ON GENERALIZED 2 - RECURRENT SPECIAL KAWAGUCHI SPACE

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A generalized 2-recurrent special Kawaguchi space is defined and the properties exhibited by such an n-space are discussed. A necessary and sufficient condition for a special Kawaguchi space with generalized 2-recurrent tensor field K^i_{jk} to be a generalized 2-recurrent special Kawaguchi space is also established.

S. Kawaguchi [3] [1)] has introduced the recurrent curvature tensor in a special Kawaguchi space and investigated necessary conditions of a special Kawaguchi space of recurrent curvature. A recurrent special Kawaguchi space of second order has been defined and studied by S.P. Singh [4]. In the present paper, we define a generalized 2-recurrent special Kawaguchi space and discuss the properties exhibited by such an n-space. We have also found a necessary and sufficient condition for a special Kawaguchi space with generalized 2-recurrent tensor field $K_{jk}{}^i$ to be a generalized 2-recurrent special Kawaguchi space.

1. Introduction : An n-dimensional metric space equipped with the special integral

$$S = \int (A_i (x,x')\, x''^i + B(x,x'))^{1/p}\, dt\,, \tag{1.1}$$

where s is the arc-length of a curve $x^i = x^i(t)$ in the above space, is called an n-dimensional special Kawaguchi space. A. Kawaguchi ([1], [2]) has defined a connection in this space by applying the Craig vector of the function $F = A_i x''^i + B$. The Craig vector is defined by

$$T_i = (A_{k(i)} - 2\, A_{i(k)})\, x''^k - 2\, A_{ik}\, x'^k + B_{(i)}\,, \tag{1.2}$$

1) Numbers in square brackets refer to the references at the end of the paper.

where

$$A_{k(i)} = \frac{\partial A_k}{\partial x'^i}\ ,\ A_{ik} = \frac{\partial A_i}{\partial x^k}\ ,\ B_{(i)} = \frac{\partial B}{\partial x'^i}\ .$$

When $2p \neq 3$, we have the relation

$$F = A_i\, x^{[2]i} \tag{1.3}$$

where

$$x^{[2]i} = x''^i + 2\Gamma^i\ ,\ 2\Gamma^i = (2A_{lk}\, x'^k - B_{(l)})\, G^{li}\ ,\ G_{ik} = 2A_{i(k)} - A_{k(i)}\ ,\ G_{ik}\, G^{il} = \delta^l{}_k\ .$$

The covariant derivative of a vector field v^i which is homogeneous of degree zero with respect to x'^i is given by

$$\nabla_j\, v^i = \frac{\partial v^i}{\partial x^j} - \frac{\partial v^i}{\partial x'^k}\, \Gamma^k{}_{(j)} + \Gamma^i{}_{(k)(j)}\, v^k\ ,\ \nabla_j'\, v^i = \frac{\partial v^i}{\partial x'^j}\ . \tag{1.4}$$

It follows from (1.4) that

$$\nabla_j\, x'^i = 0\ . \tag{1.4^1}$$

From the parenthesis of Poisson for covariant derivatives we find the curvature tensors as follows :

$$(\nabla_j \nabla_k - \nabla_k \nabla_j)\, v^i = -\, R_{jkl}^{\cdot\cdot\cdot i}\, v^l + K_{jk}^{\cdot\cdot l}\, \nabla_l'\, v_l^i\ , \tag{1.5}$$

$$(\nabla_j \nabla_k' - \nabla_k' \nabla_j)\, v^i = -\, B_{jkl}^{\cdot\cdot\cdot i}\, v^l\ , \tag{1.6}$$

where

$$B_{jkl}^{\cdot\cdot\cdot i} = \Gamma^i{}_{(j)(k)(l)}\ , \tag{1.7}$$

$$R_{jkl}^{\cdot\cdot\cdot i} = \frac{\partial \Gamma^i{}_{(l)(j)}}{\partial x^k} - \frac{\partial \Gamma^i{}_{(l)(k)}}{\partial x^j} + \Gamma^h{}_{(l)(j)}\, \Gamma^i{}_{(k)(h)} -$$

