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ANALYTIC SOLUTION OF ONE-DIMENSIONAL PROBLEM FOR PARTIAL INTEGRO-DIFFERANTIAL EQUATIONS WHICH HAVE PARTIAL CONTINUOUS COEFFICIENTS IN THERMOVISCOELASTICITY THEORY[1]

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Summary: In this paper, a non–stationary problem on thermomechanic wave propagation is solved in an environment, which is, consists of a finite thick plate connected with a semiinfinite space. Materials of the plate and the space are in conformity with linear viscoelasticity laws. Mathematical model of the problem consists of: linear equations of viscoelasticity and heat transfer for each environment independently, initial conditions and on the connection surface of environments conditions of increasing temperature and normal stress, depending only on time which are given as known functions. It is assumed that temperature and mechanical fields depend on each other. As a system, parabolic type partial integro-differential equation of temperature and hyperbolic type partial integro-differential equation of wave are solved. It is assumed that kernels of integral operators are difference kernels. Depending on boundary conditions, functions of temperature and mechanical magnitudes become only functions of time and a space axis, which is perpendicular to free surface. In this case the problem turns out to be a one-dimensional one.

1.INTRODUCTION

Determination of the reaction of the bodies changing from to dynamical forces is one of important problems in the field of mechanical science. We face to these problems in daily practice frequently.

Although pioneering works of Euler, Bernoulli, Sofi-Jerman and others were initialized in 18^{th} century, in the field , it is gained scientific form almost in 20^{th} century. Dynamics of elastic bodies has important application in seismology and in other branches of technology. Path breaking research accelerated after 40's of 20^{th} century, reached important results many articles and book were published. We will mention some of the published materials that are concerned with our research.

Heat is one of the important magnitude which effects waves in bodies changing forms. Effect of heat is high in Composite and Polymer materials. Solution of the thermoelastic wave propagation problem in homogenous bar became classical and exists in many books. These problems may be classified into two; one is dependent and the other is independent. In the first, heat and stress-

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deformation fields are mutually effected and can not be determined independently. Mathematically, in this case, motion and heat equations constitute an interdependent system and solved commonly. In independent dynamical problems field of temperature is determined initially then temperature is added to motion equation as a defined function. It is clear that, in this case, the effect of stress-deformation field is neglected. As it is seen, in second type solution of the problem became easier, mathematically is reduced to the one of one dimensional problem which is finding solution of non-homogenous hyperbolic type equation with initial and boundary conditions. The most widely known of analytical solution methods of linear problems is integral transform method. Interesting and difficult part of this method is calculation of inverse transforms. Evens in the most simple dependent problem inverse Laplace transforms can not be find. Owing to this fact in the solution of similar problems, we consider asymptotically situations, that is very large and small values of Laplace transform parameter, or initially solution of independent problem is taken and perturbation method is applied.

In this paper we research one dependent problem of propagation of mechanical waves in an environment which consist of viscoelastical plate and semiinfinite space. Considering partial constant coefficient in addition to what explained above makes even more complex. Problem is solved by Laplace integral transform, and system which is obtained by substation of boundary conditions is solved analytically in Computer by using software "Mathematica 2.0". Inverse Laplace transforms are calculated for very large and small values of Laplace parameter.

Classical solutions of thermodynamically problems for elastic and viscoelastic materials are in [9] and in several books.

In [31] perturbation method is developed to investigate nonstationary temperature field and stress field in viscoelastic bodies of properties depend on temperature. In solutions of thermoelastic wave propagation in cylinder of infinite length and in sphere, and viscoelastic plates are given as infinite series and determined convergence conditions for these series.

In [21] termoviscoelastic problem is approached in a general form by using perturbation method with temperature-time analog, and same problems are investigated which are emphasizing effects of temperature field on properties of viscoelastic bodies.

In [10] nonstationary oscillations of viscoelastic bar are investigated under the pressure of periodical variable force considering temperature effect on properties of materials. In addition under the effect of harmonic load, lengthwise oscillations are studied for viscoelastic bars. Mechanical properties of bar depend on temperature. Because of continuos transformation of mechanical energy to heat, temperature of bar is increased and this in turn changes speed of the propagation of waves. This problem is solved numerically.

In articles [12,13,14,15] and[23,24] effects of temperature and stress fields on propagation of waves in homogenous half-space are studied.

In articles [1,2,3,5,6,7,16,17,18,19,20,22,25,26,27,28,29,30,32,33,34] related studies are performed.

2.FORMULATION OF PROBLEM

Let us consider a non-stationary problem that resulted from mechanical and temperature strokes in a semiinfinite stratified environment.

Let us take origin point on free surface and let us take x-axis perpendicular to the free surface downward. Let us assume termomechanical effect, which is in the boundary (x=0) uniform for other coordinates, only depends on the time. In this situation determination of stress, deformation and temperature fields is reduced to solution of one-dimensional problem. Magnitudes which belong to plate are indexed by "1" and those which belong to semiinfinite space are indexed by "2". Thickness of the plate is shown by "h".

Let us consider the linear problem of the related thermoviscoelasticity theory and assume that conditions of the environment are not depend on temperature . In this situation we obtain following displacement, heat conduction and state equations.

$$\frac{\partial \sigma_i(x,t)}{\partial x} = \rho_i \frac{\partial^2 u_i(x,t)}{\partial t^2} \qquad (1)$$

$$\frac{\chi_i}{T_0}\frac{\partial^2 \theta_i(x,t)}{\partial x^2} = \frac{\partial}{\partial t}\int_0^t m_i(t-\tau)d\theta_i + \frac{\partial}{\partial t}\int_0^t \psi_i(t-\tau)de_i \qquad (2)$$

$$\sigma_i(x,t) = \int_0^t \left[R_i(t-\tau) + \frac{2}{3}Q_i(t-\tau)\right]de_i - \int_0^t \psi_i(t-\tau)d\theta_i \qquad (3)$$

Above, i=1,2, σ_i (x,t) is stress, u_i (x,t) is displacement, ρ_i is density, T_i (x,t) is temperature, T_0 is initial temperature, χ_i is temperature diffusion coefficient, m_i (t-τ), ψ(t-τ), R_i(t-τ) and Q_i(t-τ) are functions which denote mechanical properties of materials. R_i (t-τ) is called volume-relaxation function, Q_i(t-τ) is called slide-relaxation function. Integrations in (2) and (3) are Stiljes type and

$$de_i = \frac{\partial e_i(x,t)}{\partial \tau}d\tau, \quad d\theta_i = \frac{\partial \theta_i(x,t)}{\partial \tau}d\tau, \qquad \theta_i(x,t)=T_i(x,t)-T_0$$

and

$$e_i(x,t) = \frac{\partial u_i(x,t)}{\partial x} \qquad (4)$$

is deformation.

When we examine the linear problem we assume that e_i and $\frac{\theta_i}{T_0}$ are infinitesimal and the same order. Beside let us assume following relation between extension resulting from temperature and mechanical properties of materials.

$$\psi_i(t)=\varepsilon_i R_i(t) \qquad (5)$$

where ε_i is coefficient of extension resulting from temperature which is independent of the time.

Firstly using (1) and (3) then (4) and then (2) and (4) we obtain

$$\int_0^t G_i(t-\tau)\frac{\partial^3 u_i(x,\tau)}{\partial \tau \partial x^2}d\tau - \int_0^t \psi_i(t-\tau)\frac{\partial^2 \theta_i(x,\tau)}{\partial \tau \partial x}d\tau = \rho_i \frac{\partial^2 u_i(x,t)}{\partial t^2} \qquad (6)$$

$$\frac{\chi_i}{T_0}\frac{\partial^2\theta_i(x,t)}{\partial x^2}=\frac{\partial}{\partial t}\int_0^t m_i(t-\tau)\frac{\partial\theta_i(x,\tau)}{\partial\tau}d\tau+\frac{\partial}{\partial t}\int_0^t \psi_i(t-\tau)\frac{\partial^2 u_i(x,\tau)}{\partial\tau\partial x}d\tau \qquad (7)$$

where

$$G_i(t)=R_i(t)+\frac{2}{3}Q_i(t)$$

and there is no summation according to repeating index. (6) and (7) are partial integro-differential equations. u_i (x,t) and $\theta_i(x,t)$ will be solved from this equations. If viscoelasticity properties of materials have instantaneous elasticity property then (6) is hyperbolic and (7) is parabolic.

Relaxation function above are continuos functions of the variable $\tau \geq 0$ and when $\tau < 0$

$$R_i(\tau)\equiv 0,\quad Q_i(\tau)\equiv 0,\quad m_i(\tau)\equiv 0 \qquad \psi_i(\tau)\equiv 0$$

Initial conditions are,

$$u_i(x,0)=0 \qquad (8)$$

$$\left.\frac{\partial u_i(x,t)}{\partial t}\right|_{t=0}=0 \qquad (9)$$

$$\theta_i(x,0)=0 \qquad (10)$$

In case $x=0$

$$\sigma_1(0,t)=f(t) \qquad (11)$$
$$\theta_1(0,t)=\varphi(t) \qquad (12)$$

In case $x=h$,

$$u_1(h,t)=u_2(h,t) \qquad (13)$$

$$\sigma_1(h,t)=\sigma_2(h,t) \qquad (14)$$
$$\theta_1(h,t)=\theta_2(h,t) \qquad (15)$$

$$\chi_1 \frac{\partial \theta_1}{\partial x}\bigg|_{x=h} = \chi_2 \frac{\partial \theta_2}{\partial x}\bigg|_{x=h} \tag{16}$$

and for $x \to \infty$

$$\lim_{x \to \infty} \sigma_2(x,t) < +\infty \tag{17}$$

$$\lim_{x \to \infty} \theta_2(x,t) < +\infty \tag{18}$$

Conditions (13),(14),(15),(16) show displacement, stress, temperature and temperature diffusion values are equal on the boundary of the space and the plate.

3.SOLUTION OF THE PROBLEM BY LAPLACE TRANSFORM

We will solve the problem (6)-(18) by Laplace integral transform according to time. If we apply Laplace integral transform, with the initial conditions (8),(9),(10),to the equations (6),(7) we obtain following differential equation system.

$$\frac{d^2\overline{u_i}}{dx^2} - \frac{\rho_i p}{\overline{G_i}}\overline{u_i} - \frac{\overline{\psi_i}}{\overline{G_i}}\frac{d\overline{\theta_i}}{dx} = 0 \tag{19}$$

$$\frac{d^2\overline{\theta_i}}{dx^2} - \frac{T_0 p^2 \overline{m_i}}{\chi_i}\overline{\theta_i} - \frac{T_0 p^2 \overline{\psi_i}}{\chi_i}\frac{d\overline{u_i}}{dx} = 0 \tag{20}$$

where $\overline{u}$ is the Laplace transform of u.

$$\overline{f}(p) = \int_0^\infty f(t)\, e^{-pt} dt \tag{21}$$

(21) is the Laplace transform of f(t) function. p is the parameter fi the transform..

In order to determine $\overline{u_i}$ (x,p) and $\overline{\theta_i}$ (x,p) functions, a system of equations is found which is second order and have constant coefficients. Characteristic equation of the system is

$$K^4 - A_i K^2 + B_i = 0 \tag{22}$$

where,

$$A_i = \frac{\rho_i p}{G_i} + \frac{T_0 p^2 \overline{m_i}}{\chi_i} + \frac{T_0 p^2 \overline{\psi_i}}{G_i \chi_i}, B_i = \frac{\rho_i p^3 T_0 \overline{m_i}}{G_i \chi_i}$$

Solutions of the equation (22):

$$K_{1i} = -\left[\frac{A_i}{2} + \frac{1}{2}\left(A_i^2 - 4B_i\right)^{1/2}\right]^{1/2} \tag{23}$$

$$K_{2i} = \left[\frac{A_i}{2} + \frac{1}{2}\left(A_i^2 - 4B_i\right)^{1/2}\right]^{1/2} \tag{24}$$

$$K_{3i} = -\left[\frac{A_i}{2} - \frac{1}{2}\left(A_i^2 - 4B_i\right)^{1/2}\right]^{1/2} \tag{25}$$

$$K_{4i} = \left[\frac{A_i}{2} - \frac{1}{2}\left(A_i^2 - 4B_i\right)^{1/2}\right]^{1/2} \tag{26}$$

where, i=1,2. Since the numbers K_{ji} j=1,2,3,4 , are functions of the Laplace parameter p, they are complex numbers. But for real p:

$Re(K_{1i},K_{3i})<0$ (27)
$Re(K_{2i},K_{4i})>0$ (28)

Following properties exist among K_{ji} solutions.

$K_{1i}K_{2i}K_{3i}K_{4i}=B_i$ (29)
$K_{1i}K_{2i}\ K_{3i}K_{4i}=\sqrt{B_i}$ (30)
$K_{1i}=-K_{2i}$ (31)
$K_{3i}=-K_{4i}$ (32)
$K_{1i}+K_{3i}=-(K_{2i}+K_{4i})$ (33)

General solution of the system (19),(20) is as follows.

$$\overline{u_i}(x,p) = E_{1i}e^{K_{1i}x} + E_{2i}e^{K_{2i}x} + E_{3i}e^{K_{3i}x} + E_{4i}e^{K_{4i}x} \tag{34}$$

$$\overline{\theta_i}(x,p) = F_{1i}e^{K_{1i}x} + F_{2i}e^{K_{2i}x} + F_{3i}e^{K_{3i}x} + F_{4i}e^{K_{4i}x} \tag{35}$$

When this values are replaced to (19) or (20), following relations among F_{ji} and E_{ji} are obtained.

$$F_{ji} = \frac{K_{ji}^2 - a_i}{b_i K_{ji}} E_{ji} \qquad j=1,2,3,4, \quad i=1,2 \tag{36}$$

where $a_i = \frac{\rho_i p}{\overline{G_i}}, b_i = \frac{\overline{\psi_i}}{\overline{G_i}}$

From (17),(18),(27) and (28)

$$E_{22}=E_{42}=0 \tag{37}$$

and in that way solutions of the system (19),(20) which are limited in infinity as follows:

$$\overline{u_1}(x,p) = E_{11}e^{K_{11}x} + E_{21}e^{K_{21}x} + E_{31}e^{K_{31}x} + E_{41}e^{K_{41}x} \tag{38}$$

$$\overline{u_2}(x,p) = E_{12}e^{K_{12}x} + E_{32}e^{K_{32}x} \tag{39}$$

$$\overline{\theta_1}(x,p) = \frac{K_{11}^2 - a_1}{b_1 K_{11}} E_{11}e^{K_{11}x} + \frac{K_{21}^2 - a_1}{b_1 K_{21}} E_{21}e^{K_{21}x} + \frac{K_{31}^2 - a_1}{b_1 K_{31}} E_{31}e^{K_{31}x} + \frac{K_{41}^2 - a_1}{b_1 K_{41}} E_{41}e^{K_{41}x} \tag{40}$$

$$\overline{\theta_2}(x,p) = \frac{K_{12}^2 - a_2}{b_2 K_{12}} E_{12}e^{K_{12}x} + \frac{K_{31}^2 - a_2}{b_2 K_{32}} E_{32}e^{K_{32}x} \tag{41}$$

As shown solution depends on six constants (E_{11},E_{21},E_{31},E_{41},E_{12},E_{32}) and these constants will be calculated from (12),(13),(14),(15),(16). When we apply Laplace transform to (3)

$$\overline{\sigma_i}(x,p) = p\overline{G_i}\frac{d\overline{u_i}}{dx} - p\overline{\psi_i \theta_i} \tag{42}$$

is obtained. Using (42) from (38),(39),(40),(41)

$$\overline{\sigma_1}(x,p) = \frac{\rho_1 p^2}{K_{11}} E_{11}e^{K_{11}x} + \frac{\rho_1 p^2}{K_{21}} E_{21}e^{K_{21}x} + \frac{\rho_1 p^2}{K_{31}} E_{31}e^{K_{31}x} + \frac{\rho_1 p^2}{K_{41}} E_{41}e^{K_{41}x} \tag{43}$$

$$\overline{\sigma_2}(x,p)=\frac{\rho_2 p^2}{K_{12}}E_{12}e^{K_{12}x}+\frac{\rho_2 p^2}{K_{32}}E_{32}e^{K_{32}x} \tag{44}$$

are obtained. When we apply (38),(39),(40),(41) and (43),(44) equations to Laplace transforms of conditions (11),(12),(13),(14),(15),(16) we obtain following system of linear equations, which consist of six equations.

$$\frac{E_{11}}{K_{11}}+\frac{E_{31}}{K_{21}}+\frac{E_{31}}{K_{31}}+\frac{E_{31}}{K_{41}}=\frac{\overline{f}(p)}{\rho_1 p^2} \tag{45}$$

$$\frac{K_{11}^2-a_1}{K_{11}}E_{11}+\frac{K_{21}^2-a_1}{K_{21}}E_{21}+\frac{K_{31}^2-a_1}{K_{31}}E_{31}+\frac{K_{41}^2-a_1}{K_{41}}E_{41}=\overline{\varphi}(p)h_1 \tag{46}$$

$$E_{11}e^{K_{11}h}+E_{21}e^{K_{21}h}+E_{31}e^{K_{31}h}+E_{41}e^{K_{41}h}-E_{12}e^{K_{12}h}-E_{32}e^{K_{32}h}=0 \tag{47}$$

$$\frac{E_{11}}{K_{11}}e^{K_{11}h}+\frac{E_{21}}{K_{21}}e^{K_{21}h}+\frac{E_{31}}{K_{31}}e^{K_{31}h}+\frac{E_{41}}{K_{41}}e^{K_{41}h}-\frac{\rho_2 E_{12}}{\rho_1 K_{12}}e^{K_{12}h}-\frac{\rho_2 E_{32}}{\rho_1 K_{32}}e^{K_{32}h}=0 \tag{48}$$

$$\frac{K_{11}^2-a_1}{K_{11}}E_{11}e^{K_{11}h}+\frac{K_{21}^2-a_1}{K_{21}}E_{21}e^{K_{21}h}+\frac{K_{31}^2-a_1}{K_{31}}E_{31}e^{K_{31}h}+\frac{K_{41}^2-a_1}{K_{41}}E_{41}e^{K_{41}h}-$$
$$\frac{b_1(K_{12}^2-a_2)}{b_2K_{12}}E_{12}e^{K_{12}h}-\frac{b_1(K_{32}^2-a_2)}{b_2K_{32}}E_{32}e^{K_{32}h}=0 \tag{49}$$

$$(K_{11}^2-a_1)E_{11}e^{K_{11}h}+(K_{21}^2-a_1)E_{21}e^{K_{21}h}+(K_{31}^2-a_1)E_{31}e^{K_{31}h}+(K_{41}^2-a_1)E_{41}e^{K_{41}h}-$$
$$\frac{\chi_2 b_1(K_{12}^2-a_2)}{\chi_1 b_2}E_{12}e^{K_{12}h}-\frac{\chi_2 b_1(K_{32}^2-a_2)}{\chi_1 b_2}E_{32}e^{K_{32}h}=0 \tag{50}$$

