THE COMMUTATION FORMULAE IN n-DIMENSIONAL SPECIAL KAWAGUCHI SPACE +)

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Another kind of covariant derivative (∇^*) is introduced in n-dimensional special Kawaguchi space and Ricci identities involving this derivative is given and commutation formulae involving this covariant derivative and the usual one (∇) are established.

Introduction. The theory of n-dimensional special Kawaguchi space in which the arc length of a curve $x^i = x^i(t)$ is given by $S = \int F^{\frac{1}{p}} dt$, where $F = (A_i x''^i + B)$, $x'^i = dx^i/dt$, $x''^i = d^2 x^i/dt^2$ and the functions A_i and B are differentiable functions of x^i and x'^i, has been established by A. KAWAGUCHI [1] 1). The functions A_i and B are homogeneous functions of degree $(p - 2)$ and p respectively. Since the arc length S is a scalar, evidently A_i is a vector. If $p \neq 3/2$ and the determinant of the tensor $G_{ik} \overset{\text{def}}{=} (2A_{i(k)} - A_{k(i)})$ does not vanish identically, we have

$$\overset{[2]i}{x} = T_j G^{ji} = x''^i + 2\Gamma^i, \tag{1.1}$$

where

$$2\Gamma^i = (2A_{lk} x'^k - B_{(l)}) G^{li}, \quad G_{ik} G^{il} = \delta^l_k, \tag{1.2}$$

$$A_{ij} = \partial A_i / \partial x^j = \partial_j A_i \quad \text{and} \quad B_{(j)} = \partial B / \partial x'^j = \partial'_j B.$$

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1) The numbers in brackets refer to the references given at the end of the paper.

In special Kawaguchi space the covariant derivatives for a tensor field $T^i_j(x, x')$ are defined as [1] :

(1.3) $$\nabla_k T^i_j = \partial_k T^i_j - (\partial'_m T^i_j) \Gamma^m_{(k)} + T^m_j \Gamma^i_{(m)(k)} - T^i_m \Gamma^m_{(j)(k)}$$

and

(1.4) $$\tilde{\nabla}'_k T^i_j = \partial'_k T^i_j + T^m_j C^i_{mk} - T^i_m C^m_{jk},$$

where the tensor C^i_{jk} [1] is symmetric in its lower indices.

We define another covariant derivative for the connection parameter Π^i_{jk} as follows :

(1.5) $$\nabla^*_k T^i_j = \partial_k T^i_j - (\partial'_m T^i_j) \Gamma^m_{pk} x'^p + T^m_j \Pi^i_{mk} - T^i_m \Pi^m_{jk},$$

where the connection parameter Π^i_{jk} [4] is given by

(1.6) $$\Pi^i_{jk} \overset{\text{def}}{=} \Gamma^i_{(j)(k)} - \frac{1}{n+1} \Gamma^h_{(j)(k)(h)} x'^i.$$

The function Π^i_{jk} is homogeneous of degree zero with respect to x'^i.

From (1.3) and (1.5) we have

(1.7) $$\nabla^*_k T^i_j = \nabla_k T^i_j - \frac{1}{n+1} T^m_j \Gamma^r_{(m)(k)(r)} x'^i + \frac{1}{n+1} T^i_m \Gamma^r_{(j)(k)(r)} x'^m.$$

The commutation formulae involving the curvature tensor fields are as follows [1]

(1.8) $$(\nabla_j \nabla_k - \nabla_k \nabla_j) X^i = - R^i_{jkl} X^l + K^l_{jk} \nabla'_l X^i,$$

(1.9) $$(\nabla_j \tilde{\nabla}'_k - \tilde{\nabla}'_k \nabla_j) X^i = - \tilde{B}^i_{jkl} X^l + C^l_{jk} \nabla_l X^i$$

and

(1.10) $$(\nabla_j \nabla'_k - \nabla'_k \nabla_j) X^i = - B^i_{jkl} X^l,$$

where

$$(1.11)\qquad R^i_{jkl} = \partial_k \Gamma^i_{(l)(j)} - \partial_l \Gamma^i_{(l)(k)} + \Gamma^h_{(l)(j)} \Gamma^i_{(k)(h)} - \Gamma^h_{(l)(k)} \Gamma^i_{(j)(h)} + \\ + \Gamma^h_{(j)} \Gamma^i_{(l)(k)(h)} - \Gamma^h_{(k)} \Gamma^i_{(l)(j)(h)},$$

