SOME REMARKS ON JACOBIAN PROBLEM

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A polynomial self-map $F: k^n \to k^n$ of an affine space k^n is said to be invertible if there is a polynomial map $G: k^n \to k^n$ satisfying $F \circ G = G \circ F = \mathrm{id}_{k^n}$. The Jacobian Problem (JP) consists in asking whether or not a polynomial map $F: k^n \to k^n$ is invertible if the Jacobian $J(F)$ is invertible in the matrix algebra (of size n) over the polynomial ring $k[\underset{\sim}{x}] = k[x_1, \ldots, x_n]$. It is known that the answer to this question is in the negative if the characteristic $\chi(k) \neq 0$ (Nousiainen, 1981). On the other hand, for the case $\chi(k) = 0$, it is generally believed that the answer will be in the positive. In this talk I point out that the problem JP can be reduced to the following one which will be called the Jacobian Surjectiveness Problem (JSP) : Is a polynomial map $F: k^n \to k^n$ surjective, if $J(F)$ is invertible? This problem can further be reduced to the following one : Let $G: k^n \to k^n$ be a polynomial map whose Jacobian $J(G)$ is nilpotent. Is it possible to find a coordinate transformation under which

$$G(\underset{\sim}{x}) = (G_1(\underset{\sim}{x}), \ldots, G_n(\underset{\sim}{x}))$$

takes the form

$$G_i(x) = \text{a polynomial in } x_1, \ldots, x_{i-1}$$

for $i = 1, \ldots, n$.

REFERENCE

[1] BASS, H., CONNELL, E.H. and WRIGHT, D. : *The Jacobian Conjecture*, Bull. AMS, 2 (1982).

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