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A Study On The Hyper Darboux Lines In A Generalised Weyl Space

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In the previous works, some properties of the Hyper Darboux lines which are the generalized form of Darboux lines defined in 3- dimensional Euclid space were defined in Riemannian and Kaehlerian space [6], [7]. In this work, some properties of a Hyper Darboux line of various orders in GW_n hyperspace of GW_{n+1} space are obtained. In addition to this, a Hyper Darboux line of order zero in GW_n hyperspace of GW_{n+1} space is considered and the equations involving the second fundamental tensor of the subspace are deduced.

1. INTRODUCTION

An n- dimensional manifold GW_n is said to be a generalized Weyl space if it has an asymmetric conformal metric tensor g_{ij} and asymmetric connection ∇_k satisfying the compatibility condition given by the equation

$$\nabla_k g_{ij} = 2T_k g_{ij} \tag{1.1}$$

where T_k denotes a covariant vector field and ∇_k denotes the usual covariant derivative.

Under a renormalization of the fundamental tensor of the form $\breve{g}_{ij} = \lambda^2 g_{ij}$ the covariant vector T_k is transformed by the law $\breve{T}_k = T_k + \partial_k \ln \lambda$, where λ is a scalar function defined on GW_n.

Let L^i_{jk} denote the coefficients of the asymmetric connection ∇_k. So, a generalized Weyl space is shortly written as $GW_n(L^i_{jk}, g_{ij}, T_k)$.

The main properties of $GW_n(L^i_{jk}, g_{ij}, T_k)$ can be expressed as follows

$$g_{ij} = g_{(ij)} + g_{[ij]} \tag{1.2}$$

$$\nabla_k\, g_{(ij)} = 2g_{(ij)}T_k \tag{1.3}$$

$$\nabla_k\, g_{[ij]} = 2g_{[ij]}T_k \tag{1.4}$$

$$g_{(ik)}g^{(kl)} = \delta^l_i \tag{1.5}$$

$$\nabla_k\, g^{(ij)} = -2T_k g^{(ij)} \tag{1.6}$$

where $g_{(ij)}$ and $g_{[ij]}$ denote symmetric and antisymmetric part of g_{ij}, respectively.

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The symmetric part of connection coefficients L^i_{jk} are given as ([1], [2], [3])

$$L^i_{jk} = W^i_{jk} = \begin{bmatrix} i \\ jk \end{bmatrix} - (\delta^i_j T_k + \delta^i_k T_j - g_{jk} g^{mi} T_m) \tag{1.7}$$

where $\begin{bmatrix} i \\ jk \end{bmatrix}$ are second kind Christoffel symbols defined by

$$\begin{bmatrix} i \\ jk \end{bmatrix} = \frac{1}{2} g^{(ir)} \left[\frac{\partial g_{(jr)}}{\partial x^k} + \frac{\partial g_{(kr)}}{\partial x^j} - \frac{\partial g_{(jk)}}{\partial x^r} \right]. \tag{1.8}$$

A quantity A is called a satellite of weight $\{p\}$ of the tensor g_{ij}, if it admits a transformation of the form

$$\breve{A} = \lambda^p A. \tag{1.9}$$

The prolonged covariant derivative of a satellite A of the tensor g_{ij} of weight $\{p\}$ is defined by

$$\dot{\nabla}_k A = \nabla_k A - p T_k A. \tag{1.10}$$

Let $C : x^i = x^i(s)$ be curve in GW_n. The generalized covariant derivative along the curve C of the tensor field T, is defined by

$$\frac{\overset{\sqcap}{\delta} T}{\delta s} = \xi^k_{(1)} \overset{\sqcap}{\nabla}_k T \tag{1.11}$$

where $\xi^k_{(1)}$ are the components of the tangent vector of the curve C.

The Frenet equations of C may be written as [4].

$$\frac{\dot{\delta} \xi^i_{(\alpha)}}{\delta s} = \kappa_{(\alpha)} \xi^i_{(\alpha+1)} - \kappa_{(\alpha-1)} \xi^i_{(\alpha-1)} \quad (\alpha = 1, 2, ..., n; \kappa_{(0)} = \kappa_{(n)} = 0) \tag{1.12}$$

In the above equation $\xi^i_{(\alpha)} (\alpha = 2, ..., n)$ denote the $\alpha -$th curvature of weight $\{-1\}$ normalized by the condition

$$g_{ij} \xi^i_{(\alpha)} \xi^j_{(\alpha)} = 1 \tag{1.13}$$

of the curve C and $\kappa_{(\alpha)} (\alpha = 1, ..., n-1)$ denote the $\alpha -$th curvature of weight $\{-1\}$ of the curve C.

