ON THE EXISTENCE OF SPECIAL PROJECTIVE AFFINE MOTION IN A RECURRENT FINSLER SPACE [+)]

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An n-dimensional affinely connected Finsler space F_n whose curvature tensor satisfies a certain relation is called a special projective recurrent Finsler space. The subject of this paper is such a space admitting an infinitesimal point transformation with a certain condition.

1. Introduction. Let us consider an n-dimensional affinely connected Finsler space F_n [1] [1)] with $2n$ line elements $(x^i, \dot{x}^i)$, $(i = 1, 2, \dots n)$, and a positively homogeneous metric function $F(x, \dot{x})$ of degree one in $\dot{x}^i$. The fundamental metric tensor of the space is defined by

$$g_{ij}(x, \dot{x}) = \tfrac{1}{2}\, \dot{\partial}_i\, \dot{\partial}_j\, F^2(x, \dot{x}), \quad (\dot{\partial}_i \equiv \partial/\partial \dot{x}^i). \tag{1.1}$$

The projective covariant derivative [8] of any tensor field $T^i_j(x, \dot{x})$ with respect to x^k is given by

$$T^i_{j((k))} = \partial_h\, T^i_j - (\dot{\partial}_m\, T^i_j)\, \Pi^m_{kr}\, \dot{x}^r + T^h_j\, \Pi^i_{hk} - T^i_h\, \Pi^h_{jk}\,, \tag{1.2}$$

where

$$\Pi^i_{hk}(x, \dot{x}) \stackrel{\text{def}}{=} \left\{ G^i_{hk} - \frac{1}{(n+1)}\, (2\, \delta^i_{(h}\, G^r_{k)r} + \dot{x}^i\, G^r_{rkh}) \right\} \tag{1.3}$$

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1) The numbers in square brackets refer to the references given at the end of the paper.

2) $2\, A_{[hk]} = A_{hk} - A_{kh}$ and $2\, A_{(hk)} = A_{hk} + A_{kh}$.

are called the projective connection coefficients and satisfy the following relations :

(1.4) a) $\Pi^i_{hkr} = \dot\partial_h \Pi^i_{kr}$, b) $\Pi^i_{hjk} \dot x^h = 0$ and c) $\Pi^i_{kk} = \Pi^i_{kh}$.

Involving the projective covariant derivative, we have the following commutation formulae :

$$\dot\partial_h(T^i_{j((k))}) - (\dot\partial_h T^i_j)_{((k))} = T^s_j \Pi^i_{shk} - T^i_s \Pi^s_{jhk} \tag{1.5}$$

and

$$2\, T^i_{j[((h))((k))]} = - \dot\partial_r T^i_j Q^r_{shk} \dot x^s + T^s_j Q^i_{shk} - T^i_s Q^s_{jhk} , \tag{1.6}$$

where

$$Q^i_{hjk}(x, \dot x) \overset{\text{def}}{=} 2\,\{\partial_{[k} \Pi^i_{j]h} - \Pi^i_{rh[j} \Pi^r_{k]} + \Pi^r_{h[j} \Pi^i_{k]r}\} \tag{1.7}$$

is called the projective entity and satisfies the following identities :

$$Q^i_{hjk((s))} + Q^i_{hks((j))} + Q^i_{hsj((k))} = 0 \tag{1.8}$$

and

$$\text{a)}\ Q^i_{hjk} = - Q^i_{hkj} \quad \text{and} \quad \text{b)}\ Q^i_{hjk} \dot x^h = Q^i_{jk} . \tag{1.9}$$

If the curvature tensor $Q^i_{hjk}(x, \dot x)$ of the space satisfies the relation

$$Q^i_{hjk((s))} = \mu_s Q^i_{hjk} , \tag{1.10}$$

where $\mu_s(x)$ means a non-zero covariant vector, the space is called a special projective recurrent Finsler space or $S\text{-}PR\ F_n$.

