LARMOR RADIUS AND COLLISIONAL EFFECTS ON THERMAL-CONVECTIVE INSTABILITY OF A COMPOSITE STELLAR ATMOSPHERE [1)]

R. C. SHARMA

The problem of thermal-convective instability of a hydromagnetic composite stellar atmosphere has been studied to include simultaneously the effects of finite LARMOR radius and the frictional effects with neutrals. The effect of a uniform rotation has also been included. It is found that the criterion for monotonic instability holds good in the presence of the effects due to rotation, finite LARMOR radius and frictional effects with neutrals.

1. **Introduction.** The convective instability (in which motions are driven by buoyancy forces) of a thermally unstable atmosphere has been termed as «thermal-convective instability» by DEFOUW (1970). He has generalized the SCHWARZSCHILD criterion for convection to include departures from adiabatic motion and has shown that a thermally unstable atmosphere is also convectively unstable, irrespective of the atmospheric temperature gradient.

DEFOUW (1970) has found that a stellar atmosphere is unstable if

$$D = \frac{1}{C_p}(L_T - \varrho\, \alpha\, L_\varrho) + \varkappa\, k^2 < 0, \tag{1}$$

where L is the heat-loss function and α, $\varkappa$, k, L_T, L_ϱ denote respectively the coefficient of thermal expansion, the coefficient of thermometric conductivity, the wave number of the perturbation, the partial derivative of L with respect to temperature T and the partial derivative of L with respect to density ϱ, both evaluated in the equilibrium state. C_p is is the specific heat at constant pressure.

The effects of a uniform rotation and a uniform magnetic field on thermal-convective instability of a stellar atmosphere have been studied, separately by DEFOUW (1970) and simultaneously by BHATIA (1971). The effects of the finiteness of the ion LARMOR radius. which exhibits itself in the form of a magnetic viscosity in the fluid equations, on plasma instabilities have been studied by several authors (e.g. ROBERTS and TAYLOR, 1962 ; ROSENBLUTH, KRALL and ROSTOKER, 1962; JUKES, 1964; SHARMA, 1972). Recently SHARMA and PRAKASH (1977) have studied the finite LARMOR radius effect on thermal-convective instability of a stellar atmosphere. It has been found in the above studies that inequality (1) is a sufficient condition for monotonic instability, for situations of astrophysical interest. In the above studies, a fully ionized plasma has been considered. Quite frequently the plasma is not fully ionized and may, instead, be permeated with neutral atoms. The collisional effects on the stability of superposed media have been studied by HANS (1968). BHATIA and STEINER (1972) have considered the LARMOR radius and collisional effects on the dynamic stability of a composite medium. They idealized a plasma, which may not be fully ionized but also permeated with neutral atoms, as a composite mixture of a hydromagnetic (ionized) component and a neutral component, the two interacting through mutual collisions.

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It may, therefore, be of importance and is the object of the present paper to study the result of the simultaneous inclusion of finite LARMOR radius and collisional effects on the thermal-convective instability of a composite stellar atmosphere. The effect of a uniform rotation has also been included.

Consider an infinite horizontal composite layer consisting of a finitely conducting hydromagnetic incompressible fluid of density ϱ and a neutral gas of density ϱ_d, which is in a state of uniform rotation $\vec{\Omega} = (0, 0, \Omega)$; acted on by a vertical magnetic field $\mathbf{H} = (0, 0, H)$ and gravity force $\mathbf{g} = (0, 0, -g)$. This layer is heated such that a steady temperature gradient $\beta (= dT/dz)$ is maintained. Regarding the model under consideration we assume that both the ionized fluid and the neutral gas behave like continuum fluids and that the effects on the neutral component resulting from the presence of gravity and pressure are neglected. The magnetic field interacts with the ionized component only.

2. Perturbation equations. The first law of thermodynamics may be written as

$$c_v \frac{dT}{dt} = -L + \frac{K}{\varrho} \nabla^2 T + \frac{p}{\varrho^2} \frac{d\varrho}{dt}, \tag{2}$$

where c_v, K, T and t denote respectively the specific heat at constant volume, the thermal conductivity, the temperature and the time.