When this system is solved by Cramer method;

$$E_{11}=\frac{\Delta_1}{\Delta} \tag{51}$$

$$E_{21}=\frac{\Delta_2}{\Delta} \tag{52}$$

$$E_{31}=\frac{\Delta_3}{\Delta} \tag{53}$$

$$E_{41}=\frac{\Delta_4}{\Delta} \tag{54}$$

$$E_{12}=\frac{\Delta_5}{\Delta} \tag{55}$$

$$E_{32}=\frac{\Delta_6}{\Delta} \tag{56}$$

are obtained. Here , Δ is the determinant of coefficients of system Δ_i, i=1,2,3,4,5,6 are determinant which are obtained by deleting i^{th} column replacing right side values instead. Right side:

$$\left(\frac{\overline{f}(p)}{\rho_1 p^2}, \overline{\varphi}(p)h_1, 0,0,0,0\right) \tag{57}$$

Let us show the coefficient matrix of the system (45),(46),(47),(48),(49),(50) by *i,j*=1,2,3,4,5,6 where

$$\alpha_{11}=\frac{1}{K_{11}}, \alpha_{12}=\frac{1}{K_{21}}, \alpha_{13}=\frac{1}{K_{31}}, \alpha_{14}=\frac{1}{K_{41}}, \alpha_{15}=\alpha_{16}=0$$

$$\alpha_{21}=\frac{K_{11}^2-a_1}{K_{11}}, \alpha_{22}=\frac{K_{21}^2-a_1}{K_{21}}, \alpha_{23}=\frac{K_{31}^2-a_1}{K_{31}}, \alpha_{24}=\frac{K_{41}^2-a_1}{K_{41}},$$

$$\alpha_{25}=\alpha_{26}=0, \alpha_{31}=e^{K_{11}h}, \alpha_{32}=e^{K_{21}h}$$

$$\alpha_{33}=e^{K_{31}h}, \alpha_{34}=e^{K_{41}h}, \alpha_{35}=-e^{K_{12}h}, \alpha_{36}=-e^{K_{32}h},$$

$$\alpha_{41}=\frac{e^{K_{11}h}}{K_{11}}, \alpha_{42}=\frac{e^{K_{21}h}}{K_{21}}, \alpha_{43}=\frac{e^{K_{31}h}}{K_{31}}, \alpha_{44}=\frac{e^{K_{41}h}}{K_{41}},$$

$$\alpha_{45}=-\frac{\rho_2}{\rho_1 K_{12}}e^{K_{12}h}, \alpha_{46}=-\frac{\rho_2}{\rho_1 K_{32}}e^{K_{32}h},$$

$$\alpha_{51}=\frac{K_{11}^2-a_1}{K_{11}}e^{K_{11}h}, \alpha_{52}=\frac{K_{21}^2-a_1}{K_{21}}e^{K_{21}h},$$

$$\alpha_{53}=\frac{K_{31}^2-a_1}{K_{31}}e^{K_{31}h}, \alpha_{54}=\frac{K_{11}^2-a_1}{K_{41}}e^{K_{41}h},$$

$$\alpha_{55}=-\frac{h_1(K_{12}-a_2)}{h_2 K_{12}}e^{K_{12}h}, \alpha_{56}=-\frac{h_1(K_{32}^2-a_2)}{h_2 K_{32}}e^{K_{32}h},$$

$$\alpha_{61}=(K_{11}^2-a_1)e^{K_{11}h}, \alpha_{62}=(K_{21}^2-a_1)e^{K_{21}h},$$

$$\alpha_{63}=(K_{31}^2-a_1)e^{K_{31}h}, \alpha_{64}=(K_{41}^2-a_1)e^{K_{41}h},$$

$$\alpha_{65}=-\frac{\chi_2 h_1}{\chi_1 h_2}(K_{12}^2-a_2)e^{K_{12}h}, \alpha_{66}=-\frac{\chi_2 h_1}{\chi_1 h_2}(K_{32}^2-a_2)e^{K_{32}h}$$

Coefficients determinant of the system and the other determinants are calculated by the formula

$$\Lambda = \sum (-1)^{i+j}\Lambda_{ij} M_{ij} \tag{58}$$

where Λ_{ij} are subdeterminant of 2 × 2 which are subdeterminants of the matrix of 2 × 6 which is obtained from first two rows. While M_{ij} are respecting 4 × 4 minor determinats. Minus sing in front of summation stems from calculation of determinant with respect to first two row.

When we express Λ and Λ_i determinants by help of M_{ij}'s

$$\Delta = \frac{\sqrt{A_1^2 - 4B_1}}{B_1\sqrt{B_2}} e^{(K_{11}+K_{31})h} (M_{13} - M_{14} - M_{23} + M_{24}), \tag{59}$$

$$\Delta_1 = \frac{K_{11}}{B_1\sqrt{B_2}} e^{(K_{11}+K_{31})h} \quad (F_1 M_{12} - F_3 M_{13} + F_3 M_{14}), \tag{60}$$

$$\Delta_2 = -\frac{K_{11}}{B_1\sqrt{B_2}} e^{(K_{11}+K_{31})h} (-F_1 M_{12} + F_3 M_{23} - F_3 M_{24}), \tag{61}$$

$$\Delta_3 = \frac{K_{31}}{B_1\sqrt{B_2}} e^{(K_{11}+K_{31})h} (F_1 M_{13} - F_1 M_{23} - F_3 M_{24}), \tag{62}$$

$$\Delta_4 = \frac{K_{31}}{B_1\sqrt{B_2}} e^{(K_{11}+K_{31})h} (F_1 M_{14} - F_1 M_{24} + F_3 M_{34}), \tag{63}$$

$$\Delta_5 = \frac{K_{12}}{B_1\sqrt{B_2}} e^{K_{12}h} (F_1 M_{15} - F_1 M_{25} + F_3 M_{35} - F_3 M_{45}), \tag{64}$$

$$\Delta_6 = \frac{K_{32}}{B_1\sqrt{B_2}} e^{K_{32}h} (-F_1 M_{16} + F_1 M_{26} - F_3 M_{36} + F_3 M_{46}). \tag{65}$$

are obtained, where

$$F_1 = \frac{\overline{f}}{\rho_1 p^2}(K_{11}^2 - a_1) - \frac{\overline{\varphi}}{b_1},\ F_3 = \frac{\overline{f}(K_{31}^2 - a_1)}{\rho_1 p^2} - \frac{\overline{\varphi}}{b_1}$$

Solution of so stated problem by the help of Laplace transform, considering (51),(52),(53),(54),(55),(56) and (59),(60),(61),(62),(63),(64),(65) only consist of (38),(39),(40),(41),(43),(44). Essential solution is obtained by calculation of inverse Laplace transforms.

Since calculation of inverse Laplace transform are very difficult we will find approximate solutions are asymptotic solutions. These are solutions, for very small and large values of Laplace transform parameter p.

Firstly, let us assume materials of semi infinite space and the plate are elastic. In that case Laplace transforms of material functions verify following conditions. m_i, G_i, R_i, $i=1,2$ are constants and

$$p\overline{m_i} = m_i \tag{66}$$

$$p\overline{G_i} = G_i \tag{67}$$

$$p\overline{R_i} = R_i \tag{68}$$

4.SOLUTIONS FOR VERY SMALL VALUES OF LAPLACE PARAMETER

By using (66), (67), (68) and (22)

$$A_i = \frac{p^2}{\xi_i^2} + \frac{T_0 p m_i}{\chi_i} + \frac{\varepsilon_i R_i T_0 p}{G_i \chi_i}, B_i = \frac{p^3 T_0 m_i}{\xi_i^2 \chi_i}.$$

where $\xi_i^2 = \frac{G_i}{\rho_i}$

$$A_i - 4B_i = \frac{p^4}{\xi_i^4} + \frac{T_0^2 m_i^2 p^2}{\chi_i^2} + \frac{\varepsilon_i^4 R_i^4 T_0^2 p^2}{G_i^2 \chi_i^2} - 2\frac{T_0 m_i}{\xi_i^2 \chi_i} p^3 + 2\frac{\varepsilon_i^2 R_i^2 T_0}{\xi_i^2 G_i \chi_i} p^3 + 2\frac{T_0^2 m_i \varepsilon_i^2 R_i^2}{G_i \chi_i} p^2$$

Since p^4 and p^3 are very small can be neglected and

$$A_i^2 - 4B_i = p^2 \left(\frac{T_0 m_i}{\chi_i} + \frac{\varepsilon_i^2 R_i^2 T_0}{G_i \chi_i} \right)^2$$

is obtained. When we replaced this value in (23),(24),(25),(26) and (36)

$$K_{1i} = \frac{A_i}{2} + \frac{1}{2}\sqrt{A_i^2 - 4B_i}$$
$$= \frac{p^2}{2\xi_i^2} + \frac{T_0 m_i p}{2\chi_i} + \frac{\varepsilon_i^2 R_i^2 T_0 p}{2G_i \chi_i} + \frac{p}{2}\left(\frac{T_0 m_i}{\chi_i} + \frac{\varepsilon_i^2 R_i^2 T_0}{G_i \chi_i} \right)$$

and by neglecting p^2 since it is very small;

$$K_{1i}^2 = \frac{T_0 p}{\chi_i}\left(m_i + \frac{\varepsilon_i^2 R_i^2}{G_i} \right)$$

and

$$K_{1i} = -\left[\frac{T_0 p}{\chi_i}\left(m_i + \frac{\varepsilon_i^2 R_i^2}{G_i} \right) \right]^{1/2} \tag{69}$$

$$K_{2i} = \left[\frac{T_0 p}{\chi_i}\left(m_i + \frac{\varepsilon_i^2 R_i^2}{G_i} \right) \right]^{1/2} \tag{70}$$

are obtained. From which

$$K_{3i} = -\frac{p}{\sqrt{2\xi_i}} \tag{71}$$

$$K_{4i} = \frac{p}{\sqrt{2\xi_i}} \tag{72}$$

are obtained.

$$F_{1i}=\frac{K_{1i}^2-a_i}{b_iK_{1i}}E_{1i} = \frac{\frac{T_0 p}{\chi_i}\left(m_i+\frac{\varepsilon_i^2R_i^2}{G_i}\right)-\frac{p^2}{\xi_i^2}}{-\frac{\varepsilon_iR_i}{G_i}\sqrt{\frac{T_0p}{\chi_i}\left(m_i+\frac{\varepsilon_i^2R_i^2}{G_i}\right)}}E_{1i},$$

neglecting p^2;

$$F_{1i}=-\frac{G_i}{\varepsilon_iR_i}\sqrt{p}\sqrt{\frac{T_0}{\chi_i}\left(m_i+\frac{\varepsilon_i^2R_i^2}{G_i}\right)}\qquad E_{1i} \tag{73}$$

$$F_{2i}=\frac{G_i}{\varepsilon_iR_i}\sqrt{p}\sqrt{\frac{T_0}{\chi_i}\left(m_i+\frac{\varepsilon_i^2R_i^2}{G_i}\right)}\qquad E_{2i} \tag{74}$$

$$F_{3i}=\frac{K_{3i}^2-a_i}{b_iK_{3i}}E_{3i}=\frac{G_ip}{\sqrt{2}\varepsilon_iR_i\xi_i}E_{3i} \tag{75}$$

$$F_{4i}=\frac{K_{4i}^2-a_i}{b_iK_{4i}}E_{4i}=-\frac{G_ip}{\sqrt{2}\varepsilon_iR_i\xi_i}E_{4i} \tag{76}$$

are obtained. Replacing these values in (38),(39),(40),(41),(43),(44)

$$\overline{u_1}(x,p)=e^{-a\sqrt{p}x}E_{11}+e^{a\sqrt{p}}E_{21}+e^{\frac{-px}{\sqrt{2}\xi_1}}E_{31}+e^{\frac{px}{\sqrt{2}\xi_1}}E_{41} \tag{77}$$

$$\overline{u_2}(x,p)=e^{-A\sqrt{p}x}E_{12}+e^{\frac{-px}{\sqrt{2}\xi_2}}E_{32} \tag{78}$$

$$\overline{\theta_1}(x,p)=-\frac{aG_1\sqrt{p}}{\varepsilon_1R_1}e^{-a\sqrt{p}x}E_{11}+\frac{aG_1\sqrt{p}}{\varepsilon_1R_1}e^{a\sqrt{p}x}E_{21}+\frac{G_1p}{\sqrt{2}\xi_1\varepsilon_1R_1}e^{-px/(\sqrt{2}\xi_1)}E_{31}-\frac{G_1p}{\sqrt{2}\xi_1\varepsilon_1R_1}e^{px/(\sqrt{2}\xi_1)}E_{41} \tag{79}$$

$$\overline{\theta_2}(x,p)=\frac{AG_2\sqrt{p}}{\varepsilon_2R_2}e^{-A\sqrt{p}}E_{12}+\frac{G_2p}{\sqrt{2}\xi_2\varepsilon_2R_2}e^{\frac{-px}{\sqrt{2}\xi_2}}E_{32} \tag{80}$$

$$\overline{\sigma_1}(x,p)=-\frac{p^{3/2}\rho_1}{a}e^{-a\sqrt{p}}E_{11}+\frac{p^{3/2}\rho_1}{a}e^{a\sqrt{p}}E_{21}-\sqrt{2}\xi_1\rho_1pe^{-px/(\sqrt{2}\xi_1)}E_{31}+\sqrt{2}\xi_1\rho_1pe^{px/(\sqrt{2}\xi_1)}E_{41} \tag{81}$$

$$\overline{\sigma}_2(x,p) = -\frac{p^{3/2}\rho_2}{A} e^{-A\sqrt{p}} E_{12} - \sqrt{2}\xi_2\rho_2 p e^{-px/(\sqrt{2}\xi_2)} E_{32} \tag{82}$$

where

$$a = \sqrt{\frac{T_0}{\chi_1}\left(m_1 + \frac{\varepsilon_1^2 R_1^2}{G_1}\right)},\ A = \sqrt{\frac{T_0}{\chi_2}\left(m_2 + \frac{\varepsilon_2^2 R_2^2}{G_2}\right)}$$

In that case by necessary simplification in coefficient matrix (α_{ij}) i,j=1,2,3,4,5,6;

$$\alpha_{11} = -\frac{1}{\sqrt{pa}}, \quad \alpha_{12} = \frac{1}{\sqrt{pa}}, \quad \alpha_{13} = -\frac{\sqrt{2}\xi_1}{p}, \quad \alpha_{14} = \frac{\sqrt{2}\xi_1}{p},$$

$$\alpha_{15} = \alpha_{16} = 0, \quad \alpha_{21} = -\sqrt{p}, \quad \alpha_{22} = \sqrt{p}a, \alpha_{23} = \alpha_{24} = \alpha_{25} = \alpha_{26} = 0,$$

$$\alpha_{31} = e^{-\sqrt{p}ah}, \quad \alpha_{32} = e^{\sqrt{p}ah}, \quad \alpha_{33} = e^{-\frac{ph}{\sqrt{2}\xi_1}}, \quad \alpha_{34} = e^{\frac{ph}{\sqrt{2}\xi_1}},$$

$$\alpha_{35} = -e^{-\sqrt{p}Ah}, \quad \alpha_{36} = -e^{-\frac{ph}{\sqrt{2}\xi_2}}, \quad \alpha_{41} = -\frac{e^{-\sqrt{p}ah}}{\sqrt{pa}}, \quad \alpha_{42} = \frac{e^{\sqrt{p}ah}}{\sqrt{pa}},$$

$$\alpha_{43} = -\frac{e^{-\frac{ph}{\sqrt{2}\xi_1}}}{\frac{p}{\sqrt{2}\xi_1}}, \quad \alpha_{44} = \frac{\sqrt{2}\xi_1}{p} e^{\frac{ph}{\sqrt{2}\xi_1}}, \quad \alpha_{45} = \frac{\rho_2}{\rho_1}\frac{e^{-\sqrt{p}Ah}}{\sqrt{pA}}$$

$$\alpha_{46} = \frac{\rho_2\sqrt{2}\xi_2}{\rho_1 p} e^{-\frac{ph}{\sqrt{2}\xi_2}}, \quad \alpha_{51} = -\sqrt{p}ae^{-\sqrt{p}ah}, \quad \alpha_{52} = \sqrt{p}ae^{\sqrt{p}ah},$$

$$\alpha_{53} = \alpha_{54} = 0, \quad \alpha_{55} = \frac{b_1}{b_2}\sqrt{p}Ae^{-\sqrt{p}Ah}, \quad \alpha_{56} = 0, \quad \alpha_{61} = pa^2 e^{-\sqrt{p}ah}$$

$$\alpha_{62} = pa^2 e^{\sqrt{p}ah}, \quad \alpha_{63} = \alpha_{64} = \alpha_{66} = 0, \quad \alpha_{65} = -\frac{pb_1}{\chi_1 b_2} A^2 e^{-\sqrt{p}Ah}$$

are obtained. So simplifications of solutions (51),(52),(53),(54),(55),(56) as follows.

