sedimentary rocks. Metamorphic rocks are composed of upper and lower cretaceous old schists, gneisses and limestones. Sedimentary units are represented by minerals such as Oligocene and Miocene aged gypsum, limestone, sandstone, shale. Volcanic units are seen as andesite, basalt and diabase rocks [24].

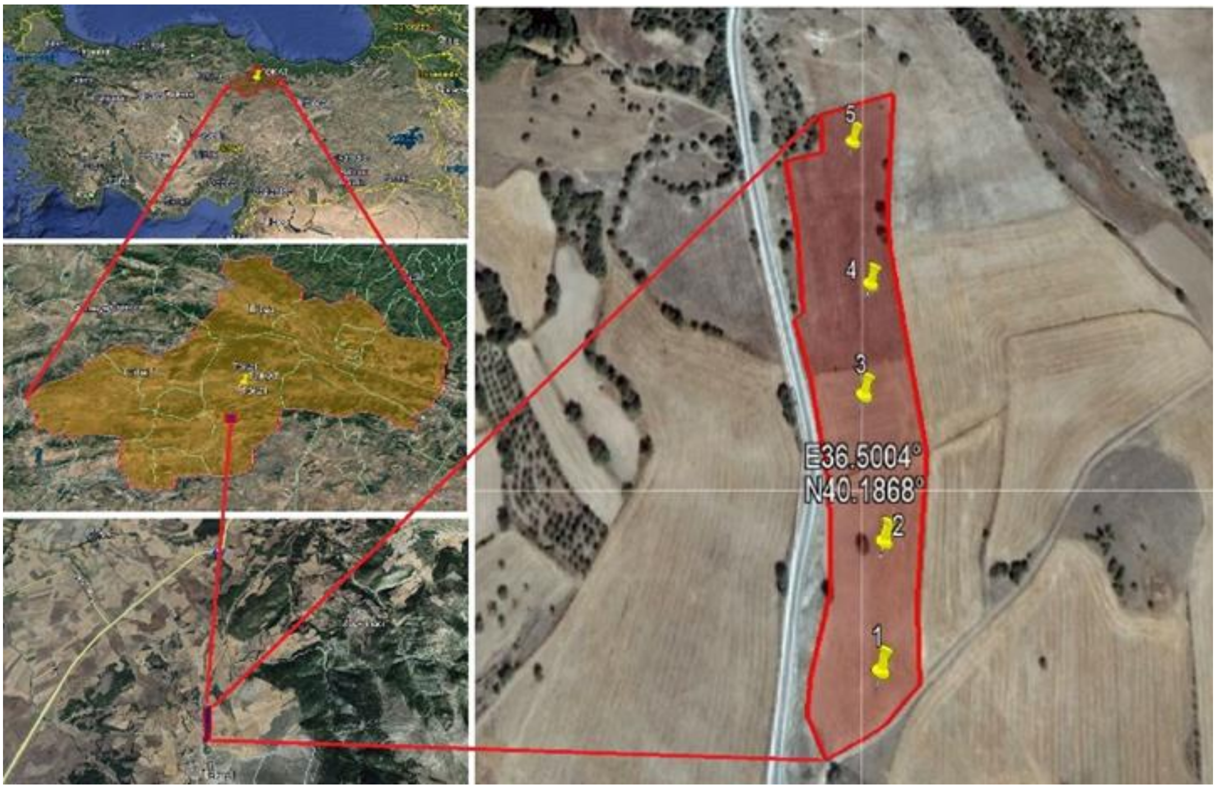

Figure 1. Site location map of the study area

2.2. Methods

To determine soil properties affecting soil erosion tendency, a total of 15 degraded surface soil samples (0-20cm) were collected from 5 different points of the slope segments of a field with a convex slope, where wheat-fallow crop rotation is carried out with three replications.

"Bouyoucos" hydrometer method was used in texture analysis of the soil [25]. The very fine sand fraction of the samples was determined by draining off the mechanical analysis suspension from a 0.105 mm sieve [26]. The organic matter was determined by the Walkey-Black method [27], and the hydraulic conductivity of the soils was found by the use of hydraulically permeable sets [28] where the water level was constant.

2.2.1. Determination of soil erosion susceptibility (Erodibility) K factor

The soil's erodibility (K) factor stands for the resistance of the soil to external erosive forces, and the probability of erosion occurs due to the physical and chemical properties of soils. Under the same external forces, this value is quite low in some soils; however, it is quite high in some others. The K factor value is calculated according to the empirical equation given below according to the results obtained from laboratory analysis:

$$100K = 2.1 \times 10^{-4}\,(12 - OM)M1.14 + 3.25(S - 2) + 2.5(P - 3)d \qquad (1)$$

Here, OM=Organic matter %, S=Soil structure classification, P=Soil permeability code, M=Grain thickness distribution parameter, d = Metric system transformation coefficient, and d= 1.292.

The following equation was used in the calculation of the M factor:

$$M = (\text{Silt}\,\% + \text{Very Fine Sand}\,\%)(100 - \text{Clay}\,\%) \qquad (2)$$

3.Results and Discussion

To determine the soil's erosion susceptibility in the study area, Equation 1 and K values were calculated by considering the textural and organic material content and water permeability values and structural characteristics of the samples in the study area. The results are presented in Table 1. The clay and sand content of soils of the study area was 50% and 35%, respectively, and the texture class was determined as Clay. The erodibility values of the clay-rich soils are very low. This is because the particles are bonded together in clay soils by various cement materials and show strong resistance to decomposition-transportation [29].

Table 1. Physical and chemical properties of soils in the study area

Location	Sample Point	Sand (%)	Clay (%)	Silt (%)	Classification	Organic Matter (%)	Very Fine Sand (%)	Hydraulic Conductivity (%)	Soil Erodibility (K) tha^{-1} Mj mm^{-1}
Summit	1	32	54	14	Clay	2.07	3.48	3	0.10
	2	26	60	14	Clay	2.40	3.78	7.8	0.09
	3	34	56	10	Clay	2.01	3.22	1.7	0.11
Shoulder	4	38	46	16	Clay	2.20	3.1	2.3	0.08
	5	34	52	14	Clay	2	2.6	8.4	0.07
	6	34	54	12	Clay	2.36	4.78	2.3	0.09
Back Slope	7	34	56	10	Clay	3.16	3.78	1.3	0.11
	8	28	56	16	Clay	3.02	3.36	3.5	0.10
	9	30	58	12	Clay	1.46	2.68	1.7	0.12
Foot Slope	10	38	46	16	Clay	3.20	3.78	8.3	0.08
	11	36	52	12	Clay	2.54	2.96	2.8	0.09
	12	40	46	14	Clay	1.93	3.62	7.2	0.08
Toe Slope	13	34	52	14	Clay	2.68	3.36	3.6	0.10
	14	38	50	12	Clay	1.74	3.02	3.6	0.09
	15	38	52	10	Clay	2.54	3	2.2	0.08

Erosion erodibility values of the samples taken from various locations of the sloping land are given in Table 2. The K values range between 0.07 and 0.12 tha^{-1} Mj mm^{-1}, and the study area soils fall into the slightly erosive class. The study conducted in China found that soil erosion values with the organic matter content ranging between 2.5 and 5.5% were between 0.02 and 0.04 [30]. Besides, in his study in which he investigated the erosion susceptibility of the clay soils in Nigeria, Okorafor [31] found that K values ranged between 0.060-0.067 investigated the erosion susceptibility of soils in the Yamchi basin in the north of Iran [32]. K values of the soils taken from 0-20 cm depth were found between 0.442 and 0.0076. The study results indicated that the erosion susceptibility of soils increased with the decrease of organic matter content. In this study, the organic matter contents of soils no. 9, 12, and 14 were determined as 1.46, 1.93, and 1.74, respectively. These values obtained resulted in an increase in the K value in these locations and were classified as moderately eroded soils (Table 2).

Table 2. Erosion susceptibility (erodibility) degrees of soils in the study area

Sample Location	K Value	Classification
1	0.10	Very low soil erodibility
2	0.09	Very low soil erodibility
3	0.10	Very low soil erodibility
4	0.09	Very low soil erodibility
5	0.07	Very low soil erodibility
6	0.09	Very low soil erodibility
7	0.10	Very low soil erodibility
8	0.10	Very low soil erodibility
9	0.12	Low soil erodibility
10	0.08	Very low soil erodibility
11	0.09	Very low soil erodibility
12	0.11	Low soil erodibility
13	0.10	Very low soil erodibility
14	0.11	Low soil erodibility
15	0.08	Very low soil erodibility

Statistical analyses were performed to determine the relationship between erosion susceptibility and soil properties. Results are presented in Table 3. When the standard deviation, skewness, and kurtosis values of the K factor are examined, it is seen that the results were close to the mean, and the data were normally distributed.

The relationship between the K factor and soil properties was determined by correlation analysis. There is a negative relationship (-0.658) between K and the permeability of the soil, and the relationship between them was significant (p<0.01). [33] found similar results in his study. Similarly, [33] found a negative (-0.882) relationship between the K-factor and the hydraulic conductivity of soils in the study, which investigated the erosion susceptibility of soils. A negative relationship was found between the K factor and the soil's clay content (-0.616) and the amount of organic matter (-0.249). Both soil properties have colloid binding properties. They increase the erosion susceptibility of soils. [33] found a negative relationship between clay content and k factor in their study.

Table 3. Descriptive Statistics

	K Factor	Clay %	Sand %	Organic Material %	Hydraulic Conductivity mm/hr	Very Fine Sand %
Mean	0.09	52	34.53	2.49	3.98	3.37
Standard Deviation	0.01	5.01	4.81	0.72	2.57	0.55
Kurtosis	-0.46	-.0.05	-0.2	1.64	-0.78	2.09
Skewness	0.38	-0.57	0.03	1.05	0.94	1.05

Table 4. Correlation analysis between the K factor and some soil properties

	K Factor	Clay %	Sand %	Organic Material %	Hydraulic Conductivity mm/hr	Very Fine Sand %
K	1					
Clay	-.616	1				
Sand	-.473	-.818	1			
Organic matter	-.249	.358	-.446	1		
Hydraulic Conductivity	-.632	-.832	.447	-.085	1	
Very Fine Sand	.006	-.023	-.148	.192	.042	1

4. Conclusion

The K factor of the USLE model is very closely related to soil losses and is a key factor used in predicting soil erosion. The soil erosion values of the hillside land of the Tekneli village were found in the moderately erosive soil group. As a result of the analysis, it was observed that the erosion value is closely related to the organic matter and clay content. The clay content of the study area soils is very high. Since clay particles form aggregates resistant to decomposition, the soil erosion value of the region is reduced. Erosion degree depends only on soil properties. It is not associated with slope, precipitation, vegetation, and management practices.

Author Contributions

All authors contributed equally to this work. They all read and approved the last version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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Research Article

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Evaluation of temperature parameters in Kayseri province with CLIGEN

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Keywords:
CLIGEN,
Mann-Kendall Test,
Kayseri

Abstract — One of the critical consequences of climate change, expecting in the future but beginning to appear nowadays, is the increase in average earth temperatures. The Mediterranean basin we live in is one of the regions that this climate change will most affect. Therefore, simulation studies using climate models gain importance. In this study, Kayseri station's 39-year temperature changes between the 1980-2018 years were simulated using the CLIGEN climate model. The relationship between observed and predicted temperatures was determined utilizing the Mann-Kendall statistical method. CLIGEN estimated the annual average, minimum and maximum average temperatures above the detected value. These values have shown that the study area may encounter a drought problem and be affected by climate change soon.