$$-\, \Gamma^h{}_{(l)(k)}\, \Gamma^i{}_{(j)(h)} + \Gamma^h{}_{(j)}\, \Gamma^i{}_{(l)(k)(h)} - \Gamma^h{}_{(k)}\, \Gamma^i{}_{(l)(j)(h)}\ , \tag{1.8}$$

$$K_{jk}^{\cdot\cdot i} = \frac{\partial \Gamma^i{}_{(j)}}{\partial x^k} - \frac{\partial \Gamma^i{}_{(k)}}{\partial x^j} + \Gamma^h{}_{(j)}\, \Gamma^i{}_{(k)(h)} - \Gamma^h{}_{(k)}\, \Gamma^i{}_{(j)(h)}\ . \tag{1.9}$$

These curvature tensor fields satisfy the identities

$$R_{jkl}{}^i + R_{kjl}^{\cdot\cdot\cdot i} = 0\ ,\quad R_{jkl}^{\cdot\cdot\cdot i} + R_{klj}^{\cdot\cdot\cdot i} + R_{ljk}^{\cdot\cdot\cdot i} = 0\ , \tag{1.10}$$

$$K_{jk}^{\cdot\cdot i} = R_{jkl}^{\cdot\cdot\cdot i}\, x'^l\ ,\quad R_{jkl}^{\cdot\cdot\cdot i} = K_{jk(l)}^{\cdot\cdot i} = \nabla_l'\, K_{jk}^{\cdot\cdot i}\ , \tag{1.11}$$

$$B_{jkl}^{\cdot\cdot\cdot i}\, x'^l = 0\ . \tag{1.12}$$

The Bianchi identities satisfied by $R_{jkl}^{\cdot\cdot\cdot i}$ and $K_{jk}^{\cdot\cdot i}$ are given by

$$\nabla_h R_{jkl}^{\cdot\cdot\cdot i} + \nabla_j R_{khl}^{\cdot\cdot\cdot i} + \nabla_k R_{hjl}^{\cdot\cdot\cdot i} + K_{hj}^{\cdot\cdot\gamma} B_{kl\gamma}^{\cdot\cdot\cdot i} + K_{jk}^{\cdot\cdot\gamma} B_{hl\gamma}^{\cdot\cdot\cdot i} + K_{kh}^{\cdot\cdot\gamma} B_{jl\gamma}^{\cdot\cdot\cdot i} = 0\,, \tag{1.13}$$

$$\nabla_h K_{jk}^{\cdot\cdot i} + \nabla_j K_{kh}^{\cdot\cdot i} + \nabla_k K_{hj}^{\cdot\cdot i} = 0\,. \tag{1.14}$$

2. Generalized 2-Recmrent Special Kawaguchi Space : Let us assume that $R_{jkh}^{\cdot\cdot\cdot i} \neq 0$ and $\nabla_m R_{jkh}^{\cdot\cdot\cdot i} \neq 0$. Let the curvature tensor field $R_{jkh}^{\cdot\cdot\cdot i}$ satisfy the relation

$$\nabla_l \nabla_m R_{jkh}^{\cdot\cdot\cdot i} = a_{lm} R_{jkh}^{\cdot\cdot\cdot i} + \lambda_l \nabla_m R_{jkh}^{\cdot\cdot\cdot i} \tag{2.1}$$

where a_{lm} is a non-zero tensor field and λ_l is a non-zero vector field. We call a_{lm} and λ_l the associated recurrent tensor and vector fields respectively. An n-dimensional special Kawaguchi space with the condition (2.1) will be called generalized 2-recurrent special Kawaguchi space. If λ_l is zero, the condition (2.1) reduces to the second order recurrence condition.

