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MAXIMUM TERM FUNCTION OF ENTIRE DIRICHLET SERIES

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Let $f(s) = \sum_{n \in \mathbf{N}} a_n e^{s\lambda_n}$ be an entire function defined by an everywhere convergent Dirichlet series whose exponents are subjected to the condition that $\limsup_{n \to +\infty} \frac{\log n}{\lambda_n} = D \in \mathbf{R}_+ \cup \{0\}$ ($\mathbf{R}_+$ is the set of positive reals), and let $\mu(\sigma, f) = \sup_{n \in \mathbf{N}} \{ |a_n| e^{\sigma\lambda_n} \}$ be the maximum term, for $\operatorname{Re}(s) = \sigma$, in the Dirichlet series defining $f(s)$. We study a few results involving the function μ.

1. Let E be the set of mappings $f : \mathbf{C} \to \mathbf{C}$ ($\mathbf{C}$ is the complex plane) such that the image under f of an element $s \in \mathbf{C}$ is $f(s) = \sum_{n \in \mathbf{N}} a_n e^{s\lambda_n}$ with $\limsup_{n \to +\infty} \frac{\log n}{\lambda_n} = D \in \mathbf{R}_+ \cup \{0\}$ ($\mathbf{R}_+$ is the set of positive reals), and $\sigma_c^f = +\infty$ (σ_c^f is the abscissa of convergence of the Dirichlet series defining f), $\mathbf{N}$ is the set of natural numbers $0, 1, 2, \ldots$, $< \lambda_n \mid n \in \mathbf{N} >$ is a strictly increasing unbounded sequence of nonnegative reals, $s = \sigma + it$, σ, $t \in \mathbf{R}$ ($\mathbf{R}$ is the field of reals), and $< a_n \mid n \in \mathbf{N} >$ is a sequence in $\mathbf{C}$. Since the Dirichlet series defining f converges for each $s \in \mathbf{C}$, f is an entire function. Also, since $D \in \mathbf{R}_+ \cup \{0\}$, we have ([1], p. 168), $\sigma_a^f = +\infty$ (σ_a^f is the abscissa of absolute convergence of the Dirichlet series defining f), and that f is bounded on each vertical line $\operatorname{Re}(s) = \sigma_0$.

Let

$$M(\sigma, f) = \sup_{n \in \mathbf{R}} \{ |f(\sigma + it)| \}, \ \forall \sigma < \sigma_c^f, \tag{1.1}$$

be the maximum modulus of an entire function $f \in E$ on any vertical line $\operatorname{Re}(s) = \sigma$,

$$\mu(\sigma, f) = \sup_{n \in \mathbf{N}} \{ |a_n| e^{\sigma\lambda_n} \}, \ \forall \sigma < \sigma_c^f, \tag{1.2}$$

be the maximum term, for $\operatorname{Re}(s) = \sigma$, in the Dirichlet series defining f, and

$$\nu(\sigma, f) = \sup_{n \in \mathbf{N}} \{ n \mid \mu(\sigma, f) = |a_n| e^{\sigma\lambda_n} \}, \ \forall \sigma < \sigma_c^f, \tag{1.3}$$

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be the rank of the maximum term.

In this paper, we study a few results involving the function μ.

2. We first define a function A_p, $p \in \mathbf{Z}_+$ ($\mathbf{Z}_+$ is the set of positive integers), for every entire function $f \in E$, as

$$A_p(\sigma, f) = \frac{\mu_p(\sigma, f^{(p)})}{\mu(\sigma, f)}, \ \forall \sigma < \sigma_c^f, \tag{2.1}$$

and establish a result regarding it. We call A_p the quotient function of p-th order of f.

Teorem 1. For every entire function $f \in E$,

$$\limsup_{\sigma \to +\infty} \frac{(A_p(\sigma, f))^{1/p}}{\lambda_{\nu(\sigma, f^{(p)})}} \leq 1 \leq \liminf_{\sigma \to +\infty} \frac{(A_p(\sigma, f))^{1/p}}{\lambda_{\nu(\sigma, f)}}, \ \forall p \in \mathbf{Z}_+. \tag{2.2}$$

Proof. We know ([2], lemma 2) that, for any $p \in \mathbf{Z}_+$,

$$\lambda_{\nu(\sigma, f)} \leq \left(\frac{\mu_p(\sigma, f^{(p)})}{\mu(\sigma, f)} \right)^{1/p} \leq \lambda_{\nu(\sigma, f^{(p)})}. \tag{2.3}$$

Dividing both sides of the first inequality in (2.3) by $\lambda_{\nu(\sigma, f)}$, and proceeding to limits, we get

$$\liminf_{\sigma \to +\infty} \frac{(A_p(\sigma, f))^{1/p}}{\lambda_{\nu(\sigma, f)}} \geq 1, \tag{2.4}$$

and dividing both sides of the second inequality in (2.3) by $\lambda_{\nu(\sigma, f^{(p)})}$, and proceeding to hmits, we get

$$\limsup_{\sigma \to +\infty} \frac{(A_p(\sigma, f))^{1/p}}{\lambda_{\nu(\sigma, f^{(p)})}} \leq 1. \tag{2.5}$$

Combining (2.4) and (2.5), we get (2.2).