Subject Classification (2020):

1. Introduction

The performance of solar energy system changes based on wind velocity, ambient temperature and clamminess. These factors are identified according to their change over time [1]. Air temperature is expressed as the amount of moisture retained in the atmosphere [2]. Individual precipitation events and increases in precipitation intensity happen based on the increases in temperature [1].

Surface air temperature is one of the most important factors [3]. Global climate change indicators are used to indicate the change of surface temperature over time. These are (1) positive recycling between ambient temperature and carbon cycle [4]. (2) earth temperature, which controls soil air and soil failure [5], cause and effect between global warming and decreasing bio-diversity [6], the changes in plant phenology [7] and growing season [8].

Temperature is an essential parameter in many environmental factors [9]. These models use the average temperature over a certain period. In the general directorate of meteorology, temperature data of the past 150 years read automatically with digital tools. These tools evaluate temperature actuarially

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[10,11]. Although the daily maximum and minimum temperature data show normal distribution, it has been observed that it does not show the normal distribution in many conditions. Data shows distribution below or above normal, but they are moving away from normal.

The method used to evaluate the daily maximum and minimum temperature data is crucial. Because the temperature values found as a result of the simulation must be close to the observed temperature values. LARS-WG and CLIGEN provided in [12] performed well to simulate long-term climate data in the Western Lake Erie Basin (WLEB). CLIGEN calculates temperature values that are not affected by precipitation. There was no statistically significant relationship between observed and expected values as a consequence of the analysis. Therefore, the climate model must be suitable for the climatic conditions of the area [12]. There are many studies on the changes in daily temperature values due to climate change during the 20th century and at present. In these works, the effects of temperature changes on agricultural, forest, environment and human were evaluated with a climate model. Several stochastic weather generators (SWGs) have been developed over the last few decades, such as the Weather GENerator (WGEN) [13, 14], the CLIMate GENerator (CLIMGEN) [15], the CLImate GENerator (CLIGEN) [16, 17] and the Long Ashton Research Station-Weather Generator (LARS-WG) [18]. They have been widely used to simulate daily weather time series for impact studies [18, 19, 20, 21, 22].

According to the preliminary research, there is no study on the trend of long-term temperature data simulated with the CLIGEN climate model in Kayseri and the effects on agricultural production. This study aimed to use the CLIGEN climate model to simulate temperature values obtained from the Kayseri Meteorological Station from 1980 to 2018 and compare observed and simulated temperature values with annual, monthly, and seasonal evaluations of the model's performance in Kayseri climate conditions.

2. Material

The sea rises to a height of 1050 meters. It is Central Anatolia's third-largest city. Kayseri has many steppe climate characteristics. Summers in Kayseri are hot and dry, while winters are cold and snowy. Erciyes Mountain, at 3.916 meters, is the province's highest peak, and it encompasses a significant portion of the province, and volcanic soils make up a significant portion of the agricultural region. The average annual air temperature is 18.21°C, with 399.6 mm of precipitation recorded at the Turkish State Meteorological Service's Kayseri Meteorological Station between 1980 and 2018. The wettest months are July and August, with the least amount of rain falling in May.

Kayseri is located in the central Kızılırmak area of Central Anatolia (Figure 1). The height of the sea is 1050 m. It is the third-largest city in Central Anatolia. There are many steppe climate characteristics in Kayseri. In Kayseri, summers are hot and dry; winters are cold and snowy. The highest mountain of the province is Erciyes Mountain, with a height of 3.916 meters, and a large part of the agricultural area is made up of volcanic soils. The average annual air temperature is 18.21°C, and the annual amount of precipitation between 1980 and 2018 was measured 399.6 mm in the Kayseri Meteorological Station of the Turkish State Meteorological Service. The lower precipitations are found in July and August, and the highest precipitations are in May.

Figure 1. Geographic position of the studied province to Turkey.

3. Methods

3.1 Meteorological Data

This work contains an analysis of surface air temperature trends obtained from Kayseri meteorological stations. The locations of the stations are presented in Figure 1, and their main parameters are given in Table 1 by the meteorological Service of Kayseri.

3.2 CLIGEN

CLIGEN is a climate model that simulates climatic parameters such as precipitation, maximum and minimum temperature, solar radiation, relative humidity, wind direction and intensity (Figure 2). It makes daily weather forecasts using the Markow chain, which predicts the probability of a wet day P (W|W) following a wet day and a dry day P (W/D) following a wet day. The simulations use the amount of precipitation on a wet day and the skewed normal distribution. [16]. The predicted air temperature with CLIGEN may be higher than the temperature of the dry day following a dry day and may be lower than the temperature of the wet day following a wet day. [23,13]. WEPP model estimates the temperature using the equation given below:

$$T_{\max} \coloneqq T_{\mathrm{mx}} + (ST_{\mathrm{mx}}) * v * B \tag{1}$$

$$T_{\max} \coloneqq T_{mn} + (ST_{mn}) * v * B \tag{2}$$

Here, $T_{\max}$ and $T_{\max}$ are the simulated maximum and minimum temperatures. Tmx and Tmn are the maximum and minimum temperatures observed in each month. STmx and STmn are the standard deviation values of the observed maximum and minimum temperatures. 'v' is the normal standard deviation, and B is the probability of being wet-dry. The B value is calculated according to the formulas given below:

$$T_{\max} \coloneqq T_{\mathrm{mx}} + (ST_{\mathrm{mx}}) * v * B \tag{1}$$

$$T_{\max} \coloneqq T_{mn} + (STmn) * v * B \tag{2}$$

$$B\ (W/D) = 1 - (P(W/D))/PF \tag{3}$$

$$B\ (W/W) = 1 - (P(W/W))/PF \tag{4}$$

$$B\ (D/D) = (P(D/D))/PF \quad (5)$$

$$B\ (D/W) = (P(D/W))/PF \quad (6)$$

P (W/D) is the wet days after a dry day, and P (W/W) is the wet day after a wet day. PF is a factor based on the probability of wet and dry and is calculated by the formula given below:

$$PF\ = P(W\backslash D)(1-(W\backslash D))\ +\ P(W\backslash W)(l-P(W\backslash WO) \quad (7)$$

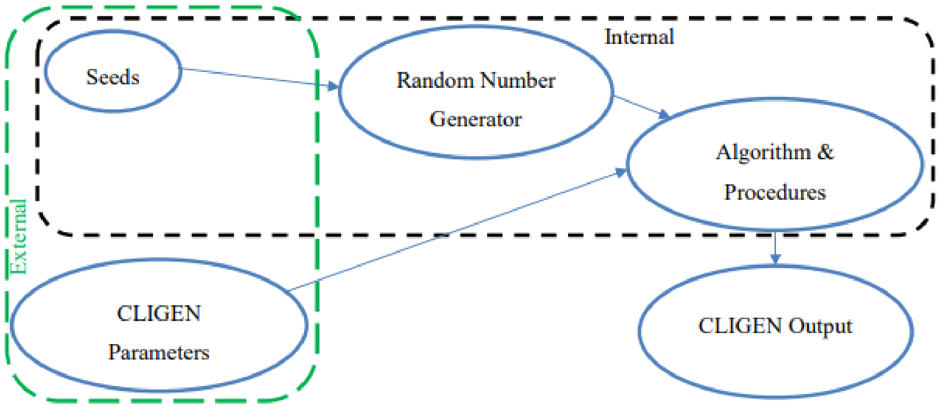

Figure 2. Mechanism of daily temperatures process in CLIGEN

Descriptive statistical data such as mean, standard error, median value, minimum and maximum for data series and average standard error statistics were used in determining which years some measured climatic data showed excess. An evaluation of the trends in climatic variables is essential for understanding the effect of climate change on temperature, precipitation which has a direct and adverse impact on hydrological, agricultural and economic. Various statistical methods are available to determine trends in climatic and hydrologic variables [24-27]. In hydro-meteorological data, the non-normal distribution and the censored character are typical, and the Mann-Kendall can handle such issues [27,28]. Therefore, in the present research, these methods were selected to detect in variation the annual and seasonal precipitation measured in the Kayseri station. A detailed description of the methods used is given below.

3.3 Mann-Kendall Test

This study's statistical approach used the Mann–Kendall test [29,30] to indicate statistically significant trends. The Mann–Kendall test is widely used in the analysis of climatologic time series; for example, temperature and precipitation [31], extreme temperatures [32], hail [33,34], aridity [35], evapotranspiration [36], and atmospheric deposition [37], and also in hydrological time series [38] and other geophysical time series, such as soil freezing and thawing [39] because it is simple and robust and can overcome values below the detection limit and missing values.

In using the Mann–Kendall test to define statistically significant trends, two hypotheses were tested: the null hypothesis H_0, that there is no trend in the time series and the alternative hypothesis H_a, that there is a trend in the time series for a given significance level. Probability p in per cent [31,40] was calculated to determine the level of confidence in the hypothesis. If the computed value p is lower than the chosen significance level α (e.g., α = 5 %), the H_0 (there is no trend) should be rejected, and the Ha (there is a significant trend) should be accepted, and if p is greater than the significance level α, then the H_0 (there is a significant trend) is accepted (or cannot be rejected). For calculating probability p and hypothesis testing, XLSTAT statistical analysis software was employed (Internet 2).

It is considered that accepting the H_a indicates that a trend is statistically significant. On the other hand, the acceptance of H_0 implies that there is no trend (no change), whereas, in practice, the trend equation usually indicates the opposite, that is, a trend. Therefore, to reduce the contradictions in analysing the temperature trends between two independent statistical approaches -the trend equation and the Mann-Kendall test- the modified interpretation of the Mann-Kendall test will be offered. Moreover, this interpretation makes it possible to obtain more diverse results.

It is quite clear that, with decreasing probability p, statistical confidence in the H_0 decreases, and confidence in the Ha increases, and vice versa. Because probability p takes values between 0% and 100%, for this study, a modified interpretation of the Mann–Kendall test was introduced, and four levels of confidence were defined. When the computed probability p is: (1) less or equal to 5%, the trend is significantly positive/negative; (2) greater than 5% and less than or equal to 30%, the trend is moderately positive/negative; (3) greater than 30% and less than or equal to 50%, the trend is slightly positive/negative; and (4) greater than 50%, there is no trend. As can be seen, in cases (1) and (4), both interpretations of the Mann–Kendall test have the same meaning: there is a significant trend and no trend. Differences occur in cases (2) and (3), where the Mann–Kendall test claims there is no trend, and the modified Mann–Kendall test allows a trend with reduced levels of confidence.

4. Results and Discussion

4.1 Annual Average Temperatures

Observed and simulated annual average temperatures were determined as 21.91 and 18.21°C, respectively. The relationship between them is given graphically in Figure 3. The determination coefficient was R^2: 0.83, which was a very high value. The model has simulated the annual average temperatures above the observed value. The data distribution above the 1:1 line also indicates this result (Figure 3). The global warming caused by the greenhouse effect strengthened because of the greenhouse gas accumulation in the atmosphere, became more evident, especially after the 1980s, and reached its highest value in the 1990s [41]. These climate changes cause hydrological cycle fluctuations, increasing the extreme hydrological events' severity and frequency. These events, which occur depending on the annual average temperature, also affect the soil structure and quality. Although it shows a positive effect in the short term, it causes a deterioration in the long term.