Following DEFOUW (1970), the linearized perturbation form of Equation (2) is

$$\frac{\partial \theta}{\partial t} + \frac{1}{c_p}(L_T - \varrho \varkappa L_\varrho)\,\theta - \varkappa \nabla^2 \theta = -\left(\beta + \frac{g}{c_p}\right) w, \tag{3}$$

where θ is the perturbation in temperature. In obtaining (3), use has been made of the BOUSSINESQ equation of state,

$$\delta \varrho = -\alpha \varrho \theta. \tag{4}$$

Let $\mathbf{q}(u, v, w)$, $\mathbf{h}(h_x, h_y, h_z)$, $\delta\varrho$ and δp denote the perturbations in velocity, magnetic field $\mathbf{H}$, density ϱ and pressure p respectively; g, ν, η, $\mathbf{q}_d$, ν_c denote, respectively, the gravitational acceleration, the kinematic viscosity, the resistivity, the velocity of the neutral gas and the collision frequency between the two components of the composite medium.

The linearized perturbation equations governing the motion of the mixture of the hydromagnetic fluid and a neutral gas are:

$$\varrho \frac{\partial \mathbf{q}}{\partial t} = -\nabla \delta p - \nabla \overleftrightarrow{P} + \varrho \nu \nabla^2 \mathbf{q} + \frac{1}{4\pi}(\nabla \times \mathbf{h}) \times \mathbf{H} + \mathbf{g}\,\delta\varrho + 2\varrho(\mathbf{q} \times \vec{\Omega}) + \varrho_d \nu_c (\mathbf{q}_d - \mathbf{q}), \tag{5}$$

$$\frac{\partial \mathbf{q}_d}{\partial t} = -\nu_c (\mathbf{q}_d - \mathbf{q}), \tag{6}$$

$$\frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{q} \times \mathbf{H}) + \eta \nabla^2 \mathbf{h}, \tag{7}$$

$$\nabla \cdot \mathbf{q} = 0, \quad \nabla \cdot \mathbf{h} = 0. \tag{8}$$

For the vertical magnetic field $\mathbf{H}$ (0, 0, H) the stress tensor components $\overleftrightarrow{P}$, taking into account the finite ion gyration, have the-components (SHARMA, 1972)

$$P_{xx} = -\varrho \nu_0 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad P_{xy} = P_{yx} = \varrho \nu_0 \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right),$$

(9) $$P_{xz} = P_{zx} = -2\varrho \nu_0 \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \quad P_{yy} = \varrho \nu_0 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),$$

$$P_{yz} = P_{zy} = 2\varrho \nu_0 \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \quad P_{zz} = 0.$$

In Equation (9), $\varrho \nu_0 = NT/4\omega_H$, where ω_H is the ion-gyration frequency, while N and T denote, respectively, the number density and the ion temperature.

We consider the case in which both the boundaries are free and the medium adjoining the fluid is non-conducting. The case of two free boundaries is the most appropriate for stellar atmosphere (SPIEGEL, 1965).

The boundary conditions appropriate for the problem are (CHANDRASEKHAR, 1961)

(10) $$w = 0, \quad \theta = 0, \frac{\partial^2 w}{\partial z^2} = 0, \frac{\partial \zeta}{\partial z} = 0$$

$\xi = 0$ and h_x, h_y, h_z are continuous with an external vacuum field. Here ζ and ξ denote the z-components of vorticity and current density respectively.

3. Dispersion relation. Analyzing in terms of normal modes, we seek solutions whose dependence on space and time coordinates is of the form

(11) $$\exp\,[ik_x x + ik_y y + nt] \sin k_z z,$$

where k_z is an integral multiple of π divided by the thickness of the fluid layer,

$$k = (k_x^2 + k_y^2 + k_z^2)^{1/2}$$

is the wave number of the perturbation and n is the growth rate.