$$E_{11} = \varepsilon_1(\chi_1 - \chi_2)\left[\left(\xi_1\xi_2\rho_1 - \xi_2\ \rho_2\right)e^{2ah\sqrt{p}} + \left(\xi_1^2\rho_1 + \xi_1\xi_2\rho_2\right)\cdot e^{-\frac{\sqrt{2}h}{\xi_1}p + 2ah\sqrt{p}}\right]\cdot \tag{83}$$
$$R_1\overline{\varphi}/\left[2aG_1\sqrt{p}\chi_1\left(-\xi_1\xi_2\rho_1 + \xi_1^2\rho_1 + \xi_2^2\rho_2 + \xi_1\xi_{2\rho_2}\right)\right]$$

$$E_{21} = \frac{\varepsilon_1(\chi_1 + \chi_2)R_1\overline{\varphi}}{2aG_1\chi_1\sqrt{p}} \tag{84}$$

$$E_{31} = \left[\xi_2\rho_1\left(A\xi_2\varepsilon_1G_2\chi_2\rho_2R_1 + a\xi_2\varepsilon_2G_1\chi_1\rho_2R_2\right)\frac{\overline{\varphi}}{\sqrt{p}} + aAG_1G_2\chi_1\left(\xi_1\rho_1 + \xi_2\rho_2\right)\frac{\overline{f}}{\sqrt{2}p}\right] /$$

$$\left[aAG_1G_2\rho_1\chi_1\left(\xi_1\xi_2\rho_1 - \xi_1^{\,2}\rho_1 - \xi_2^{\,2}\rho_2 - \xi_1\xi_2\rho_2\right)\right] \tag{85}$$

$$E_{41} = \left[\rho_1\rho_2\xi_1\xi_2\left(\varepsilon_1G_2\chi_1R_1 + \varepsilon_2G_1\chi_1R_2\right)\frac{\overline{\varphi}}{\sqrt{p}} + G_1G_2\chi_1\left(\xi_1\rho_1 - \xi_2\rho_2\right)\frac{a\overline{f}}{\sqrt{2}}\right] / \left[a\rho_1G_1G_2\chi_1\left(\xi_1\xi_2\rho_1 - \xi_1^{\,2}\rho_1 - \xi_2^{\,2}\rho_2 - \xi_1\xi_2\rho_2\right)\right] \tag{86}$$

$$E_{12} = -\frac{\varepsilon_2R_2\overline{\varphi}e^{(a+A)h\sqrt{p}}}{AG_2\sqrt{p}} \tag{87}$$

$$E_{32} = \frac{\sqrt{2}\xi_1\overline{f}}{\left(\xi_1\xi_2\rho_1 - \xi_1^{\,2}\rho_1 - \xi_2^{\,2}\rho_2 - \xi_1\xi_2\rho_2\right)p} \tag{88}$$

5.CALCULATIONS OF INVERSE TRANSFORMS

By using these let us obtain inverse transform. If

$$\overline{u}_{11}(x,p) = E_{11}e^{-a\sqrt{p}x}$$
$$\overline{u}_{21}(x,p) = E_{21}e^{a\sqrt{p}x}$$
$$\overline{u}_{31}(x,p) = E_{31}e^{-\frac{px}{\sqrt{2}\xi_1}}$$
$$\overline{u}_{41}(x,p) = E_{41}e^{\frac{px}{\sqrt{2}\xi_1}}$$

are defined accordingly then

$$\overline{u_1}(x,p) = \sum_{i=1}^{4}\overline{u}_{i1}(x,p)$$

$$u_1(x,t) = \sum_{i=1}^{4}\overline{u}_{i1}(x,t)$$

Inverse Laplace transforms of $\overline{u}_{i1}(x,p)$ are

$$u_{11}(x,t) = -\frac{\varepsilon_1(\chi_1-\chi_2)(\xi_1\xi_2\rho_1 - \xi_2^2\rho_2 + \xi_1^2\rho_1 + \xi_1\xi_2\rho_2)R_1}{2aG_1\chi_1(\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2)\sqrt{\pi}}\int_0^t \varphi(t-\tau)\frac{d\tau}{\sqrt{\tau}} \tag{89}$$

$$u_{21}(x,t) = \frac{\varepsilon_1 R_1(\chi_1+\chi_2)}{2aG_1\chi_1}\cdot\frac{1}{\sqrt{\pi}}\cdot\int_0^t \varphi(t-\tau)\frac{d\tau}{\sqrt{\tau}} \tag{90}$$

$$u_{31}(x,t) = \frac{\xi_2(A\xi_2\varepsilon_1 G_2\chi_2\rho_2 R_1 + a\xi_2\varepsilon_2 G_1\chi_1\rho_2 R_2)}{aAG_1G_2\chi_1(\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2)\sqrt{\pi}}\cdot\int_0^t \varphi(t-\tau)\frac{d\tau}{\sqrt{\tau}} + \cdot\frac{(\xi_1\rho_1+\xi_2\rho_2)}{\sqrt{2}\rho_1(\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2)}\cdot\int_0^t f(\tau)d\tau \tag{91}$$

$$u_{41}(x,t) = \frac{\rho_2\xi_1\xi_2(\varepsilon_1 G_2\chi_2 R_1 + \varepsilon_2 G_1\chi_1 R_2)}{aG_1G_2\chi_1(\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2)}\cdot\frac{1}{\sqrt{\pi}}\cdot\int_0^t \varphi(t-\tau)\frac{d\tau}{\sqrt{\tau}} + \frac{\xi_1\rho_1 - \xi_2\rho_2}{\sqrt{2}\rho_1(\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2)}\cdot\int_0^t f(\tau)d\tau \tag{92}$$

If we define

$$\bar{u}_{12}(x,p) = E_{12}e^{-A\sqrt{p}x}$$
$$\bar{u}_{32}(x,t) = E_{32}e^{-px/(\sqrt{2}\xi_1)}$$

then

$$\bar{u}_2(x,p) = \bar{u}_{12}(x,p) + \bar{u}_{32}(x,p)$$
$$u_2(x,t) = u_{12}(x,t) + u_{32}(x,t)$$

Inverse Laplace transforms of $\bar{u}_{12}(x,p)$ and $\bar{u}_{32}(x,p)$ functions are

$$u_{12}(x,t) = -\frac{\varepsilon_2 R_2}{AG_2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau}}e^{-\frac{A^2x^2}{4\tau}}\cdot\varphi(t-\tau)d\tau \tag{93}$$

$$u_{32}(x,t) = \frac{\sqrt{2}\xi_1}{\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2}\cdot\int_0^{t-\frac{x}{\sqrt{2}\xi_1}} f(\tau)d\tau \tag{94}$$

If we define

$$\bar{\theta}_{11}(x,p) = -\frac{aG_1\sqrt{p}}{\varepsilon_1 R_1}e^{-a\sqrt{p}x}E_{11}$$

$$\bar{\theta}_{21}(x,p) = \frac{aG_1\sqrt{p}}{\varepsilon_1 R_1}e^{a\sqrt{p}x}E_{21}$$

$$\bar{\theta}_{31}(x,p)=\frac{G_1 p}{\sqrt{2}\xi_1\varepsilon_1 R_1}\cdot e^{-\frac{px}{\sqrt{2\xi_1}}}E_{31}$$

$$\bar{\theta}_{41}(x,p)=-\frac{G_1 p}{\sqrt{2}\xi_1\varepsilon_1 R_1}\cdot e^{\frac{px}{\sqrt{2\xi_1}}}E_{41}$$

then

$$\bar{\theta}_1(x,p)=\sum_{i=1}^{4}\bar{\theta}_{i1}(x,p)$$

$$\theta_1(x,t)=\sum_{i=1}^{4}\theta_{i1}(x,t)$$

Inverse Laplace transforms of $\bar{\theta}_{i1}(x,p)$ i=1,2,3,4 are

$$\theta_{11}(x,t)=\frac{(\chi_1-\chi_2)(\xi_1\xi_2\rho_1-\xi_2^{\,2}\rho_2+\xi_1^{\,2}\rho_1+\xi_1\xi_2\rho_2)}{2\chi_1(\xi_1\xi_2\rho_1-\xi_1^{\,2}\rho_1-\xi_2^{\,2}\rho_2-\xi_1\xi_2\rho_2)}\varphi(t) \tag{95}$$

$$\theta_{21}(x,t)=\frac{\chi_1+\chi_2}{2\chi_1}\varphi(t) \tag{96}$$

$$\theta_{31}(x,t)=\frac{G_1(\xi_1\rho_1+\xi_2\rho_2)}{2\xi_1\varepsilon_1 R_1\rho_1(\xi_1\xi_2\rho_1-\xi_1^{\,2}\rho_1-\xi_2^{\,2}\rho_2-\xi_1\xi_2\rho_1)}f(t)+$$
$$\frac{\xi_2(A\xi_2\varepsilon_1 G_2\chi_2\rho_2 R_1+a\xi_2\varepsilon_2 G_1\chi_1\rho_2 R_2)}{\sqrt{2\pi}\xi_1\varepsilon_1 R_1 aAG_2\chi_1(\xi_1\xi_2\rho_1-\xi_1^{\,2}\rho_1-\xi_2^{\,2}\rho_2-\xi_1\xi_2\rho_2)}\int_0^t\varphi'(t-\tau)\frac{d\tau}{\sqrt{\tau}} \tag{97}$$

$$\theta_{41}(x,t)=-\frac{G_1(\xi_1\rho_1-\xi_2\rho_2)}{2\xi_1\varepsilon_1 R_1\rho_1(\xi_1\xi_2\rho_1-\xi_1^{\,2}\rho_1-\xi_2^{\,2}\rho_2-\xi_1\xi_2\rho_2)}f(t)-$$
$$\frac{\rho_2\xi_2(\varepsilon_1 G_2\chi_2 R_1+\varepsilon_2 G_1\chi_1 R_2)}{\sqrt{2\pi}\varepsilon_1 R_1 aG_2\chi_1(\xi_1\xi_2\rho_1-\xi_1^{\,2}\rho_1-\xi_2^{\,2}\rho_2-\xi_1\xi_2\rho_2)}\cdot\int_0^t\varphi'(t-\tau)\frac{d\tau}{\sqrt{\tau}} \tag{98}$$

If we define

$$\bar{\theta}_{12}(x,p)=\frac{AG_2\sqrt{p}}{\varepsilon_2 R_2}e^{-A\sqrt{p}x}E_{12}$$

$$\bar{\theta}_{32}(x,p)=\frac{G_2 p}{\sqrt{2}\xi_2\varepsilon_2 R_2}e^{-\frac{px}{\sqrt{2\xi_2}}}E_{32}$$

then

$$\overline{\theta_2}(x,p)=\bar{\theta}_{12}(x,p)+\bar{\theta}_{32}(x,p)$$
$$\theta_2(x,t)=\theta_{12}(x,t)+\theta_{32}(x,t)$$

Inverse Laplace transforms of $\bar{\theta}_{12}(x,p)$ and $\bar{\theta}_{32}(x,p)$ are

$$\theta_{12}(x,t) = -\int_0^t Erf\left(\frac{Ax}{2\sqrt{\tau}}\right) f'(t-\tau)\,d\tau \tag{99}$$

where, Erf(x)=1-erf(x) ; $\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-s^2}\,ds$

$$\theta_{32}(x,t) = \frac{G_2\xi_1}{\xi_2\varepsilon_2 R_2\left(\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2\right)} f\left(t - \frac{x}{\sqrt{2}\xi_2}\right) \tag{100}$$

If we define

$$\bar{\sigma}_{11}(x,p) = -\frac{p^{3/2}\rho_1}{a} e^{-a\sqrt{p}x} E_{11}$$

$$\bar{\sigma}_{21}(x,p) = \frac{p^{3/2}\rho_1}{a} e^{a\sqrt{p}x} E_{21}$$

$$\bar{\sigma}_{31}(x,p) = -\sqrt{2}\xi_1\rho_1 p e^{-px/(\sqrt{2}\xi_1)} E_{31}$$

$$\bar{\sigma}_{41}(x,p) = \sqrt{2}\xi_1\rho_1 p e^{px/(\sqrt{2}\xi_1)} E_{41}$$

then

$$\bar{\sigma}_1(x,p) = \sum_{i=1}^{4} \bar{\sigma}_{i1}(x,p)$$

$$\sigma_1(x,t) = \sum_{i=1}^{4} \sigma_{i1}(x,t)$$

Inverse Laplace transforms of σ_{i1} i=1,2,3,4, are

$$\sigma_{11}(x,t) = \frac{\rho_1\varepsilon_1(\chi_1-\chi_2)\left(\xi_1\xi_2\rho_1 - \xi_2^2\rho_2 + \xi_1^2\rho_1 + \xi_1\xi_2\rho_2\right)R_1}{2a^2G_1\chi_1\left(\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2\right)}\varphi'(t) \tag{101}$$

$$\sigma_{21}(x,t) = \frac{\rho_1\varepsilon_1(\chi_1+\chi_2)R_1}{2a^2G_1\chi_1}\varphi'(t) \tag{102}$$

$$\sigma_{31}(x,t) = -\frac{\xi_1(\xi_1\rho_1 + \xi_2\rho_2)}{\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2} f(t) - \frac{\sqrt{2}\xi_1\xi_2\rho_1\left(A\xi_2\varepsilon_1G_1\chi_2\rho_2R_1 + a\xi_2\varepsilon_2G_1\chi_1\rho_2R_2\right)}{aAG_1G_2\chi_1\left(\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2\right)\sqrt{\pi}} \cdot \int_0^t \varphi'(t-\tau)\frac{d\tau}{\sqrt{\tau}} \tag{103}$$

$$\sigma_{41}(x,t) = \frac{\xi_1(\xi_1\rho_1 - \xi_2\rho_2)}{\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2} f(t) + \frac{\sqrt{2}\rho_1\rho_2\xi_1^2\xi_2\left(\varepsilon_1G_2\chi_2R_1 + \varepsilon_2G_1\chi_1R_2\right)}{\sqrt{\pi}aG_1G_1\chi_1\left(\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2\right)} \cdot \int_0^t \varphi'(t-\tau)\frac{d\tau}{\tau} \tag{104}$$

If we define

$$\overline{\sigma}_{12}(x,p) = -\frac{p^{3/2}\rho_2}{A} e^{-A\sqrt{p}x} E_{12}$$

$$\overline{\sigma}_{32}(x,p) = -\sqrt{2}\xi_2\rho_2 p e^{-\frac{px}{\sqrt{2}\xi_2}} E_{32}$$

then

$$\overline{\sigma}_2(x,p) = \overline{\sigma}_{12}(x,p) + \overline{\sigma}_{32}(x,p)$$

$$\sigma_2(x,t) = \sigma_{12}(x,t) + \sigma_{32}(x,t)$$

Inverse Laplace transforms of $\overline{\sigma}_{12}(x,p)$ and $\overline{\sigma}_{32}(x,p)$ functions are

$$\sigma_{12}(x,t) = \frac{\rho_2\varepsilon_2 R_2}{A^2 G_2}\int_0^t Erf\left(\frac{Ax}{2\sqrt{\pi}}\right)\varphi''(t-\tau)d\tau \tag{105}$$

$$\sigma_{32}(x,t) = -\frac{2\xi_1\xi_2\rho_2}{\xi_1\xi_2\rho_1 - \xi_1^2\rho_1 - \xi_2^2\rho_2 - \xi_1\xi_2\rho_2} f\left(t - \frac{x}{\sqrt{2}\xi_2}\right) \tag{106}$$

6.SOLUTIONS FOR VERY LARGE VALUES OF LAPLACE PARAMETER

$$A_i^2 - 4B_i = \frac{p^4}{\xi_i^4} + \frac{T_0 m_i^2 p^2}{\chi_i^2} + \frac{\varepsilon_i^4 R_i^4 T_0^2 p^2}{G_i^2\chi_i^2} - \frac{2T_0 m_i}{\xi_i^2\chi_i} p^3 + \frac{2\varepsilon_i^2 R_i^4 T_0 p^3}{\xi_i^2 G_i \chi i_i} + \frac{2T_0^2 m_i \varepsilon_i^2 R_i^2 p^2}{G_i \chi_i^2}$$

neglecting p^2

$$A_i^2 - 4B_i = \frac{p^4}{\xi_i^4} + \frac{2p^3 T_0}{\xi_i^2\chi_i}\left(\frac{\varepsilon_i^2 R_i^2}{G_i} - m_i\right)$$

$$\sqrt{A_i^2 - 4B_i} = \frac{1}{\xi_i^2}\left[p + \frac{\xi_i^2 T_0}{2\chi_i}\left(\frac{\varepsilon_i^2 R_i^2}{G_i} - m_i\right)\right]^2$$

$$K_{1i}^2 = \frac{p^2}{2\xi_i^2} + \frac{T_0 m_i p}{2\chi_i} + \frac{\varepsilon_i^2 R_i^2 T_0 p}{2G_i\chi_i} + \frac{1}{2\xi_i^2}\left[p^2 + \frac{pT_0\xi_i^2}{\chi_i}\left(\frac{\varepsilon_i^2 R_i^2}{G_i} - m_i\right) + \left(\frac{\xi_i^2 T_0}{2\chi_i}\left(\frac{\varepsilon_i^2 R_i^2}{G_i} - m_i\right)\right)^2\right]$$

neglecting ***p***

$$K_{1i}^2 = \frac{p^2}{\xi_i^2} + \frac{\varepsilon_i^2 R_i^2 T_0 p}{G_i\chi_i} = \left(\frac{p}{\xi_i} + \frac{\varepsilon_i^2 R_i^2 T_0 \xi_i}{2G_i\chi_i}\right)^2,$$

So

$$K_{1i} = -\left(\frac{p}{\xi_i} + \frac{\varepsilon_i^{\,2} R_i^{\,2} T_0 \xi_i}{2G_i \chi_i}\right) \tag{107}$$

$$K_{2i} = -K_{1i} \tag{108}$$

Again by (25)

$$K_{3i}^{\,2} = \frac{p^2}{2\xi_i^{\,2}} + \frac{T_0 m_i p}{2\chi_i} + \frac{\varepsilon_i^{\,2} R_i^{\,2} T_0 p}{2G_i \chi_i} - \frac{1}{2\xi_i^{\,2}}\left[p^2 + \frac{pT_0 \xi_i^{\,2}}{\chi_i}\left(\frac{\varepsilon_i^{\,2} R_i^{\,2}}{G_i} - m_i\right) + \left(\frac{\xi_i^{\,2} T_0}{2\chi_i}\left(\frac{\varepsilon_i^{\,2} R_i^{\,2}}{G_i} - m_i\right)\right)^2\right.$$

neglecting p^0

$$K_{3i}^{\,2} = \frac{T_0 m_i p}{\chi_i}$$

and so

$$K_{3i}^{\,2} = -\sqrt{\frac{T_0 m_i p}{\chi_i}} \tag{109}$$

$$K_{4i}^{\,2} = \sqrt{\frac{T_0 m_i p}{\chi_i}} \tag{110}$$

are obtained.