Remark. If f is of Ritt order $\rho \in \mathbf{R}_+^* \cup \{0\}$ ($\mathbf{R}_+^*$ is the set of extended positive reals) and lower order $\lambda \in \mathbf{R}_+^* \cup \{0\}$, it follows from (2.3) and the following result ([3], Theorem 2.7 and 2.8)

$$\frac{\rho}{\lambda} = \lim_{\sigma \to +\infty} \frac{\sup}{\inf} \frac{\log \log M(\sigma, f)}{\sigma} = \lim_{\sigma \to +\infty} \frac{\sup}{\inf} \frac{\log \lambda_{\nu(\sigma, f)}}{\sigma} \tag{2.6}$$

that

$$\lim_{\sigma \to +\infty} \frac{\sup}{\inf} \frac{\log (\mu_{p(\sigma, f^{(p)})} / \mu(\sigma, f))^{1/p}}{\sigma} = \frac{\rho}{\lambda}; \tag{2.7}$$

a result stated without proof by Srivastava ([4], p. 89), and proved by Kamthan ([5], Theorem E) adopting a lengthy method.

Next we improve upon the following Theorem of Srivastava ([4], Theorem 3):

Theorem A. If $f \in E$ is an entire function of Ritt order $\rho \in \mathbf{R}_+^* \cup \{0\}$ and lower order $\lambda \in \mathbf{R}_+^* \cup \{0\}$, then

$$\liminf_{\sigma \to +\infty} \frac{\log \mu(\sigma, f)}{\lambda_{\nu(\sigma,f)}} \leq \frac{1}{\rho} \leq \frac{1}{\lambda} \leq \limsup_{\sigma \to +\infty} \frac{\log \mu(\sigma, f)}{\lambda_{\nu(\sigma,f)}}. \tag{2.8}$$

Remark. Theorem A has been proved under the condition that $D = 0$, but it is true in general.

We show that :

Theorem 2. For every entire function $f \in E$ of Ritt order $\rho \in \mathbf{R}_+^* \cup \{0\}$, and lower order $\lambda \in \mathbf{R}_+^* \cup \{0\}$, and for any $p \in \mathbf{N}$,

$$\liminf_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} \leq \frac{1}{\rho} \leq \frac{1}{\lambda} \leq \limsup_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}}. \tag{2.9}$$

That Theorem 2 improves upon (2.8) follows from the fact ([6], Theorem 3) that

$$\mu(\sigma, f) \leq \mu_1(\sigma, f^{(1)}) \leq \ldots \leq \mu_p(\sigma, f^{(p)}) \leq \ldots .$$

Proof. We have, from (2.3),

$$\left\{ \begin{aligned} \log \lambda_\nu(\sigma, f) &\leq \frac{1}{p} (\log \mu_p(\sigma, f^{(p)}) - \log \mu(\sigma, f)) \\ &\leq \log \lambda_\nu(\sigma, f^{(p)}) . \end{aligned} \right. \tag{2.10}$$

From the first inequality in (2.10), we get

$$\begin{aligned} p \left(\lim_{\sigma \to +\infty} \frac{\log \lambda_\nu(\sigma, f)}{\lambda_{\nu(\sigma,f)}} \right) &\leq \lim_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} - \lim_{\sigma \to +\infty} \frac{\log \mu(\sigma, f)}{\lambda_{\nu(\sigma,f)}} \\ &\leq \lim_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} - \frac{1}{\lambda}, \end{aligned} \tag{2.11}$$

in view of (2.8). Since $\lambda_{\nu(\sigma,f)}$ tends to infinity with σ, it follows, from (2.11), that

$$\limsup_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} \geq \frac{1}{\lambda}. \tag{2.12}$$

Also, from the second inequality in (2.10), we have

$$\begin{aligned} p \left(\limsup_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} \right) &\geq \liminf_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} - \liminf_{\sigma \to +\infty} \frac{\log \mu(\sigma, f)}{\lambda_{\nu(\sigma,f)}} \\ &\geq \liminf_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} - \frac{1}{\rho}, \end{aligned} \tag{2.13}$$

in view of (2.8). Since, from (2.6),

$$\limsup_{\sigma \to +\infty} \frac{\log \lambda_{\nu(\sigma, f^{(p)})}}{\sigma} = \mathrm{p},$$

it follows, from (2.13), that

$$\liminf_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} \leq \frac{1}{\rho}. \tag{2.14}$$

Combining (2.12) and (2.14) we get (2.9).