Eliminating $\mathbf{q}_d$ between Equations (5) and (6), Equations (3) and (5)-(8) give

(12) $$n'(\nabla^2 w) = g\alpha \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + \nu \nabla^4 w + \frac{H}{4\pi\varrho} \frac{\partial}{\partial z} \nabla^2 h_z - 2\Omega \frac{\partial \zeta}{\partial z} + \nu_0 \left(\nabla^2 - 3\frac{\partial^2}{\partial z^2}\right) \frac{\partial \zeta}{\partial z},$$

(13) $$n'\zeta = \nu \nabla^2 \zeta + 2\Omega \frac{\partial w}{\partial z} + \frac{H}{4\pi\varrho} \frac{\partial \xi}{\partial z} - \nu_0 \left(\nabla^2 - 3\frac{\partial^2}{\partial z^2}\right) \frac{\partial w}{\partial z},$$

(14) $$(n - \eta \nabla^2) h_z = H \frac{\partial w}{\partial z},$$

(15) $$(n - \eta \nabla^2)\xi = H \frac{\partial \zeta}{\partial z},$$

(16) $$(n + D)\theta = -\left(\beta + \frac{g}{c_p}\right) w,$$

where

(17) $$n' = n\left(1 + \frac{\alpha_0 \nu_c}{n + \nu_c}\right) \text{ and } \alpha_0 = \varrho_d/\varrho.$$

Eliminating θ, ζ, h_z and ξ from Equations (12)-(16) and using (11), we obtain the dispersion relation

$$n^7 + A_6 n^6 + A_5 n^5 + A_4 n^4 + A_3 n^3 + A_2 n^2 + A_1 n + A_0 = 0, \tag{18}$$

where

$$A_6 = D + 2k^2(\nu + \eta) + 2\nu_c(\alpha_0 + 1),$$

$$A_5 = 2k_z^2 V^2 + \Gamma\left(\beta + \frac{g}{c_p}\right) + 2\nu k^2(D + \nu_c) + 2\nu_c(\alpha_0 + 1)D + 2\eta k^2\{D + \nu k^2 + \nu_c(\alpha_0 + 1)\}$$
$$+ \{k^2(\nu + \eta) + \nu_c(\alpha_0 + 1)\}^2 + \frac{k_z^2}{k^2}(2\Omega + \nu_0 \overline{k^2 - 3k_z^2})^2,$$

$$A_4 = \nu_c\left\{\nu k^2 D + \Gamma\left(\beta + \frac{g}{c_p}\right) + k_z^2 V^2 + \nu\eta k^4\right\} + \eta k^2\left\{\nu k^2(D + \nu_c) + \nu_c(\alpha_0 + 1)D + \right.$$
$$\left.\Gamma\left(\beta + \frac{g}{c_p}\right)\right\} + \{k^2(\nu + \eta) + \nu_c(\alpha_0 + 1)\}\left[\nu k^2(D + \nu_c) + \nu_c D(\alpha_0 + 1) + \Gamma\left(\beta + \frac{g}{c_p}\right) + \right.$$
$$\left. k_z^2 V^2 + \eta k^2\{D + \nu k^2 + \nu_c(\alpha_0 + 1)\}\right] + \{D + k^2(\nu + \eta) + \nu_c(\alpha_0 + 1)\}\{\nu k^2 \nu_c(\alpha_0 + 1) +$$
$$\nu\eta k^4 + k_z^2 V^2\} + k_z^2 V^2(D + \nu_c) + \frac{k_z^2}{k^2}(2\Omega + \nu_0 \overline{k^2 - 3k_z^2})^2\{D + 2(\nu_c + \eta k^2)\}, \tag{19}$$