Transvecting (2.1) by x'^h and using (1.4)[1] and (1.11), we have

$$\nabla_l \nabla_m K_{jk}^{\cdot\cdot i} = a_{lm} K_{jk}^{\cdot\cdot i} + \lambda_l \nabla_m K_{jk}^{\cdot\cdot i}\,. \tag{2.2}$$

We shall now prove the following theorems :

Theorem (2.1). In a generalised 2-recurrent special Kawaguchi space, we have

$$\{(\nabla_h' a_{lm}) K_{jk}^{\cdot\cdot i} + (\nabla_h' \lambda_l)(\nabla_m K_{jk}^{\cdot\cdot i})\}\, x'^h = 0\,. \tag{2.3}$$

Proof. Applying ∇_h' to (2.2) and noting (1.11), we have

$$\begin{aligned}\nabla_h' \nabla_l \nabla_m K_{jk}^{\cdot\cdot i} = (\nabla_h' a_{lm}) K_{jk}^{\cdot\cdot i} + a_{lm} R_{jkh}^{\cdot\cdot\cdot i} + \\ + (\nabla_h' \lambda_l)(\nabla_m K_{jk}^{\cdot\cdot i}) + \lambda_l (\nabla_h' \nabla_m K_{jk}^{\cdot\cdot i})\, h\,.\end{aligned} \tag{2.4}$$

With the help of the commutation formula (1.6) and the equation (2.1), the equation (2.4) yields

$$\begin{aligned}(\nabla_h' a_{lm}) K_{jk}^{\cdot\cdot i} + (\nabla_h' \lambda_l)(\nabla_m K_{jk}^{\cdot\cdot i}) = \nabla_l (K_{jk}^{\cdot\cdot\gamma} B_{mh\gamma}^{\cdot\cdot\cdot i} \\ - K_{\gamma k}^{\cdot\cdot i} B_{mhj}^{\cdot\cdot\cdot\gamma} - K_{j\gamma}^{\cdot\cdot i} B_{mhk}^{\cdot\cdot\cdot\gamma}) + (\nabla_m K_{jk}^{\cdot\cdot\gamma}) B_{lh\gamma}^{\cdot\cdot\cdot i} \\ - (\nabla_\gamma K_{jk}^{\cdot\cdot i}) B_{lhm}^{\cdot\cdot\cdot\gamma} - (\nabla_m K_{\gamma k}^{\cdot\cdot i}) B_{lhj}^{\cdot\cdot\cdot\gamma} - (\nabla_m K_{j\gamma}^{\cdot\cdot i}) B_{lhk}^{\cdot\cdot\cdot\gamma} \\ - \lambda_l (K_{jk}^{\cdot\cdot\gamma} B_{mh\gamma}^{\cdot\cdot\cdot i} - K_{\gamma k}^{\cdot\cdot i} B_{mhj}^{\cdot\cdot\cdot\gamma} - K_{j\gamma}^{\cdot\cdot i} B_{mhk}^{\cdot\cdot\cdot\gamma})\,.\end{aligned} \tag{2.5}$$

Transvecting (2.5) by x'^h and noting (1.4)[1] and (1.12), we have (2.3).

Theorem (2.2). In a generalized 2-recurrent special Kawaguchi space, we have

$$(\nabla_h' a_{lm} - \nabla_m' a_{lh}) K_{jk}^{\cdot\cdot i} + \{(\nabla_h' \lambda_l)(\nabla_m K_{jk}^{\cdot\cdot i}) - (\nabla_m' \lambda_l)(\nabla_h K_{jk}^{\cdot\cdot i})\}$$
$$- \{(\nabla_m K_{jk}^{\cdot\cdot\gamma}) B_{lh\gamma}^{\cdot\cdot\cdot i} - (\nabla_h K_{jk}^{\cdot\cdot\gamma}) B_{lm\gamma}^{\cdot\cdot\cdot i}\} + (\nabla_m K_{\gamma k}^{\cdot\cdot i}) B_{lhj}^{\cdot\cdot\cdot\gamma} \tag{2.6}$$
$$- (\nabla_h K_{\gamma k}^{\cdot\cdot i}) B_{lmj}^{\cdot\cdot\cdot\gamma}\} + \{(\nabla_m K_{j\gamma}^{\cdot\cdot i}) B_{lhk}^{\cdot\cdot\cdot\gamma} - (\nabla_h K_{j\gamma}^{\cdot\cdot i}) B_{lmk}^{\cdot\cdot\cdot\gamma}\} = 0\,.$$