Let an n-dimentional hypersurface GW_n given by the equations $y^\alpha = y^\alpha(x^i)$ $(\alpha = 1, ..., n+1; i = 1, 2, ..., n)$ be immersed in a generalized Weyl space GW_{n+1}.

The prolonged covariant derivative of the satellite A, relative to GW_{n+1} and GW_n are related by

$$\dot{\nabla}_k A = x^\gamma_k \dot{\nabla}_\gamma A \tag{1.14}$$

where $x^\gamma_k = \dfrac{\partial y^\gamma}{\partial x^k}$.

The components of any vector U relative to GW_{n+1} and GW_n are related by

$$U^{\alpha} = x_i^{\alpha} U^i . \tag{1.15}$$

The prolonged covariant derivative x_i^{α} is given by

$$\dot{\nabla}_j x_i^{\alpha} = w_{ij} N^{\alpha} + A_{ij}^h x_h^{\alpha} \tag{1.16}$$

where w_{ij} are the components of second fundamental form of GW_n defined by

$$w_{ij} = g_{(\alpha\beta)} N^{\beta} \dot{\nabla}_j x_i^{\alpha} \tag{1.17}$$

and A_{ij}^h are defined by

$$A_{ij}^h = g_{(\alpha\beta)} \dot{\nabla}_j x_i^{\alpha} x_l^{\beta} g^{(hl)} . \tag{1.18}$$

The components q^{α} and p^i of the first curvature vectors of the curve $C : x^i = x^i(s)$ with respect to GW_{n+1} and GW_n are given by [5]

$$q^{\alpha} = \frac{\dot{\delta} \xi_{(1)}^{\alpha}}{\delta s} = \kappa_{(n)} N^{\alpha} + p^i x_i^{\alpha} + I^h x_h^{\alpha} \tag{1.19}$$

where

$$\kappa_{(n)} = w_{ij} \frac{dx^i}{ds} \frac{dx^j}{ds} \tag{1.20}$$

and I^h are the components of intrinsic curvature vector of the curve C in the hypersurface defined by [5]

$$I^h = A_{ij}^h \frac{dx^i}{ds} \frac{dx^j}{ds} . \tag{1.21}$$

The prolonged covariant derivative of the unit normal is given by

$$\dot{\nabla}_i N^{\alpha} = -g^{(jk)} w_{ij} B_k^{\alpha} \tag{1.22}$$

2.HYPER DARBOUX LINES OF ORDER H

Let us take an n-dimensional GW_n defined by the equations $y^{\alpha} = y^{\alpha}(x^i)$ $(\alpha = 1, ..., n+1; i = 1, ..., n)$ which is immersed in a generalized Weyl Space GW_{n+1}.

Let us take $C : x^i = x^i(s)$ which is a curve (not a geodesic of the enveloping space) of the hypersurface .

The components $\xi_{(0)}^{\alpha}, \xi_{(1)}^{\alpha}$ and $\xi_{(r)}^{\alpha}$ are of the unit tangent vectors, of the principal normal vector and of the $(r-1)-$th binormal vectors at every point of the curve, respectively. These vectors define an orthogonal system unit vectors at every point of the curve. We consider a congruence of curves given by unit vector field λ in GW_{n+1} as

$$\lambda^{\alpha} = q^i x_i^{\alpha} + r N^{\alpha} \tag{2.1}$$

where N^a are the components of the unit normal vector of the hyper space.

Definition 2.1: If the surface spanned by the vectors $\xi^{\alpha}_{(0)}$ and $R_{(h+1)}\xi^{\alpha}_{(h+1)} + R_{(h+2)}\frac{\dot{\delta} R_{(h+1)}}{\delta s}\xi^{\alpha}_{(h+2)}$ ($R_{(\alpha)} \equiv \frac{1}{\kappa_{(\alpha)}}$ and $\kappa_{(\alpha)}$ is the curvature of the $\alpha-$th order) contains the vector λ^a. The curve C is said to be a hyper D-line of order h $(0 \le h \le (n+1)-3)$.