$$(1.12)\qquad K^i_{jk} = \partial_k \Gamma^i_{(j)} - \partial_j \Gamma^i_{(k)} + \Gamma^h_{(j)} \Gamma^i_{(k)(h)} - \Gamma^h_{(k)} \Gamma^i_{(j)(h)},$$

$$(1.13)\qquad B^i_{jkl} = \Gamma^i_{(j)(k)(l)}$$

and

$$(1.14)\qquad \widetilde{B}^i_{jkl} = \Gamma^i_{(l)(j)(k)} + C^i_{hk} \Gamma^h_{(l)(j)} - \partial_j C^i_{lk} + C^i_{lk(h)} \Gamma^h_{(j)} - \\ - \Gamma^i_{(h)(j)} C^h_{lk} + \Gamma^h_{(j)(k)} C^i_{lh}.$$

Also the commutation formula involving curvature tensor R^{*i}_{jkh} [3] is given by

$$(1.15)\qquad (\nabla^*_k \nabla^*_h - \nabla^*_h \nabla^*_k) X^i = -(\partial'_m X^i) R^{*m}_{hkl} x'^l + X^m R^{*i}_{hkm},$$

where

$$(1.16)\qquad R^{*i}_{jkh} = \partial_k \Pi^i_{hj} - \partial_j \Pi^i_{hk} + \Pi^l_{hj} \Pi^i_{kl} - \Pi^l_{hk} \Pi^i_{jl} + \\ + \Pi^l_j \Pi^i_{hk(l)} - \Pi^l_k \Pi^i_{hj(l)}.$$

2. Ricci identities involving covariant derivative of type (1.5). We have proved following theorems with respect to covariant derivative (1.5).

Theorem (2.1). *The Ricci identity for a contravariant tensor of rank 2 is given by*

$$(2.1)\qquad (\nabla^*_h \nabla^*_k - \nabla^*_k \nabla^*_h) T^{ij} = -(\partial'_m T^{ij}) R^{*m}_{khl} x'^l + T^{mj} R^{*i}_{khm} + T^{im} R^{*j}_{khm}.$$

Proof. Let $V_i(x, x')$ be any arbitrary covariant vector field such that its inner product with a tensor field $T^{ij}(x, x')$ is given by

$$(2.2)\qquad X^i(x, x') \stackrel{\text{def}}{=} T^{ij}(x, x') V_j(x, x').$$

Analogous to the commutation formula (1.15) we have

$$(2.3) \qquad (\overset{*}{\nabla}_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \overset{*}{\nabla}_h) V_i = -(\partial'_m V_l) R^{*m}_{khl}\, x'^l - V_m R^{*m}_{khi} .$$

Eliminating $X^i(x, x')$ from the equations (1.15) and (2.2) and using (2.3) we obtain

$$(2.4) \qquad V_j [(\overset{*}{\nabla}_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \overset{*}{\nabla}_h) T^{ij} + (\partial'_m T^{ij}) R^{*m}_{khl}\, x'^l - T^{mj} R^{*i}_{khm} - T^{im} R^{*j}_{khm}] = 0.$$

Since the vector $V_j(x, x')$ is an arbitrary therefore (2.4) establish the equation (2.1).

In consequence of the theorem (2.1) we have

Theorem (2.2). *The Ricci identity for a contravariant tensor* $T^{i_1 \dots i_r}$ *rank is given by*

$$(2.5) \qquad (\overset{*}{\nabla}_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \overset{*}{\nabla}_h) T^{i_1 \dots i_r} = -(\partial'_m T^{i_1 \dots i_r}) R^{*m}_{khl}\, x'^l + \sum_{a=1}^{r} T^{i_1 \dots i_{a-1}\, m\, i_{a+1} \dots i_r} R^{*i_a}_{khm} .$$

Theorem (2.3). *The Ricci identity for a covariant tensor* T_{ij} *is given by*

$$(2.6) \qquad (\overset{*}{\nabla}_k \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \overset{*}{\nabla}_h) T_{ij} = -(\partial'_m T_{ij}) R^{*m}_{khl}\, x'^l - T_{mj} R^{*m}_{khi} - T_{im} R^{*m}_{khj} .$$