$$F_{1i} = \frac{K_{1i}^{\,2} - a_i}{b_i K_{1i}} E_{1i} = \frac{\dfrac{p^2}{\xi_i^{\,2}} + \dfrac{\varepsilon_i^{\,2} R_i^{\,2} T_0 p}{G_i \chi_i} - \dfrac{p^2}{\xi_i^{\,2}}}{-\dfrac{\varepsilon_i R_i}{G_i}\left(\dfrac{p^2}{\xi_i} + \dfrac{\varepsilon_i^{\,2} R_i^{\,2} T_0 \xi_i}{2G_i \chi_i}\right)} E_{1i}$$

$$= -\frac{R_i T_0 p \varepsilon_i}{\dfrac{\chi_i p}{\xi_i} + \dfrac{\varepsilon_i^{\,2} R_i^{\,2} T_0 \xi_i}{2G_i}} E_{1i}$$

$$= -\frac{R_i T_0 \xi_i \varepsilon_i}{\chi_i} \cdot \frac{1}{1 + \dfrac{\varepsilon_i^{\,2} R_i^{\,2} T_0 \xi_i^{\,2}}{2G_i \chi_i p}} E_{1i}$$

Since for very large values of p $\quad \left|\dfrac{\varepsilon_i^{\,2} R_i^{\,2} T_0 \xi_i^{\,2}}{2G_i \chi_i p}\right| < 1$

$$F_{1i} = -\frac{R_i T_0 \xi_i \varepsilon_i}{\chi_i} \sum_{i=0}^{\infty} (-1)^n \left(\frac{\varepsilon_i^{\,2} R_i^{\,2} T_0 \xi_i^{\,2}}{2G_i \chi_i p}\right)^n E_{1i}$$

Taking first two terms of the series

$$F_{1i} = -\frac{R_i T_0 \xi_i \varepsilon_i}{\chi_i}\left(1 - \frac{\varepsilon_i^2 R_i^2 T_0 \xi_i^2}{2G_i \chi_i p}\right)E_{1i} \tag{111}$$

$$F_{2i} = -\left(F_{1i}/E_{1i}\right)E_{2i} \tag{112}$$

$$F_{3i} = \frac{K_{3i}^2 - a_i}{b_i K_{3i}} E_{3i} = -\frac{\frac{T_0 m_i p}{\chi_i} - \frac{p^2}{\xi_i^2}}{\frac{\varepsilon_i R_i}{G_i}\sqrt{\frac{T_0 m_i p}{\chi_i}}} E_{3i} \tag{113}$$

$$F_{4i} = \left(-F_{3i}/E_{3i}\right)E_{4i} \tag{114}$$

Replacing this values in the solutions (38),(39),(40),(41),(43),(44)

$$\overline{u_1}(x,p) = e^{-\left(b+\frac{p}{\xi_1}\right)x} E_{11} + e^{\left(b+\frac{p}{\xi_1}\right)x} E_{21} + e^{-x\sqrt{\frac{T_0 m_1 p}{\chi_1}}} E_{31} + e^{x\sqrt{\frac{T_0 m_1 p}{\chi_1}}} E_{41} \tag{115}$$

$$\overline{u_2}(x,p) = e^{-\left(B+\frac{p}{\xi_2}\right)x} E_{12} + e^{-x\sqrt{\frac{T_0 m_2 p}{\chi_2}}} E_{32} \tag{116}$$

$$\overline{\theta_1}(x,p) = -\frac{\varepsilon_1 R_1 T_0 \xi_1}{\chi_1}\left(1 - \frac{b\xi_1}{p}\right)e^{-\left(b+\frac{p}{\xi_1}\right)x} E_{11} + \frac{\varepsilon_1 R_1 T_0 \xi_1}{\chi_1}\left(1 - \frac{b\xi_1}{p}\right)e^{\left(b+\frac{p}{\xi_1}\right)x} E_{21} - \frac{\frac{T_0 m_1 p}{\chi_1} - \frac{p^2}{\xi_1^2}}{\frac{\varepsilon_1 R_1}{G_1}\sqrt{\frac{T_0 m_1 p}{\chi_1}}} e^{-x\sqrt{\frac{T_0 m_1 p}{\chi_1}}} E_{31} + \frac{\frac{T_0 m_1 p}{\chi_1} - \frac{p^2}{\xi_1^2}}{\frac{\varepsilon_1 R_1}{G_1}\sqrt{\frac{T_0 m_1 p}{\chi_1}}} e^{x\sqrt{\frac{T_0 m_1 p}{\chi_1}}} E_{41} \tag{117}$$

$$\overline{\theta_2}(x,p) = -\frac{\varepsilon_2 R_2 T_0 \xi_2}{\chi_2}\left(1 - \frac{B\xi_2}{p}\right)e^{-\left(\frac{p}{\xi_2}+B\right)x} E_{12} - \frac{\frac{T_0 m_2 p}{\chi_2} - \frac{p^2}{\xi_2^2}}{\frac{\varepsilon_2 R_2}{G_2}\sqrt{\frac{T_0 m_2 p}{\chi_2}}} e^{-x\sqrt{\frac{T_0 m_2 p}{\chi_2}}} E_{32} \tag{118}$$

$$\overline{\sigma_1}(x,p) = -\frac{\rho_1 p^2}{\left(b + \frac{p}{\xi_1}\right)} e^{-\left(b+\frac{p}{\xi_1}\right)x} E_{11} + \frac{\rho_1 p^2}{\left(b + \frac{p}{\xi_1}\right)} e^{\left(b+\frac{p}{\xi_1}\right)x} E_{21} - \frac{\rho_1 p^2}{\sqrt{\frac{T_0 m_1 p}{\chi_1}}} \cdot e^{-x\sqrt{\frac{T_0 m_1 p}{\chi_1}}} E_{31} + \frac{\rho_1 p^2}{\sqrt{\frac{T_0 m_1 p}{\chi_1}}} e^{x\sqrt{\frac{T_0 m_1 p}{\chi_1}}} E_{41} \tag{119}$$

$$\overline{\sigma_2}(x,p) = -\frac{\rho_2 p^2}{B+\frac{p}{\xi_2}} e^{-\left(B+\frac{p}{\xi_2}\right)} E_{12} + \frac{\rho_2 p^2}{\sqrt{\frac{T_0 m_2 p}{\chi_2}}} e^{-x\sqrt{\frac{T_0 m_2 p}{\chi_2}}} E_{32} \tag{120}$$

where,

$$b = \frac{\varepsilon_1^{\,2} R_1^{\,2} T_0 \xi_1}{2G_1 \chi_1} \quad , \quad B = \frac{\varepsilon_2^{\,2} R_2^{\,2} T_0 \xi_2}{2G_2 \chi_1}$$

Making necessary simplifications in coefficient matrix (α_{ij}), $i,j = 1,2,3,4,5,6$, for large values of p.

$$\alpha_{11} = -\frac{\xi_1}{p+b\xi_1} \quad , \quad \alpha_{12} = 0 \quad , \quad \alpha_{13} = -\sqrt{\frac{\chi_1}{T_0 m_1 p}} \quad , \quad \alpha_{14} = \alpha_{15} = \alpha_{16} = 0$$

$$\alpha_{21} = 2b\left(\frac{b\xi_1}{p} - 1\right) \quad , \quad \alpha_{22} = 0 \quad , \quad \alpha_{23} = \left(\frac{p}{\xi_1^{\,2}} - \frac{T_0 m_1 p}{\chi_1}\right)\sqrt{\frac{\chi_1}{T_0 m_1 p}}$$

$$\alpha_{24} = \alpha_{25} = \alpha_{26} = 0 \quad , \quad \alpha_{31} = 0 \quad , \quad \alpha_{32} = e^{\frac{p}{\xi_1} + bh} \quad , \quad \alpha_{33} = 0$$

$$\alpha_{34} = e^{h\sqrt{\frac{T_0 m_1 p}{\chi_1}}} \quad , \quad \alpha_{35} = -e^{-\frac{ph}{\xi_2} - Bh} \quad , \quad \alpha_{36} = e^{-h\sqrt{\frac{T_0 m_2 p}{\chi_2}}} \quad , \quad \alpha_{41} = 0$$

$$\alpha_{42} = \frac{e^{\frac{ph}{\xi_1} + bh}}{\frac{p}{\xi_1} + b} \quad , \quad \alpha_{43} = 0 \quad , \quad \alpha_{44} = \frac{e^{h\sqrt{\frac{T_0 m_1 p}{\chi_1}}}}{\sqrt{\frac{T_0 m_1 p}{\chi_1}}} \quad , \quad \alpha_{45} = \frac{\rho_2}{\rho_1}\frac{e^{-\frac{ph}{\xi_2} - Bh}}{\frac{p}{\xi_2} + B}$$

$$\alpha_{46} = \frac{\rho_2}{\rho_1}\frac{e^{h\sqrt{\frac{T_0 m_2 p}{\chi_2}}}}{\sqrt{\frac{T_0 m_2 p}{\chi_2}}} \quad , \quad \alpha_{51} = 0 \quad , \quad \alpha_{52} = 2b\left(1 - \frac{b\xi_1}{p}\right)e^{\frac{ph}{\xi_1} + bh}$$

$$\alpha_{53} = \frac{\frac{p^2}{\xi_1^{\,2}} - \frac{T_0 m_1 p}{\chi_1}}{\sqrt{\frac{T_0 m_1 p}{\chi_1}}} e^{h\sqrt{\frac{T_0 m_1 p}{\chi_1}}} \quad , \quad \alpha_{54} = \frac{\frac{T_0 m_1 p}{\chi_1} - \frac{p^2}{\xi_1^{\,2}}}{\sqrt{\frac{T_0 m_1 p}{\chi_1}}} e^{h\sqrt{\frac{T_0 m_1 p}{\chi_1}}} ,$$

$$\alpha_{55} = \frac{\varepsilon_1 \varepsilon_2 \xi_2 R_1 R_2 T_0}{G_1 \chi_2}\left(1 - \frac{\xi_2 B}{p}\right)e^{-\frac{hp}{\xi_2} - Bh} \qquad \alpha_{56} = \frac{\varepsilon_1 R_1 G_2\left(\frac{T_0 m_2 p}{\chi_2} - \frac{p^2}{\xi_2^{\,2}}\right)}{\varepsilon_2 R_2 G_1 \sqrt{\frac{T_0 m_2 p}{\chi_2}}} e^{-h\sqrt{\frac{T_0 m_2 p}{\chi_2}}}$$

$$\alpha_{61} = 0 , \alpha_{62} = \frac{\varepsilon_1^{\,2} R_1^{\,2} T_0 p}{G_1 \chi_1} e^{\frac{ph}{\xi_1} + bh} \quad , \quad \alpha_{63} = \left(\frac{T_0 m_1 p}{\chi_2} - \frac{p^2}{\xi_1^{\,2}}\right)e^{-h\sqrt{\frac{T_0 m_1 p}{\chi_1}}} \quad ,$$

$$\alpha_{64} = \left(\frac{T_0 m_1 p}{\chi_1} - \frac{p^2}{\xi_1^{\,2}}\right)e^{h\sqrt{\frac{T_0 m_1 p}{\chi_1}}} \quad , \quad \alpha_{65} = -\frac{\varepsilon_1 R_1 T_0 \varepsilon_2 R_2 p}{G_1 \chi_1} e^{-\frac{ph}{\xi_2} - Bh}$$

$$\alpha_{66} = -\frac{\chi_2 \varepsilon_1 R_1 G_2}{\chi_1 \varepsilon_2 R_2 G_1}\left(\frac{T_0 m_2 p}{\chi_2} - \frac{p^2}{\xi_2^{\,2}}\right) e^{h\sqrt{\frac{T_0 m_2 p}{\chi_2}}}$$

In that case let us calculate solutions of (51),(52),(53),(54),(55),(56).
If we expend Δ, Δ_1, Δ_2, Δ_3, Δ_4, Δ_5, Δ_6 determinants with respect to first two rows

$$\Delta = -(\alpha_{11}\alpha_{23} - \alpha_{21}\alpha_{13})M_{13}$$

$$\Delta_1 = -\left(\frac{\overline{f}}{\rho_1 p^2}\alpha_{23} - \overline{\varphi} h_1 \alpha_{13}\right)M_{13}$$

$$\Delta_2 = \left(\alpha_{11}\overline{\varphi} h_1 - \frac{\overline{f}}{\rho_1 p^2}\alpha_{21}\right)M_{12}$$

$$\Delta_3 = -\left(\alpha_{11}\overline{\varphi} h_1 - \alpha_{21}\frac{\overline{f}}{\rho_1 p^2}\right)M_{13}$$

$$\Delta_4 = \left(\alpha_{11}\overline{\varphi} h_1 - \alpha_{21}\frac{\overline{f}}{\rho_1 p^2}\right)M_{14}$$

$$\Delta_5 = -\left(\alpha_{11}\overline{\varphi} h_1 - \alpha_{21}\frac{\overline{f}}{\rho_1 p^2}\right)M_{15}$$

$$\Delta_6 = \left(\alpha_{11}\overline{\varphi} h_1 - \alpha_{21}\frac{\overline{f}}{\rho_1 p^2}\right)M_{16}$$

are obtained. Since exponents are the same in each column of the matrix 4x6 which consists of last four rows of the matrix (α_{ij}) i,j=1,2,3,4,5,6, using properties of determinant

$$M_{13} = e^{\frac{ph}{\xi_1} + bh + h\sqrt{\frac{T_0 m_1 p}{\chi_1}} - \frac{ph}{\xi_2} - Bh - h\sqrt{\frac{T_0 m_2 p}{\chi_2}}}\ \overset{*}{M}_{13}$$

$$M_{12} = e^{h\sqrt{\frac{T_0 m_1 p}{\chi_1}} - \frac{ph}{\xi_2} - Bh - h\sqrt{\frac{T_0 m_1 p}{\chi_1}} - h\sqrt{\frac{T_0 m_2 p}{\chi_2}}}\ \overset{*}{M}_{12}$$

$$M_{14} = e^{\frac{ph}{\xi_1} + bh - \frac{ph}{\xi_2} - Bh - h\sqrt{\frac{T_0 m_1 p}{\chi_1}} - h\sqrt{\frac{T_0 m_2 p}{\chi_2}}}\ \overset{*}{M}_{14}$$

$$M_{15} = e^{\frac{ph}{\xi_1} + bh - h\sqrt{\frac{T_0 m_2 p}{\chi_2}}}\ \overset{*}{M}_{15}$$

$$M_{16} = e^{\frac{ph}{\xi_1} + bh - \frac{ph}{\xi_2} - Bh}\ \overset{*}{M}_{16}$$

are obtained.

If we define

$$Z = -\frac{\alpha_{11}\overline{\varphi}h_1 - \frac{\overline{f}}{\rho_1 p^2}\alpha_{21}}{\alpha_{11}\alpha_{23} - \alpha_{21}\alpha_{13}}$$

after same simplifications, we obtain

$$Z = \frac{\xi_1\chi_1\left(2h^3\xi_1^{\ 2}\overline{f}G_1 - 2h\overline{f}G_1 p^2 + \xi_1\varepsilon_1\rho_1 R_1\overline{\varphi}p^3\right)}{G_1\rho_1\sqrt{\frac{\chi_1}{T_0 m_1}}\,p^{3/2}\left(2h^3\xi_1\chi_1 - 2h\xi_1\chi_1 p^2 - \chi_1 p^3 + \xi_1^{\ 2}m_1 T_0 p^2\right)}$$

So,

$$E_{11} = \frac{\frac{\overline{f}}{\rho_1 p^2}\alpha_{23} - \overline{\varphi}h_1\alpha_{13}}{\alpha_{11}\alpha_{23} - \alpha_{21}\alpha_{13}}$$

$$= \frac{(h\xi_1 + p)\left(\overline{f}G_1\chi_1 p - \xi_1^{\ 2}\overline{f}G_1 m_1 T_0 + \xi_1^{\ 2}\varepsilon_1\chi_1\rho_1 R_1\overline{\varphi}p\right)}{\xi_1 G_1\rho_1\left(2h^3\xi_1^{\ 3}\chi_1 - 2h\xi_1\chi_1 p^2 - \chi_1 p^3 + \xi_1^{\ 2}m_1 T_0 p^2\right)} \tag{121}$$

$$E_{21} = Ze^{-\left(\frac{ph}{\xi_1} + bh\right) - h\sqrt{\frac{T_0 m_1 p}{\chi_1}}}\ \frac{\overset{*}{M}_{12}}{\overset{*}{M}_{13}} \tag{122}$$

$$E_{31} = -Z \tag{123}$$

$$E_{41} = Ze^{-2h\sqrt{\frac{T_0 m_1 p}{\chi_1}}}\ \frac{\overset{*}{M}_{14}}{\overset{*}{M}_{13}} \tag{124}$$

$$E_{12} = -Ze^{\frac{ph}{\xi_2} + Bh - h\sqrt{\frac{T_0 m_1 p}{\chi_1}}}\ \frac{\overset{*}{M}_{15}}{\overset{*}{M}_{13}} \tag{125}$$

$$E_{32} = Ze^{h\sqrt{\frac{T_0 m_2 p}{\chi_2}} - h\sqrt{\frac{T_0 m_1 p}{\chi_1}}}\ \frac{\overset{*}{M}_{12}}{\overset{*}{M}_{13}} \tag{126}$$