From this definition, for a hyper darboux line of order h we can write

$$\lambda^a = \underset{(h)}{p}\left[R_{(h+1)}\xi^a_{(h+1)} + R_{(h+2)}\frac{\dot{\delta} R_{(h+1)}}{\delta s}\xi^a_{(h+2)} \right] + \underset{(h)}{q}\,\xi^a_{(0)}. \tag{2.2}$$

From (1.12) we have

$$\begin{aligned}\frac{\dot{\delta}^2 \xi^a_{(r)}}{\delta s^2} &= -\frac{\dot{\delta}\kappa_{(r)}}{\delta s}\xi^a_{(r-1)} - \kappa_{(r)}\frac{\dot{\delta}\xi^i_{(r-1)}}{\delta s} + \\ &\quad + \frac{\dot{\delta}\kappa_{(r+1)}}{\delta s}\xi^a_{(r+1)} + \kappa_{(r+1)}\frac{\dot{\delta}\xi^a_{(r+1)}}{\delta s}\end{aligned} \tag{2.3}$$

Using the (1.12) in this equation:

$$\begin{aligned}\frac{\dot{\delta}^2 \xi^a_{(r)}}{\delta s^2} &= -\frac{\dot{\delta}\kappa_{(r)}}{\delta s}\xi^a_{(r-1)} - \kappa_{(r)}(-\kappa_{(r-1)}\xi^a_{(r-2)} + \kappa_{(r)}\xi^a_{(r)}) + \\ &\quad + \frac{\dot{\delta}\kappa_{(r+1)}}{\delta s}\xi^a_{(r+1)} + \kappa_{(r+1)}(-\kappa_{(r+1)}\xi^a_{(r)} + \kappa_{(r+2)}\xi^a_{(r+2)}) \\ &= \kappa_{(r)}\kappa_{(r-1)}\xi^a_{(r-2)} - \frac{\dot{\delta}\kappa_{(r)}}{\delta s}\xi^a_{(r-1)} - (\kappa_{(r)}{}^2 + \kappa_{(r+1)}{}^2)\xi^a_{(r)} + \\ &\quad + \frac{\dot{\delta}\kappa_{(r+1)}}{\delta s}\xi^a_{(r+1)} + \kappa_{(r+1)}\kappa_{(r+2)}\xi^a_{(r+2)}.\end{aligned} \tag{2.4}$$

This equation gives

$$g_{\underline{ab}}\left(\frac{\dot{\delta}^2 \xi^a_{(r)}}{\delta s^2}\right)\left[R_{(h+1)}\xi^b_{(h+1)} + R_{(h+2)}\frac{\dot{\delta} R_{(h+1)}}{\delta s}\xi^b_{(h+2)} \right] = 0. \tag{2.5}$$

Multiplying λ^a by $g_{\underline{ab}}\xi^b_{(0)}$ and $g_{\underline{ab}}\dfrac{\overset{\bullet}{\delta}{}^2\xi^b_{(h)}}{\delta s^2}$ and using the last equation, we get

$$g_{\underline{ab}}\lambda^a\xi^b_{(0)} = \underset{(h)}{q}, \tag{2.6}$$

$$g_{\underline{ab}}\frac{\overset{\bullet}{\delta}{}^2\xi^b_{(h)}}{\delta s^2}\lambda^a = \underset{(h)}{q}\, g_{\underline{ab}}\frac{\overset{\bullet}{\delta}{}^2\xi^b_{(h)}}{\delta s^2}\xi^b_{(0)} \tag{2.7}$$

and using the definition $\lambda_{(h)} = g_{\underline{ab}}\lambda^a\xi^b_{(h)}$ we obtain

$$\underset{(h)}{q} = \lambda_{(h)} \tag{2.8}$$

and

$$g_{\underline{ab}}\frac{\overset{\bullet}{\delta}{}^2\xi^b_{(h)}}{\delta s^2}\lambda^a = \lambda_{(h-2)}\kappa_{(h)}\kappa_{(h-1)} - \lambda_{(h-1)}\frac{\overset{\bullet}{\delta}\kappa_{(h)}}{\delta s} + \\ -\lambda_{(h)}\left(\kappa^2_{(h)} + \kappa^2_{(h+1)}\right) + \lambda_{(h+1)}\frac{\overset{\bullet}{\delta}\kappa_{(h+1)}}{\delta s} + \lambda_{(h+2)}\kappa_{(h+1)}\kappa_{(h+2)} = 0. \tag{2.9}$$