Let us consider an infinitesimal point transformation

$$\bar x^i = x^i + v^i(x)\, dt, \tag{1.11}$$

where $v^i(x)$ is any vector field and dt is an infinitesimal point constant. The above transformation which is considered at each point in the space is called a special projective affine motion when and only when, we have

(1.12) $$\pounds_v \, \Pi^i_{jk} = 0,$$

where $\pounds_v$ denote the well known Lie-derivative with respect to the infinitesimal transformation given above. In view of (1.11) and the projective covariant derivative the Lie-derivatives of $T^i_j(x, \dot{x})$ and the projective connection coefficients $\Pi^i_{jk}(x, \dot{x})$ are given by [2] :

(1.13) $$\pounds_v \, T^i_j = T^i_{j((h))} \, v^h + T^i_h \, v^h_{((j))} - T^h_j \, v^i_{((h))} + \dot{\partial}_h \, T^i_j \, v^h_{((r))} \, \dot{x}^r$$

and

(1.14) $$\pounds_v \, \Pi^i_{jk} = v^i_{((j))((k))} + Q^i_{jkh} \, v^h + \Pi^i_{sjk} \, v^s_{((r))} \, \dot{x}^r$$

respectively.

We have also the following commutation formulae :

(1.15) $$\pounds_v(\dot{\partial}_l \, T^i_j) - \dot{\partial}_l(\pounds_v \, T^i_j) = 0,$$

(1.16) $$(\pounds_v \, T^i_{jk((l))}) - (\pounds_v \, T^i_{jl})_{((k))} = T^s_{jk} \, \pounds_v \, \Pi^i_{sl} - T^i_{sk} \, \pounds_v \, \Pi^s_{jl} - T^i_{js} \, \pounds_v \, \Pi^s_{kl}$$

and

(1.17) $$(\pounds_v \, \Pi^i_{jh})_{((k))} - (\pounds_v \, \Pi^i_{kh})_{((j))} = \pounds_v \, Q^i_{hjk} + 2 \dot{x}^s \, \Pi^i_{rh[j} \, \pounds_v \, \Pi^r_{k]s} \, .$$

Hence, from (1.16), we can easily see that for a special projective affine motion, the operators $\pounds_v$ and $((k))$ are commutative with each other.

By virtue of the equations (1.12) and (1.17), we obtain

(1.18) $$\pounds_v \, Q^i_{hjk} = 0.$$

Applying $\pounds_v$ to the both side of (1.10) and using the equations (1.12), (1.16) and (1.18), we get

(1.19) $$(\pounds_v \, \mu_s) \, Q^i_{hjk} = 0.$$

Since the space is a non-flat one (i.e. $Q^i_{hjh} \neq 0$), we have

(1.20) $$\pounds_v \mu_s = 0$$

i.e. the recurrence vector μ_s of the space must be Lie-invariant one.

In what follows, we shall study a space *S-PR* F_n admitting an infinitesimal point transformation $\overline{x}^i = x^i + v^i(x)dt$ which satisfies (1.20). We shall call such a restricted space, for brevity, a F_n^* space.

2. The vanishing of $\pounds_v Q^i_{hjk}(x, \dot{x})$. First of all, let us prove the following:

Lemma (2.1). *In an F_n^* space, the recurrence vector $\mu_s(x)$ is a gradient one, we have $\mu_s v^s = const.$*

Proof. For brevity, let us put

(2.1) $$\beta = \mu_s v^s.$$

Then, from the conditions (1.13) and (1.20), we get

(2.2) $$\pounds_v \mu_m = \mu_{m((s))} v^s + \mu_s v^s_{((m))} = 0$$

and the assumption $\mu_{m((s))} = \mu_{s((m))}$, we can see straightly $\beta_{((s))} = 0$. This completes the proof.

In view of (1.13), the Lie-derivative of the curvature tensor Q^i_{hjk} is given by

(2.3) $$\pounds_v Q^i_{hjk} = Q^i_{hjk((s))} v^s + Q^i_{sjk} v^s_{((h))} + Q^i_{hsk} v^s_{((j))} + \\ + Q^i_{hjs} v^s_{((k))} - Q^s_{hjk} v^i_{((s))} + \dot{\partial}_s Q^i_{hjk} v^s_{((r))} \dot{x}^r.$$