After calculating $\overset{*}{M}_{1j}$, j=2,3,4,5,6, and making some simplifications we have

$$\overset{*}{M}_{13} = \frac{p^{3/2}}{\xi_1^{\ 2}\xi_2^{\ 2}\varepsilon_2 G_1\rho_1 R_2 T_0\sqrt{\chi_1\chi_2 m_1 m_2}}\left(\phi_1 p - \phi_2\right)$$

$$\phi_1 = \varepsilon_1 G_2\chi_2 R_1\left(\xi_1\rho_1\sqrt{T_0 m_1\chi_1} + \xi_2\rho_2\sqrt{T_0 m_1\chi_1} + \xi_1\rho_1\sqrt{T_0 m_2\chi_2} + \xi_2\rho_2\sqrt{T_0 m_2\chi_2}\right)$$

$$\phi_2 = G_2\varepsilon_1 R_1 T_0\left(\xi_1\xi_2^{\,2} m_2\rho_1\sqrt{T_0 m_1\chi_1} + \xi_2^{\,3} m_2\rho_2\sqrt{T_0 m_1\chi_1} + \xi_1^{\,3} m_1\rho_1\chi_2\sqrt{\frac{T_0 m_1}{\chi_1}} + \right.$$

$$\left. \xi_1^{\,2}\xi_2 m_1\rho_2\chi_2\sqrt{\frac{T_0 m_1}{\chi_1}} + \frac{\xi_1^{\,3}\chi_2^{\,2} m_1\rho_1}{\chi_1}\sqrt{\frac{T_0 m_2}{\chi_2}} + \xi_1\xi_2^{\,2} m_2\rho_1\sqrt{T_0 m_2\chi_2} + \xi_2^{\,3} m_2\rho_2\sqrt{T_0 m_2\chi_2} \right)$$

$$\dot{M}_{12} = \frac{-p + \dfrac{\xi_1^{\,2} m_1 T_0}{\chi_1}}{\chi_1\xi_1^{\,4}\xi_2^{\,2}\varepsilon_2 G_1 m_1\rho_1 R_2 T_0\sqrt{\dfrac{T_0 m_2}{\chi_2}}}\left(\mu_1 p^2 + \mu_2 p^{3/2} + \mu_3 p \right)$$

where

$$\mu_1 = \xi_1^{\,2}\varepsilon_1 G_2\chi_2\rho_1 R_1\sqrt{T_0 m_1\chi_1} - 2\xi_2^{\,2}\varepsilon_2 G_1\chi_2\rho_2 R_2\sqrt{T_0 m_1\chi_1} - \xi_1^{\,2}\varepsilon_1 G_2\chi_2\rho_1 R_1\sqrt{T_0 m_2\chi_2}$$

$$\mu_2 = \xi_1^{\,2}\xi_2\varepsilon_1 G_2\chi_2 m_1\rho_2 R_1 T_0 - \xi_1^{\,2}\xi_2\varepsilon_1 G_2\chi_2^{\,2}\rho_2 R_1 T_0\sqrt{\frac{m_1 m_2}{\chi_1\chi_2}} - 2\xi_2^{\,3}\varepsilon_2 G_1\chi_1\chi_2\rho_2 R_2 T_0\sqrt{\frac{m_1 m_2}{\chi_1\chi_2}}$$

$$\mu_3 = -\xi_1^{\,2}\xi_2^{\,2}\varepsilon_1 G_2 m_2\rho_1 R_1 T_0\sqrt{T_0 m_1\chi_1} + \xi_1^{\,2}\xi_2^{\,2}\varepsilon_2 G_2\chi_1\chi_2\rho_2 R_2\left(\frac{T_0 m_1}{\chi_1}\right)^{3/2} + \xi_1^{\,2}\xi_2^{\,3}\varepsilon_1 G_2\chi_2^{\,2}\rho_1 R_1\left(\frac{T_0 m_2}{\chi_2}\right)$$

$$\dot{M}_{14} = \frac{-p + \dfrac{\xi_1^{\,2} m_1 T_0}{\chi_1}}{\xi_1^{\,2}\xi_2^{\,2}\varepsilon_2 G_1\rho_1 R_2 T_0\sqrt{m_1 m_2\chi_1\chi_2}}\left(\lambda_1 p^{3/2} + \lambda_2\sqrt{p} \right)$$

where

$$\lambda_1 = -\xi_1\varepsilon_1 G_1\chi_1\chi_2\rho_1 R_1\sqrt{\frac{T_0 m_1}{\chi_1}} - \xi_2\varepsilon_1 G_2\chi_1\chi_2\rho_2 R_1\sqrt{\frac{T_0 m_1}{\chi_1}} +$$

$$\xi_1\varepsilon_1 G_2\chi_2^{\,2}\rho_1 R_1\sqrt{\frac{T_0 m_2}{\chi_2}} + \xi_2\varepsilon_1 G_2\chi_2^{\,2}\rho_2 R_1\sqrt{\frac{T_0 m_2}{\chi_2}}$$

$$\lambda_2 = \xi_1\xi_2^{\,2}\varepsilon_1 G_2\chi_1 m_2\rho_1 R_1 T_0\sqrt{\frac{T_0 m_1}{\chi_1}} + \xi_2^{\,3}\varepsilon_1 G_2\chi_1 m_2\rho_2 R_1 T_0\sqrt{\frac{T_0 m_1}{\chi_1}} -$$

$$\xi_1\xi_2^{\,2}\varepsilon_1 G_2\chi_2\rho_1 R_1\left(\frac{T_0 m_2}{\chi_2}\right)^{3/2} - \xi_2^{\,3}\varepsilon_1 G_2\chi_2^{\,2}\rho_2 R_1\left(\frac{T_0 m_2}{\chi_2}\right)^{3/2}$$

$$\dot{M}_{15}=\frac{-p+\dfrac{\xi_1^{\,2}m_1T_0}{\chi_1}}{\xi_1^{\,4}\xi_2^{\,2}\varepsilon_2G_1m_1\rho_1R_2T_0\sqrt{\dfrac{T_0m_2}{\chi_2}}}\left(\gamma_1p^2+\gamma_2p^{3/2}+\gamma_3p\right)$$

$$\gamma_1=-\xi_1^{\,2}\varepsilon_1G_2\rho_1R_1\sqrt{T_0m_1\chi_1}+2\xi_2^{\,2}\varepsilon_2G_1\rho_2R_2\sqrt{T_0m_1\chi_1}+\xi_1^{\,2}\varepsilon_1G_2\rho_1R_1\sqrt{T_0m_2\chi_2}$$

$$\gamma_2=\xi_1^{\,3}\varepsilon_1G_2m_1\rho_1R_1T_0-\xi_1^{\,3}\varepsilon_1G_2\chi_2\rho_1R_1T_0\sqrt{\frac{m_1m_2}{\chi_1\chi_2}}-2\xi_1\xi_2^{\,2}\varepsilon_1G_1\chi_1\rho_1R_2T_0\sqrt{\frac{m_1m_2}{\chi_1\chi_2}}$$

$$\gamma_3=\frac{\chi_1}{\chi_2}\xi_1^{\,2}\xi_2^{\,2}\varepsilon_1G_2m_2\rho_1R_1T_0\sqrt{\frac{T_0m_1}{\chi_1}}-2\xi_1^{\,2}\xi_2\varepsilon_2G_1\chi_1\rho_2R_2\left(\frac{T_0m_1}{\chi_1}\right)^{3/2}-\xi_1^{\,2}\xi_2^{\,2}\varepsilon_1G_2\chi_2\rho_1R_1\left(\frac{T_0m_2}{\chi_2}\right)^{3/2}$$

and

$$\dot{M}_{16}=\frac{2\chi_1(\xi_1\rho_1+\xi_2\rho_2)\sqrt{\dfrac{T_0m_1}{\chi_1}}}{\xi_1^{\,4}m_1\rho_1T_0}\left(-p+\frac{\xi_1^{\,2}m_1T_0}{\chi_1}\right)\left(p^{3/2}-\frac{T_0m_1\xi_1^{\,2}}{\chi_1}\sqrt{p}\right)$$

Moreover,

$$E_{11}=-\frac{G_1\overline{\Gamma}+\xi_1^{\,2}\varepsilon_1\rho_1R_1\overline{\varphi}}{\xi_1G_1\rho_1}\left(\frac{1}{p}+\frac{\xi_1^{\,2}m_1T_0}{\chi_1p^2}\right)\tag{127}$$

$$E_{21}=\frac{-2bG_1\xi_1\chi_2\overline{f}+\xi_1^{\,2}\varepsilon_1\rho_1R_1\chi_2\overline{\varphi}p}{G_1\rho_1\chi_1\xi_1^{\,2}\phi_1}e^{Bh}e^{\frac{hb}{\xi_1}-h\sqrt{\frac{T_0m_1p}{\chi_1}}}\left[\frac{1}{p^2}\mu_1+\frac{1}{p^{5/2}}\mu_2+\frac{1}{p^3}(\mu_3+\frac{\mu_1\phi_2}{\phi_1})\right]\tag{128}$$

$$E_{31}=\frac{-2bG_1\xi_1\overline{f}+\xi_1^{\,2}\varepsilon_1\rho_1R_1\overline{\varphi}p}{G_1\rho_1\sqrt{\dfrac{\chi_1}{m_1T_0}}}\left(\frac{1}{p^{5/2}}+\frac{\xi_1^{\,2}m_1T_0}{\chi_1p^{7/2}}\right)\tag{129}$$

$$E_{11}=\frac{-2bG_1\xi_1\overline{f}+\xi_1^{\,2}\varepsilon_1\rho_1\overline{\varphi}p}{G_1\rho_1\sqrt{\dfrac{\chi_1}{m_1T_0}}\phi_1}e^{-2h\sqrt{\frac{T_0m_1p}{\chi_1}}}\cdot\left[\frac{\lambda_1}{p^{5/2}}+\frac{1}{p^{7/2}}\left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}\right)\right]\tag{130}$$

$$E_{12}=\frac{2b\xi_1G_1\chi_2\overline{f}-\xi_1^{\,2}\varepsilon_1\rho_1R_1\chi_2\overline{\varphi}p}{G_1\rho_1\xi_1^{\,2}\phi_1}e^{\frac{ph}{\xi_2}+Bh-h\sqrt{\frac{T_0m_1p}{\chi_1}}}\cdot\left[\frac{\gamma_1}{p^2}+\frac{\gamma_2}{p^{5/2}}+\frac{1}{p^3}\left(\gamma_3+\frac{\gamma_1\phi_2}{\phi_1}\right)\right]$$

(131)

$$E_{32}=\frac{-4bG_1\xi_1(\xi_1\rho_1+\xi_2\rho_2)\overline{f}+2\xi_1^{2}\varepsilon_1\rho_1R_1(\xi_1\rho_1+\xi_2\rho_2)\overline{\varphi}p}{\rho_1\xi_1^{2}\phi_1}T_0\xi_2^{2}\varepsilon_2R_2\cdot$$

$$\sqrt{m_1m_2\chi_1\chi_2}\cdot\left[\frac{1}{p^{5/2}}+\frac{1}{p^{7/2}}\left(\frac{\phi_2}{\phi_1}-\frac{T_0m_1\xi_{12}}{\chi_1}\right)\right]\cdot e^{h\sqrt{\frac{T_0m_2p}{\chi_2}}-h\sqrt{\frac{T_0m_1p}{\chi_1}}} \tag{132}$$

7.CALCULATIONS OF IN INVERSE TRANSFORM

By using above formulas, let us obtain inverse Laplace transforms of solutions.

$$\overline{u}_{11}(x,p)=e^{-\left(b+\frac{p}{\xi_1}\right)x}E_{11}$$

$$\overline{u}_{21}(x,p)=e^{\left(b+\frac{p}{\xi_1}\right)x}E_{21}$$

$$\overline{u}_{31}(x,p)=e^{-x\sqrt{\frac{T_0m_1p}{\chi_1}}}E_{31}$$

$$\overline{u}_{41}(x,p)=e^{x\sqrt{\frac{T_0m_1p}{\chi_1}}}E_{41}$$

are defined accordingly then

$$\overline{u}_1(x,p)=\sum_{i=1}^{4}\overline{u}_{i1}(x,p)$$

$$u_1(x,t)=\sum_{i=1}^{4}\overline{u}_{i1}(x,t)$$

By using inverse Laplace transform theory and Convolution theory, inverse Laplace transforms of $\overline{u}_{i1}(x,p)$ as follows.

$$u_{11}(x,t)=-\frac{e^{-bx}}{\xi_1\rho_1}\left[\int_0^{t-x/\xi_1}f(\tau)d\tau+\frac{\xi_1^{2}m_1T_0}{\chi_1}\int_0^{t-x/\xi_1}f(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau+\right.$$

$$\left.\frac{\xi_1^{4}m_1^{2}T_0^{2}}{2\chi_1^{2}}\int_0^{t-x/\xi_1}f(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)^2d\tau\right]-\frac{\xi_1\varepsilon_1R_1}{G_1}\left[\int_0^{t-x/\xi_1}\varphi(\tau)d\tau+\right.$$

$$\left.\frac{\xi_1^{2}m_1T_0}{\chi_1}\int_0^{t-x/\xi_1}\varphi(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau+\frac{\xi_1^{4}m_1^{2}T_0^{2}}{2\chi_1^{2}}\cdot\int_0^{t-x/\xi_1}\varphi(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)^2d\tau\right] \tag{133}$$

$$u_{21}(x,t)=-\frac{2b\chi_2}{\rho_1\chi_1\xi_1\phi_1}e^{b(h-x)}\left[\mu_1\int_0^{t-(h-x)/\xi_1}f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)d\tau+\right.$$

$$\frac{1}{2}\left(\mu_2+\frac{\mu_1\phi_2}{\phi_1}\right)\int_0^{t-(h-x)/\xi_1}f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)^2d\tau+$$

$$\frac{\mu_2}{\Gamma\left(\frac{5}{2}\right)}\int_0^{t-(h-x)/\xi_1} f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)^{3/2} d\tau\Bigg] + \frac{\varepsilon_1 R_1 \chi_2}{G_1\chi_1\phi_1}\cdot$$

$$\Bigg[\mu_1\int_0^{t-(h-x)/\xi_1}\varphi(\tau)d\tau + \left(\mu_3+\frac{\mu_1\phi_2}{\phi_1}\right)\int_0^{t-(h-x)/\xi_1}\varphi(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)d\tau +$$

$$\frac{\mu_2}{\Gamma\left(\frac{3}{2}\right)}\int_0^{t-(h-x)/\xi_1}\varphi(\tau)\sqrt{t-\frac{h-x}{\xi_1}-\tau}\, d\tau\Bigg] \tag{134}$$

$$u_{31}(x,t) = -\frac{2b\xi_1}{\rho_1\sqrt{\frac{\chi_1}{m_1 T_0}}}\left[\int_0^t f(\tau)(t-\tau)d\tau + \frac{\xi_1{}^2 m_1 T_0}{2\chi_1}\int_0^t f(\tau)(t-\tau)^2 d\tau\right]*$$

$$\frac{1}{\sqrt{\pi t}}e^{-\frac{T_0 m_1 x^2}{4\chi_1 t}} + \frac{\xi_1{}^2\varepsilon_1 R_1}{G_1\sqrt{\frac{\chi_1}{m_1 T_0}}}\left[\int_0^t\varphi(\tau)d\tau + \frac{\xi_1{}^2 m_1 T_0}{\chi_1}\int_0^t\varphi(\tau)(t-\tau)d\tau\right]*$$

$$\frac{1}{\sqrt{\pi t}}e^{-\frac{T_0 m_1 x^2}{4\chi_1 t}} \tag{135}$$

where

$$F(t)*\varphi(t) = \int_0^t F(t-s)\varphi(s)ds$$

which is called convolution of F(t) and φ (t)

$$u_{41}(x,t) = -\frac{2b\xi_1}{\rho_1\sqrt{\frac{\chi_1}{m_1 T_0}}\phi_1}\left[\lambda_1\int_0^t f(\tau)(t-\tau)d\tau + \frac{1}{2}\left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}\right)\cdot\right.$$

$$\left.\int_0^t f(\tau)(t-\tau)^2 d\tau\right]*\frac{1}{\sqrt{\pi t}}e^{-\frac{T_0 m_1(2h-x)^2}{4\chi_1 t}} + \frac{\xi_1{}^2\varepsilon_1 R_1}{G_1\sqrt{\frac{\chi_1}{m_1 T_0}}\phi_1}\left[\lambda_1\int_0^t\varphi(\tau)d\tau + \right.$$

$$\left.\left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}\right)\int_0^t\varphi(\tau)(t-\tau)d\tau\right]*\frac{1}{\sqrt{\pi t}}e^{-\frac{T_0 m_1(2h-x)^2}{4\chi_1 t}} \tag{136}$$

If we define

$$\bar{u}_{12}(x,p) = e^{-\left(B+\frac{p}{\xi_2}\right)x}E_{12}$$

$$\bar{u}_{22}(x,p) = e^{-x\sqrt{\frac{T_0 m_2 p}{\chi_2}}}E$$

then

$$\bar{u}_2(x,p) = \bar{u}_{12}(x,p) + \bar{u}_{32}(x,p)$$
$$u_2(x,t) = \bar{u}_{12}(x,t) + \bar{u}_{32}(x,t)$$

Inverse Laplace transforms of $\bar{u}_{12}(x,p)$ and $\bar{u}_{32}(x,p)$ has been calculated as follows.