$$A_3 = \eta k^2 \nu_c\left\{\nu k^2 D + \Gamma\left(\beta + \frac{g}{c_p}\right)\right\} + \nu_c(k_z^2 V^2 + \nu\eta k^4)\{D + k^2(\nu + \eta) + \nu_c(\alpha_0 + 1)\} +$$
$$k_z^2 V^2 D\nu_c + \{k^2(\nu + \eta) + \nu_c(\alpha_0 + 1)\}\left[\nu k^2 D\nu_c + \nu_c\Gamma\left(\beta + \frac{g}{c_p}\right) + k_z^2 V^2(D + \nu_c) + \right.$$
$$\left.\eta k^2\left\{\nu k^2(D + \nu_c) + \nu_c(\alpha_0 + 1)D + \Gamma\left(\beta + \frac{g}{c_p}\right)\right\}\right] + \{\nu k^2 \nu_c + \eta k^2(\alpha_0 + 1)\nu_c + \nu\eta k^4 +$$
$$k_z^2 V^2\}\left[k_z^2 V^2 + \Gamma\left(\beta + \frac{g}{c_p}\right) + \nu k^2(D + \nu_c) + \nu_c(\alpha_0 + 1)D + \eta k^2\{D + \nu k^2 + \right.$$
$$\left.\nu_c(\alpha_0 + 1)\}\right] + \frac{k_z^2}{k^2}(2\Omega + \nu_0 \overline{k^2 - 3k_z^2})^2(\eta^2 k^4 + \nu_c^2 + 4\nu_c \eta k^2 + 2\eta k^2 D + 2\nu_c D),$$

$$A_2 = \{k^2(\nu + \eta) + \nu_c(\alpha_0 + 1)\}\left[\eta k^2 \nu_c\left\{\nu k^2 D + \Gamma\left(\beta + \frac{g}{c_p}\right)\right\} + k_z^2 V^2 D\nu_c\right]$$
$$\{\nu k^2 \nu_c + \eta k^2 \nu_c(\alpha_0 + 1) + \nu\eta k^4 + k_z^2 V^2\}\left[\nu k^2 D\nu_c + \Gamma\left(\beta + \frac{g}{c_p}\right)\nu_c + \right.$$
$$\left. k_z^2 V^2(D + \nu_c) + \eta k^2\left\{\nu k^2(D + \nu_c) + \nu_c(\alpha_0 + 1)D + \Gamma\left(\beta + \frac{g}{c_p}\right)\right\}\right] + \nu_c(k_z^2 V^2 +$$
$$\nu\eta k^4)\left[\nu k^2(D + \nu_c) + \nu_c(\alpha_0 + 1)D + \Gamma\left(\beta + \frac{g}{c_p}\right) + \eta k^2\{D + \nu k^2 + \nu_c(\alpha_0 + 1)\} + k_z^2 V^2\right] +$$
$$\frac{k_z^2}{k^2}(2\Omega + \nu_0 \overline{k^2 - 3k_z^2})^2\{2\nu_c \eta^2 k^4 + 2\nu_c^2 \eta k^2 + D(\eta^2 k^4 + \nu_c^2 + 4\nu_c \eta k^2)\},$$

$$A_1 = \{\nu k^2 \nu_0 + \eta k^2 \nu_c(\alpha_0 + 1) + \nu\eta k^4 + k_z^2 V^2\}\left[\eta k^2 \nu_c\left\{\nu k^2 D + \Gamma\left(\beta + \frac{g}{c_p}\right)\right\} + k_z^2 V^2 D\nu_c\right] +$$
$$(k_z^2 V^2 + \nu\eta k^4)\nu_c\left[\nu k^2 D\nu_c + \Gamma\left(\beta + \frac{g}{c_p}\right)\nu_c + k_z^2 V^2(D + \nu_c) + \eta k^2\left\{\nu k^2(D + \nu_c) + \right.\right.$$

$$+ \nu_c(\alpha_0 + 1) D + \Gamma\left(\beta + \frac{g}{c_p}\right)\Big\}\Big] + \frac{k_z^2}{k^2}(2\Omega + \nu_0 \overline{k^2 - 3k_z^2})^2 \{\eta^2 k^4 \nu_c^2 + 2\nu_c \eta k^2 D(\eta k^2 + \nu_c)\},$$

$$A_0 = \nu_c(k_z^2 V^2 + \nu\eta k^4)\left[\eta k^2 \nu_c \left\{\nu k^2 D + \Gamma\left(\beta + \frac{g}{c_p}\right)\right\} + k_z^2 V^2 D \nu_c\right]$$

$$+ \frac{k_z^2}{k^2}(2\Omega + \nu_0 \overline{k^2 - 3k_z^2})^2 \eta^2 k^4 \nu_c^2 D,$$

and

$$\Gamma = \frac{g\,\alpha(k_x^2 + k_y^2)}{k^2}; \quad V^2 = \frac{H^2}{4\pi\varrho}.$$