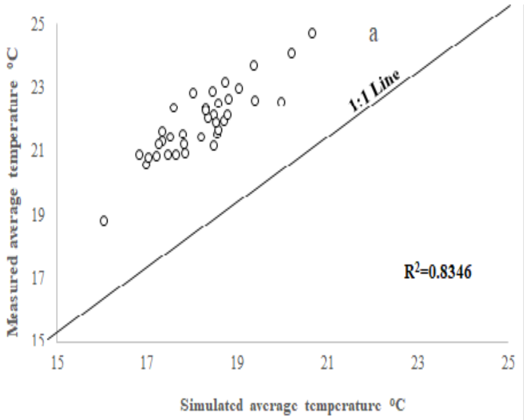

Figure 3. Relationship between observed and simulated annual average temperature

Since climate systems have variable and complex structures, it is very challenging to make accurate predictions. Climate change simulations are used to make climate projections despite the difficulties it entails. However, today's simulation studies may be inadequate due to the lack of reliable data on soil properties and soil management practices [42]. Global-scale statistical analyses cannot be reliable due to the insufficient and unreliability agricultural data having been obtained in some regions. Besides, analyses based on more reliable data obtained from another world region cannot be sufficient to create global simulations [43]. Because climate models such as CLIGEN complete the missing data using statistical analysis, they have a higher performance than other climate models. The temperature trend increasing since the mid-1990s has also been observed in the working area temperatures (Figure a). An increasing trend is in question, especially since the early 2000s. CLIGEN simulated the annual average data very close to the observed values. There is an increase in observed and simulated annual average temperatures after 2012 (Figure 4 a, b).

In [44], the CLIGEN climate model simulates the long-term average temperature data for Kayseri, Sivas, and Yozgat meteorological stations. As a result of the study, an increase in temperatures was observed. These values vary depending on the region and season.

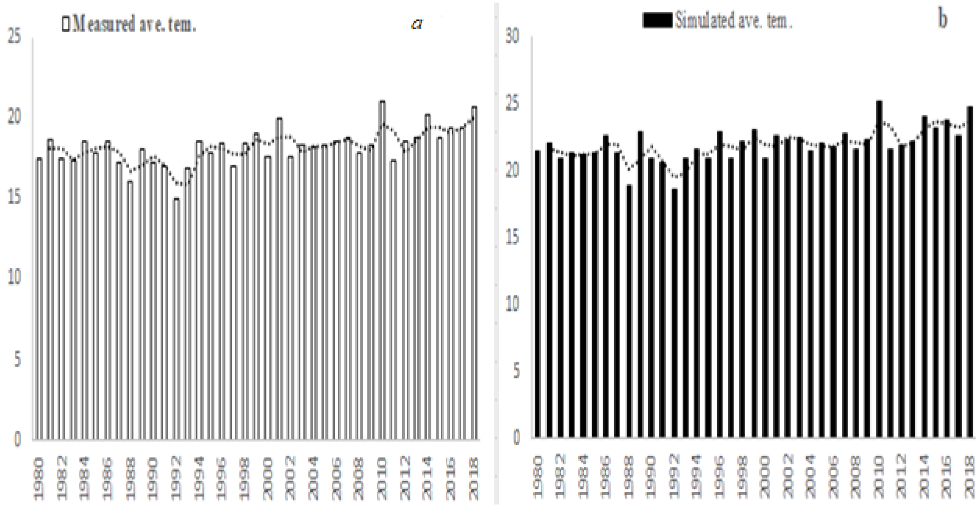

Figure 4. Trend change of observed and simulated annual mean temperatures

According to the Mann-Kendall method, trend analysis results of the annual average temperature data are given in Table 1. Figure 4 exhibits that there is a trend among the annual average temperature data.

Table 1. Mann Kendall analysis result for the annual average temperature

Parameters	Average	Standard deviation	Lower Limit	Upper Limit	Kendall's tau	p
CLIGEN	21.922	1.320	18.541	25.118	0.414	0.000
OBSERVED	18.224	1.183	14.955	21.058	0.385	0.001

H_0: There is no trend in the series. H_a: There is a trend in the series. Since the p values calculated for both variables are less than $\alpha = 0.05$, the H_0 hypothesis should be rejected, and the alternative hypothesis, the H_a hypothesis, should be accepted. So, there is a trend in the series.

4.2 Minimum Average Temperatures

Observed and simulated minimum temperatures were determined and graphically shown in Figure 5. The temperatures are 3.55 and 6.42°C, respectively, and the determination coefficient is 0.95. When Figure 4a is examined, the data are observed to show a distribution above the 1:1 line. The graphic showing the model performance by months is given in Figure 5b. The model has predicted the temperature values for especially March and November months, which are very low ordinarily, as very high. The precipitation seen in these months is quite variable, and the number of wet days is high. Therefore, the model has exaggerated the minimum temperature values (Figure 5). Besides, the model made close estimates to the observed value in July, August and September. These months are relatively dry in Kayseri province. Therefore, the model water budget does not change. When the minimum temperature changes by month are examined, the difference is observed as the lowest in January, February, November, and December and the highest in June and July (Figure 5b).

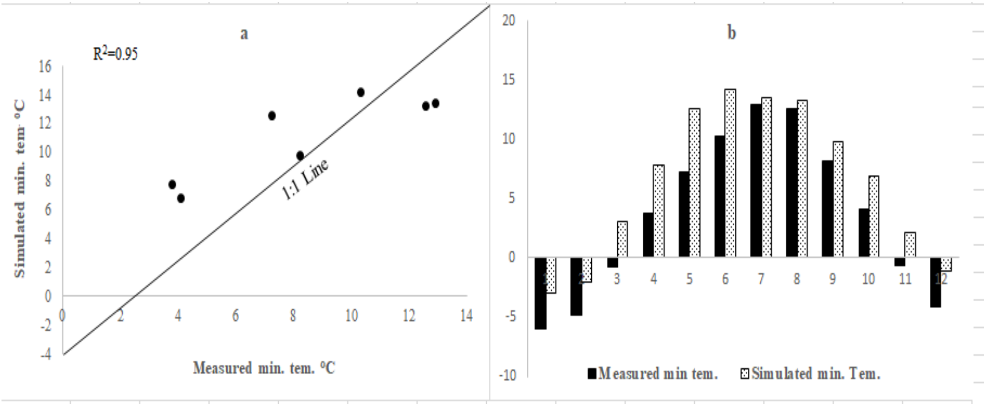

Figure 5. a) Relationship between observed and simulated minimum temperatures, b) Variation of observed and simulated minimum temperatures to months

The H_0 hypothesis cannot be rejected since the calculated p values for both variables are greater than α = 0.05. Thus, there is no trend in the series (Table 1).

Table 2. Mann Kendall analysis result for the minimum temperature

Parameters	Average	Standard deviation	Lower Limit	Upper Limit	Kendall's tau	p
CLIGEN	4.424	6.382	-4.833	12.903	0.091	0.755
OBSERVED	7.279	6.010	-2.050	14.220	-0.091	0.755

4.3 Maximum Temperatures

The observed and simulated maximum temperatures are 18.15 and 21.86°C, respectively. The relationship between them is shown graphically in the Figure. The determination coefficient is R^2=0.96. The model has overestimated the maximum temperatures compared to observed values (Figure 6a). The data disperse above the 1:1 line. In particular, the model has estimated the temperature values of January and December very high (Figure 66b). The rainfall reduction in the subtropical zone has become efficient in Turkey and the eastern Mediterranean basin since the 1970s [45,46]. The significant downward tendency in precipitation and drought events emerges more obviously in winters. Therefore, these reductions in precipitation cause the model to over-predict its temperatures.

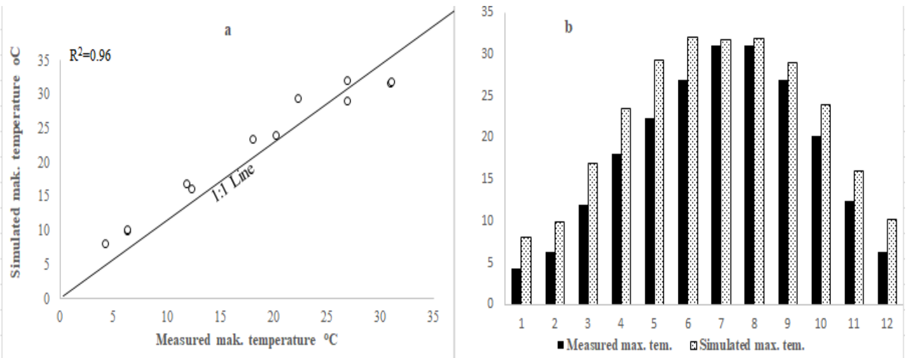

Figure 6. a) Relationship between observed and simulated maximum temperatures, b) Variation of observed and simulated maximum temperatures to months

The H_0 hypothesis cannot be rejected since the calculated p values for both variables are greater than $\alpha = 0.05$. So, there is no trend in the series (Table 3).

Table 3. Mann Kendall analysis result for the maximum temperature

Parameters	Average	Standard deviation	Lower Limit	Upper Limit	Kendall's tau	p
CLIGEN	23.106	8.546	9.964	31.993	-0.018	1.000
OBSERVED	19.408	9.164	6.330	31.040	0.127	0.640

5. Conclusion and Discussion

Climate change is considered to be one of the most critical environmental problems of today. Today, the climate change problem, affecting every phase of our lives, including nature, city life, industry, economy, technology, human rights, agriculture, food, clean water, and health, obliges the governments for a solution.

In parallel with the rapid growth trend that started after the industrial revolution, a significant warming trend is observed in global average surface temperatures due to CO_2 and other greenhouse gases accumulating in the atmosphere. According to the most recent international assessments, there has been an increase in global average surface temperatures of about 0.4-0.8°C in the last century. This warming trend became more evident after the 1980s, and in this period, high-temperature records were broken almost every year. The year 1998 was recorded as the hottest year globally averages since 1860 when instrumental temperature observations were started. Climate models predict that the global average surface temperature will increase between 1 and 3.5 °C until the year 2100 compared to 1990, and depending on this increase, the observed changes in the climate continue.

Besides, mostly as in the world's largest cities in the last 35-40 years, also in large cities in Turkey, where air pollution, rapid population growth, and intense urbanization are widespread, heating at night temperatures, cooling in daytime temperatures, and a decrease in daily temperature widths are observed generally. These trends are particularly evident in the hot, dry, cloudless summer seasons.

A CLIGEN climate model is a novel model that has recently been used in our country. In many regions globally, the model's performance has been evaluated, and very successful results have been obtained. It is significant to increase the simulation works performed with regional climate models to conduct more realistic climate forecasts in Turkey. Climate models such as CLIGEN consider the climate and the hydrological properties of the soil. To reduce soil losses, policies and measures can be determined through projection studies carried out with these models. As a result of these precautions, significant contributions can be made to the country's economy.

Author Contributions

All authors contributed equally to this work. They all read and approved the last version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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Some results on soft topological notions

Tuğçe Aydın[1], Serdar Enginoğlu[2]

Keywords:
Soft sets,
Soft open sets,
Soft topology,
Soft α-open sets,
Soft α-T_0 space

Abstract — Recently, the generalizations of soft open sets have become a popular subject. These generalizations define based on the concepts of the soft interior and soft closure. Therefore, the properties related to these concepts play an essential role in propositions concerning the generalizations. To this end, we consider the soft interior and soft closure through the concept of the soft element, and thus we clarify the relationships between a soft topological space and its soft subspace topologies. Afterwards, we mention soft α-open sets, soft α-closed sets, and soft α-T_0 space via soft elements. Finally, we discuss soft α-separation axioms for further research.