Proof. Let us assure $X^i(x, x')$ be arbitrary contravariant vector field such that its inner product with a tensor field $T_{ij}(x, x')$ is given by

$$(2.7) \qquad V_i(x, x') \overset{\text{def}}{=} T_{ij}(x, x')\, X^j(x, x').$$

Eliminating $V_i(x, x')$ from (2.3) and (2.7) and using the equation (1.15) we obtain

$$(2.8) \qquad X^j [(\overset{*}{\nabla}_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \overset{*}{\nabla}_h) T_{ij} + (\partial'_m T_{ij}) R^{*m}_{khl}\, x'^l + T_{mj} R^{*m}_{khi} + T_{im} R^{*m}_{khj}] = 0.$$

Since $X^i(x, x')$ is an arbitrary vector therefore equation (2.8) yield (2.6).

Similarly, we can prove the following theorems :

Theorem (2.4). *The Ricci identity for a covariant tensor of order p is given by*

(2.9) $$(\overset{*}{\nabla}_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \overset{*}{\nabla}_h)\, T_{j_1 \ldots j_p} = \partial'_m(T_{j_1 \ldots j_p})\, R^{*m}_{klh}\, x'^l - \\ - \sum_{\beta=1}^{p} T_{j_1 \ldots j_{\beta-1}\, m j_{\beta+1} \ldots j_p}\, R^{*m}_{khj_\beta} .$$

Theorem (2.5). *The Ricci identity for a mixed tensor of order (r, p) is given by*

(2.10) $$(\overset{*}{\nabla}_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \overset{*}{\nabla}_h)\, T^{i_1 \ldots i_r}_{j_1 \ldots j_p} = -(\partial'_m T^{i_1 \ldots i_r}_{j_1 \ldots j_p})\, R^{*m}_{khl}\, x'^l + \\ + \sum_{a=1}^{r} T^{i_1 \ldots i_{a-1}\, m i_{a+1} \ldots i_r}_{j_1 \ldots \ldots \ldots \ldots \ldots \ldots j_p}\, R^{*i_a}_{khm} - \\ - \sum_{\beta=1}^{p} T^{i_1 \ldots \ldots \ldots \ldots \ldots \ldots i_r}_{j_1 \ldots j_{\beta-1}\, m j_{\beta+1} \ldots j_p}\, R^{*m}_{khj_\beta} .$$

3. Commutation formulae involving the covariant derivatives of the types (1.3) and (1.5). The commutation formulae for the covariant derivatives of the type (1.3) and (1.5) are given by the following theorems.

Theorem (3.1). *For a contravariant vector $X^i(x, x')$ the operators ∇ and $\overset{*}{\nabla}$ commute according to*

(3.1) $$(\nabla_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \nabla_h)\, X^i = K^l_{hk}\, \nabla'_l\, X^i - X^l\, Z^i_{hkl} - \frac{1}{n+1}\, \nabla_m\, X^i\, B_{hk}\, x'^m ,$$

where

(3.2) $$Z^i_{hkl} \overset{\text{def}}{=} R^i_{hkl} + \frac{1}{n+1}\, (\nabla_h\, B_{lk})\, x'^i \quad \text{and} \quad B_{hk} \overset{\text{def}}{=} B^r_{hkr} .$$

Proof. The relation (1.7) for a contravariant vector X^i assumes the form

$$(3.3) \qquad \overset{*}{\nabla}_k X^i = \nabla_k X^i - \frac{1}{n+1} X^m B_{mk} x'^i.$$

Differentiating (3.3) covariantly in view of (1.3) with respect to x^h we have

$$(3.4) \qquad \nabla_h \overset{*}{\nabla}_k X^i = \nabla_h \nabla_k X^i - \frac{1}{n+1} \{(\nabla_h X^m) B_{mk} x'^i + X^m (\nabla_h B_{mk}) x'^i\}.$$

We may assume that $(\nabla_h X^i)$ is a mixed tensor of the type (1.1) and therefore by virtue of (1.7) we get

$$(3.5) \qquad \overset{*}{\nabla}_k \nabla_h X^i = \nabla_k \nabla_h X^i - \frac{1}{n+1} (\nabla_h X^m) B_{mk} x'^i + $$
$$+ \frac{1}{n+1} (\nabla_m X^i) B_{hk} x'^m .$$

Subtracting (3.5) from (3.4) and using (1.8) we obtain (3.1).

