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THE NECESSARY CONDITIONS OF OPTIMALITY IN A NON SMOOTH CONTROL PROBLEM FOR DISCRETE SYSTEMS

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Abstract

In this paper a necessary conditions for optimality of the non smooth control problem in discrete systems is obtained.

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Let the control process in "discrete interval" $T=[t_0,t_0+1,...]$ be described by the system of non-linear difference equations

$$x(t+1)=f(t,x(t),u(t)),\quad t\in T\setminus\{t_0\},\ x(t_0)=x_0, \tag{1}$$

where n-dimensional vector function $f(t,x,u)$ is continuous together with its the first order partial derivatives with respect to x, $u(t)$ is r-dimensional control function with values from a given bounded domain U,

$$u(t)\in U\subset R^r,\ t\in T \tag{2}$$

It needs to minimize the functional

$$S_0(u)=\Phi_0(x(t_1)) \tag{3}$$

defined on the solition of the system (1) under the conditions

$$S_i(u)=\Phi_i(x(t_1))\le 0,\qquad i=1,2,...,p. \tag{4}$$

Let the given scalar functions $\Phi_i(x), i=1,2,...,p.$ satisfy the Lipshitz condition and have a derivatives on the arbitrary direction. The problem of minimization of the functional (3) within the conditions (1),(2),(4) problem is called the (1)-(4).

Let a matrix function $F(t,\tau)$ is a solution of the difference equation.

$F(t,\tau-1)=F(t,\tau)A(\tau), \quad F(t,t-1)=E$

E is unit matrix Suppose that $(x(t),u(t))$ is fixed prosses. Let us denole the function

$$\ell(v)=\sum_{t=t_0}^{t_1-1} F'(t_1,t)[f(t,x(t),v)-f(t,x(t),u(t))],$$

$$I(u)=\{i:\Phi_i(x(t_1))=0,\ i=1,2,...,p\},$$

$$J=\{0\}\cup I.$$

Theorem 1. Suppose that the set of admisiable speeds of the system (1), i.e, the set

$$F(t,x(t),U)=\{y:y=f(t,x(t),v), v\in U\}$$

is convex along the prosses. İf the admissible control u(t) is optimal for problem (1)-(4) then the unequality hold for the every $v(t)\in U, t\in T$

$$\max_{i\in J(u)} \frac{\partial\Phi_i(x(t_i))}{\partial\ell(v)} \geq 0 \qquad (5)$$

holds for the every $v(t)\in U, t\in T$.

Proof of an analoguns theorem is in[8].

Now suppose that in the problem (1)-(4) the the function f is linear with respect to x, i.e,

$$f(t,x,u)=A(t)x+b(t,u), \qquad (6)$$

where $A(t)$ is given $(n\times n)$ matrix function, $b(t,u)$ is a n-dimensionel vector.

We suppose additionaly that $\Phi_i(x),\ i=0,1,2,...,p$ have second derivatives on the arbitrary direction. Set

$$g(v)=\sum_{t=t_0}^{t_1-1}F(t_1,t)[b(t,v(t))-b(t,u(t))],$$

where $F(t,\tau)$ is a solution of the problem

$$F(t,\tau-1)=F(t,\tau)A(\tau),\ \ F(t,t-1)=E.$$

Theorem 2. Let the set

$$b(t,u)=\{y:y=b(t,v),\ v\in U\}$$

be convex. If $(x(t),u(t))$ for the optimal solution of the problem (1)-(4), (6) then the unequality

$$\frac{\partial\Phi_i(x(t_1))}{\partial g(v)}\geq 0 \tag{7}$$

hold for every $v(t)\in U$, $t\in T$ such that

$$\max_{i\in I(u)}\frac{\partial\Phi_i(x(t_1))}{\partial g(v)}<0.$$

As we see in difference from Theorem 1 when we prove the Theorem 2 we need not that $\Phi_i(x), i=1,2,\dots,p$ satisbu the Lipshitz condition. İt is a result of the linearity of the righthand side of the system (1) with respect to x.

Definition 1. We call the control $u(t)$ special in the problem (1)-(4),(6) if for all $v(t)\in U$, $t\in T$

$$\frac{\partial\Phi_0(x(t_1))}{\partial g(v)}=0 \tag{8}$$

In this case next Theorem is true

Theorem 3. For the optimality of the special in the ginee of (8) control $u(t)$ in the problem (1)-(4),(6) it is necessary that

$$\frac{\partial^2\Phi_0(x(t_1))}{\partial g^2(v)}\geq 0 \tag{9}$$

hold for all $v(t)\in U$, $t\in T$ such that

$$\max_{i \in I(u)} \frac{\partial \Phi_i(x(t_i))}{\partial g(v)} < 0 .$$

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