Following corollaries are immediate from (2.9):

Corollary 1. If f is of infinite Ritt order, then

$$\liminf_{\sigma \to +\infty} \frac{\log \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} = 0. \tag{2.15}$$

Corollary 2. If f is of lower order zero, then

$$\limsup_{\sigma \to +\infty} \frac{\text{iog}\, \mu_p(\sigma, f^{(p)})}{\lambda_{\nu(\sigma,f)}} = +\infty. \tag{2.16}$$

We now obtain a majorant for the quantity $\limsup_{\sigma \to +\infty} \frac{\log \mu(\sigma, f)}{\lambda_{\nu(\sigma,f)}} = \alpha$ (say),

with the help of growth numbers of f The growth number $\gamma \in \mathbf{R}_+^* \cup \{0\}$ and lower growth number $\delta \in \mathbf{R}_+^* \cup \{0\}$ of an entire function $f \in E$ of **R**itt order $\mathrm{p} \in \mathbf{R}_+$ are defined [7] as

$$\begin{matrix}\gamma \\ \delta\end{matrix} = \lim_{\sigma \to +\infty} \begin{matrix}\sup \\ \inf\end{matrix} \frac{\lambda_{\nu(\sigma,f)}}{e^{\rho\sigma}}. \tag{2.17}$$

One majorant for α has already been obtained by Srivastava and Gupta who have shown ([7], Theorem 2) that :

Theorem B. If $f \in E$ is an entire function of **R**itt order $\mathrm{p} \in \mathbf{R}_+$, lower order $\lambda \in \mathbf{R}_+ \cup \{0\}$, growth number $\gamma \in \mathbf{R}_+^* \cup \{0\}$, and lower growth number $\delta \in \mathbf{R}_+^* \cup \{0\}$, then

$$\frac{\delta}{\rho\gamma} \leq \text{iim}\inf_{\sigma \to +\infty} \frac{\log \mu(\sigma, f)}{\lambda_{\nu(\sigma,f)}} \leq \frac{1}{\rho} \leq \frac{1}{\lambda} \leq \limsup_{\sigma \to +\infty} \frac{\log \mu(\sigma, f)}{\lambda_{\nu(\sigma,f)}} \leq \frac{\gamma}{\delta\,\mathrm{p}}. \tag{2.18}$$

We however show that :

Theorem 3. Under the hypothesis of Theorem **B**,

$$\alpha \leq \frac{1}{\rho}\left(1 + \log \frac{\gamma}{\delta}\right). \tag{2.19}$$

Proof. It is known ([8], p. 67) that

$$\log \mu(\sigma, f) = 0(1) + \int_{\sigma_0}^{\sigma} \lambda_{\nu(x,f)}\, dx . \tag{2.20}$$

We choose a $k \in \mathbf{R}_+$ and get, from (2.20),

$$\log \mu\left(\sigma + \frac{k}{\rho}, f\right) = 0(1) + \int_{\sigma_0}^{\sigma} \lambda_{\nu(x,f)}\, dx + \int_{\sigma}^{\sigma+\frac{k}{\rho}} \lambda_{\nu(x,f)}\, dx .$$

This gives, in view of (2.17), for any $\varepsilon \in \mathbf{R}_+$ and sufficiently large σ,

$$\log \mu\left(\sigma + \frac{k}{\rho}, f\right) < 0(1) + \frac{\gamma+\varepsilon}{\rho}\left(e^{\rho\sigma} - e^{\rho\sigma_0}\right) + \lambda_{\nu\left(\sigma+\frac{k}{\rho}, f\right)} \frac{k}{\rho} .$$