Subject Classification (2020): 54A05, 54A99

1. Introduction

The topological notions of the soft sets [1] first were introduced in two different studies in 2011. In the first of these studies, Çağman et al. [2] have defined soft topology on a soft set. They have proposed basic topological concepts, such as soft open set, soft interior, soft closure, soft limit point, and soft boundary with elements of a soft set and investigated some of their basic properties. On the other, Shabir and Naz [3] have described the concept of soft topology on a universal set. Moreover, they have suggested the basic definitions and properties of the concept regarding a universal set's elements. Afterwards, Enginoğlu et al. [4] have updated the definition of the soft closed set provided in [2] and several theorems related to it to eliminate inconsistencies between definitions and theorems. So far, many researchers have conducted studies [5-14] on various topological concepts ranging from soft separation axioms to soft compactness.

Recently, the researchers have focused on soft α-open sets [15], soft pre-open sets [16], soft semi-open sets [17], and soft β-open sets [16]. Akdağ and Özkan [15] have defined soft α-continuous and soft α-open functions and investigated their relationships among the other soft continuous structures. After that, Akdağ and Özkan [18] have introduced soft α-separation axioms by using the elements of a universal set. Moreover, they have studied some of their fundamental properties and compared the soft α-separation axioms with the soft separation axioms. Khattak et al. [19] have propounded soft α-separation axioms in terms of soft points. Saleh and Sur [20] have proposed novel separation axioms called soft α-R_0, soft α-symmetric, and soft α-R_1 using soft points.

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The present study's primary goal is to study some relationships concerning the soft interior and soft closure of a soft set in a soft topological space and its soft subspace to avail from them in the next research. The second one is to define soft α-T_0 space by using the soft element's concept [5] and the soft topological notions provided in [2,4] and analyse some of its fundamental properties.

In Section 2 of the present paper, we present the soft topological notions, such as soft element, soft open set, soft interior, and soft closure, and some of their basic properties to be utilised in the following sections. In Section 3, we study the relationships related to soft interior and soft closure concepts between soft topological space and its soft subspace. In Section 4, as different from the literature, we propose some properties related to soft α-open sets and soft α-closed sets via the concept of the soft element. Furthermore, we describe soft α-T_0 space via the soft element concept and investigate several of its basic properties. In Section 5, we discuss soft α-separation axioms for further research.

2. Preliminaries

In this section, firstly, we present the concepts of soft sets [1,21] and soft element [5] and some of their basic properties [4,8,21] to be employed in the following sections. Throughout this study, let U be a universal set, E be a parameter set, and $P(U)$ be the power set of U.

Definition 2.1. [1] Let f be a function from E to $P(U)$. Then, the set $F \coloneqq \{(x, f(x)) : x \in E\}$ is called a soft set parameterized via E over U (or briefly over U).

Definition 2.2. [21] Let $A \subseteq E$ and f_A be a function from E to $P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Then, the set $F_A \coloneqq \{(x, f_A(x)) : x \in E\}$ is called a soft set parameterized via E over U (or briefly over U).

In the present paper, the set of all the soft sets over U is denoted by $S(U)$. Besides, as long as it causes no confusion, we do not display the elements $(x, \emptyset)$ in a soft set.

Definition 2.3. [21] Let $F_A \in S(U)$. For all $x \in A$, if $f_A(x) = \emptyset$, then F_A is called empty soft set and is denoted by F_Φ, and if $f_A(x) = U$, then F_A is called A-universal soft set and is denoted by $F_{\tilde{A}}$. Moreover, E-universal soft set is called universal soft set and is denoted by $F_{\tilde{E}}$.

Definition 2.4. [21] Let $F_A, F_B \in S(U)$. For all $x \in E$, if $f_A(x) \subseteq f_B(x)$ then, F_A is called a soft subset of F_B and is denoted by $F_A \tilde{\subseteq} F_B$, and if $f_A(x) = f_B(x)$, then F_A and F_B are called equal soft sets and is denoted by $F_A = F_B$.

Proposition 2.5. [21] Let $F_A, F_B, F_C \in S(U)$. Then, $F_A \tilde{\subseteq} F_{\tilde{E}}$, $F_\Phi \tilde{\subseteq} F_A$, $F_A \tilde{\subseteq} F_A$, $(F_A \tilde{\subseteq} F_B \ \wedge \ F_B \tilde{\subseteq} F_C) \Rightarrow F_A \tilde{\subseteq} F_C$, $(F_A = F_B \ \wedge \ F_B = F_C) \Rightarrow F_A = F_C$, and $(F_A \tilde{\subseteq} F_B \ \wedge \ F_B \tilde{\subseteq} F_A) \Leftrightarrow F_A = F_B$.

Definition 2.6. [21] Let $F_A, F_B, F_C \in S(U)$. For all $x \in E$, if $f_C(x) = f_A(x) \cup f_B(x)$, then F_C is called soft union of F_A and F_B and is denoted by $F_A \tilde{\cup} F_B$, and if $f_C(x) = f_A(x) \cap f_B(x)$, then F_C is called soft intersection of F_A and F_B and is denoted by $F_A \tilde{\cap} F_B$.

Definition 2.7. [21] Let $F_A, F_B, F_C \in S(U)$. For all $x \in E$, if $f_C(x) = f_A(x) \setminus f_B(x)$, then F_C is called soft difference between F_A and F_B and is denoted by $F_A \tilde{\setminus} F_B$.

Definition 2.8. [21] Let $F_A, F_B \in S(U)$. For all $x \in E$, if $f_B(x) = f_A^c(x)$, then F_B is called soft complement of F_A and is denoted by $F_A^{\tilde{c}}$. Here, $f_A^c(x) \coloneqq f_{\tilde{E}}(x) \setminus f_A(x)$.

Proposition 2.9. [21] Let $F_A, F_B, F_C \in S(U)$. Then,

i. $F_A \tilde{\cup} F_A = F_A$ and $F_A \tilde{\cap} F_A = F_A$

ii. $F_A \tilde{\cup} F_\Phi = F_A$ and $F_A \tilde{\cap} F_{\tilde{E}} = F_A$

iii. $F_A \tilde{\cup} F_{\tilde{E}} = F_{\tilde{E}}$ and $F_A \tilde{\cap} F_\Phi = F_\Phi$

iv. $F_A \tilde{\cup} F_B = F_B \tilde{\cup} F_A$ and $F_A \tilde{\cap} F_B = F_B \tilde{\cap} F_A$

v. $(F_A \tilde{\cup} F_B) \tilde{\cup} F_C = F_A \tilde{\cup} (F_B \tilde{\cup} F_C)$ and $(F_A \tilde{\cap} F_B) \tilde{\cap} F_C = F_A \tilde{\cap} (F_B \tilde{\cap} F_C)$

vi. $F_A \tilde{\cup} (F_B \tilde{\cap} F_C) = (F_A \tilde{\cup} F_B) \tilde{\cap} (F_A \tilde{\cup} F_C)$ and $F_A \tilde{\cap} (F_B \tilde{\cup} F_C) = (F_A \tilde{\cap} F_B) \tilde{\cup} (F_A \tilde{\cap} F_C)$

vii. $F_A \tilde{\cup} F_A^{\tilde{c}} = F_{\tilde{E}}$ and $F_A \tilde{\cap} F_A^{\tilde{c}} = F_\Phi$

viii. $(F_A \tilde{\cup} F_B)^{\tilde{c}} = F_A^{\tilde{c}} \tilde{\cap} F_B^{\tilde{c}}$ and $(F_A \tilde{\cap} F_B)^{\tilde{c}} = F_A^{\tilde{c}} \tilde{\cup} F_B^{\tilde{c}}$

ix. $\left(F_A^{\tilde{c}}\right)^{\tilde{c}} = F_A$

x. $F_A \tilde{\setminus} F_B = F_A \tilde{\cap} F_B^{\tilde{c}}$

Proposition 2.10. [4,8] Let $F_A, F_B, F_C, F_D \in S(U)$. Then,

i. $(F_A \tilde{\subseteq} F_B \wedge F_C \tilde{\subseteq} F_D) \Rightarrow F_A \tilde{\cap} F_C \tilde{\subseteq} F_B \tilde{\cap} F_D$

ii. $(F_A \tilde{\subseteq} F_B \wedge F_C \tilde{\subseteq} F_D) \Rightarrow F_A \tilde{\cup} F_C \tilde{\subseteq} F_B \tilde{\cup} F_D$

iii. $(F_B, F_C \tilde{\subseteq} F_A \wedge F_B \tilde{\cap} F_C = F_\Phi) \Leftrightarrow F_B \tilde{\subseteq} \left(F_A \tilde{\setminus} F_C\right)$

Example 2.11. [2] Let $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$ such that $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, and $A = \{x_1, x_2\}$. Then, all the soft subsets of F_A are as follows:

$F_{A_1} = \{(x_1, \{u_1\})\}$	$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$
$F_{A_2} = \{(x_1, \{u_2\})\}$	$F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$
$F_{A_3} = \{(x_1, \{u_1, u_2\})\}$	$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$
$F_{A_4} = \{(x_2, \{u_2\})\}$	$F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\}$
$F_{A_5} = \{(x_2, \{u_3\})\}$	$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$
$F_{A_6} = \{(x_2, \{u_2, u_3\})\}$	$F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_3\})\}$
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$	$F_{A_{15}} = F_A$
$F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_3\})\}$	$F_{A_{16}} = F_\Phi$

Definition 2.12. [22] Let $F_A \in S(U)$ and $F_B \tilde{\subseteq} F_A$. If, for $x_0 \in B$, $f_B(x_0) \neq \emptyset$, and for all $y \in B \setminus \{x_0\}$, $f_B(y) = \emptyset$, then F_B is called a soft point of F_A and is denoted by $F_B \tilde{\in} F_A$.

Definition 2.13. [5] Let $F_A \in S(U)$ and $F_B \tilde{\subseteq} F_A$. If, for $x_0 \in B$, $f_B(x_0)$ is a single point set and for all $y \in B \setminus \{x_0\}$, $f_B(y) = \emptyset$, then F_B is called a soft element of F_A and is denoted by $F_B \tilde{\in} F_A$.

This study exploits Definition 2.13. Throughout this paper, the set of all the soft elements of F_A is denoted by $\mathcal{S}(F_A)$ or briefly $\mathcal{S}$. It is clear that $F_B \tilde{\in} F_A$ and $F_B \in \mathcal{S}(F_A)$ are the same.

Example 2.14. For Example 2.11, $\{(x_1, \{u_1\})\}, \{(x_1, \{u_2\})\}, \{(x_2, \{u_2\})\}, \{(x_2, \{u_3\})\} \tilde{\in} F_A$ and $\{(x_1, \{u_1\})\}, \{(x_2, \{u_2\})\}, \{(x_2, \{u_3\})\} \tilde{\in} F_{A_9}$.