With the help of the equations (1.10) and (2.1), the above relation reduces to

(2.4) $$\pounds_v Q^i_{hjk} = \beta Q^i_{hjk} + Q^i_{sjk} v^s_{((h))} + Q^i_{hsk} v^s_{((j))} + Q^i_{hjs} v^s_{((h))} - \\ - Q^s_{hjk} v^i_{((s))} + \dot{\partial}_s Q^i_{hjk} v^s_{((r))} \dot{x}^r .$$

Applying the commutation formula (1.6) to the curvature tensor $Q^i_{hjk}(x, \dot{x})$, we obtain

$$(2.5) \qquad 2\, Q^i_{hjk[((l))((m))]} = - \dot\partial_r\, Q^i_{hjk}\, Q^r_{slm}\, \dot x^s + Q^s_{hjk}\, Q^i_{slm} - Q^i_{sjk}\, Q^s_{hlm} - - Q^i_{hsk}\, Q^s_{jlm} - Q^i_{hjs}\, Q^s_{klm}$$

which in view of the definition (1.10) reduces to

$$(2.6) \qquad (\mu_{l((m))} - \mu_{m((l))})\, Q^i_{hjk} = - \dot\partial_r\, Q^i_{hjk}\, Q^r_{slm}\, \dot x^s + Q^s_{hjk}\, Q^i_{slm} - - Q^i_{sjk}\, Q^s_{hlm} - Q^i_{hsk}\, Q^s_{jlm} - Q^i_{hjs}\, Q^s_{klm}\,.$$

Next, let us assume that β is not a constant. Then, from the Lemma (2.1), we can see

$$(2.7) \qquad E_{lm}(x) \overset{\text{def}}{=} (\mu_{l((m))} - \mu_{m((l))}) \neq 0.$$

Let us take

$$(2.8) \qquad Q^i_{hjk}\; p^{jk} = v^i_{((h))}$$

for a suitable non-symmetric tensor p^{jh}, then multiplying (2.6) by p^{lm} and summing over l and m, we get

$$(2.9) \qquad E_{lm}\, p^{lm}\, Q^i_{hjk} = - \dot\partial_r\, Q^i_{hjk}\, v^r_{((s))}\, \dot x^s + Q^s_{hjk}\, v^i_{((s))} - Q^i_{sjk}\, v^s_{((h))} - - Q^i_{hsk}\, v^s_{((j))} - Q^i_{hjs}\, v^s_{((k))}\,.$$

Comparing the last equation with (2.4), we obtain

$$(2.10) \qquad \pounds_v\, Q^i_{hjk} = Q^i_{hjk}(\beta - p^{lm}\, E_{lm}),$$

which vanishes when and only when $\beta = p^{lm}\, E_{lm}$.

For $\beta \neq$ const. and $E_{lm} \neq 0$, from (2.4) and (2.6), we can make the following identity

$$(2.11) \qquad E_{lm}\, \pounds_v\, Q^i_{hjk} = Q^s_{hjk}(\beta\, Q^i_{slm} - E_{lm}\, v^i_{((s))}) - Q^i_{sjk}(\beta\, Q^s_{hlm} - E_{lm}\, v^s_{((h))}) - - Q^i_{hsk}(\beta\, Q^s_{jlm} - E_{lm}\, v^s_{((j))}) - Q^i_{hjs}(\beta\, Q^s_{klm} - E_{lm}\, v^s_{((k))}) - - \dot\partial_r\, Q^i_{hjk}(\beta\, Q^r_{slm} - E_{lm}\, v^r_{((s))})\, \dot x^s.$$

Thus, for $\pounds_v\, Q^i_{hjk} = 0$, the above relation easily yields [7]

$$\beta\, Q^i_{hjk} = E_{jk}\, v^i_{((k))}\,, \tag{2.12}$$

where v^i does not mean a parallel vector.