Therefore,

$$\frac{\log \mu\left(\sigma + \frac{k}{\rho}, f\right)}{\lambda_{\nu\left(\sigma+\frac{k}{\rho}, f\right)}} < 0(1) + \frac{\gamma+\varepsilon}{\rho} \frac{e^{\rho\left(\sigma+\frac{k}{\rho}\right)}}{e^k \lambda_{\nu\left(\sigma+\frac{k}{\rho}, f\right)}}(1 - o(1)) + \frac{k}{\rho} ,$$

or

$$\limsup_{\sigma\to+\infty} \frac{\log \mu\left(\sigma + \frac{k}{\rho}, f\right)}{\lambda_{\nu\left(\sigma+\frac{k}{\rho}, f\right)}} \le \frac{k}{\rho} + \frac{\gamma}{\rho e^k} \limsup_{\sigma\to+\infty} \frac{e^{\rho\left(\sigma+\frac{k}{\rho}\right)}}{\lambda_{\nu\left(\sigma+\frac{k}{\rho}, f\right)}} ,$$

which gives $\alpha \geq \dfrac{k}{\rho} + \dfrac{\gamma}{\rho e^k} \cdot \dfrac{1}{\delta}$. Taking $k = \log \dfrac{\gamma}{\delta}$, we get

$$\alpha \le \frac{1}{\rho}\left(1 + \log \frac{\gamma}{\delta}\right),$$

proving (2.19).

Remarks. (i) Since $1 + \log x \le x$ for $x \geq 1$, it follows that

$$\frac{1}{\rho}\left(1 + \log \frac{\gamma}{\delta}\right) \le \frac{\gamma}{\rho\,\delta} .$$

Thus the majorant for α given by (2.19) is better than the one given by (2.18).

(ii) Since, for $x \geq 1$, $x - (1 + \log x)$ is nonnegative nondecreasing function and has the maximum at $x = 1$, it follows that if $\gamma \neq \delta$ then $\alpha < \dfrac{\gamma}{\rho\,\delta}$.

(iii) The minorant for the quantity $\liminf\limits_{\sigma\to+\infty} \dfrac{\log \mu(\sigma, f)}{\lambda_{\nu(\sigma,f)}}$ given by (2.18) could not be improved by our method of investigation.

3. Srivastava and Gupta ([2], p. 241) have defined a difference function χ_p for every entire function $f \in E$, as

$$\chi_p(\sigma, f) = \chi(\sigma, p) = \lambda_{\nu(\sigma, f^{(p)})} - \lambda_{\nu(\sigma, f)}, \ \forall\, \sigma < \sigma_c^f, \tag{3.1}$$

and have proved ([7], Theorem 1) that :

Theorem C. If $f \in E$ is an entire function of Ritt order $\rho \in \mathbf{R}_+$, growth number $\gamma \in \mathbf{R}_+^* \cup \{0\}$, lower growth number $\delta \in \mathbf{R}_+^* \cup \{0\}$, and $\limsup_{\sigma \to +\infty} \chi_p(\sigma, f)$ is finite, then

$$\lim_{\sigma \to +\infty} \begin{matrix} \sup \\ \inf \end{matrix} \frac{(A_p(\sigma, f))^{1/p}}{e^{\rho\sigma}} = \begin{matrix} \gamma \\ \delta \end{matrix}. \tag{3.2}$$

We, however, prove

Theorem 4. Let $f_1, f_2 \in E$ be two entire functions, respectively, of Ritt orders $\rho, \rho' \in \mathbf{R}_+$, growth numbers $\gamma, \gamma' \in \mathbf{R}_+^* \cup \{0\}$ and lower growth numbers $\delta, \delta' \in \mathbf{R}_+^* \cup \{0\}$, and let

$$\lim_{\sigma \to +\infty} \begin{matrix} \sup \\ \inf \end{matrix} (\lambda_{\nu(\sigma, f_2)} - \lambda_{\nu(\sigma, f_1)}) = \begin{matrix} \alpha \\ \beta \end{matrix}.$$

If $\alpha, \beta \in \mathbf{R}$, and $\limsup_{\sigma \to +\infty} \chi_p(\sigma, f_1)$ and $\limsup_{\sigma \to +\infty} \chi_p(\sigma, f_2)$ are finite, then

$$\frac{\delta}{\gamma} \le \liminf_{\sigma \to +\infty} \frac{(A_p(\sigma, f_1))^{1/p}}{(A_q(o, f_2))^{1/q}} \le \limsup_{\sigma \to +\infty} \frac{(A_p(\sigma, f_1))^{1/p}}{(A_q(\sigma, f_2))^{1/q}} \le \frac{\gamma}{\delta}. \tag{3.3}$$

Proof. It is known (([2], Theorem 4) and ([9], Theorem 3)) that under the hypothesis of the theorem $\rho = \rho'$, $\gamma = \gamma'$ and $\delta = \delta'$. Making use of (3.2) for f_1 and f_2, respectively, we get, for any $\varepsilon \in \mathbf{R}_+$ and sufficiently large σ,

$$(\delta - \varepsilon)\, e^{\rho\sigma} < (A_p(\sigma, f_1))^{1/p} < (\gamma + \varepsilon)\, e^{\rho\sigma},$$

and

$$(\delta - \varepsilon)\, e^{\rho\sigma} < (A_q(\sigma, f_2))^{1/q} < (\gamma + \varepsilon)\, e^{\rho\sigma}.$$

Therefore, for any $\varepsilon \in \mathbf{R}_+$ and sufficiently large σ,

$$\frac{\delta - \varepsilon}{\gamma + \varepsilon} < \frac{(A_p(\sigma, f_1))^{1/p}}{(A_q(\sigma, f_2))^{1/q}} < \frac{\gamma + \varepsilon}{\delta - \varepsilon}.$$

Now proceeding to limits, we get (3.3).