The above equation is valid for $h = 3, ..., (n+1)-3$.
In the case of $h = 2$ it is written as

$$g_{\underline{ab}}\frac{\delta^2\overset{\bullet}{\xi}{}^b_{(2)}}{\delta s^2}\lambda^a = \lambda_{(0)}g_{\underline{ab}}\xi^a_{(0)}\frac{\delta^2\overset{\bullet}{\xi}{}^b_{(2)}}{\delta s^2} = \kappa_{(2)}\kappa_{(1)}\lambda_{(0)} \tag{2.10}$$

and so

$$-\frac{\overset{\bullet}{\delta}\kappa_{(2)}}{\delta s}\lambda_{(1)} - \left(\kappa^2_{(2)} + \kappa^2_{(3)}\right)\lambda_{(2)} + \frac{\overset{\bullet}{\delta}\kappa_{(3)}}{\delta s}\lambda_{(3)} + \kappa_{(3)}\kappa_{(4)}\lambda_{(4)} = 0. \tag{2.11}$$

In the case of $h = 1$ it can be written that

$$g_{\underline{ab}}\frac{\overset{\bullet}{\delta}{}^2\xi^b_{(1)}}{\delta s^2}\lambda^a = \underset{(0)}{q}\, g_{\underline{ab}}\frac{\overset{\bullet}{\delta}{}^2\xi^b_{(1)}}{\delta s^2}\xi^a_{(0)} = \lambda_{(0)}g_{\underline{ab}}\xi^a_{(0)}\frac{\overset{\bullet}{\delta}{}^2\xi^b_{(1)}}{\delta s^2} \tag{2.12}$$

and so

$$-\left(\kappa^2_{(1)} + \kappa^2_{(2)}\right)\lambda_{(1)} + \frac{\overset{\bullet}{\delta}\kappa_{(2)}}{\delta s}\lambda_{(2)} + \kappa_{(2)}\kappa_{(3)}\lambda_{(3)} = 0. \tag{2.13}$$

In the case of $h = 0$ it can be written that

$$g_{\underline{ab}}\frac{\overset{\bullet}{\delta}{}^2\xi^b_{(0)}}{\delta s^2}\lambda^a = \underset{(0)}{q}\, g_{\underline{ab}}\frac{\overset{\bullet}{\delta}{}^2\xi^b_{(0)}}{\delta s^2}\xi^a_{(0)} = \lambda_{(0)}g_{\underline{ab}}\xi^a_{(0)}\frac{\overset{\bullet}{\delta}{}^2\xi^b_{(0)}}{\delta s^2} \tag{2.14}$$

and so

$$\frac{\overset{\bullet}{\delta}\kappa_{(1)}}{\delta s}\lambda_{(1)}+\kappa_{(1)}\kappa_{(2)}\lambda_{(2)}=0. \tag{2.15}$$

These equations show the hyper D-lines of various orders.

Theorem 2.1: If the congruence λ^a is along the $h-$th binormal $\xi^a_{(h+1)}$ of a curve then the necessary and sufficient to be a hyper D-line of order h is its being a curve of constant $(h+1)-$th curvature.

Proof: Let the congruence λ^a is along the $h-$th binormal $\xi^a_{(h+1)}$ of a curve. That is $\lambda^a=c\cdot\xi^a_{(h+1)}$, where c is a constant. From the equations (2.6) and (2.8), we get

$$\lambda_{(h)}=g_{\underline{ab}}\lambda^a\xi^b_{(h)}=cg_{\underline{ab}}\xi^a_{(h+1)}\xi^b_{(h)}=\underset{(h)}{q}=0 \tag{2.16}$$

and so, $\lambda_{(i)}=c\,\delta_i^{h+1}$.

If this curve is a hyper D-line of order h, from (2.9) it is

$$\begin{aligned}&\lambda_{(h-2)}\kappa_{(h)}\kappa_{(h-1)}-\lambda_{(h-1)}\frac{\overset{\bullet}{\delta}\kappa_{(h)}}{\delta s}-\lambda_{(h)}\left(\kappa^2_{(h)}+\kappa^2_{(h+1)}\right)+\\&\qquad+\lambda_{(h+1)}\frac{\overset{\bullet}{\delta}\kappa_{(h+1)}}{\delta s}+\lambda_{(h+2)}\kappa_{(h+1)}\kappa_{(h+2)}=0\end{aligned} \tag{2.17}$$

From the last equation, it is found that

$$\kappa_{(h+1)}=\text{const.} \tag{2.18}$$

Conversely, if the $\kappa_{(h+1)}$ is a constant then $\lambda_{(h+1)}\dfrac{\overset{\bullet}{\delta}\kappa_{(h+1)}}{\delta s}=0$ may be written and the equation (2.9) is satisfied and so, the curve is a hyper D-line.