Proof. Interchanging the indices m and h in (2.5) and subtracting from it the result thus obtained, we get (2.6) because of the symmetry of $B_{jkl}^{\cdot\cdot\cdot i}$ in j, k, l.

Corollary (2.1). In a generalized 2-recurrent special Kawaguchi space, if $B_{jkl}^{\cdot\cdot\cdot i} = 0$, then

$$(\nabla_h' a_{lm}) K_{jk}^{\cdot\cdot i} + (\nabla_h' \lambda_l)(\nabla_m K_{jk}^{\cdot\cdot i}) = 0\,. \tag{2.7}$$

Proof. It is obvious from (2.5).

Theorem (2.3). In a generalized 2-recurrent special Kawaguchi space, we have

$$[\{(\nabla_k' a_{lm}) P_{jb} - (\nabla_j' a_{lm}) P_{kb}\} + a_{lm}(\nabla_k' P_{jb} - \nabla_j' P_{kb}) + \{(\nabla_k' \lambda_l)(\nabla_m P_{jb}) -$$
$$- (\nabla_j' \lambda_l)(\nabla_m P_{kb})\} + \lambda_l(\nabla_k' \nabla_m P_{jb} - \nabla_j' \nabla_m P_{kb}) + \{(\nabla_\gamma P_{jb}) B_{lkm}^{\cdot\cdot\cdot\gamma} - \tag{2.8}$$
$$- (\nabla_\gamma P_{kb}) B_{ljm}^{\cdot\cdot\cdot\gamma}\}]\, x'^b = 0\,,$$

where

$$P_{jk} \overset{\text{def}}{=} \frac{n}{n^2 - 1} R_{jak}^{\cdot\cdot\cdot a} + \frac{1}{n^2 - 1} R_{kaj}^{\cdot\cdot\cdot a}\,. \tag{2.9}$$

Proof. Let us put

$$Q_{jk} = \nabla_k' (P_{jb}\, x'^b)\,. \tag{2.10}$$

It can be shown that the tensors P_{jk} and Q_{jk} satisfy the following relation:

$$P_{jk} - P_{kj} = Q_{jk} - Q_{kj}\,. \tag{2.11}$$

With the help of (2.9) and (2.1), we have

$$\nabla_l \nabla_m P_{jk} = a_{lm} P_{jk} + \lambda_l \nabla_m P_{jk}\,. \tag{2.12}$$

Hence from (2.11), we have

$$\nabla_l \nabla_m (Q_{jk} - Q_{kj}) = a_{lm}(Q_{jk} - Q_{kj}) + \lambda_l \nabla_m (Q_{jk} - Q_{kj})\,. \tag{2.13}$$

On the other hand, from (2.10), we have

$$\nabla_l \nabla_m Q_{jk} = \{(\nabla_k' a_{lm}) P_{jb} + a_{lm} (\nabla_k' P_{jb}) + (\nabla_k' \lambda_l)(\nabla_m P_{jb}) + \lambda_l (\nabla_k' \nabla_m P_{jb}) + (\nabla_\gamma P_{jb}) B_{lkm}^{\cdots\gamma} + (\nabla_m P_{\gamma b}) B_{lkj}^{\cdots\gamma} + \nabla_l (P_{\gamma b} B_{mkj}^{\cdots\gamma})\} x'^b + a_{lm} P_{jk} + \lambda_l \nabla_m P_{jk}\,, \tag{2.14}$$

where we have used (2.12), (1.6) and (1.12).