$$u_{12}(x,t) = \frac{2h\chi_2}{\rho_1\xi_1\phi_1} e^{B(x-h)} \Bigg[\gamma_1 \int_0^{t-(x-h)/\xi_2} f(\tau)\left(t - \frac{x-h}{\xi_2} - \tau\right) d\tau +$$
$$\frac{\gamma_2}{\Gamma\left(\frac{3}{2}\right)} \int_0^{t-(x-h)/\xi_2} f(\tau)\left(t - \frac{x-h}{\xi_2} - \tau\right)^{3/2} d\tau + \frac{1}{2}\left(\gamma_3 + \frac{\gamma_1\phi_2}{\phi_1}\right) \cdot$$
$$\int_0^{t-(x-h)/\xi_2} f(\tau)\left(t - \frac{x-h}{\xi_2} - \tau\right)^2 d\tau \Bigg] - \frac{\varepsilon_1 R_1 \chi_2}{G_1\phi_1}\Bigg[\gamma_1 \int_0^{t-(x-h)/\xi_2} \varphi[\tau] d\tau +$$
$$\frac{\gamma_2}{\Gamma\left(\frac{1}{2}\right)} \int_0^{t-(x-h)/\xi_2} \varphi(\tau)\sqrt{t - \frac{x-h}{\xi_2} - \tau}\, d\tau + \left(\gamma_3 + \frac{\gamma_1\phi_2}{\phi_1}\right)$$
$$\int_0^{t-(x-h)/\xi_2} \varphi(\tau)\left(t - \frac{x-h}{\xi_2} - \tau\right) dt \Bigg] \tag{137}$$

$$u_{32}(x,t) = -\frac{4hG_1(\xi_1\rho_1 + \xi_2\rho_2)T_0\xi_2^2\varepsilon_2 R_2\sqrt{m_1 m_2 \chi_1 \chi_2}}{\rho_1\xi_1\phi_1} \cdot$$
$$\left[\int_0^t f(\tau)(t-\tau)d\tau + \frac{1}{2}\left(\frac{\phi_2}{\phi_1} - \frac{T_0 m_1 \xi_1^2}{\chi_1}\right)\int_0^t f(\tau)(t-\tau)^2 d\tau\right] *$$
$$\frac{1}{\sqrt{\pi t}} e^{-\frac{\left[\sqrt{\frac{T_0 m_2}{\chi_2}}(x-h)+h\sqrt{\frac{T_0 m_1}{\chi_1}}\right]^2}{4t}} +$$
$$\frac{2\varepsilon_1 R_1(\xi_1\rho_1 + \xi_2\rho_2)T_0\xi_2^2\varepsilon_2 R_2\sqrt{m_1 m_2 \chi_1 \chi_2}}{\phi_1} \cdot$$
$$\left[\int_0^t \varphi(\tau)d\tau + \left(\frac{\phi_2}{\phi_1} - \frac{T_0 m_1 \xi_1^2}{\chi_1}\right)\int_0^t \varphi(\tau)(t-\tau)d\tau\right] *$$
$$\frac{1}{\sqrt{\pi t}} e^{-\frac{\left[\sqrt{\frac{T_0 m_2}{\chi_2}}(x-h)+h\sqrt{\frac{T_0 m_1}{\chi_1}}\right]^2}{4t}} \tag{138}$$

If we define

$$\bar{\theta}_{11}(x,p) = -\frac{\varepsilon_1 R_1 T_0 \xi_1}{\chi_1}\left(1-\frac{b\xi_1}{p}\right)e^{-\left(b+\frac{p}{\xi_1}\right)x}E_{11}$$

$$\bar{\theta}_{21}(x,p) = -\frac{\varepsilon_1 R_1 T_0 \xi_1}{\chi_1}\left(1-\frac{b\xi_1}{p}\right)e^{\left(b+\frac{p}{\xi_1}\right)x}E_{21}$$

$$\bar{\theta}_{31}(x,p) = -\frac{\frac{T_0 m_1 p}{\chi_1}-\frac{p^2}{\xi_1^{\,2}}}{\frac{\varepsilon_1 R_1}{G_1}\sqrt{\frac{T_0 m_1 p}{\chi_1}}}e^{-\sqrt{\frac{T_0 m_1 p}{\chi_1}}x}E_{31}$$

$$\bar{\theta}_{41}(x,p) = -\frac{\frac{T_0 m_1 p}{\chi_1}-\frac{p^2}{\xi_1^{\,2}}}{\frac{\varepsilon_1 R_1}{G_1}\sqrt{\frac{T_0 m_1 p}{\chi_1}}}e^{\sqrt{\frac{T_0 m_1 p}{\chi_1}}x}E_{41}$$

then

$$\bar{\theta}_1(x,p) = \sum_{i=1}^{4}\bar{\theta}_{i1}(x,p)$$

$$\theta_1(x,t) = \sum_{i=1}^{4}\bar{\theta}_{i1}(x,t)$$

Inverse Laplace transforms of $\bar{\theta}_{i1}(x,p)$ i=1,2,3,4, has been calculated as follows.

$$\theta_{11}(x,t) = \frac{\varepsilon_1 R_1 T_0}{\chi_1 \rho_1}e^{-bx}\left[\int_0^{t-x/\xi_1} f(\tau)d\tau + \left(\frac{\xi_1^{\,2} m_1 T_0}{\chi_1}-b\xi_1\right)\int_0^{t-x/\xi_1} f(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau\right]+$$
$$\frac{\varepsilon_1^{\,2} R_1^{\,2} T_0 \xi_1^{\,2}}{\chi_1 G_1}e^{-bx}\left[\int_0^{t-x/\xi_1}\varphi(\tau)d\tau + \left(\frac{\xi_1^{\,2} m_1 T_0}{\chi_1}-b\xi_1\right)\int_0^{t-x/\psi_1}\varphi(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau\right] \tag{139}$$

$$\theta_{21}(x,t) = \frac{-2h\chi_2\varepsilon_1 R_1 T_0}{\rho_1 \chi_1^{\,2}\phi_1}e^{bx}\left[\mu_1\int_0^{t-(h-x)/\xi_1} f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)d\tau+\right.$$
$$\left.\frac{\mu_2}{\Gamma\left(3/2\right)}\int_0^{t-(h-x)/\xi_1} f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)^{3/2}d\tau+\frac{1}{2}\left(\mu_3+\frac{\mu_1\phi_2}{\phi_1}-b\xi_1\mu_1\right)\cdot\int_0^{t-(h-x)/\xi_1} f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)^2 d\tau\right]+$$

$$\frac{\varepsilon_1{}^2 R_1{}^2 T_0 \xi_1 \chi_2}{G_1 \chi_1{}^2 \phi_1} e^{hx}\left[\mu_1 \int_0^{t-(h-x)/\xi_1} \varphi(\tau) d\tau + \frac{\mu_2}{\Gamma\left(1/2\right)} \int_0^{t-(h-x)/\xi_1} \varphi(\tau)\sqrt{t - \frac{h-x}{\xi_1} - \tau}\, d\tau + \right.$$

$$\left.\left(\mu_3 + \frac{\mu_1 \phi_2}{\phi_1} - b\xi_1\mu_1\right) \int_0^{t-(h-x)/\xi_1} \varphi(\tau)\left(t - \frac{h-x}{\xi_1} - \tau\right) d\tau\right] \tag{140}$$

$$\theta_{31}(x,t) = -\frac{2bG_1}{\rho_1 \varepsilon_1 R_1 \xi_1}\left[\left(1 - \frac{\xi_1{}^2 T_0 m_1}{\chi_1}\right)\int_0^t f(t-\tau)\,\mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right) d\tau + \right.$$

$$\left.\frac{\xi_1{}^2 T_0 m_1}{\chi_1}\int_0^t f(\tau) d\tau * \mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{t}}\right)\right] + \left(1 - \frac{\xi_1{}^2 T_0 m_1}{\chi_1}\right)\cdot$$

$$\int_0^t \varphi'(t-\tau)\cdot \mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right) d\tau + \frac{\xi_1{}^2 m_1 T_0}{\chi_1}\cdot \int_0^t \varphi(t-\tau)\cdot \mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right) d\tau \tag{141}$$

$$\theta_{41}(x,t) = \frac{2bG_1}{\varepsilon_1 R_1 \xi_1 \rho_1 \phi_1}\left[\lambda_1 \int_0^t f(t-\tau)\cdot Erf\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\tau}\right) d\tau + \right.$$

$$\left.\left(\lambda_2 + \frac{\lambda_1 \phi_2}{\phi_1} - \frac{T_0 m_1 \xi_1{}^2 \lambda_1}{\chi_1}\right)\int_0^t f(\tau) d\tau * \mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{t}}\right)\right] -$$

$$\frac{\lambda_1}{\phi_1}\int_0^t \varphi'(t-\tau)\cdot \mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right) d\tau - \frac{1}{\phi_1}\left(\lambda_2 + \frac{\lambda_1 \phi_2}{\phi_1} - \frac{T_0 m_1 \xi_1{}^2 \lambda_1}{\chi_1}\right)$$

$$\int_0^t \varphi(t-\tau)\cdot \mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right) d\tau \tag{142}$$

If we define

$$\overline{\theta}_{12}(x,p)=-\frac{\varepsilon_2 R_2 T_0 \xi_2}{\chi_2}\left(1-\frac{B\xi_2}{p}\right)e^{-\left(\frac{p}{\xi_2}+B\right)x}E_{12}$$

$$\overline{\theta}_{32}(x,p)=-\frac{\left(\frac{T_0 m_2 p}{\chi_2}-\frac{p}{\xi_2{}^2}\right)}{\frac{\varepsilon_2 R_2}{G_2}\sqrt{\frac{T_0 m_2 p}{\chi_2}}}e^{-\sqrt{\frac{T_0 m_2 p}{\chi_2}}x}E_{32}$$

then

$$\overline{\theta}_2(x,p)=\overline{\theta}_{12}(x,p)+\overline{\theta}_{32}(x,p)$$
$$\theta_2(x,t)=\theta_{12}(x,t)+\theta_{32}(x,t)$$

Inverse Laplace transforms of $\overline{\theta}_{12}(x,p)$ and $\overline{\theta}_{32}(x,p)$ has been calculated as follows.

$$\theta_{12}(x,t)=-\frac{2\varepsilon_2 R_2 T_0 \xi_2 b}{\rho_1 \xi_1 \phi_1}e^{-B(x-h)}\cdot\left[\gamma_1\int_0^{t-(x-h)/\xi_2} f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)d\tau+\right.$$
$$\frac{\gamma_2-B\xi_2\gamma_1}{\Gamma\left(3/2\right)}\int_0^{t-(x-h)/\xi_2} f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)^{3/2}d\tau+\frac{1}{2}\left(\gamma_3+\frac{\lambda_1\phi_2}{\phi_1}\right)\cdot$$
$$\left.\int_0^{t-(x-h)/\xi_2} f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)^2 d\tau\right]+\frac{\varepsilon_2 R_2 T_0 \xi_2 \varepsilon_1 R_1}{G_1\phi_1}e^{-B(x-h)}\cdot$$
$$\left[\gamma_1\int_0^{t-(x-h)/\xi_2}\varphi(\tau)d\tau+\frac{\gamma_2-B\xi_2\gamma_1}{\Gamma\left(1/2\right)}\int_0^{t-(x-h)/\xi_2}\varphi(\tau)\sqrt{t-\frac{x-h}{\xi_2}-\tau}\,d\tau+\right.$$
$$\left.\left(\gamma_3+\frac{\gamma_1\phi_2}{\phi_1}\right)\int_0^{t-(x-h)/\xi_2}\varphi(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)d\tau\right] \tag{143}$$

$$\theta_{32}(x,t)=-\frac{4bG_1G_2\chi_2\sqrt{T_0 m_1\chi_1}}{\rho_1\xi_1\phi_1}\left\{\int_0^t f(t-\tau)\mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0 m_2}{\chi_2}}+h\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right]d\tau+\right.$$

$$\left(\frac{\phi_2}{\phi_1}-\frac{T_0 m_1 \xi_1^{\,2}}{\chi_1}-\frac{T_0 m_2 \xi_2^{\,2}}{\chi_2}\right)\int_0^t f(\tau)d\tau * \mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0 m_2}{\chi_2}}+h\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{t}}\right]\Bigg\}+$$

$$\frac{2\varepsilon_1 R_1 G_2 \chi_2(\xi_1\rho_1+\xi_2\rho_2)\sqrt{T_0 m_1 \chi_1}}{\phi_1}\cdot\Big\{\int_0^t \varphi'(t-\tau)$$

$$Erf\left[\frac{(x-h)\sqrt{\frac{T_0 m_2}{\chi_2}}+h\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right]d\tau\left(\frac{\phi_2}{\phi_1}-\frac{T_0 m_1 \xi_1^{\,2}}{\chi_1}-\frac{T_0 m_2 \xi_2^{\,2}}{\chi_2}\right)$$

$$\int_0^t \varphi(t-\tau)\mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0 m_2}{\chi_2}}+h\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right]d\tau\Big\} \qquad (144)$$

If we define

$$\bar{\sigma}_{11}(x,p)=-\frac{\rho_1 p^2}{\left(b+\frac{p}{\xi_1}\right)}e^{-\left(b+\frac{p}{\xi_1}\right)x}E_{11}$$

$$\bar{\sigma}_{21}(x,p)=\frac{\rho_1 p^2}{\left(b+\frac{p}{\xi_1}\right)}e^{\left(b+\frac{p}{\xi_1}\right)x}E_{21}$$

$$\bar{\sigma}_{31}(x,p)=-\frac{\rho_1 \rho^2}{\sqrt{\frac{T_0 m_1 p}{\chi_1}}}e^{-x\sqrt{\frac{T_0 m_1 p}{\chi_1}}}E_{31}$$

$$\bar{\sigma}_{41}(x,p)=\frac{\rho_1 p^2}{\sqrt{\frac{T_0 m_1 p}{\chi_1}}}e^{x\sqrt{\frac{T_0 m_1 p}{\chi_1}}}E_{41}$$

then

$$\bar{\sigma}_1(x,p)=\sum_{i=1}^{4}\bar{\sigma}_{i1}(x,p)$$

$$\sigma_1(x,t)=\sum_{i=1}^{4}\sigma_{i1}(x,t)$$

Inverse Laplace transforms of $\overline{\sigma}_{i1}(x,p)$ has been calculated as follows.

$$\sigma_{11}(x,t)=e^{-bx}\left[f\left(t-\frac{x}{\xi_1}\right)+\frac{\xi_1^{\,2}m_1T_0-b\xi_1\chi_1}{\chi_1}\int_0^{t-x/\xi_1}f(\tau)d\tau-\frac{\xi_1^{\,3}m_1T_0b}{\chi_1}\int_0^{t-x/\xi_1}f(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau\right]+\frac{\xi_1^{\,2}\varepsilon_1\rho_1R_1}{G_1}e^{-bx}\left[\varphi\left(t-\frac{x}{\xi_1}\right)+\frac{\xi_1^{\,2}m_1T_0-b\xi_1\chi_1}{\chi_1}\int_0^{t-x/\xi_1}\varphi(\tau)d\tau-\frac{\xi_1^{\,3}m_1T_0b}{\chi_1}\int_0^{t-x/\xi_1}\varphi(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau\right] \tag{145}$$

$$\sigma_{21}(x,t)=-\frac{2b\chi_2}{\chi_1\phi_1}e^{-b(h-x)}\left[\mu_1\int_0^{t-\frac{h-x}{\xi_1}}f(\tau)d\tau+\frac{\mu_2}{\Gamma\left(\frac{1}{2}\right)}\int_0^{t-\frac{h-x}{\xi_1}}f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)^{1/2}d\tau+\left(\mu_3+\frac{\mu_1\phi_2}{\phi_1}-b\xi_1\mu_1\right)\int_0^{t-\frac{h-x}{\xi_1}}f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)d\tau\right]+\frac{\xi_1\varepsilon_1\rho_1R_1\chi_2}{G_1\chi_1\phi_1}\cdot e^{-b(h-x)}\left[\mu_1\varphi\left(t-\frac{h-x}{\xi_1}-\tau\right)+\frac{\mu_2}{\sqrt{\pi}}\int_0^{t-\frac{h-x}{\xi_1}}\varphi\left(t-\frac{h-x}{\xi_1}-\tau\right)\frac{d\tau}{\sqrt{\tau}}+\left(\mu_3+\frac{\mu_1\phi_2}{\phi_1}-b\xi_1\mu_1\right)\int_0^{t-\frac{h-x}{\xi_1}}\varphi(\tau)d\tau\right] \tag{146}$$

$$\sigma_{31}(x,t)=2b\xi_1\left[\int_0^t f(t-\tau)\mathrm{Erf}\left(\frac{x\sqrt{T_0m_1/\chi_1}}{2\tau}\right)d\tau+\frac{\xi_1^{\,2}m_1T_0}{\chi_1}\cdot\int_0^t f(\tau)d\tau*\mathrm{Erf}\left(\frac{x\sqrt{T_0m_1/\chi_1}}{2\tau}\right)\right]-\frac{\xi_1^{\,2}\varepsilon_1\rho_1R_1}{G_1}\cdot\left[\int_0^t\varphi'(t-\tau)\mathrm{Erf}\left(\frac{x\sqrt{T_0m_1/\chi_1}}{2\tau}\right)d\tau+\frac{\xi_1^{\,2}m_1T_0}{\chi_1}\int_0^t\varphi(t-\tau)\mathrm{Erf}\left(\frac{x\sqrt{T_0m_1/\chi_1}}{2\tau}\right)d\tau\right] \tag{147}$$

$$\sigma_{41}(x,t)=-\frac{2b\xi_1}{\phi_1}\left\{\lambda_1\int_0^t f(t-\tau)Erf\left[\frac{(2h-x)\sqrt{T_0m_1/\chi_1}}{2\tau}\right]d\tau+\left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}\right)\cdot\right.$$

$$\int_0^t f(\tau)d\tau * \mathrm{Erf}\left[\frac{(2h-x)\sqrt{T_0 m_1/\chi_1}}{2t}\right]\Bigg\} + \frac{\xi_1^2\varepsilon_1\rho_1 R_1}{G_1\phi_1}\Bigg\{\lambda_1\int_0^t \varphi'(t-\tau)$$

$$\mathrm{Erf}\left[\frac{(2h-x)\sqrt{T_0 m_1/\chi_1}}{2\tau}\right]d\tau + \left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}\right)\int_0^t \varphi(t-\tau)\,\mathrm{Erf}\left[\frac{(2h-x)\sqrt{T_0 m_1/\chi_1}}{2\tau}\right]d\tau\Bigg\} \qquad (148)$$

If we define

$$\bar{\sigma}_{12}(x,p) = -\frac{\rho_2 p^2}{\left(B+\frac{p}{\xi_2}\right)} e^{-\left(B+\frac{p}{\xi_2}\right)x} E_{12}$$

$$\bar{\sigma}_{32}(x,p) = -\frac{\rho_2 p^2}{\sqrt{\frac{T_0 m_2 p}{\chi_2}}} e^{-x\sqrt{\frac{T_0 m_2 p}{\chi_2}}} E_{32}$$

then

$$\bar{\sigma}_2(x,p) = \bar{\sigma}_{12}(x,p) + \bar{\sigma}_{32}(x,p)$$

$$\sigma_2(x,t) = \sigma_{12}(x,t) + \sigma_{32}(x,t)$$

Inverse Laplace transforms of $\bar{\sigma}_{12}$ and $\bar{\sigma}_{32}$ has been calculated as follows.