Theorem (3.2). *For a covariant vector v_i the operators ∇ and $\overset{*}{\nabla}$ commute according to*

$$(3.6) \qquad (\nabla_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \nabla_h) V_i = K^l_{hk} (\nabla'_l V_i) + V_l Z^l_{hki} - \frac{1}{n+1} (\nabla_m V_i) B_{hk} x'^m .$$

Proof. Let $V_i(x, x')$ be a covariant vector field in $K_n^{(1)}$, for which the equation (1.7) takes the form :

$$(3.7) \qquad \overset{*}{\nabla}_k V_i = \nabla_k V_i + \frac{1}{n+1} V_m B_{ik} x'^m.$$

Using equations (1.3), (1.7), (1.8), (3.7) and proceeding on the same pattern as the proof given for the theorem (3.1), we obtain (3.6).

Theorem (3.3). *For a contravariant tensor T^{ij} the operators ∇ and $\overset{*}{\nabla}$ commute according to*

$$(3.8)\qquad (\nabla_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \nabla_h)\, T^{ij} = K^l_{hk}(\nabla'_l\, T^{ij}) - T^{lj} Z^i_{hkl} - T^{il} Z^j_{hkl} - \frac{1}{n+1}(\nabla_m\, T^{ij})\, B_{hk}\, x'^m.$$

Proof. Eliminating $X^i(x, x')$ from the equations (2.2) and (3.1) and using (3.6) we obtain

$$(3.9)\qquad V_j \left[(\nabla_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \nabla_h)\, T^{ij} + T^{lj} Z^i_{hkl} + T^{il} Z^j_{hkl} - K^l_{hk}(\nabla'_l\, T^{ij}) + \frac{1}{n+1}(\nabla_m\, T^{ij})\, B_{hk}\, x'^m \right] = 0.$$

Since the vector $V_j(x, x')$ is arbitrary, therefore from (3.9) we immediately obtain (3.8).

Similarly for a contravariant tensor $T^{i_1 \cdots i_r}$, we have

Theorem (3.4). *For a contravariant tensor* $T^{i_1 \cdots i_r}$ *of rank r the operators* ∇ *and* $\overset{*}{\nabla}$ *commute according to*

$$(3.10)\qquad (\nabla_k \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \nabla_h)\, T^{i_1 \cdots i_r} = K^l_{hk}(\nabla'_l\, T^{i_1 i_2 \cdots i_r}) - \sum_{a=1}^{r} T^{i_1 \cdots i_{a-1}\, l\, i_{+1} \cdots i_r} Z^{i_a}_{hkl} - \frac{1}{n+1}(\nabla_l\, T^{i_1 \cdots i_r})\, B_{hk}\, x'^l.$$

Theorem (3.5). *For a covariant tensor* T_{ij} *the operators* ∇ *and* $\overset{*}{\nabla}$ *commute according to*

$$(3.11)\qquad (\nabla_h \overset{*}{\nabla}_k - \overset{*}{\nabla}_k \nabla_h)\, T_{ij} = K^l_{hk}(\nabla'_l\, T_{ij}) + T_{mj} Z^m_{hki} + T_{im} Z^m_{hkj} - \frac{1}{n+1}(\nabla_m\, T_{ij})\, B_{hk}\, x'^m.$$

Proof. Eliminating $V_i(x, x')$ from (2.7) and (3.6) we obtain an equation of the type (3.9) for a covariant tensor T_{ij}. Combining this equation with that of (3.1) we obtain (3.11).

Accordingly, we can prove the following theorems :

Theorem (3.6). *For a tensor field* $T_{j_1 \dots j_p}(x, x')$ *the operators* ∇ *and* ∇^* *commute according to*

$$(3.12)\qquad (\nabla_h \nabla_k^* - \nabla_k^* \nabla_h) T_{j_1 \dots j_p} = K_{hk}^l (\nabla'_l T_{j_1 \dots j_p}) + \sum_{\beta=1}^{p} T_{j_1 \dots j_{\beta-1} m j_{\beta+1} \dots j_p} Z_{hkj_\beta}^m - \frac{1}{n+1} (\nabla_m T_{j_1 \dots j_p}) B_{hk} x'^m.$$