Theorem 2.2: If the congruence λ^a is along the $(h-2)$th binormal $\xi^a_{(h-1)}$ of a curve then the necessary and sufficient condition to be a hyper D-line of order h $(h\geq 2)$ is its being a curve of constant $h-$th curvature.

Proof: Let the congruence λ^a be along the $(h-2)$th binormal $\xi^a_{(h-1)}$ of a curve. That is $\lambda^a=c\cdot\xi^a_{(h-1)}$, where c is a constant. From the equation (2.6) and (2.8), we get

$$\lambda_{(h)}=g_{\underline{ab}}\lambda^a\xi^b_{(h)}=cg_{\underline{ab}}\xi^a_{(h-1)}\xi^b_{(h)} \tag{2.19}$$

and so, $\lambda_{(i)}=c\,\delta_i^{h-1}$

If this curve is a hyper D-line of order h $(h\geq 2)$, it is

$$\lambda_{(h-2)}\kappa_{(h)}\kappa_{(h-1)} - \lambda_{(h-1)}\frac{\overset{\bullet}{\delta}\kappa_{(h)}}{\delta s} - \lambda_{(h)}\left(\kappa_{(h)}^2 + \kappa_{(h+1)}^2\right) + + \lambda_{(h+1)}\frac{\overset{\bullet}{\delta}\kappa_{(h+1)}}{\delta s} + \lambda_{(h+2)}\kappa_{(h+1)}\kappa_{(h+2)} = 0 \tag{2.20}$$

From (2.19) and the last equation, it is found that

$$\kappa_{(h)} = \text{const.} \tag{2.21}$$

Conversely, if the $\kappa_{(h)}$ is a constant then $\lambda_{(h-1)}\frac{\overset{\bullet}{\delta}\kappa_{(h)}}{\delta s} = 0$ may be written and the equation ...... is satisfied, and so, the curve is a hyper D-line.

HYPER D-LINES

A hyper D-line of order zero is called the hyper D-line of the hyperspace. This curve is represented by

$$\frac{\overset{\bullet}{\delta}\kappa_{(1)}}{\delta s}\lambda_{(1)} + \kappa_{(1)}\kappa_{(2)}\lambda_{(2)} = 0 \tag{3.1}$$

Theorem 3.1: If the congruence lies along the first binormal $\xi_{(2)}^a$ then the necessary and the sufficient condition that the curve be a hyper D- line is that it be the curve of zero torsion $\left(\kappa_{(2)} = 0\right)$.

Proof: Let the congruence λ^a be along the first binormal $\xi_{(2)}^a$ of a curve. So, we have $\lambda^a = c \cdot \xi_{(2)}^a$ .From the equations (2.6) and (2.8), we get

$$\lambda_{(h)} = g_{\underline{ab}}\lambda^a\xi_{(2)}^b = cg_{\underline{ab}}\xi_{(2)}^a\xi_{(h)}^b \tag{3.2}$$

And so, $\lambda_{(i)} = c\,\delta_i^2$.

If this curve is a hyper D-line, it is

$$\frac{\overset{\bullet}{\delta}\kappa_{(1)}}{\delta s}\lambda_{(1)} + \kappa_{(1)}\kappa_{(2)}\lambda_{(2)} = 0 \tag{3.3}$$

From (3.2) and the last equation, it is found that $\kappa_{(1)}\kappa_{(2)} = 0$. Since $\kappa_{(1)} \neq 0$, $\kappa_{(2)}$ is equal to zero.

Conversely, if the $\kappa_{(2)}$ is zero then $\frac{\overset{\bullet}{\delta}\kappa_{(1)}}{\delta s}\lambda_{(1)} = 0$ may be written and the equation (3.1) is satisfied. And so, the curve is a hyper D-line.