From (2.14), we can deduce

$$\nabla_l \nabla_m (Q_{jk} - Q_{kj}) = [\{(\nabla_k' a_{lm}) P_{jb} - (\nabla_j' a_{lm}) P_{kb}\} + a_{lm} (\nabla_k' P_{jb} - \nabla_j' P_{kb}) + \{(\nabla_k' \lambda_l)(\nabla_m P_{jb}) - (\nabla_j' \lambda_l)(\nabla_m P_{kb})\} + \lambda_l (\nabla_k' \nabla_m P_{jb} - \nabla_j' \nabla_m P_{kb}) + \{(\nabla_\gamma P_{jb}) B_{lkm}^{\cdots\gamma} - (\nabla_\gamma P_{kb}) B_{ljm}^{\cdots\gamma}\}] x'^b + a_{lm} (Q_{jk} - Q_{kj}) + \lambda_l \nabla_m (Q_{jk} - Q_{kj})\,. \tag{2.15}$$

Substituting (2.13) into (2.15), we get (2.8).

Theorem (2.4). In a generalized 2-recurrent special Kawaguchi space, we have

$$\nabla_l \nabla_m (\nabla_p' R_{jkh}^{\cdots i}) = a_{lm} (\nabla_p' R_{jkh}^{\cdots i}) + \lambda_l \nabla_m (\nabla_p' R_{jkh}^{\cdots i}) \tag{2.16}$$

if and only if

$$(\nabla_p' a_{lm}) R_{jkh}^{\cdots i} + (\nabla_p' \lambda_l)(\nabla_m R_{jkh}^{\cdots i}) + \lambda_l (B_{mp\gamma}^{\cdots i} R_{jkh}^{\cdots\gamma} - B_{mpj}^{\cdots\gamma} R_{\gamma kh}^{\cdots i} - B_{mpk}^{\cdots\gamma} R_{j\gamma h}^{\cdots i} - B_{mph}^{\cdots\gamma} P_{jk\gamma}^{\cdots i}) - \nabla_l (B_{mp\gamma}^{\cdots i} R_{jkh}^{\cdots\gamma} - B_{mpj}^{\cdots\gamma} R_{\gamma kh}^{\cdots i} - B_{mpk}^{\cdots\gamma} R_{j\gamma h}^{\cdots i} - B_{mph}^{\cdots\gamma} R_{jk\gamma}^{\cdots i}) - B_{lp\gamma}^{\cdots i} (\nabla_m R_{jkh}^{\cdots\gamma}) + B_{lpm}^{\cdots\gamma} (\nabla_\gamma R_{jkh}^{\cdots i}) + B_{lpj}^{\cdots\gamma} (\nabla_m R_{\gamma kh}^{\cdots i}) + B_{lpk}^{\cdots\gamma} (\nabla_m R_{j\gamma h}^{\cdots i}) + B_{lph}^{\cdots\gamma} (\nabla_m R_{jk\gamma}^{\cdots i}) = 0\,. \tag{2.17}$$

Proof. Applying ∇_p' to (2.1), we get

$$\nabla_p' \nabla_l \nabla_m R_{jkh}^{\cdots i} = (\nabla_p' a_{lm}) R_{jkh}^{\cdots i} + a_{lm} (\nabla_p' R_{jkh}^{\cdots i}) + (\nabla_p' \lambda_l)(\nabla_m R_{jkh}^{\cdots i}) + \lambda_l (\nabla_p' \nabla_m R_{jkh}^{\cdots i})\,. \tag{2.18}$$