$$\sigma_{12}(x,t) = -\frac{2b\chi_2\xi_2\rho_2}{\rho_1\xi_1\phi_1} e^{-B(x-h)}\Bigg[\gamma_1\int_0^{t-\frac{x-h}{\xi_2}} f(\tau)d\tau + \frac{\gamma_2}{\Gamma(1/2)}$$

$$\int_0^{t-\frac{x-h}{\xi_2}} f(\tau)\sqrt{t-\frac{x-h}{\xi_2}-\tau}\,d\tau + \left(\gamma_3+\frac{\lambda_1\phi_2}{\phi_1}-\xi_2 B\gamma_1\right)$$

$$\int_0^{t-\frac{x-h}{\xi_2}} f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)d\tau\Bigg] + \frac{\varepsilon_1 R_1\chi_2\xi_2\rho_2}{G_1\phi_1} e^{-B(x-h)}$$

$$[\gamma_1\varphi\left(t-\frac{x-h}{\xi_2}\right) + \frac{\gamma_2}{\sqrt{\pi}}\int_0^{t-\frac{x-h}{\xi_2}} \varphi(t-\tau)\frac{d\tau}{\sqrt{\tau}} + \left(\gamma_3+\frac{\lambda_1\phi_2}{\phi_1}-\xi_2 B\gamma_1\right)$$

$$\int_0^{t-\frac{x-h}{\xi_2}} \varphi(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)d\tau\,] \qquad (149)$$

$$\sigma_{32}(x,t) = -\frac{4hG_1\rho_2\chi_2}{\rho_1\xi_1\phi_1}(\rho_1\xi_1+\rho_2\xi_2)\sqrt{T_0 m_1\chi_1}\xi_2^{\,2}\varepsilon_2 R_2$$

$$\left\{\int_0^t f(t-\tau)\,\mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0 m_2}{\chi_2}}+h\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right]d\tau + \left(\frac{\phi_2}{\phi_1}-\frac{T_0 m_1\xi_1^{\,2}}{\chi_1}\right)\right.$$

$$\left.\int_0^t f(\tau)d\tau * \mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0 m_2}{\chi_2}}+h\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{t}}\right]\right\} + \frac{2\xi_1 R_1\rho_2\chi_2}{\phi_1}$$

$$(\rho_1\xi_1+\rho_2\xi_2)\sqrt{T_0 m_1\chi_1}\xi_2^{\,2}\varepsilon_2 R_2\cdot$$

$$\left\{\int_0^t \varphi'(t-\tau)\,\mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0 m_2}{\chi_2}}+h\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right]d\tau + \left(\frac{\phi_2}{\phi_1}-\frac{T_0 m_1\xi_1^{\,2}}{\chi_1}\right)\cdot\right.$$

$$\int_0^t \varphi(t-\tau)\,\mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0 m_2}{\chi_2}}+h\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right]d\tau \qquad (150)$$

8.SOLUTIONS FOR VISCOELASTIC MATERIALS FOR LARGE VALUES OF TIME

We mentioned before how difficult it was finding solutions of problems for viscoelastic bodies. One of difficulties stem from functions $\overline{m}_i$, $\overline{\psi}_i$, $\overline{R}_i$, $\overline{G}_i$ which are included in solutions of Laplace integral transforms. In order to reduce difficulties partially it is assumed that ratios of these functions mutually do not depend Laplace transforms parameter. In that case for very small values of p, and taking m,n constants,

$$\overline{m}_i p = m_i(1+pm)$$
$$\overline{G}_i p = G_i(1+pn)$$

formulas are used. So using solutions in part 5 solutions in large values of time for viscoelastic materials are found as follows. Again symbols in part 5 will be used.

$$u_1(x,t) = \sum_{i=1}^{4} u_{i1}(x,t)$$

$$u_{11}(x,t)=-\frac{\varepsilon_1(\chi_1-\chi_2)(\xi_1\xi_2\rho_1+\xi_1^2\rho_1-\xi_2^2\rho_2+\xi_1\xi_2\rho_2)R_1}{2aG_1\chi_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2+\xi_1\xi_2\rho_2)}\cdot\frac{1}{\sqrt{\pi}}\int_0^t\varphi(t-\tau)-\frac{m}{2}\varphi'(t-\tau)\Big]\frac{d\tau}{\sqrt{\pi}} \quad (151)$$

$$u_{21}(x,t)=\frac{\varepsilon_1R_1(\chi_1+\chi_2)}{2aG_1\chi_1}\frac{1}{\sqrt{\pi}}\int_0^t\left[\varphi(t-\tau)-\frac{m}{2}\varphi'(t-\tau)\right]\frac{d\tau}{\sqrt{\pi}} \quad (152)$$

$$u_{31}(x,t)=\frac{\xi_2(A\xi_2\varepsilon_1G_2\chi_2\rho_2R_1+a\xi_2\varepsilon_2G_1\chi_1\rho_2R_2)}{aAG_1G_2\chi_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\frac{1}{\sqrt{\pi}}\int_0^t\left[\varphi(t-\tau)-\frac{m}{2}\varphi'(t-\tau)\right]\frac{d\tau}{\sqrt{\pi}}+$$

$$\frac{\xi_1\rho_1+\xi_2\rho_2}{\sqrt{2}\rho_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\left[\int_0^t f(\tau)d\tau-\frac{n}{2}f(t)\right] \quad (153)$$

$$u_{41}(x,t)=\frac{\rho_2\xi_1\xi_2(\xi_1G_2\chi_2\rho\ \ +\varepsilon_2G_1\chi_1R_2)}{aG_1G_2\chi_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\frac{1}{\sqrt{\pi}}\int_0^t\left[\varphi(t-\tau)-\frac{m}{2}\varphi'(t-\tau)\right]\frac{d\tau}{\sqrt{\pi}}+$$

$$\frac{\xi_1\rho_1-\xi_2\rho_2}{\sqrt{2}\rho_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\left[\int_0^t f(\tau)d\tau-\frac{n}{2}f(t)\right] \quad (154)$$

$$u_2(x,t)=u_{12}(x,t)+u_{32}(x,t)$$

$$u_{12}(x,t)=-\frac{\varepsilon_2R_2}{AG_2\sqrt{\pi}}\int_0^t\frac{1}{\tau}e^{-\frac{A^2\chi^2}{4\tau}}\left[\varphi(t-\tau)-\frac{m}{2}\varphi'(t-\tau)\right]d\tau \quad (155)$$

$$u_{32}(x,t)=\frac{\sqrt{2}\xi_1}{\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2}\left[\int_0^{t-\frac{x}{\sqrt{2}\xi_1}}f(\tau)d\tau-\frac{n}{2}f\left(t-\frac{x}{\sqrt{2}\xi_1}\right)\right] \quad (156)$$

$$\theta_1(x,t)=\sum_{i=1}^{4}\theta_{i1}(x,t)$$

$$\theta_{11}(x,t)=\frac{(\chi_1-\chi_2)(\xi_1\xi_2\rho_1+\xi_1^2\rho_1-\xi_2^2\rho_2+\xi_1\xi_2\rho_2)}{2\chi_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2+\xi_1\xi_2\rho_2)}\varphi(t) \quad (157)$$

$$\theta_{21}(x,t)=\frac{\chi_1+\chi_2}{2\chi_1}\varphi(t) \quad (158)$$

$$\theta_{31}(x,t)=\frac{G_1(\xi_1\rho_1+\xi_2\rho_2)}{2\xi_1\varepsilon_1R_1\rho_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\left[f(t)-nf'(t)-nf(0)\right]+$$

$$\frac{\xi_2\rho_1(A\xi_2\varepsilon_1G_2\chi_2\rho_2R_1+a\xi_2\varepsilon_2G_1\chi_1\rho_2R_2)}{\sqrt{2\pi}\xi_1\varepsilon_1R_1aAG_2\rho_1\chi_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\cdot\int_0^t\left[\varphi'(t-\tau)-\frac{m+n}{2}\varphi''(t-\tau)\right]\frac{d\tau}{\sqrt{\tau}} \quad (159)$$

$$\theta_{41}(x,t)=-\frac{G_1(\xi_1\rho_1-\xi_2\rho_2)}{2\xi_1\varepsilon_1R_1\rho_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\left[f(t)-nf'(t)\right]-$$

$$\frac{\xi_2\rho_2(\varepsilon_1G_2\chi_2R_1+\varepsilon_2G_1\chi_1R_2)}{\sqrt{2\pi}\varepsilon_1R_1aG_2\chi_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\cdot\int_0^t\left[\varphi'(t-\tau)-\frac{m+n}{2}\varphi''(t-\tau)\right]\frac{d\tau}{\sqrt{\tau}} \quad (160)$$

$$\theta_2(x,t)=\theta_{12}(x,t)+\theta_{32}(x,t)$$

$$\theta_{12}(x,t)=-\int_0^t \mathrm{Erf}\left(\frac{Ax}{2\sqrt{\tau}}\right)f'(t-\tau)d\tau \quad (161)$$

$$\theta_{32}(x,t)=\frac{G_2\xi_1}{\xi_2\varepsilon_2R_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\cdot\left[f\left(t-\frac{x}{\sqrt{2}\xi_2}\right)-nf'\left(t-\frac{x}{\sqrt{2}\xi_2}\right)\right] \quad (162)$$

$$\sigma_1(x,t)=\sum_{i=1}^{4}\sigma_{i1}(x,t)$$

$$\sigma_{11}(x,t)=\frac{\rho_1\varepsilon_1(\chi_1-\chi_2)(\xi_1\xi_2\rho_1+\xi_1^2\rho_1-\xi_2^2\rho_2+\xi_1\xi_2\rho_2)}{2a^2G_1\chi_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\left[\varphi'(t)-m\varphi''(t)\right] \quad (163)$$

$$\sigma_{21}(x,t)=\frac{\rho_1\varepsilon_1(\chi_1+\chi_2)R_1}{2a^2G_1\chi_1}\left[\varphi'(t)-m\varphi''(t)\right] \quad (164)$$

$$\sigma_{31}(x,t)=-\frac{\xi_1(\xi_1\rho_1+\xi_2\rho_2)}{\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2}f(t)-$$

$$\frac{\sqrt{2}\xi_1\xi_2\rho_1(A\xi_2\varepsilon_1G_1\chi_2\rho_2R_1+a\xi_2\varepsilon_2G_1\chi_1\rho_2R_2)}{\sqrt{\pi}aAG_1G_2\chi_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\cdot\int_0^t\left[\varphi'(t-\tau)+\frac{n-m}{2}\varphi''(t-\tau)\right]\frac{d\tau}{\sqrt{\tau}} \quad (165)$$

$$\sigma_{41}(x,t)=\frac{\xi_1(\xi_1\rho_1-\xi_2\rho_2)}{\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2}f(t)+$$

$$\frac{\sqrt{2}\rho_1\rho_2\,\xi_1^2\xi_2(\varepsilon_1G_2\chi_2R_1+\varepsilon_2G_1\chi_1R_2)}{\sqrt{\pi}aAG_1G_2\chi_1(\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2)}\cdot\int_0^t\left[\varphi'(t-\tau)+\frac{n-m}{2}\varphi''(t-\tau)\right]\frac{d\tau}{\sqrt{\tau}} \quad (166)$$

$$\sigma_2(x,t)=\sigma_{12}(x,t)+\sigma_{32}(x,t)$$

$$\sigma_{12}(x,t)=\frac{\rho_2\varepsilon_2R_2}{A^2G_2}\int_0^t \mathrm{Erf}\left(\frac{Ax}{2\sqrt{\tau}}\right)\left[\varphi''(t-\tau)+\frac{n-m}{2}\varphi'''(t-\tau)\right]d\tau \quad (167)$$

$$\sigma_{32}(x,t)=-\frac{2\xi_1\xi_2\rho_2}{\xi_1\xi_2\rho_1-\xi_1^2\rho_1-\xi_2^2\rho_2-\xi_1\xi_2\rho_2}f\left(t-\frac{x}{\sqrt{2}\xi_2}\right) \quad (168)$$

9.SOLUTIONS FOR VISCOELASTIC MATERIALS FOR SMALL VALUES OF TIME

In this case for very large values of p, and taking M,N constants

$$\overline{m}_i p = m_i\left(1+\frac{M}{p}\right)$$
$$\overline{G}_i p = G_i\left(1+\frac{N}{p}\right)$$

formulas are used. So using solutions in part 7 solutions in small values of time for viscoelastic materials are found as follows. Again symbols in part 7 will be used.

$$u_1(x,t)=\sum_{i=1}^{4}u_{i1}(x,t)$$

$$u_{11}(x,t)=-\frac{e^{-x\left(b+\frac{N}{2\xi_1}\right)}}{\xi_1\rho_1}\left[\int_0^{t-\frac{x}{\xi_1}} f(\tau)d\tau+\frac{\xi_1{}^2 m_1 T_0}{\chi_1}t-\int_0^{t-\frac{x}{\xi_1}} f(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau+\right.$$

$$\left.\frac{\xi_1{}^4 m_1{}^2 T_0{}^2}{2\chi_1{}^2}\int_0^{t-\frac{x}{\xi_1}} f(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)^2 d\tau\right]-\frac{\xi_1\varepsilon_1 R_1}{G_1}\left[\int_0^{t-\frac{x}{\xi_1}}\varphi(\tau)d\tau+\right.$$

$$\left.\frac{\xi_1{}^2 m_1 T_0}{\chi_1}\int_0^{t-\frac{x}{\xi_1}}\varphi(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau+\frac{\xi_1{}^4 m_1{}^2 T_0{}^2}{2\chi_1{}^2}\int_0^{t-\frac{x}{\xi_1}}\varphi(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)^2 d\tau\right] \qquad (169)$$

$$u_{21}(x,t)=-\frac{2b\chi_1}{\rho_1\chi_1\xi_1\phi_1}e^{-(h-x)\left(b+\frac{N}{2\xi_1}\right)}\left[\mu_1\int_0^{t-(h-x)/\xi_1} f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)d\tau+\right.$$

$$\frac{1}{2}\left(\mu_3+\frac{\mu_1\phi_2}{\phi_1}\right)\int_0^{t-(h-x)/\xi_1} f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)^2 d\tau+\frac{\mu_2}{\Gamma(5/2)}$$

$$\int_0^{t-(h-x)/\xi_1} f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)^{3/2} d\tau+\frac{\varepsilon_1 R_1\chi_2}{G_1\chi_1\phi_1}\left[\mu_1\int_0^{t-(h-x)/\xi_1}\varphi(\tau)d\tau+\right.$$

$$\left(\mu_3+\frac{\mu_1\phi_2}{\phi_1}\right)\int_0^{t-(h-x)/\xi_1}\varphi(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)d\tau+\frac{\mu_2}{\Gamma(3/2)}$$

$$\left.\int_0^{t-(h-x)/\xi_1}\varphi(\tau)\sqrt{t-\frac{h-x}{\xi_1}-\tau}\,d\tau\right] \tag{170}$$

$$u_{31}(x,t)=-\frac{2b\xi_1}{\rho_1\sqrt{\frac{\chi_1}{m_1T_0}}}\left[\int_0^t f(\tau)(t-\tau)d\tau+\frac{\xi_1{}^2m_1T_0}{2\chi_1}\int_0^t f(\tau)(t-\tau)^2d\tau\right]*$$

$$\frac{1}{\sqrt{\pi t}}e^{-\frac{T_0m_1x^2}{4\chi_1t}}+\frac{\xi_1{}^2\varepsilon_1R_1}{G_1\sqrt{\frac{\chi_1}{T_0m_1}}}\left[\int_0^t\varphi(\tau)d\tau+\frac{\xi_1{}^2m_1T_0}{\chi_1}\int_0^t\varphi(\tau)(t-\tau)d\tau\right]*\frac{1}{\sqrt{\pi t}}e^{-\frac{T_0m_1x^2}{4\chi_1t}} \tag{171}$$

$$u_{41}(x,t)=-\frac{2b\xi_1}{\rho_1\phi_1\sqrt{\frac{\chi_1}{m_1T_0}}}\left[\lambda_1\int_0^t f(\tau)(t-\tau)d\tau+\frac{1}{2}\left(\lambda_{2+}+\frac{\lambda_1\phi_2}{\phi_1}\right)\int_0^t f(\tau)(t-\tau)^2d\tau\right]*$$

$$\frac{1}{\sqrt{\pi t}}e^{-\frac{T_0m_1(2h-x)^2}{4\chi_1t}}+\frac{\xi_1{}^2\varepsilon_1R_1}{G_1\phi_1\sqrt{\frac{\chi_1}{T_0m_1}}}$$

$$\left[\lambda_1\int_0^t\varphi(\tau)d\tau+\left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}\right)\int_0^t\varphi(\tau)(t-\tau)d\tau\right]*\frac{1}{\sqrt{\pi t}}e^{-\frac{T_0m_1(2h-x)^2}{4\chi_1t}} \tag{172}$$

$$u_2(x,t)=u_{12}(x,t)+u_{32}(x,t)$$

$$u_{12}(x,t)=\frac{2h\chi_2}{\rho_1\xi_1\phi_1}e^{-(x-h)\left(B+\frac{N}{2\xi_2}\right)}\left[\gamma_1\int_0^{t-(x-h)/\xi_2}f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)d\tau+\frac{\gamma_2}{\Gamma(3/2)}\right.$$

$$\int_0^{t-(x-h)/\xi_2}f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)^{3/2}d\tau+\frac{1}{2}\left(\gamma_3+\frac{\gamma_1\phi_2}{\phi_1}\right)$$

$$\int_{0}^{t-(x-h)/\xi_2} f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)^2 d\tau\Bigg] - \frac{\varepsilon_1 R_1 \chi_2}{G_1 \phi_1} e^{-(x-h)N/(2\xi_2)}$$