Proposition 2.15. [8] Let $F_A, F_B \in S(U)$. Then, $F_A \tilde{\subseteq} F_B$ iff $\varepsilon \tilde{\in} F_B$, for all $\varepsilon \tilde{\in} F_A$. Besides, $\mathcal{S}(F_A \tilde{\cup} F_B) = \{\varepsilon : \varepsilon \tilde{\in} F_A \vee \varepsilon \tilde{\in} F_B\}$. Namely, $\varepsilon \tilde{\in} F_A \tilde{\cup} F_B \Leftrightarrow \varepsilon \tilde{\in} F_A \vee \varepsilon \tilde{\in} F_B$. Similarly, $\varepsilon \tilde{\in} F_A \tilde{\cap} F_B \Leftrightarrow \varepsilon \tilde{\in} F_A \wedge \varepsilon \tilde{\in} F_B$, $\varepsilon \tilde{\in} F_A \tilde{\setminus} F_B \Leftrightarrow \varepsilon \tilde{\in} F_A \wedge \varepsilon \tilde{\notin} F_B$, and $\varepsilon \tilde{\in} F_A^{\tilde{c}} \Leftrightarrow \varepsilon \tilde{\notin} F_A \wedge \varepsilon \tilde{\in} F_{\tilde{E}}$.

Secondly, we provide some of the soft topological notions [2,4,5,7,15,23], such as soft open sets, soft interior, soft closure, and soft α-open sets and some of their basic properties to be used in the next sections.

Definition 2.16. [2] Let $F_A \in S(U)$ and $\tilde{\tau}$ be a collection of soft subsets of F_A. If

i. $F_\Phi, F_A \in \tilde{\tau}$

ii. Countable soft unions of the soft subsets of $\tilde{\tau}$ belong to $\tilde{\tau}$.

iii. Finite soft intersections of the soft subsets of $\tilde{\tau}$ belong to $\tilde{\tau}$.

then $\tilde{\tau}$ is called a soft topology on F_A. Moreover, the ordered pair $(F_A, \tilde{\tau})$ is referred to as a soft topological space.

Here, this study utilizes the arbitrary soft unions instead of the countable soft unions provided in Definition 2.16.

Definition 2.17. [2,4] Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called a $\tilde{\tau}$-soft open set (or briefly soft open). Furthermore, if $F_B \in \tilde{\tau}$, then $F_A \tilde{\setminus} F_B$ is referred to as a $\tilde{\tau}$-soft closed set (or briefly soft closed).

From now on, set of all the soft closed sets of $(F_A, \tilde{\tau})$ is denoted by $\tilde{\tau}^k$.

Proposition 2.18. [2,4] In $(F_A, \tilde{\tau})$, F_Φ and F_A are both soft open and soft closed.

Example 2.19. For Example 2.11, $\tilde{\tau} = \{F_\Phi, F_A, F_{A_1}, F_{A_7}, F_{A_{14}}\}$ is a soft topology on F_A and so $(F_A, \tilde{\tau})$ is a soft topological space.

Definition 2.20. [2] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the collection $\{F_{A_i} \tilde{\cap} F_B : F_{A_i} \in \tilde{\tau},\ i \in I\}$ is called a soft subspace topology on F_B and is denoted by $\tilde{\tau}_{F_B}$. Moreover, $(F_B, \tilde{\tau}_{F_B})$ is referred to as a soft topological subspace of $(F_A, \tilde{\tau})$.

Proposition 2.21. [2] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, a soft subspace topology on F_B is a soft topology on F_B.

Example 2.22. For Example 2.19, $\tilde{\tau}_{F_{A_7}} = \{F_\Phi, F_{A_7}, F_{A_1}\}$ and $\tilde{\tau}_{F_{A_{12}}} = \{F_\Phi, F_{A_{12}}, F_{A_4}, F_{A_{11}}\}$ are soft subspace topologies on F_{A_7} and $F_{A_{12}}$, respectively. Therefore, $(F_{A_7}, \tilde{\tau}_{F_{A_7}})$ and $(F_{A_{12}}, \tilde{\tau}_{F_{A_{12}}})$ are soft topological subspaces of $(F_A, \tilde{\tau})$.

Theorem 2.23. [4] Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$. If $F_C \in \tilde{\tau}_{F_B}$, then there exists at least one $F_D \in \tilde{\tau}$ such that $F_C \tilde{\subseteq} F_D$.

Theorem 2.24. [4] Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, $\tilde{\tau}^k$ provides the following conditions:

i. F_Φ and F_A are soft closed.

ii. Arbitrary soft intersections of the soft closed sets are soft closed.

iii. Finite soft unions of the soft closed sets are soft closed.

Definition 2.25. [5] Let $(F_A, \tilde{\tau})$ be a soft topological space, $F_B \tilde{\subseteq} F_A$, and $\varepsilon \tilde{\in} F_B$. If there exists at least one $F_C \in \tilde{\tau}$ such that $\varepsilon \tilde{\in} F_C$ and $F_C \tilde{\subseteq} F_B$, then ε is called a $\tilde{\tau}$-soft interior point (or briefly soft interior point) of F_B. Moreover, the soft union of all the soft interior points of F_B is called $\tilde{\tau}$-soft interior (or briefly soft interior) of F_B and is denoted by ${F_B}^\circ$.

Definition 2.26. [2] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the soft intersection of all the soft closed sets containing F_B is called $\tilde{\tau}$-soft closure (or briefly soft closure) of F_B and is denoted by $\overline{F_B}$.

Proposition 2.27. [2] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the soft interior of F_B is soft union all the soft open subsets of F_B. In other words, ${F_B}^\circ$ is the biggest soft open set contained by F_B. Moreover, $\overline{F_B}$ is the smallest soft closed set containing F_B.

Proposition 2.28. [2] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then,

i. $F_B \in \tilde{\tau} \Leftrightarrow F_B = {F_B}^\circ$

ii. $F_B \in \tilde{\tau}^k \Leftrightarrow F_B = \overline{F_B}$

iii. $({F_B}^\circ)^\circ = {F_B}^\circ$ and $\overline{(\overline{F_B})} = \overline{F_B}$

iv. $(F_B \tilde{\subseteq} F_C \Rightarrow {F_B}^\circ \tilde{\subseteq} {F_C}^\circ)$ and $(F_B \tilde{\subseteq} F_C \Rightarrow \overline{F_B} \tilde{\subseteq} \overline{F_C})$

v. ${F_B}^\circ \tilde{\cap} {F_C}^\circ = (F_B \tilde{\cap} F_C)^\circ$ and $\overline{(F_B \tilde{\cap} F_C)} \tilde{\subseteq} \overline{F_B} \tilde{\cap} \overline{F_C}$

vi. $F_B^{\circ} \tilde{\cup} F_C^{\circ} \tilde{\subseteq} (F_B \tilde{\cup} F_C)^{\circ}$ and $\overline{F_B} \tilde{\cup} \overline{F_C} = \overline{(F_B \tilde{\cup} F_C)}$

vii. $F_B^{\circ} \tilde{\subseteq} F_B \tilde{\subseteq} \overline{F_B}$

Theorem 2.29. [2,4] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then, $\varepsilon \tilde{\in} \overline{F_B}$ iff, for all $F_C \in \tilde{\tau}$ such that $\varepsilon \tilde{\in} F_C$, $F_B \tilde{\cap} F_C \neq F_{\Phi}$.

Proposition 2.30. [7] Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B \tilde{\subseteq} F_A$.

i. $F_C \in \tilde{\tau}_{F_B}{}^k$ iff there exists at least one $F_D \in \tilde{\tau}^k$ such that $F_C = F_D \tilde{\cap} F_B$.

ii. If $F_C \in \tilde{\tau}$, then $F_C \in \tilde{\tau}_{F_B}$.

iii. If $F_C \in \tilde{\tau}^k$, then $F_C \in \tilde{\tau}_{F_B}{}^k$.

Proposition 2.31. [23] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. If F_B is soft open, then $F_B \tilde{\cap} \overline{F_C} \tilde{\subseteq} \overline{F_B \tilde{\cap} F_C}$.

Definition 2.32. [15] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. If $F_B \tilde{\subseteq} \left(\overline{F_B^{\circ}}\right)^{\circ}$, then F_B is called a soft α-open set in $\tilde{\tau}$. If F_B is a soft α-open set, then $F_A \tilde{\setminus} F_B$ is called a soft α-closed set in $\tilde{\tau}$.

Hereinafter, the set of all the soft α-open sets and all the soft α-closed sets in $(F_A, \tilde{\tau})$ are denoted by $S\alpha O(\tilde{\tau})$ and $S\alpha C(\tilde{\tau})$, respectively.

Proposition 2.33. [15] In a soft topological space, the arbitrary soft unions of soft α-open sets are soft α-open set. Moreover, the arbitrary soft intersections of soft α-closed sets are soft α-closed set.

Definition 2.34. [15] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, $\tilde{\tau}$-soft α-interior (or briefly soft α-interior) of F_B is defined by $\tilde{\bigcup}\{F_C \in S\alpha O(\tilde{\tau}) : F_C \tilde{\subseteq} F_B\}$ and is denoted by $(F_B)^{\circ}_{\alpha}$, and $\tilde{\tau}$-soft α-closure (or briefly soft α-closure) of F_B is defined by $\tilde{\bigcap}\{F_C \in S\alpha C(\tilde{\tau}) : F_B \tilde{\subseteq} F_C\}$ and is denoted by $\overline{(F_B)}_{\alpha}$.

Proposition 2.35. [15] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then,

i. $F_B \in S\alpha C(\tilde{\tau}) \Leftrightarrow \overline{(F_B)}_{\alpha} = F_B$

ii. $F_B \tilde{\subseteq} F_C \Rightarrow \overline{(F_B)}_{\alpha} \tilde{\subseteq} \overline{(F_C)}_{\alpha}$

Theorem 2.36. [15] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, $F_B \in S\alpha C(\tilde{\tau})$ iff $\overline{(\overline{F_B})^{\circ}} \tilde{\subseteq} F_B$.

Theorem 2.37. [23] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then, $F_B \in S\alpha O(\tilde{\tau})$ iff there exists at least one $F_C \in \tilde{\tau}$ such that $F_C \tilde{\subseteq} F_B \tilde{\subseteq} (\overline{F_C})^{\circ}$.

3. Relationships between Soft Interior and Soft Closure in Soft Topological Spaces and Their Soft Subspaces

This section studies several properties containing relationships between the soft interior and soft closure of a soft set according to a soft topological space and its soft subspace.

Theorem 3.1. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$.

i. If F_B is soft open, then $F_B \tilde{\subseteq} \overline{F_B^{\circ}}$.

ii. If F_B is soft closed, then $(\overline{F_B})^{\circ} \tilde{\subseteq} F_B$.

Proof. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$.

i. If F_B is soft open, then $F_B = F_B^{\circ}$. Moreover, from Proposition 2.28 (vii), $F_B \tilde{\subseteq} \overline{F_B}$. Thus, $F_B \tilde{\subseteq} \overline{F_B^{\circ}}$.

ii. If F_B is soft closed, then $F_B = \overline{F_B}$. Moreover, from Proposition 2.28 (vii), $F_B^{\circ} \tilde{\subseteq} F_B$. Thus, $(\overline{F_B})^{\circ} \tilde{\subseteq} F_B$.

The converse of the propositions provided in Theorem 3.1 is not always correct. This situation is proved with the following example.

Example 3.2. For Example 2.19, $\overline{F_{A_9}}^{\circ} = \overline{F_{A_7}} = F_A$. Thus, $F_{A_9} \tilde{\subseteq} \overline{F_{A_9}}^{\circ}$, but F_{A_9} is not soft open. Similarly, $\left(\overline{F_{A_5}}\right)^{\circ} = {F_{A_{11}}}^{\circ} = F_{\Phi}$. Thus, $\left(\overline{F_{A_5}}\right)^{\circ} \tilde{\subseteq} F_{A_5}$, but F_{A_5} is not soft closed.