Theorem (3.7). *For any tensor* $T_{j_1 \dots j_p}^{i_1 \dots i_r}$ *of order* (r, p) *the operators* ∇ *and* ∇^* *commute according to*

$$(3.13)\qquad (\nabla_h \nabla_k^* - \nabla_k^* \nabla_h) T_{j_1 \dots j_p}^{i_1 \dots i_r} = K_{hk}^l (\nabla'_l T_{j_1 \dots j_p}^{i_1 \dots i_r}) - \sum_{a=1}^{r} T_{j_1 \dots \dots \dots j_p}^{i_1 \dots i_{a-1} m j_{a+1} \dots i_r} Z_{hkm}^{i_a} + \sum_{\beta=1}^{p} T_{j_1 \dots j_{\beta-1} m j_{\beta+1} \dots j_p}^{i_1 \dots \dots \dots i_r} Z_{hkj_\beta}^m - \frac{1}{n+1} (\nabla_m T_{j_1 \dots j_p}^{i_1 \dots i_r}) B_{hk} x'^m.$$

4. The commutation formulae involving covariant derivatives of the Types (1.4) and (1.5).

The commutation rules for the operators $\tilde{\nabla}'_k$ and ∇^*, given by (1.4) and (1.5), obey the following theorems :

Theorem (4.1). *The commutation formula for a contravariant vector* X^i *is given by*

$$(4.1)\qquad (\nabla_h^* \tilde{\nabla}'_k - \tilde{\nabla}'_k \nabla_h^*) X^i = - X^l \tilde{Z}_{hkl}^i + (\nabla_l X^i) C_{hk}^l + \frac{1}{n+1} (\tilde{\nabla}'_m X^i) B_{hk} x'^m,$$

where

$$(4.2)\qquad \tilde{Z}^i_{hkl} \overset{\text{def}}{=} \tilde{B}^i_{hkl} - \frac{1}{n+1}\{(\tilde{\nabla}'_k B_{lh})\, x'^i + B_{lh}\, \delta^i_k\}.$$

Proof. Differentiating (3.3) covariantly with respect to x^h in the sense of (1.4), we obtain

$$(4.3)\qquad \tilde{\nabla}'_k \overset{*}{\nabla}_h X^i = \tilde{\nabla}'_k \nabla_h X^i - \frac{1}{n+1}(\tilde{\nabla}'_k X^l \Gamma^r_{(l)(h)(r)}\, x'^i + X^l \tilde{\nabla}'_k \Gamma^r_{(l)(h)(r)}\, x'^i + X^l \Gamma^r_{(l)(h)(r)}\, \delta^i_k).$$

Differentiating $\tilde{\nabla}'_k X^i$ covariantly with respect to x^h in the sence of (1.7), we get

$$(4.4)\qquad \overset{*}{\nabla}_h \tilde{\nabla}'_k X^i = \nabla_h \tilde{\nabla}'_k X^i - \frac{1}{n+1}(\tilde{\nabla}'_k X^m)\, \Gamma^r_{(m)(h)(r)}\, x'^i + \frac{1}{n+1}(\tilde{\nabla}'_m X^i)\, \Gamma^r_{(k)(h)(r)}\, x'^m.$$

Using (1.9), (4.2), (4.3) and (4.4), we get the result.

Similarly for covariant vector V_i, we have

Theorem (4.2). *The commutation formula involving the operators* $\tilde{\nabla}'$ *and* $\overset{*}{\nabla}$ *for a covariant vector is given by*

$$(4.5)\qquad (\overset{*}{\nabla}_h \tilde{\nabla}'_k - \tilde{\nabla}'_k \overset{*}{\nabla}_h)\, V_i = V_l\, \tilde{Z}^l_{hki} + (\nabla_l V_i)\, C^l_{hk} + \frac{1}{n+1}(\tilde{\nabla}'_m V_i)\, B_{kh}\, x'^m.$$

Theorem (4.3). *The commutation formula for a contravariant tensor of order 2 is given by*

$$(4.6)\qquad (\overset{*}{\nabla}_h \tilde{\nabla}'_k - \tilde{\nabla}'_k \overset{*}{\nabla}_h)\, T^{ij} = -T^{lj}\, \tilde{Z}^i_{hkl} - T^{il}\, \tilde{Z}^j_{hkl} + (\nabla_l T^{ij})\, C^l_{hk} + \frac{1}{n+1}(\tilde{\nabla}'_m T^{ij})\, B_{hk}\, x'^m.$$