We put here the

Definition (2.1). *An $F_n{}^*$ space satisfying $\mu_m v^m \neq$ const. is called a special one of the first kind.*

Next, let us turn back again to the case $\mu_m v^m =$ const. of the foregoing Lemma (2.1). Then, (2.6) is replaced by

$$\begin{aligned} -\dot{\partial}_r\, Q^i_{hjk}\, Q^r_{slm}\, \dot{x}^s + Q^s_{hjk}\, Q^i_{slm} - Q^i_{sjk}\, Q^s_{hlm} - \\ - Q^i_{hsk}\, Q^s_{jlm} - Q^i_{hjs}\, Q^s_{klm} = 0. \end{aligned} \tag{2.13}$$

By virtue of (2.8) transvecting the above relation by p^{lm} and summing over l and m, we get

$$\begin{aligned} -\dot{\partial}_r\, Q^i_{hjk}\, v^r_{((s))}\, \dot{x}^s + Q^s_{hjk}\, v^i_{((s))} - Q^i_{sjk}\, v^s_{((h))} - \\ - Q^i_{hsk}\, v^s_{((j))} - Q^i_{hjs}\, v^s_{((k))} = 0. \end{aligned} \tag{2.14}$$

Introducing the last equation into (2.4), we obtain

$$\pounds_v\, Q^i_{hjk} = \beta\, Q^i_{hjk}\,. \tag{2.15}$$

Hence, when the arbitrary constant β vanishes, we have $\pounds_v\, Q^i_{hjk} = 0$.

We put the

Definition (2.2). *When $\mu_m v^m =$ const. holds good, an $F_n{}^*$ is called a special one of the second kind.*

Then, summarising the above results, we have the following theorems.

Theorem (2.1). *In an $F_n{}^*$ space of the first kind, if the space has the resolved curvature tensor $Q^i_{hjk}(x, \dot{x})$ of the form (2.12), $\pounds_v\, Q^i_{hjk} = 0$ holds good.*

Theorem (2.2). *In an F_n^* space of the second kind, if the arbitrary constant $\mu_m v^m$ vanishes, we have $\pounds_v Q^i_{hjk} = 0$.*

From Theorem (2.2) as a case of $\mu_m = 0$, the definition (1.10) yields

Corollary (2.1). *In a projective symmetric* FINSLER *space* [9] *(i.e. $Q^i_{hjk((s))} = 0$), $\pounds_v Q^i_{hjk} = 0$ is satisfied identically.*

3. Complete condition. We shall obtain a necessary and sufficient condition for (2.12). From the assumption (1.20), we have

$$\pounds_v \mu_m = \mu_{m((s))} v^s + (\mu_s v^s)_{((m))} - \mu_{s((m))} v^s = 0. \tag{3.1}$$

With the help of the equations (2.1) and (2.7), the above relation reduces to

$$\beta_{((m))} + E_{ms} v^s = 0. \tag{3.2}$$

In view of the equation (1.13), the Lie-derivative of $E_{lm}(x)$ is given by

$$\pounds_v E_{lm} = E_{lm((s))} v^s + E_{sm} v^s_{((l))} + E_{ls} v^s_{((m))} . \tag{3.3}$$

By virtue of the commutation formula (1.16), we have

$$\pounds_v(\mu_{m((s))}) - (\pounds_v \mu_m)_{((s))} = - \mu_r \pounds_v \Pi^r_{ms} \tag{3.4}$$

which in view of the equations (1.4)c, (1.20) and (2.7) reduces to

$$\pounds_v E_{ms} = 0. \tag{3.5}$$

Differentiating (2.6) projective covariantly with respect to x^n and using the equations (1.5), (1.10), (2.6) and (2.7), we obtain

$$\begin{aligned} E_{lm((n))} Q^i_{hjk} = \mu_n E_{lm} Q^i_{hjk} + Q^r_{alm}(Q^s_{hjk} \Pi^i_{srn} - \\ - Q^i_{sjk} \Pi^s_{hrn} - Q^i_{hsk} \Pi^s_{jrn} - Q^i_{hjs} \Pi^s_{krn}) \dot{x}^a. \end{aligned} \tag{3.6}$$

Transvecting the last relation by $\dot{x}^n$ and using (1.4)b, we get after little simplifications :

$$E_{lm((n))} = \mu_n E_{lm} . \tag{3.7}$$

Thus, with the help of the equations (3.3), (3.5) and (3.7), we obtain

$$\beta E_{lm} + E_{sm} v^s_{((l))} + E_{ls} v^s_{((m))} = 0. \tag{3.8}$$

Next, from (3.2), we get

$$2 \beta_{[((m))((n))]} = -(E_{ms} v^s)_{((n))} + (E_{ns} v^s)_{((m))} , \tag{3.9}$$