Theorem 3.3. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. If F_B is soft open, then $\overline{F_B \tilde{\cap} \overline{F_C}} = \overline{F_B \tilde{\cap} F_C}$.

Proof. Let $(F_A, \tilde{\tau})$ be a soft topological space, $F_B, F_C \tilde{\subseteq} F_A$, and F_B be a soft open. Then, from Proposition 2.31, $F_B \tilde{\cap} \overline{F_C} \tilde{\subseteq} \overline{F_B \tilde{\cap} F_C}$ and from Proposition 2.28 (iv), $\overline{F_B \tilde{\cap} \overline{F_C}} \tilde{\subseteq} \overline{\left(\overline{F_B \tilde{\cap} F_C}\right)}$. Therefore, $\overline{F_B \tilde{\cap} \overline{F_C}} \tilde{\subseteq} \overline{F_B \tilde{\cap} F_C}$. Moreover, since F_B is soft open, then $F_B \tilde{\subseteq} {F_B}^{\circ}$. From Proposition 2.28 (vii), $F_C \tilde{\subseteq} \overline{F_C}$. Therefore, $F_B \tilde{\cap} F_C \tilde{\subseteq} {F_B}^{\circ} \tilde{\cap} \overline{F_C}$ and so $\overline{F_B \tilde{\cap} F_C} \tilde{\subseteq} \overline{F_B \tilde{\cap} \overline{F_C}}$. Consequently, $\overline{F_B \tilde{\cap} \overline{F_C}} = \overline{F_B \tilde{\cap} F_C}$.

Theorem 3.4. Let $\left(F_B, \tilde{\tau}_{F_B}\right)$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B \tilde{\subseteq} F_A$.

i. If $F_B \in \tilde{\tau}$ and $F_C \in \tilde{\tau}_{F_B}$, then $F_C \in \tilde{\tau}$.

ii. If $F_B \in \tilde{\tau}^k$ and $F_C \in {\tilde{\tau}_{F_B}}^{k}$, then $F_C \in \tilde{\tau}^k$.

Proof. Let $\left(F_B, \tilde{\tau}_{F_B}\right)$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B \tilde{\subseteq} F_A$.

i. Let $F_B \in \tilde{\tau}$ and $F_C \in \tilde{\tau}_{F_B}$. Then, there exists at least one $F_D \in \tilde{\tau}$ such that $F_C = F_D \tilde{\cap} F_B$. Moreover, since $F_B, F_D \in \tilde{\tau}$, then $F_D \tilde{\cap} F_B \in \tilde{\tau}$. Therefore, $F_C \in \tilde{\tau}$.

ii. Let $F_B \in \tilde{\tau}^k$ and $F_C \in {\tilde{\tau}_{F_B}}^{k}$. Then, from Proposition 2.30 (i), there exists at least one $F_D \in \tilde{\tau}^k$ such that $F_C = F_D \tilde{\cap} F_B$. Moreover, since $F_B, F_D \in \tilde{\tau}^k$, then $F_D \tilde{\cap} F_B \in \tilde{\tau}^k$. Therefore, $F_C \in \tilde{\tau}^k$.

Henceforth, ${F_B}^{\circ\tilde{\tau}_1}$ and $\overline{F_B}^{\tilde{\tau}_1}$indicate $\tilde{\tau}_1$-soft interior and $\tilde{\tau}_1$-soft closure of F_B, respectively.

Theorem 3.5. Let $(F_A, \tilde{\tau}_1)$ and $(F_A, \tilde{\tau}_2)$ be two soft topological spaces and $F_B \tilde{\subseteq} F_A$. Then,

i. $\tilde{\tau}_1 \subseteq \tilde{\tau}_2 \Leftrightarrow {F_B}^{\circ\tilde{\tau}_1} \tilde{\subseteq} {F_B}^{\circ\tilde{\tau}_2}$

ii. $\tilde{\tau}_1 \subseteq \tilde{\tau}_2 \Leftrightarrow \overline{F_B}^{\tilde{\tau}_2} \tilde{\subseteq} \overline{F_B}^{\tilde{\tau}_1}$

Proof. Let $(F_A, \tilde{\tau}_1)$ and $(F_A, \tilde{\tau}_2)$ be two soft topological spaces and $F_B \tilde{\subseteq} F_A$.

i. $(\Rightarrow)$: Let $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ and $\varepsilon \tilde{\in} {F_B}^{\circ\tilde{\tau}_1}$. Then, there exists at least one $F_C \in \tilde{\tau}_1$ such that $\varepsilon \tilde{\in} F_C$ and $F_C \tilde{\subseteq} F_B$. Since $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$, then $F_C \in \tilde{\tau}_2$. That is, there exists at least one $F_C \in \tilde{\tau}_2$ such that $\varepsilon \tilde{\in} F_C$ and $F_C \tilde{\subseteq} F_B$. In other words, $\varepsilon \tilde{\in} {F_B}^{\circ\tilde{\tau}_1} \Rightarrow \varepsilon \tilde{\in} {F_B}^{\circ\tilde{\tau}_2}$. Hence, ${F_B}^{\circ\tilde{\tau}_1} \tilde{\subseteq} {F_B}^{\circ\tilde{\tau}_2}$.

$(\Leftarrow)$: Let ${F_B}^{\circ\tilde{\tau}_1} \tilde{\subseteq} {F_B}^{\circ\tilde{\tau}_2}$, for all $F_B \tilde{\subseteq} F_A$. Then, for all $F_C \in \tilde{\tau}_1$, the hypothesis is valid. Since $F_C = {F_C}^{\circ\tilde{\tau}_1}$, then $F_C \tilde{\subseteq} {F_C}^{\circ\tilde{\tau}_2}$. Besides, ${F_C}^{\circ\tilde{\tau}_2} \tilde{\subseteq} F_C$. That is, ${F_C}^{\circ\tilde{\tau}_2} = F_C$. Therefore, $F_C \in \tilde{\tau}_1 \Rightarrow F_C \in \tilde{\tau}_2$. Hence, $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$.

ii. $(\Rightarrow)$: Let $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ and $\varepsilon \tilde{\in} \overline{F_B}^{\tilde{\tau}_2}$. Then, from Theorem 2.29, for all $F_C \in \tilde{\tau}_2$ such that $\varepsilon \tilde{\in} F_C$, $F_B \tilde{\cap} F_C \neq F_{\Phi}$. Since $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$, then for all $F_C \in \tilde{\tau}_1$ such that $\varepsilon \tilde{\in} F_C$, $F_B \tilde{\cap} F_C \neq F_{\Phi}$. Therefore, $\varepsilon \tilde{\in} \overline{F_B}^{\tilde{\tau}_1}$. Hence, $\overline{F_B}^{\tilde{\tau}_2} \tilde{\subseteq} \overline{F_B}^{\tilde{\tau}_1}$.

$(\Leftarrow)$: Let $\overline{F_B}^{\tilde{\tau}_2} \tilde{\subseteq} \overline{F_B}^{\tilde{\tau}_1}$, for all $F_B \tilde{\subseteq} F_A$. Then, for all $F_A \tilde{\setminus} F_C \in \tilde{\tau}_1$, the hypothesis is valid. Since $F_C = \overline{F_C}^{\tilde{\tau}_1}$, then $\overline{F_C}^{\tilde{\tau}_2} \tilde{\subseteq} F_C$. Besides, $F_C \tilde{\subseteq} \overline{F_C}^{\tilde{\tau}_2}$. That is, $\overline{F_C}^{\tilde{\tau}_2} = F_C$. Therefore, $F_A \tilde{\setminus} F_C \in \tilde{\tau}_2$. Hence, $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$.

Theorem 3.6. Let $\left(F_B, \tilde{\tau}_{F_B}\right)$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B$. Then,

i. ${F_C}^{\circ} \tilde{\subseteq} {F_C}^{\circ\tilde{\tau}_{F_B}}$

ii. $\overline{F_C}^{\tilde{\tau}_{F_B}} \tilde{\subseteq} \overline{F_C}$

Proof. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B$.

i. Let $\varepsilon \tilde{\in} F_C{}^{\circ}$. Then, there exists at least one $F_D \in \tilde{\tau}$ such that $\varepsilon \tilde{\in} F_D$ and $F_D \tilde{\subseteq} F_C$. Therefore, from Proposition 2.30 (ii), there exists at least one $F_D \in \tilde{\tau}_{F_B}$ such that $\varepsilon \tilde{\in} F_D$ and $F_D \tilde{\subseteq} F_C$. Therefore, $\varepsilon \tilde{\in} F_C{}^{\circ\tilde{\tau}_{F_B}}$. Hence, $F_C{}^{\circ} \tilde{\subseteq} F_C{}^{\circ\tilde{\tau}_{F_B}}$.

ii. Let $\varepsilon \tilde{\in} \overline{F_C}^{\tilde{\tau}_{F_B}}$. Then, from Theorem 2.29, for all $F_D \in \tilde{\tau}_{F_B}$ such that $\varepsilon \tilde{\in} F_D$, $F_C \tilde{\cap} F_D \neq F_\Phi$. Besides, for all $F_D \in \tilde{\tau}_{F_B}$, there exists at least one $F_K \in \tilde{\tau}$ such that $F_D = F_K \tilde{\cap} F_B$. Since $F_K \tilde{\cap} F_B \tilde{\subseteq} F_K$ and $F_K \in \tilde{\tau}$, then for all $F_K \in \tilde{\tau}$ such that $\varepsilon \tilde{\in} F_K$, $F_C \tilde{\cap} F_K \neq F_\Phi$. Therefore, $\varepsilon \tilde{\in} \overline{F_C}$. Hence, $\overline{F_C}^{\tilde{\tau}_{F_B}} \tilde{\subseteq} \overline{F_C}$.

Theorem 3.7. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B$. Then,

i. If $F_B \in \tilde{\tau}$, then $F_C{}^{\circ\tilde{\tau}_{F_B}} \tilde{\subseteq} F_C{}^{\circ}$.

ii. If $F_B \in \tilde{\tau}$, then $\overline{F_C} \tilde{\subseteq} \overline{F_C}^{\tilde{\tau}_{F_B}}$.

Proof. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B$.

i. Let $F_B \in \tilde{\tau}$ and $\varepsilon \tilde{\in} F_C{}^{\circ\tilde{\tau}_{F_B}}$. Then, there exists at least one $F_D \in \tilde{\tau}_{F_B}$ such that $\varepsilon \tilde{\in} F_D$ and $F_D \tilde{\subseteq} F_C$. Therefore, from Theorem 3.4 (i), there exists at least one $F_D \in \tilde{\tau}$ such that $\varepsilon \tilde{\in} F_D$ and $F_D \tilde{\subseteq} F_C$. Thus, $\varepsilon \tilde{\in} F_C{}^{\circ}$. Hence, $F_C{}^{\circ\tilde{\tau}_{F_B}} \tilde{\subseteq} F_C{}^{\circ}$.

ii. Let $F_B \in \tilde{\tau}$ and $\varepsilon \tilde{\in} \overline{F_C}$. Then, from Theorem 2.29, for all $F_D \in \tilde{\tau}$ such that $\varepsilon \tilde{\in} F_D$, $F_C \tilde{\cap} F_D \neq F_\Phi$. Moreover, for all $F_D \tilde{\cap} F_B \in \tilde{\tau}$ such that $\varepsilon \tilde{\in} F_D \tilde{\cap} F_B$, $F_C \tilde{\cap} (F_D \tilde{\cap} F_B) \neq F_\Phi$. Therefore, for all $F_D \tilde{\cap} F_B \in \tilde{\tau}_{F_B}$ such that $\varepsilon \tilde{\in} F_D \tilde{\cap} F_B$, $F_C \tilde{\cap} (F_D \tilde{\cap} F_B) \neq F_\Phi$. Thus, $\varepsilon \tilde{\in} \overline{F_C}^{\tilde{\tau}_{F_B}}$. Hence, $\overline{F_C} \tilde{\subseteq} \overline{F_C}^{\tilde{\tau}_{F_B}}$.