Proof. Eliminating X^i from (2.2) and (4.1), using the equation (4.2) and (4.5), we get

$$(4.7) \qquad V_j \Big[(\overset{*}{\nabla}_h \widetilde{\nabla}'_k - \widetilde{\nabla}'_k \overset{*}{\nabla}_h) T^{ij} + T^{lj} \widetilde{Z}^i_{hkl} + T^{il} \widetilde{Z}^i_{hkl} - (\nabla_l T^{ij}) C^l_{hk} - - \frac{1}{n+1} (\widetilde{\nabla}'_m T^{ij}) B_{hk} x'^m \Big] = 0.$$

Since $V_j(x, x')$ is an arbitrary vector therefore the equation (4.7) reveals (4.6).

On the similar way we have the following theorems :

Theorem (4.4). *The commutation formula for covariant tensor of order 2 is given by*

$$(4.8) \qquad (\overset{*}{\nabla}_h \widetilde{\nabla}'_k - \widetilde{\nabla}'_k \overset{*}{\nabla}_h) T_{ij} = T_{lj} \widetilde{Z}^l_{hki} + T_{il} \widetilde{Z}^l_{hkj} + (\nabla_l T_{ij}) C^l_{hk} + + \frac{1}{n+1} (\widetilde{\nabla}'_m T_{ij}) B_{hk} x'^m.$$

Theorem (4.5). *The commutation formulae for tensors* $T^{i_1 \ldots i_r}$ *and* $T_{j_1 \ldots j_p}$ *of rank r and p respectively are given by*

$$(4.9) \qquad (\overset{*}{\nabla}_h \widetilde{\nabla}'_k - \widetilde{\nabla}'_k \overset{*}{\nabla}_h) T^{i_1 \ldots i_r} = - \sum_{a=1}^{r} T^{i_1 \ldots i_{a-1} m i_{a+1} \ldots i_r} Z^{i_a}_{hkm} + + (\nabla_l T^{i_1 \ldots i_r}) C^l_{hk} + \frac{1}{n+1} (\widetilde{\nabla}'_m T^{i_1 \ldots i_r}) B_{hk} x'^m$$

and

$$(4.10) \qquad (\overset{*}{\nabla}_h \widetilde{\nabla}'_k - \widetilde{\nabla}'_k \overset{*}{\nabla}_h) T_{j_1 \ldots j_p} = \sum_{\beta=1}^{p} T_{j_1 \ldots j_{\beta-1} m j_{\beta+1} \ldots j_p} \widetilde{Z}^m_{hkj_\beta} + + (\nabla_l T_{j_1 \ldots j_p}) C^l_{hk} + \frac{1}{n+1} (\widetilde{\nabla}'_m T_{j_1 \ldots j_p}) B_{hk} x'^m.$$

Theorem (4.6). *The commutation formula for a mixed tensor* $T^{i_1 \ldots i_r}_{j_1 \ldots j_p}$ *of order (r, p) is given by*

$$(4.11)\qquad (\overset{*}{\nabla}_h \tilde{\nabla}'_k - \tilde{\nabla}'_k \overset{*}{\nabla}_h)\, T^{i_1 \dots i_r}_{j_1 \dots j_p} = - \sum_{a=1}^{r} T^{i_1 \dots i_{a-1}\, m\, i_{a+1} \dots i_r}_{j_1 \dots\dots\dots\dots\dots\dots j_p}\, \tilde{Z}^{i_a}_{hkm} +$$

$$+ \sum_{\beta=1}^{p} T^{i_1 \dots i_r}_{j_1 \dots j_{\beta-1}\, m\, j_{\beta+1} \dots j_p}\, \tilde{Z}^{m}_{hkj_\beta} + (\nabla_l\, T^{i_1 \dots i_r}_{j_1 \dots j_p})\, C^l_{hk} +$$

$$+ \frac{1}{n+1} (\tilde{\nabla}'_m\, T^{i_1 \dots i_r}_{j_1 \dots j_p})\, B_{hk}\, x'^m.$$

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ÖZET

Bu çalışmada n-boyntlu özel Kawagnchi uzayında diğer bir kovariant türev tipi (∇^*) tarif edilerek bu türevi ihtiva eden Ricci özdeşlikleri verilmekte ve bu kovariant türevle mutad kovariant türev (∇) arasında komütasyon formülleri ispatlanmaktadır.