4. Discussion. In many cases of astrophysical interest, the effects of viscosity and resistivity are negligible. Setting $\nu = \eta = 0$ in Equation (18) reduces it to

$$n^7 + \{D + 2\nu_c(\alpha_0 + 1)\} n^6 + \left[\Gamma\left(\beta + \frac{g}{c_p}\right) + 2k_z^2 V^2 + 2\nu_c D(\alpha_0 + 1) + \nu_c^2(\alpha_0 + 1)^2 + \right.$$

$$\left.\frac{k_z^2}{k^2}(2\Omega + \nu_0 \overline{k^2 - 3k_z^2})^2\right] n^5 + \left[\nu_c \Gamma\left(\beta + \frac{g}{c_p}\right) + k_z^2 V^2\{2D + \nu_c(\alpha_0 + 3)\} + \right.$$

$$\left.\nu_c(\alpha_0 + 1)\left\{k_z^2 V^2 + \Gamma\left(\beta + \frac{g}{c_p}\right) + \nu_c(\alpha_0 + 1) D\right\} + \frac{k_z^2}{k^2}(2\Omega + \nu_0 \overline{k^2 - 3k_z^2})^2 (D + 2\nu_c)\right] n^4 +$$

$$\left[k_z^2 V^2\left\{\nu_c(\alpha_0 + 3) D + \Gamma\left(\beta + \frac{g}{c_p}\right) + k_z^2 V^2 + \nu_c^2(\alpha_0 + 1)\right\} + \nu_c(\alpha_0 + 1)\left\{\Gamma\left(\beta + \frac{g}{c_p}\right)\nu_c + \right.\right.$$

$$\left.\left. k_z^2 V^2 (D + \nu_c)\right\} + \frac{k_z^2}{k^2}(2\Omega + \nu_0 \overline{k^2 - 3k_z^2})^2 \nu_c(\nu_c + 2D)\right] n^3 +$$

$$\left[k_z^2 V^2\left\{k_z^2 V^2(D + 2\nu_c) + 2\Gamma\left(\beta + \frac{g}{c_p}\right)\nu_c + 2\nu_c^2 D(\alpha_0 + 1)\right\} + \frac{k_z^2}{k^2}\nu_c^2 D(2\Omega + \right.$$

$$\left.\nu_0 \overline{k^2 - 3k_z^2})^2\right] n^2 + k_z^2 V^2 \nu_c\left\{k_z^2 V^2(2D + \nu_c) + \Gamma\left(\beta + \frac{g}{c_p}\right)\nu_c\right\} n +$$

$$k_z^4 V^4 \nu_c^2 D = 0. \tag{20}$$

In the limit of vanishing the collisional frequency ν_c, Equation (20) reduces to the result [(Eq. (18), SHARMA and PRAKASH (1977)].

When $D<0$, i.e. when inequality (1) is satisfied, the constant term in Equation (20) is negative. This means that the Equation (20) has a positive real root, leading to monotonic instability. The criterion for instability (1) is, thus, the same in the presence of the effects due to rotation, finite LARMOR radius and the frictional effects with neutrals on the thermal-convective instability of a composite stellar atmosphere.

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DEPARTMENT OF MATHEMATICS
HIMACHAL PRADESH UNIVERSITY
SIMLA-INDIA

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ÖZET

Hidromanyetik, kompozit bir yıldız atmosferindeki termik-konvektif dengesizlik problemi, sonlu LARMOR yarıçapları ve nötr cisimlerle sürtünme etkenlerinin etkisi düşünülerek incelenmiştir. Düzgün bir dönme hareketi problemin koşullarına sokulmuştur. Monoton dengesizlik kuralının dönme, sonlu LARMOR çapı ve nötrlerle sürtünmenin etkisi altında yine geçerli olduğu görülmüştür.