Corollary 3.8. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$, $F_B \in \tilde{\tau}$, and $F_C \tilde{\subseteq} F_B$. Then,

i. $F_C{}^{\circ} = F_C{}^{\circ\tilde{\tau}_{F_B}}$

ii. $\overline{F_C} = \overline{F_C}^{\tilde{\tau}_{F_B}}$

Corollary 3.9. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B \tilde{\subseteq} F_A$. Then,

i. $F_C{}^{\circ} \tilde{\cap} F_B \tilde{\subseteq} F_C{}^{\circ\tilde{\tau}_{F_B}}$

ii. $\overline{F_C}^{\tilde{\tau}_{F_B}} = \overline{F_C} \tilde{\cap} F_B$

Proof. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B \tilde{\subseteq} F_A$. Then,

i. From Theorem 3.6 (i), $F_C{}^{\circ} \tilde{\cap} F_B \tilde{\subseteq} F_C{}^{\circ} \tilde{\subseteq} F_C{}^{\circ\tilde{\tau}_{F_B}}$.

ii. From Theorem 3.6 (ii), $\overline{F_C}^{\tilde{\tau}_{F_B}} \tilde{\subseteq} \overline{F_C} \tilde{\cap} F_B$. On the other hand, let $\varepsilon \tilde{\in} \overline{F_C} \tilde{\cap} F_B$. Then, $\varepsilon \tilde{\in} \overline{F_C}$ and $\varepsilon \tilde{\in} F_B$. That is, for all $F_K \in \tilde{\tau}$ such that $\varepsilon \tilde{\in} F_K$, $F_C \tilde{\cap} F_K \neq F_\Phi$ and $\varepsilon \tilde{\in} F_B$. Therefore, for all $F_K \tilde{\cap} F_B \in \tilde{\tau}_{F_B}$ such that $\varepsilon \tilde{\in} F_K \tilde{\cap} F_B$, $F_C \tilde{\cap} (F_K \tilde{\cap} F_B) \neq F_\Phi$ and so $\varepsilon \tilde{\in} \overline{F_C}^{\tilde{\tau}_{F_B}}$. Hence, $\overline{F_C} \tilde{\cap} F_B \tilde{\subseteq} \overline{F_C}^{\tilde{\tau}_{F_B}}$. Consequently, $\overline{F_C}^{\tilde{\tau}_{F_B}} = \overline{F_C} \tilde{\cap} F_B$.

Theorem 3.10. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_B, F_C \tilde{\subseteq} F_A$. Then, $\overline{F_B \tilde{\cap} F_C}^{\tilde{\tau}_{F_B}} \tilde{\subseteq} F_B \tilde{\cap} \overline{F_C}$.

Proof. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_B, F_C \tilde{\subseteq} F_A$. We have $F_B \tilde{\cap} F_C \tilde{\subseteq} F_B \tilde{\subseteq} F_A$. Because of Corollary 3.9 (ii) and $F_B \tilde{\subseteq} \overline{F_B}$,

$$\overline{F_B \tilde{\cap} F_C}^{\tilde{\tau}_{F_B}} = \overline{F_B \tilde{\cap} F_C} \tilde{\cap} F_B \tilde{\subseteq} \overline{F_B} \tilde{\cap} \overline{F_C} \tilde{\cap} F_B = (\overline{F_B} \tilde{\cap} F_B) \tilde{\cap} \overline{F_C} = F_B \tilde{\cap} \overline{F_C}$$

Hence, $\overline{F_B \tilde{\cap} F_C}^{\tilde{\tau}_{F_B}} \tilde{\subseteq} F_B \tilde{\cap} \overline{F_C}$.

4. Soft α-open Sets and Soft α-T_0 Spaces

In this section, we introduce some properties of soft α-open sets and soft α-closed sets. Moreover, we define soft α-T_0 space and study some of its basic properties.

Theorem 4.1. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. If $F_B \in \tilde{\tau}$ and $F_C \in S\alpha O(\tilde{\tau})$, then $F_B \tilde{\cap} F_C \in S\alpha O(\tilde{\tau})$.

Proof. Let $(F_A, \tilde{\tau})$ be a soft topological space, $F_B, F_C \tilde{\subseteq} F_A$, $F_B \in \tilde{\tau}$, and $F_C \in S\alpha O(\tilde{\tau})$. Since

$$
\begin{aligned}
F_B \in \tilde{\tau} \wedge F_C \in S\alpha O(\tilde{\tau}) &\Rightarrow F_B = F_B^{\circ} \wedge F_C \tilde{\subseteq} \left(\overline{F_C^{\circ}}\right)^{\circ} \\
&\Rightarrow F_B \tilde{\cap} F_C \tilde{\subseteq} F_B^{\circ} \tilde{\cap} \left(\overline{F_C^{\circ}}\right)^{\circ} \\
&\Rightarrow F_B \tilde{\cap} F_C \tilde{\subseteq} \left(F_B \tilde{\cap} \overline{F_C^{\circ}}\right)^{\circ}, \text{ from Proposition 2.28 (v)} \\
&\Rightarrow F_B \tilde{\cap} F_C \tilde{\subseteq} \left(\overline{F_B \tilde{\cap} F_C^{\circ}}\right)^{\circ}, \text{ from Proposition 2.31} \\
&\Rightarrow F_B \tilde{\cap} F_C \tilde{\subseteq} \left(\overline{F_B^{\circ} \tilde{\cap} F_C^{\circ}}\right)^{\circ} \\
&\Rightarrow F_B \tilde{\cap} F_C \tilde{\subseteq} \left(\overline{(F_B \tilde{\cap} F_C)^{\circ}}\right)^{\circ}
\end{aligned}
$$

then $F_B \tilde{\cap} F_C \in S\alpha O(\tilde{\tau})$.

Theorem 4.2. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B \tilde{\subseteq} F_A$. If $F_C \in S\alpha O(\tilde{\tau})$, then $F_C \in S\alpha O(\tilde{\tau}_{F_B})$.

Proof. Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \tilde{\subseteq} F_B \tilde{\subseteq} F_A$. Then,

$$
\begin{aligned}
F_C \in S\alpha O(\tilde{\tau}) &\Rightarrow \exists F_D \in \tilde{\tau} \ni F_D \tilde{\subseteq} F_C \tilde{\subseteq} (\overline{F_D})^{\circ}, \text{ from Theorem 2.37} \\
&\Rightarrow \exists F_D \tilde{\cap} F_B \in \tilde{\tau}_{F_B} \ni F_D \tilde{\cap} F_B \tilde{\subseteq} F_C \tilde{\cap} F_B \tilde{\subseteq} (\overline{F_D})^{\circ} \tilde{\cap} F_B \\
&\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \ni F_D \tilde{\subseteq} F_C \tilde{\subseteq} (\overline{F_D})^{\circ} \tilde{\cap} F_B \\
&\quad (\text{Since } F_D \tilde{\subseteq} F_C \tilde{\subseteq} F_B, \text{ then } F_D \tilde{\cap} F_B = F_D \text{ and } F_C \tilde{\cap} F_B = F_C.) \\
&\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \ni F_D \tilde{\subseteq} F_C \tilde{\subseteq} (\overline{F_D})^{\circ \tilde{\tau}_{F_B}} \tilde{\cap} F_B, \text{ from Theorem 3.6 (i)} \\
&\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \ni F_D \tilde{\subseteq} F_C \tilde{\subseteq} \overline{F_D}^{\circ \tilde{\tau}_{F_B}} \tilde{\cap} F_B^{\circ \tilde{\tau}_{F_B}} \\
&\quad (\text{Since } F_B \in \tilde{\tau}_{F_B}, \text{ then } F_B = F_B^{\circ \tilde{\tau}_{F_B}}.) \\
&\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \ni F_D \tilde{\subseteq} F_C \tilde{\subseteq} (\overline{F_D} \tilde{\cap} F_B)^{\circ \tilde{\tau}_{F_B}}, \text{ from Proposition 2.28 (v)} \\
&\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \ni F_D \tilde{\subseteq} F_C \tilde{\subseteq} \left(\overline{F_D}^{\tilde{\tau}_{F_B}}\right)^{\circ \tilde{\tau}_{F_B}}, \text{ from Corollary 3.9 (ii)} \\
&\quad (\text{Since } F_D \tilde{\subseteq} F_C \tilde{\subseteq} F_B \tilde{\subseteq} F_A, \text{ then } \overline{F_D}^{\tilde{\tau}_{F_B}} = \overline{F_D} \tilde{\cap} F_B) \\
&\Rightarrow F_C \in S\alpha O(\tilde{\tau}_{F_B})
\end{aligned}
$$

The converse of the Theorem 4.2 is not always correct. In other words, a soft α-open set in a soft subspace of a soft topological space is may not soft α-open set in a soft topological space. This situation is shown in the following example.

Example 4.3. For $(F_A, \tilde{\tau})$ and $(F_{A_{12}}, \tilde{\tau}_{F_{A_{12}}})$ provided in Example 2.19 and Example 2.22, $S\alpha O(\tilde{\tau}) = \{F_\Phi, F_{A_1}, F_{A_3}, F_{A_7}, F_{A_8}, F_{A_9}, F_{A_{13}}, F_{A_{14}}, F_A\}$ and $S\alpha O\left(\tilde{\tau}_{F_{A_{12}}}\right) = \{F_\Phi, F_{A_4}, F_{A_{11}}, F_{A_{12}}\}$. Hence, $F_{A_4} \in S\alpha O\left(\tilde{\tau}_{F_{A_{12}}}\right)$ but $F_{A_4} \notin S\alpha O(\tilde{\tau})$.

Theorem 4.4. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, $F_B \in S\alpha C(\tilde{\tau})$ if and only if there exists at least one $F_C \in \tilde{\tau}^k$ such that $\overline{F_C}^{\circ} \tilde{\subseteq} F_B \tilde{\subseteq} F_C$.

Proof. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$.

($\Rightarrow$): Let $F_B \in S\alpha C(\tilde{\tau})$. From Theorem 2.36, $\overline{(\overline{F_B})^{\circ}} \tilde{\subseteq} F_B$. Since $F_B \tilde{\subseteq} \overline{F_B}$ and $\overline{F_B} \in \tilde{\tau}^k$, then $\overline{(\overline{F_B})^{\circ}} \tilde{\subseteq} F_B \tilde{\subseteq} \overline{F_B}$.

($\Leftarrow$): Let there exists at least one $F_C \in \tilde{\tau}^k$ such that $\overline{F_C}^{\circ} \tilde{\subseteq} F_B \tilde{\subseteq} F_C$. Since $F_C \in \tilde{\tau}^k$, then $\overline{F_C} = F_C$. Thus, $\overline{(\overline{F_C})^{\circ}} \tilde{\subseteq} F_B \tilde{\subseteq} \overline{F_C}$. Since $\overline{(\overline{F_C})^{\circ}} \tilde{\subseteq} F_B$, then $F_B \in S\alpha C(\tilde{\tau})$.