We shall deduce the equation involving the second fundamental tensors of the hypersurface. From (1.11) and (1.16) we get

$$\frac{\delta \xi^a}{\delta s} = (p^k + I^k)x_k^a + (w_{ij}\xi^i\xi^j)N^a \tag{3.4}$$

where p^k and I^k are given by

$$p^k = (\dot{\nabla}_j \xi^k)\xi^j , \quad I^k = A_{ij}^k \xi^i \xi^j , \tag{3.5}$$

respectively , and the equation (3.6):

$$\frac{\dot{\delta}^2 \xi^a}{\delta s^2} = \left\{ \frac{\dot{\delta}(p^m + I^m)}{\delta s} + (p^k + I^k)A_{kj}^m \xi^j - w_{ij} w_{kl} \xi^i \xi^j \xi^k g^{\underline{lm}} \right\} x_m^a +$$

$$+ \left\{ p^k (2w_{ki} + w_{ik})\xi^i + I^k \xi^j w_{kj} + (\dot{\nabla}_k w_{ij})\xi^i \xi^j \xi^k \right\} N^a .$$

By putting the last equation into the equation $g_{\underline{ab}} \frac{\dot{\delta}^2 \xi^a}{\delta s^2} \lambda^b = \lambda_{(0)} g_{\underline{ab}} \xi^b \frac{\dot{\delta}^2 \xi^a}{\delta s^2}$ and using $g_{ab} N^a N^b = 1$ and $g_{\underline{ab}} N^a x_i^b = 0$ it is found that

$$\frac{\dot{\delta}(p^m + I^m)}{\delta s} q_m + (p^k + I^k) A_{kj}^l \xi^j q_l - q^p w_{ij} w_{kp} \xi^i \xi^j \xi^k +$$

$$+ r[w_{kj} I^k \xi^j + (\dot{\nabla}_k w_{ij})\xi^i \xi^j \xi^k + (2w_{ij} + w_{ji}) p^i \xi^j] +$$

$$- \underset{(0)}{\lambda} g_{\underline{sm}} \xi^s \frac{\dot{\delta}(p^m + I^m)}{\delta s} - \underset{(0)}{\lambda} (p^k + I^k) g_{\underline{sm}} \xi^s \xi^j A_{kj}^m + (w_{ij}\xi^i \xi^j)^2 \underset{(0)}{\lambda} = 0$$

We have the first two Frenet's formulae for the subspace

$$p^m = \frac{\dot{\delta} \xi^m}{\delta s} = \kappa_{(1)} \xi_1^m , \quad \frac{\dot{\delta} \xi_1^m}{\delta s} = -\kappa_{(1)} \xi^m + \kappa_{(2)} \xi_2^m \tag{3.8}$$

where ξ_1^m and ξ_2^m are the principal and binormal vectors and $\kappa_{(1)}$, $\kappa_{(2)}$ are the first and second curvatures with respect to the subspace. From the last equations, by using the fact $\underset{(0)}{\lambda} = g_{\underline{ab}} \lambda^a \xi^b = g_{\underline{ab}} (q^i x_i^a + rN^a)\xi^b = g_{\underline{ab}} q^i \xi^j = \underset{(0)}{q}$ we get

$$\frac{\dot{\delta} p^m}{\delta s} \cdot q_m = -\kappa_{(1)}^2 q_{(0)} + \frac{\dot{\delta} \kappa_{(1)}}{\delta s} q_{(1)} + \kappa_{(1)} \kappa_{(2)} q_{(2)} \tag{3.9}$$

and

$$-\kappa_{(1)}^2 q_{(0)} + \frac{\dot{\delta} \kappa_{(1)}}{\delta s} q_{(1)} + \kappa_{(1)} \kappa_{(2)} q_{(2)} + \frac{\dot{\delta} I^m}{\delta s} q_m + p^m A_{mj}^l \xi^j q_l + I^m A_{mj}^l \xi^j q_l +$$

$$- q^p w_{ij} w_{kp} \xi^b \xi^j \xi^k + r[w_{kj} I^k \xi^j + (\dot{\nabla}_k w_{ij})\xi^i \xi^j \xi^k + w_{kj} p^k \xi^j + w_{ij} p^i \xi^j + w_{ij} p^j \xi^i] +$$

$$+ \underset{(0)}{\lambda} \kappa_{(1)} \xi_1^k g_{\underline{sm}} \xi^s \xi^j A_{kj}^m - \underset{(0)}{\lambda} I^k g_{\underline{sm}} \xi^s \xi^j A_{kj}^m + \underset{(0)}{\lambda} (w_{ij} \xi^i \xi^j)^2 = 0$$

where $q_{(1)}$ and $q_{(2)}$ are the projections of q^i in the directions ξ_1^{m} and ξ_2^{m}, respectively. This represents the hyper D-line of the subspace. The equations have been expressed in terms of the second fundamental tensors and the curvatures (of the curve) with respect to the subspace.

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