β being a non-constant scalar function, this becomes

$$E_{ms} v^s_{((n))} - E_{sn} v^s_{((m))} = - \mu_n E_{ms} v^s + \mu_m E_{ns} v^s, \tag{3.10}$$

where we have used (3.7) and $E_{ms} = - E_{sm}$. Introducing the above relation into the left hand side of (3.8) and noting the equation (3.2), we obtain

$$\beta E_{mn} = - \mu_n \beta_{((m))} + \mu_m \beta_{((n))} . \tag{3.11}$$

By virtue of the equations (1.9)a (1.10) the identity (1.8) reduces to

$$\beta Q^i_{hnm} = \mu_n Q^i_{hms} v^s - \mu_m Q^i_{hns} v^s. \tag{3.12}$$

Hence, from (3.11) and (3.12), we can make

$$\begin{aligned} \beta(\beta Q^i_{hmn} - E_{mn} v^i_{((h))}) = \mu_n(\beta Q^i_{hms} v^s + \beta_{((m))} v^i_{((h))}) \\ - \mu_m(\beta Q^i_{hns} v^s + \beta_{((n))} v^i_{((h))}). \end{aligned} \tag{3.13}$$

Consequently (2.12) follows when and only when, we have

$$\beta Q^i_{hms} v^s + \beta_{((m))} v^i_{((h))} = \mu_m N^i_h , \tag{3.14}$$

where N^i_h means a suitable tensor. Multiplying the last relation by v^m and summing over m by virtue of $Q^i_{hmn} v^m v^n = 0$ and $\beta_{((m))} v^m = 0$ derived from (3.2) we get

$$\beta N^i_h = 0, \tag{3.15}$$

where, we have used (2.1). Since $\beta \neq 0$, therefore, the last relation yields $N_h^i = 0$.

Thus, from (3.14), we get

$$Q_{hms}^i v^s + \beta_m v^i_{((h))} = 0, \ (\beta_m = \beta_{((m))} \mid \beta). \tag{3.16}$$

In this way, we have the

Theorem (3.1). *In order that we have* (2.12) (3.16) *is necessary and sufficient.*

Now the equation (3.16) suggests the concrete form of the tensor p^{lm} used in the first half of § 2. In fact $\beta_m \neq 0$ there exists a suitable vector ε^m such that

$$\beta_m \varepsilon^m = 1. \tag{3.17}$$

Then, transvecting (3.16) by ε^m and noting (3.17), we get

$$v^i_{((h))} = Q_{hsm}^i v^s \varepsilon^m. \tag{3.18}$$

If, we introduce p^{lm} by

$$p^{lm} = v^l \varepsilon^m \tag{3.19}$$

then $E_{lm} p^{lm} = E_{lm} v^l \varepsilon^m = \beta_{((m))} \varepsilon^m = \beta \beta_m \varepsilon^m = \beta$.

That is, from (3.16) and (2.12), we have

$$\beta = E_{lm} p^{lm} \tag{3.20}$$

straightly. Therefore, we can take (3.19) concretely. Hence in order to have the concrete form of p^{lm}, (3.16) should be taken as a basic condition in our theory. If this is done, we are able to have (2.12) always so $\pounds_v Q_{hjk}^i = 0$ holds good.

Thus, we have

Theorem (3.2). *If we introduce* $v^i_{((h))}$ *by* (3.16), $£_v Q^i_{hjk} = 0$ *is satisfied identically*.

4. **Appendices.** At first, in a special $F_n{}^*$ space of the first kind, we shall show concretely the existence of special projective affine motion. For this purpose, let us take up (2.12) being equivalent to (3.16) which has been introduced for the purpose of getting form of $v^i_{((h))}$. In this case, according to Theorem (2.1) or (3.2), we have $£_v Q^i_{hjk} = 0$ identically so $£_v \Pi^i_{jk} = 0$ ought to be considered. However, in what follows, we shall study this fact in detail.