Definition 4.5. Let $(F_A, \tilde{\tau})$ be a soft topological space, $F_B \tilde{\subseteq} F_A$, and $\varepsilon \tilde{\in} F_B$. If there exists at least one $F_C \in S\alpha O(\tilde{\tau})$ such that $\varepsilon \tilde{\in} F_C$ and $F_C \tilde{\subseteq} F_B$, then ε is called a $\tilde{\tau}$-soft α-interior point (or briefly soft α-interior point) of F_B.

Theorem 4.6. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then, $F_A \tilde{\setminus} \overline{(F_B)}_{\alpha} = \left(F_A \tilde{\setminus} F_B\right)^{\circ}_{\alpha}$.

Proof. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then,

$$F_A \tilde{\setminus} \overline{(F_B)}_{\alpha} = F_A \tilde{\setminus} \left(\tilde{\bigcap}_{\substack{F_B \tilde{\subseteq} F_{A_i} \\ F_A \tilde{\setminus} F_{A_i} \in S\alpha O(\tilde{\tau})}} F_{A_i} \right) = \tilde{\bigcup}_{\substack{F_A \tilde{\setminus} F_{A_i} \tilde{\subseteq} F_A \tilde{\setminus} F_B \\ F_A \tilde{\setminus} F_{A_i} \in S\alpha O(\tilde{\tau})}} \left(F_A \tilde{\setminus} F_{A_i}\right) = \left(F_A \tilde{\setminus} F_B\right)^{\circ}_{\alpha}$$

Theorem 4.7. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then, $\varepsilon \tilde{\in} \overline{(F_B)}_{\alpha}$ iff, for all $F_C \in S\alpha O(\tilde{\tau})$ such that $\varepsilon \tilde{\in} F_C$, $F_B \tilde{\cap} F_C \neq F_{\Phi}$.

Proof: Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$.

($\Rightarrow$): Let $\varepsilon \tilde{\notin} \overline{(F_B)}_{\alpha}$. Then, $\varepsilon \tilde{\in} F_A \tilde{\setminus} \overline{(F_B)}_{\alpha}$. From Theorem 4.6, $\varepsilon \tilde{\in} \left(F_A \tilde{\setminus} F_B\right)^{\circ}_{\alpha}$. Thus, there exists at least one $F_C \in S\alpha O(\tilde{\tau})$ such that $\varepsilon \tilde{\in} F_C \tilde{\subseteq} F_A \tilde{\setminus} F_B$. From Proposition 2.10 (iii), there exists at least one $F_C \in S\alpha O(\tilde{\tau})$ such that $\varepsilon \tilde{\in} F_C$ and $F_B \tilde{\cap} F_C = F_{\Phi}$.

($\Leftarrow$): Let there exists at least one $F_C \in S\alpha O(\tilde{\tau})$ such that $\varepsilon \tilde{\in} F_C$, $F_B \tilde{\cap} F_C = F_{\Phi}$. From Proposition 2.10 (iii), there exists $F_A \tilde{\setminus} F_C \in S\alpha C(\tilde{\tau})$ such that $F_B \tilde{\subseteq} F_A \tilde{\setminus} F_C$. In this case, from Proposition 2.35, $\overline{(F_B)}_{\alpha} \tilde{\subseteq} \overline{\left(F_A \tilde{\setminus} F_C\right)}_{\alpha} = F_A \tilde{\setminus} F_C$. Hence, $\varepsilon \tilde{\notin} \overline{(F_B)}_{\alpha}$.

Definition 4.8. Let $(F_A, \tilde{\tau})$ be a soft topological space. For all $\varepsilon_1, \varepsilon_2 \tilde{\in} F_A$ such that $\varepsilon_1 \neq \varepsilon_2$, if there exists at least one $F_B, F_C \in S\alpha O(\tilde{\tau})$ such that $\left(\varepsilon_1 \tilde{\in} F_B \wedge \varepsilon_2 \tilde{\notin} F_B\right)$ or $\left(\varepsilon_1 \tilde{\notin} F_C \wedge \varepsilon_2 \tilde{\in} F_C\right)$, then $(F_A, \tilde{\tau})$ is called a soft α-T_0 space.

Example 4.9. Let us consider Example 4.3. Since $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$ and $F_{A_1}, F_{A_3}, F_{A_8} \in S\alpha O(\tilde{\tau})$, then

$\{(x_1, \{u_1\})\} \tilde{\in} F_{A_1}$ and $\{(x_1, \{u_2\})\} \tilde{\notin} F_{A_1}$ $\quad$ $\{(x_1, \{u_1\})\} \tilde{\in} F_{A_1}$ and $\{(x_2, \{u_2\})\} \tilde{\notin} F_{A_1}$

$\{(x_1, \{u_1\})\} \tilde{\in} F_{A_1}$ and $\{(x_2, \{u_3\})\} \tilde{\notin} F_{A_1}$ $\quad$ $\{(x_1, \{u_2\})\} \tilde{\in} F_{A_3}$ and $\{(x_2, \{u_2\})\} \tilde{\notin} F_{A_3}$

$\{(x_1, \{u_2\})\} \tilde{\in} F_{A_3}$ and $\{(x_2, \{u_3\})\} \tilde{\notin} F_{A_3}$ $\quad$ $\{(x_2, \{u_2\})\} \tilde{\notin} F_{A_8}$ and $\{(x_2, \{u_3\})\} \tilde{\in} F_{A_8}$

Therefore, $(F_A, \tilde{\tau})$ is a soft α-T_0 space. On the other hand, for $\{(x_1, \{u_2\})\}, \{(x_2, \{u_3\})\} \tilde{\in} F_{A_{12}}$, there do not exist $F_B, F_C \in S\alpha O(\tilde{\tau})$ such that $\left(\{(x_1, \{u_2\})\} \tilde{\in} F_B \wedge \{(x_2, \{u_3\})\} \tilde{\notin} F_B\right)$ or $\left(\{(x_1, \{u_2\})\} \tilde{\notin} F_C \wedge \{(x_2, \{u_3\})\} \tilde{\in} F_C\right)$. Hence, $\left(F_{A_{12}}, \tilde{\tau}_{F_{A_{12}}}\right)$ is not a soft α-T_0 space.

Corollary 4.10. Every soft subspace of a α-T_0 space is not always a soft α-T_0 space. Therefore, being soft α-T_0 space is not a hereditary property.

Theorem 4.11. Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, $(F_A, \tilde{\tau})$ is a soft α-T_0 space if and only if for all $\varepsilon_1, \varepsilon_2 \tilde{\in} F_A$ such that $\varepsilon_1 \neq \varepsilon_2$, $\overline{(\varepsilon_1)}_\alpha \neq \overline{(\varepsilon_2)}_\alpha$.

Proof.

($\Rightarrow$): Let $(F_A, \tilde{\tau})$ be a soft α-T_0 space. Then, for all $\varepsilon_1, \varepsilon_2 \tilde{\in} F_A$ such that $\varepsilon_1 \neq \varepsilon_2$, there exists at least one $F_B, F_C \in S\alpha O(\tilde{\tau})$ such that $(\varepsilon_1 \tilde{\in} F_B \wedge \varepsilon_2 \tilde{\notin} F_B)$ or $(\varepsilon_1 \tilde{\notin} F_C \wedge \varepsilon_2 \tilde{\in} F_C)$. Let $\varepsilon_1 \tilde{\in} F_B$ and $\varepsilon_2 \tilde{\notin} F_B$. Since $\varepsilon_2 \tilde{\notin} F_B$, then $\varepsilon_2 \tilde{\cap} F_B = F_\Phi$. Thus, there exists at least one $F_B \in S\alpha O(\tilde{\tau})$ such that $\varepsilon_1 \tilde{\in} F_B$ and $\varepsilon_2 \tilde{\cap} F_B = F_\Phi$. Because of Theorem 4.7, $\varepsilon_1 \tilde{\notin} \overline{(\varepsilon_2)}_\alpha$. Moreover, since $\varepsilon_1 \tilde{\in} \overline{(\varepsilon_1)}_\alpha$, then for all $\varepsilon_1, \varepsilon_2 \tilde{\in} F_A$ such that $\varepsilon_1 \neq \varepsilon_2$, $\overline{(\varepsilon_1)}_\alpha \neq \overline{(\varepsilon_2)}_\alpha$.

($\Leftarrow$): Let for all $\varepsilon_1, \varepsilon_2 \tilde{\in} F_A$ such that $\varepsilon_1 \neq \varepsilon_2$, $\overline{(\varepsilon_1)}_\alpha \neq \overline{(\varepsilon_2)}_\alpha$. Since $\varepsilon_1 \tilde{\in} \overline{(\varepsilon_1)}_\alpha$ and $\overline{(\varepsilon_1)}_\alpha \neq \overline{(\varepsilon_2)}_\alpha$, then $\varepsilon_1 \tilde{\notin} \overline{(\varepsilon_2)}_\alpha$. Because of Theorem 4.7, there exists at least one $F_B \in S\alpha O(\tilde{\tau})$ such that $\varepsilon_1 \tilde{\in} F_B$ and $\varepsilon_2 \tilde{\cap} F_B = F_\Phi$. Thus, there exists at least one $F_B \in S\alpha O(\tilde{\tau})$ such that $\varepsilon_1 \tilde{\in} F_B$ and $\varepsilon_2 \tilde{\notin} F_B$. Therefore, $(F_A, \tilde{\tau})$ is a soft α-T_0 space.

Example 4.12. For $\left(F_{A_7}, \tilde{\tau}_{F_{A_7}}\right)$ provided in Example 2.22, $S\alpha O\left(\tilde{\tau}_{F_{A_7}}\right) = \{F_\Phi, F_{A_1}, F_{A_7}\}$ and $S\alpha C\left(\tilde{\tau}_{F_{A_7}}\right) = \{F_\Phi, F_{A_4}, F_{A_7}\}$. For $\{(x_1, \{u_1\})\}, \{(x_2, \{u_2\})\} \tilde{\in} F_{A_7}$, since $\overline{\{(x_1, \{u_1\})\}}_\alpha = F_{A_7}$ and $\overline{\{(x_2, \{u_2\})\}}_\alpha = F_{A_4}$, then $\overline{\{(x_1, \{u_1\})\}}_\alpha \neq \overline{\{(x_2, \{u_2\})\}}_\alpha$. Therefore, $\left(F_{A_7}, \tilde{\tau}_{F_{A_7}}\right)$ is a soft α-T_0 subspace of $(F_A, \tilde{\tau})$.

5. Conclusion

This paper studied relationships between the soft interior and soft closure of a soft set in soft topological spaces and their soft subspaces through the concept of the soft element. Thus, the validity of many propositions on various generalizations of soft open sets can be proved easier. We then provided a few theorems concerning soft α-open sets and soft α-closed sets. Moreover, we defined soft α-T_0 space and investigated its basic properties. We showed that every soft subspace of a soft α-T_0 space is not always a soft α-T_0 space.

In the future, researchers can study that every soft subspace of a soft α-T_0 space is a soft α-T_0 space under which condition or conditions. Moreover, they can define through the concept of the soft element the other soft α-separation axioms, i.e., soft α-T_1 space, soft α-T_2 space (soft α-Hausdorff space), soft α-regular space, and soft α-normal space.

Author Contributions

All authors contributed equally to this work. They all read and approved the last version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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