$$\Bigg[\gamma_1 \int_0^{t-(x-h)/\xi_2} \varphi(\tau) d\tau + \frac{\gamma_2}{\Gamma(1/2)} \int_0^{t-(x-h)/\xi_2} \varphi(\tau)\sqrt{t-\frac{x-h}{\xi_2}-\tau}\, d\tau +$$

$$\left(\gamma_3 + \frac{\gamma_1 \phi_2}{\phi_1}\right) \int_0^{t-(x-h)/\xi_2} \varphi(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right) d\tau\Bigg] \tag{173}$$

$$u_{32}(x,t) = -\frac{4bG_1(\xi_1\rho_1+\xi_2\rho_2)T_0\xi_2^2\varepsilon_2 R_1\sqrt{m_1 m_2 \chi_1 \chi_2}}{\rho_1 \xi_1 \phi_1}\Bigg[\int_0^t f(\tau)(t-\tau)d\tau +$$

$$\frac{1}{2}\left(\frac{\phi_2}{\phi_1} - \frac{T_0 m_1 \xi_1^2}{\chi_1}\right)\int_0^t f(\tau)(t-\tau)^2 d\tau\Bigg] * \frac{1}{\sqrt{\pi t}} e^{-\frac{\left[\sqrt{\frac{T_0 m_2}{\chi_2}}(x-h)+h\sqrt{\frac{T_0 m_1}{\chi_1}}\right]^2}{4t}} +$$

$$\frac{2\varepsilon_1 R_1(\xi_1\rho_1+\xi_2\rho_2)T_0\xi_2^2\varepsilon_2 R_2\sqrt{m_1 m_2 \chi_1 \chi_2}}{\phi_1}\Bigg[\int_0^t \varphi(\tau)d\tau +$$

$$\left(\frac{\phi_2}{\phi_1} - \frac{T_0 m_1 \xi_1^2}{\chi_1}\right)\int_0^t \varphi(\tau)(t-\tau)^2 d\tau\Bigg] * \frac{1}{\sqrt{\pi t}} e^{-\frac{\left[\sqrt{\frac{T_0 m_2}{\chi_2}}(x-h)+h\sqrt{\frac{T_0 m_1}{\chi_1}}\right]^2}{4t}} \tag{174}$$

$$\theta_1(x,t) = \sum_{i=1}^{4} \theta_{i1}(x,t)$$

$$\theta_{11}(x,t) = \frac{\varepsilon_1 R_1 T_0}{\chi_1 \rho_1} e^{-bx-\frac{xN}{2\xi_1}}\left[\int_0^{t-\frac{x}{\xi_1}} f(\tau)d\tau + \left(\frac{\xi_1^2 m_1 T_0}{\chi_1} - b\xi_1\right)\int_0^{t-\frac{x}{\xi_1}} f(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau\right] +$$

$$e^{-bx-\frac{xN}{2\xi_1}}\left[\int_0^{t-\frac{x}{\xi_1}} \varphi(\tau)d\tau + \left(\frac{\xi_1^2 m_1 T_0}{\chi_1} - b\xi_1\right)\int_0^{t-\frac{x}{\xi_1}} \varphi(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau\right] \tag{175}$$

$$\theta_{21}(x,t) = -\frac{2b\chi_2\varepsilon_1 R_1 T_0}{\rho_1\chi_1^2\phi_1} e^{xb - \frac{(h-x)X}{2\xi_1}} \left[\mu_1 \int_0^{t-\frac{(h-x)}{\xi_1}} f(\tau)\left(t - \frac{h-x}{\xi_1} - \tau\right) d\tau + \frac{\mu_2}{\Gamma\left(\frac{3}{2}\right)} \right.$$

$$\int_0^{t-\frac{(h-x)}{\xi_1}} f(\tau)\left(t - \frac{h-x}{\xi_1} - \tau\right)^{\frac{3}{2}} d\tau + \frac{1}{2}\left(\mu_3 + \frac{\mu_1\phi_2}{\phi_1} - b\xi_1\mu_1\right)$$

$$\left. \int_0^{t-\frac{(h-x)}{\xi_1}} f(\tau)\left(t - \frac{h-x}{\xi_1} - \tau\right)^2 d\tau \right] + \frac{\chi_2\varepsilon_1^2 R_1^2 T_0\xi_1}{G_1\chi_1^2\phi_1} e^{xb - \frac{(h-x)X}{2\xi_1}}$$

$$\left[\mu_1 \int_0^{t-{(h-x)}/{\xi_1}} \varphi(\tau)d\tau + \frac{\mu_2}{\Gamma\left(\frac{1}{2}\right)} \int_0^{t-{(h-x)}/{\xi_1}} \varphi(\tau)\sqrt{t - \frac{h-x}{\xi_1} - \tau}\, d\tau + \right.$$

$$\left. \left(\mu_3 + \frac{\mu_1\phi_2}{\phi_1} - b\xi_1\mu_1\right) \int_0^{t-{(h-x)}/{\xi_1}} \varphi(\tau)\left(t - \frac{h-x}{\xi_1} - \tau\right) d\tau \right] \qquad (176)$$

$$\theta_{31}(x,t) = -\frac{2bG_1}{\rho_1\varepsilon_1 R_1\xi_1} \left[\left(1 - \frac{\xi_1^2 T_0 m_1}{\chi_1}\right) + \right.$$

$$\left(1 - \frac{\xi_1^2 T_0 m_1}{\chi_1}\right) \int_0^t \left(\varphi'(t-\tau) + \varphi(0)\right) \mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right) d\tau +$$

$$\frac{\xi_1^2 T_0 m_1}{\chi_1} \int_0^t \varphi(t-\tau)\, \mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right) d\tau \qquad (177)$$

$$\theta_{41}(x,t) = \frac{2bG_1}{\varepsilon_1 R_1 \xi_1 \rho_1 \phi_1} \left[\lambda_1 \int_0^t f(t-\tau)\, \mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\tau}\right) d\tau + \right.$$

$$\left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}-\frac{T_0 m_1 \xi_1^2 \lambda_1}{\chi_1}\right)\int_0^t f(\tau)d\tau * \mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{t}}\right)d\tau\Bigg]-$$

$$\frac{\lambda_1}{\phi_1}\int_0^t \varphi'(t-\tau)\mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right)d\tau-\frac{1}{\phi_1}\left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}-\frac{T_0 m_1 \xi_1^2 \lambda_1}{\chi_1}\right)$$

$$\int_0^t \varphi(t-\tau)\mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\sqrt{\tau}}\right)d\tau \qquad (178)$$

$$\theta_2(x,t)=\theta_{12}(x,t)+\theta_{32}(x,t)$$

$$\theta_{12}(x,t)=-\frac{2\varepsilon_2 R_2 T_0 \xi_2 b}{\rho_1 \xi_1 \phi_1}e^{-(x-h)\left(B+\frac{N}{2\xi_2}\right)}\left[\gamma_1\int_0^{t-(x-h)/\xi_2} f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)d\tau+\right.$$

$$\frac{\gamma_2-B\xi_2\gamma_1}{\Gamma\left(3/2\right)}\int_0^{t-(x-h)/\xi_2} f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)^{3/2}d\tau+$$

$$\left.\frac{1}{2}\left(\gamma_3+\frac{\gamma_1\phi_2}{\phi_1}\right)\int_0^{t-(x-h)/\xi_2} f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)^2 d\tau\right]+\frac{\varepsilon_2 R_2 T_0 \xi_2 \varepsilon_1 R_1}{\phi_1 G_1}\cdot$$

$$e^{-(x-h)\left(B+\frac{N}{2\xi_2}\right)}\left[\gamma_1\int_0^{t-(x-h)/\xi_2}\varphi(\tau)d\tau+\right.$$

$$\frac{\gamma_2-B\xi_2\gamma_1}{\Gamma\left(1/2\right)}\int_0^{t-(x-h)/\xi_2}\varphi(\tau)\sqrt{t-\frac{x-h}{\xi_2}-\tau}\,d\tau+$$

$$\left(\gamma_3+\frac{\gamma_1\phi_2}{\phi_1}\right)\int_0^{t-(x-h)/\xi_2}\varphi(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)d\tau\Bigg] \tag{179}$$

$$\theta_{32}(x,t)=-\frac{4bG_1G_2\chi_2\sqrt{T_0m_1\chi_1}}{\rho_1\xi_1\phi_1}\left[\int_0^t f(t-\tau)\mathrm{Erf}\left(\frac{(x-h)\sqrt{\frac{T_0m_2}{\chi_2}}+h\sqrt{\frac{T_0m_1}{\chi_1}}}{2\sqrt{\tau}}\right)d\tau+\right.$$

$$\left.\left(\frac{\phi_2}{\phi_1}-\frac{T_0m_1\xi_1^2}{\chi_1}-\frac{T_0m_2\xi_2^2}{\chi_2}\right)\int_0^t f(\tau)d\tau*\mathrm{Erf}\left(\frac{(x-h)\sqrt{\frac{T_0m_2}{\chi_2}}+h\sqrt{\frac{T_0m_1}{\chi_1}}}{2\sqrt{t}}\right)\right]+$$

$$\frac{2\varepsilon_1R_1G_2\chi_2(\xi_1\rho_1+\xi_2\rho_2)\sqrt{T_0m_1\chi_1}}{\phi_1}\cdot$$

$$\int_0^t\varphi'(t-\tau)*\mathrm{Erf}\left(\frac{(x-h)\sqrt{\frac{T_0m_2}{\chi_2}}+h\sqrt{\frac{T_0m_1}{\chi_1}}}{2\sqrt{\tau}}\right)d\tau+$$

$$\left(\frac{\phi_2}{\phi_1}-\frac{T_0m_1\xi_1^2}{\chi_1}-\frac{T_0m_2\xi_2^2}{\chi_2}\right)\int_0^t\varphi(t-\tau)Erf\left(\frac{(x-h)\sqrt{\frac{T_0m_2}{\chi_2}}+h\sqrt{\frac{T_0m_1}{\chi_1}}}{2\sqrt{\tau}}\right)d\tau\Bigg] \tag{180}$$

$$\sigma_1(x,t)=\sum_{i=1}^{4}\sigma_{i1}(x,t)$$

$$\sigma_{11}(x,t)=e^{-bx-\frac{xN}{2\xi_1}}\left[f\left(t-\frac{x}{\xi_1}\right)+\frac{\xi_1^2m_1T_0-b\xi_1\chi_1}{\chi_1}\int_0^{t-\frac{x}{\xi_1}}f(\tau)d\tau-\frac{\xi_1^3m_1T_0b}{\chi_1}\right.$$

$$\left.\int_0^{t-\frac{x}{\xi_1}}f(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau\right]+\frac{\xi_1^2\varepsilon_1\rho_1R_1}{G_1}e^{-bx-\frac{xN}{2\xi_1}}$$

$$\left[\varphi\left(t-\frac{x}{\xi_1}\right)+\frac{\xi_1^{\,2}m_1T_0-b\xi_1\chi_1}{\chi_1}\int_0^{t-\frac{x}{\xi_1}}\varphi(\tau)d\tau-\frac{\xi_1^{\,3}m_1T_0b}{\chi_1}\cdot\right.$$

$$\left.\int_0^{t-\frac{x}{\xi_1}}\varphi(\tau)\left(t-\frac{x}{\xi_1}-\tau\right)d\tau\right] \qquad (181)$$

$$\sigma_{21}(x,t)=-\frac{2b\chi_2}{\chi_1\phi_1}e^{-(h-x)\left(b+\frac{N}{2\xi_1}\right)}\left[\mu_1\int_0^{t-\frac{h-x}{\xi_1}}f(\tau)d\tau+\frac{\mu_2}{\Gamma\left(\frac{1}{2}\right)}\cdot\right.$$

$$\int_0^{t-\frac{h-x}{\xi_1}}f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)^{1/2}d\tau+\left(\mu_3+\frac{\mu_1\phi_2}{\phi_1}-b\xi_1\mu_1\right)\cdot$$

$$\left.\int_0^{t-\frac{(h-x)}{\xi_1}}f(\tau)\left(t-\frac{h-x}{\xi_1}-\tau\right)d\tau\right]+\frac{\xi_1\varepsilon_1\rho_1R_1\chi_2}{G_1\chi_1\phi_1}e^{-(h-x)\left(b+\frac{N}{2\xi_1}\right)}\left[\mu_1\varphi\left(t-\frac{h-x}{\xi_1}\right)+\frac{\mu_2}{\sqrt{\pi}}\int_0^{t-\frac{(h-x)}{\xi_1}}\varphi\left(t-\frac{h-x}{\xi_1}-\tau\right)\frac{d\tau}{\sqrt{\tau}}\right.$$

$$\left.\left(\mu_3+\frac{\mu_1\phi_2}{\phi_1}-b\xi_1\mu_1\right)\int_0^{t-\frac{(h-x)}{\xi_2}}\varphi(\tau)d\tau\right] \qquad (182)$$

$$\sigma_{31}(x,t)=2b\xi_1\left[\int_0^t f(t-\tau)\mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0m_1}{\chi_1}}}{2\tau}\right)d\tau+\frac{\xi_1^{\,2}m_1T_0}{\chi_1}\int_0^t f(\tau)d\tau*\right.$$

$$\left.\mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0m_1}{\chi_1}}}{2t}\right)\right]-\frac{\xi_1^{\,2}\varepsilon_1R_1\rho_1}{G_1}\left[\int_0^t\varphi'(t-\tau)\mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0m_1}{\chi_1}}}{2\tau}\right)d\tau+\right.$$

$$\left.\frac{\xi_1^{\,2} m_1 T_0}{\chi_1}\int_0^t \varphi(t-\tau)\mathrm{Erf}\left(\frac{x\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\tau}\right)d\tau\right] \tag{183}$$

$$\sigma_{41}(x,t)=-\frac{2b\xi_1}{\phi_1}\left[\lambda_1\int_0^t f(t-\tau)\mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\tau}\right)d\tau+\left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}\right)\int_0^t f(\tau)d\tau *\right.$$

$$\left.\mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2t}\right)\right]+\frac{\xi_1^{\,2}\varepsilon_1\rho_1 R_1}{G_1\phi_1}\left[\lambda_1\int_0^t \varphi'(t-\tau)\mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\tau}\right)d\tau+\right.$$

$$\left.\left(\lambda_2+\frac{\lambda_1\phi_2}{\phi_1}\right)\int_0^t \varphi(t-\tau)\mathrm{Erf}\left(\frac{(2h-x)\sqrt{\frac{T_0 m_1}{\chi_1}}}{2\tau}\right)d\tau\right] \tag{184}$$

$$\sigma_2(x,t)=\sigma_{12}(x,t)+\sigma_{32}(x,t)$$

$$\sigma_{12}(x,t)=-\frac{2h\chi_2\xi_2\rho_2}{\rho_1\xi_1\phi_1}e^{-(x-h)\left(B+\frac{N}{2\xi_2}\right)}\left[\gamma_1\int_0^{t-\frac{x-h}{\xi_2}} f(\tau)d\tau+\frac{\gamma_2}{\Gamma\left(\frac{1}{2}\right)}\right.$$

$$\int_0^{t-\frac{x-h}{\xi_2}} f(\tau)\sqrt{t-\frac{x-h}{\xi_2}-\tau}\,d\tau+\left(\gamma_3+\frac{\gamma_1\phi_2}{\phi_1}-\xi_2 B\gamma_1\right)$$

$$\left.\int_0^{t-\frac{x-h}{\xi_2}} f(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)d\tau\right]+\frac{\varepsilon_1 R_1\chi_2\xi_2\rho_2}{G_1\phi_1}e^{-(x-h)\left(B+\frac{N}{2\xi_2}\right)}$$

$$\left[\gamma_1\varphi\left(t-\frac{x-h}{\xi_2'}\right)+\frac{\gamma_2}{\sqrt{\pi}}\int_0^{t-\frac{x-h}{\xi_2}}\varphi(t-\tau)\frac{d\tau}{\sqrt{\tau}}+\left(\gamma_3+\frac{\gamma_1\phi_2}{\phi_1}-\xi_2 B\gamma_1\right)\right.$$

$$\left.\int_0^{t-\frac{x-h}{\xi_2}} \varphi(\tau)\left(t-\frac{x-h}{\xi_2}-\tau\right)d\tau\right] \tag{185}$$

$$\sigma_{32}(x,t)=-\frac{4bG_1\rho_2\chi_2}{\rho_1\xi_1\phi_1}(\rho_1\xi_1+\rho_2\xi_2)\sqrt{T_0m_1\chi_1}\,\xi_2^{\,2}\varepsilon_2R_2\cdot$$

$$\left\{\int_0^t f(t-\tau)\mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0m_2}{\chi_2}}+h\sqrt{\frac{T_0m_1}{\chi_1}}}{2\sqrt{\tau}}\right]d\tau+\left(\frac{\phi_2}{\phi_1}-\frac{T_0m_1\xi_1^{\,2}}{\chi_1}\right)\right.$$

$$\left.\int_0^t f(\tau)d\tau*\mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0m_2}{\chi_2}}+h\sqrt{\frac{T_0m_1}{\chi_1}}}{2\sqrt{t}}\right]\right\}+\frac{2\xi_1R_1\rho_2\chi_2}{\phi_1}\cdot$$

$$(\rho_1\xi_1+\rho_2\xi_2)\sqrt{T_0m_1\chi_1}\,\xi_2^{\,2}\varepsilon_2\mathrm{R}_2\cdot\left\{\int_0^t\varphi'(t-\tau)\right.$$

$$Erf\left[\frac{(x-h)\sqrt{\frac{T_0m_2}{\chi_2}}+h\sqrt{\frac{T_0m_1}{\chi_1}}}{2\sqrt{\tau}}\right]d\tau+\left(\frac{\phi_2}{\phi_1}-\frac{T_0m_1\xi_1^{\,2}}{\chi_1}\right)\cdot$$

$$\left.\int_0^t\varphi(t-\tau)\mathrm{Erf}\left[\frac{(x-h)\sqrt{\frac{T_0m_2}{\chi_2}}+h\sqrt{\frac{T_0m_1}{\chi_1}}}{2\sqrt{\tau}}\right]d\tau\right\} \tag{186}$$

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