The following corollary is immediate from Theorem 4.

Corollary 3. Under the hypothesis of Theorem 4, if either of the functions f_1 and f_2, say f_1, is of strictly regular growth (i. e. $\gamma = \delta$), then the other is also of strictly regular growth and, as $\sigma \to +\infty$,

$$\left(\frac{\mu_p(\sigma, f_1^{(p)})}{\mu(\sigma, f)}\right)^{1/p} \sim \left(\frac{\mu_q(\sigma, f_2^{(q)})}{\mu(\sigma, f_2)}\right)^{1/q} ; \tag{3.4}$$

in particular

$$\frac{\mu(\sigma, f_2)}{\mu(\sigma, f_1)} \sim \frac{\mu_1(\sigma, f_2^{(1)})}{\mu_1(\sigma, f_1^{(1)})} \sim \frac{\mu_2(\sigma, f_2^{(2)})}{\mu_2(\sigma, f_1^{(2)})} \sim \dots . \tag{3.5}$$

Finaly, we rectify a result of S. N. Srivastava. He has shown ([10], p. 251) that

Theorem D. If $f \in E$ is an entire function of Ritt order $\rho \in \mathbf{R}_+^* \cup \{0\}$ and lower order $\lambda \in \mathbf{R}_+^* \cup \{0\}$, and

$$\lim_{\sigma \to +\infty} \frac{\log \lambda_{\nu(\sigma, f^{(p)})} - \log \lambda_{\nu(\sigma, f)}}{\sigma} = 0 , \tag{3.6}$$

then

$$\lim_{\sigma \to +\infty} \begin{matrix} \sup \\ \inf \end{matrix} \frac{\log (\mu_p'(\sigma, f^{(p)}) / \mu'(\sigma, f))}{\sigma} = \begin{matrix} p\,\rho \\ p\,\lambda \end{matrix} , \tag{3.7}$$

where μ' is the derivative of μ with respect to σ, and $p \in \mathbf{Z}_+$.

We find that the conditions in the hypothesis of Theorem D are contradictory as is evident from

Lemma 1. An entire function $f \in E$ is of regular growth iff

$$\lim_{\sigma \to +\infty} \frac{\log \lambda_{\nu(\sigma, f^{(p)})} - \log \lambda_{\nu(\sigma, f)}}{\sigma} = 0 . \tag{3.8}$$

The proof follows from (2.6) and the fact that the Ritt order and the lower order of f are the same as that of its p-th derivative $f^{(p)}$, $\forall\, p \in \mathbf{Z}_+$.

It would thus appear that Theorem D is true only for entire functions $f \in E$ of regular growth, in which case the condition (3.6) is superfluous. We mention this observation formally as

Theorem 5. For every entire function $f \in E$ of regular growth and Ritt order $\rho \in \mathbf{R}_+^* \cup \{0\}$, and $p \in \mathbf{Z}_+$,

$$\lim_{\sigma \to +\infty} \frac{\log (\mu_p'(\sigma, f^{(p)}) / \mu'(\sigma, f))}{\sigma} = p\,\rho . \tag{3.9}$$

The proof is the same as that of Theorem D with obvious modifications.

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ÖZET

$f(s) = \sum_{n \in \mathbb{N}} a_n e^{s\lambda_n}$ eksponentleri $\limsup_{n \to +\infty} \frac{\log n}{\lambda_n} = D \in \mathbb{R}_+ \cup \{0\}$ koşuluna uyan, her yerde yakınsak bir Dirichlet serisi ile tanımlanan bir tam fonksiyon ve $\mu(\sigma, f) = \sup_{n \in \mathbb{N}} \{ |a_n| e^{\sigma\lambda_n} \}$, $f(s)$ i tanımlayan Dirichlet serisinde $\mathrm{Re}(s) = \sigma$ koşuluna uyan maksimum terim olsun. Bu çalışmada μ fonksiyonu ile ilgili bazı sonuçlar elde edilmektedir.