In view of the equations (1.10) and (3.7), differentiating (2.12) projective covariantly with respect to x^m, we get

$$\beta_{((m))} Q^i_{hjk} = E_{jk} v^i_{((h))((m))} . \tag{4.1}$$

Multiplying the last relation by v^k and noting the equation (3.2), we obtain

$$\beta_{((m))} Q^i_{hjk} v^k = - \beta_{((j))} v^i_{((h))((m))} \tag{4.2}$$

which in view of the equation (3.16) reduces to

$$\beta_{((m))} \beta_j v^i_{((h))} = \beta_{((j))} v^i_{((h))((m))} \tag{4.3}$$

from which, because of $\beta \neq$ const., we have

$$\beta_m v^i_{((h))} = v^i_{((h))((m))} . \tag{4.4}$$

Hence by virtue of the equation (3.16) and (4.4), we get

$$v^i_{((h))((m))} + Q^i_{hms} v^s = \beta_m v^i_{((h))} - \beta_m v^i_{((h))} = 0 . \tag{4.5}$$

Introducing the above relation into (1.14), we obtain

$$£_v \Pi^i_{hm} = \Pi^i_{shm} v^s_{((r))} \dot{x}^r . \tag{4.6}$$

Thus, we have

Theorem (4.1). *An F_n^* space satisfying $\pounds_v \mu_m = 0$, $\mu_m v^m \neq const.$, and having the resolved curvature tensor Q^i_{hjk} of the form* (2.12) *admits naturally a non-special projective affine motion (i.e. $\pounds_v \Pi^i_{jk} \neq 0$).*

Secondly, let us consider the space of the second kind having $\beta = \mu_m v^m = 0$. In this case according to Theorem (2.2), we have $\pounds_v Q^i_{hjk} = 0$ necessarily. Then let us study the possibility of $\pounds_v \Pi^i_{jk} = 0$. With the help of the identity (1.8) and equation (1.10), we get

$$\mu_j Q^i_{hkl} v^l = \mu_k Q^i_{hjl} \tag{4.7}$$

from which, taking care of $\mu_j \neq 0$, we can put

$$Q^i_{hkl} v^l = E^i_h \mu_k . \tag{4.8}$$

Now, being $\mu_j \neq 0$. there exists a suitable vector ε^k such that $\mu_k \varepsilon^k = 1$. Transvecting (4.8) by ε^k, we get

$$Q^i_{hkl} \varepsilon^k v^l = E^i_h . \tag{4.9}$$

Then introducing a non-symmetric tensor p^{kl} considered in the last half of § 2 by $p^{kl} = v^k \varepsilon^l$ from (4.8), we have

$$- Q^i_{hlk} p^{kl} = E^i_h . \tag{4.10}$$

By virtue of the equation (2.8), the above relation reduces to

$$v^i_{((h))} = - E^i_h . \tag{4.11}$$

Consequently (4.8) takes the form

$$Q^i_{hkl} v^l = - \mu_k v^i_{((h))} . \tag{4.12}$$

Introducing (4.12) into (1.14), we have

$$\pounds_v \Pi^i_{jk} = v^i_{((j))((k))} - \mu_k v^i_{((j))} + \Pi^i_{sjk} v^s_{((r))} \dot{x}^r. \tag{4.13}$$

Therefore, where $v^i_{((j))}$ denotes a recurrent tensor with respect to the gradient recurrent vector μ_k, we have

$$\pounds_v \Pi^i_{jk} = \Pi^i_{sjk} v^s_{((r))} \dot{x}^r. \tag{4.14}$$

Thus, we have

Theorem (4.2). *An F_n^* space defined by a gradient recurrence vector μ_m and characterized by $£_v \mu_m = 0$ and $\mu_m v^m = 0$, admits a non-special projective affine motion when the space has a recurrent tensor $v^i_{((j))}$ with respect to μ_k.*

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ÖZET

Eğrilik Tansörü belirli bir münasebeti gerçekleyen n-boyutlu ve affin bağımlı bir Finsler uzayına bir özel projektif tekrarlamalı Finsler uzayı denir. Bu çalışmanın konusu, belirli bir şarta uyan bir infinitesimal nokta transformasyonunu kabûl eden bu tip bir uzaydır.