By the repeated application of the formula (1.6) in (2.18), we get

$$\nabla_l \nabla_m (\nabla_p' R_{jkh}^{\cdots i}) - a_{lm} (\nabla_p' R_{jkh}^{\cdots i}) - \lambda_l \nabla_m (\nabla_p' R_{jkh}^{\cdots i}) = (\nabla_p' a_{lm}) R_{jkh}^{\cdots i}$$

$$+ (\nabla_p' \lambda_l)(\nabla_m R_{jkh}^{\cdots i}) + \lambda_l (R_{mp\gamma}^{\cdots i} R_{jkh}^{\cdots \gamma} - B_{mpj}^{\cdots \gamma} R_{\gamma kh}^{\cdots i} - B_{mpk}^{\cdots \gamma} R_{j\gamma h}^{\cdots i}$$

$$- B_{mph}^{\cdots \gamma} R_{jk\gamma}^{\cdots i}) - \nabla_l (B_{mp\gamma}^{\cdots i} R_{jkh}^{\cdots \gamma} - B_{mpj}^{\cdots \gamma} R_{\gamma kh}^{\cdots i} - B_{mpk}^{\cdots \gamma} B_{j\gamma h}^{\cdots i} -$$

$$- B_{mph}^{\cdots \gamma} R_{jk\gamma}^{\cdots i}) - B_{lp\gamma}^{\cdots i} (\nabla_m R_{jkh}^{\cdots \gamma}) + B_{lpm}^{\cdots \gamma} (\nabla_\gamma R_{jkh}^{\cdots i}) +$$

$$+ B_{lpj}^{\cdots \gamma} (\nabla_m R_{\gamma kh}^{\cdots i}) + B_{lpk}^{\cdots \gamma} (\nabla_m R_{j\gamma h}^{\cdots i}) + B_{lph}^{\cdots \gamma} (\nabla_m R_{jk\gamma}^{\cdots i}) . \qquad (2.19)$$

From (2.19), the theorem (2.4) follows.

Theorem (2.5). In a generalized 2-recurrent special Kawaguchi space, we have

$$a_{mh} R_{jkl}^{\cdots i} + a_{mj} R_{khl}^{\cdots i} + a_{mk} R_{hjl}^{\cdots i} - \lambda_m (K_{hj}^{\cdot\cdot\gamma} B_{kl\gamma}^{\cdots i} + K_{jk}^{\cdot\cdot\gamma} B_{hl\gamma}^{\cdots i}$$

$$+ K_{kh}^{\cdot\cdot\gamma} B_{jl\gamma}^{\cdots i}) + \nabla_m (K_{hj}^{\cdot\cdot\gamma} B_{kl\gamma}^{\cdots i} + K_{jk}^{\cdot\cdot\gamma} B_{hl\gamma}^{\cdots i} + K_{kh}^{\cdot\cdot\gamma} B_{jl\gamma}^{\cdots i}) = 0 . \qquad (2.20)$$

Proof. Applying ∇_m to Bianchi identity (1.13) and using (2.1) and (1.13), we get (2.20).

Corollary (2.2). In a generalized 2-recurrent special Kawaguchi space, the tensor field a_{lm} satisfies the relation

$$a_{mh} K_{jk}^{\cdot\cdot i} + a_{mj} K_{kh}^{\cdot\cdot i} + a_{mk} K_{hj}^{\cdot\cdot i} = 0 . \qquad (2.21)$$

Proof. Transvecting (2.20) by x'^l and using (1.11) (1.12), we get (2.21).

Theorem (2.6). A special Kawaguchi space admitting (2.2) is a generalized 2-recurrent special Kawaguchi space if and only if the associated recurrent tensor and vector fields, a_{lm} and λ_l satisfy

$$(\nabla_h' a_{lm}) K_{jk}^{\cdot\cdot i} + \lambda_l (B_{mh\gamma}^{\cdots i} K_{jk}^{\cdot\cdot\gamma} - B_{mhj}^{\cdots\gamma} K_{\gamma k}^{\cdot\cdot i} - B_{mhk}^{\cdots\gamma} K_{j\gamma}^{\cdot\cdot i})$$

$$+ (\nabla_h' \lambda_l)(\nabla_m K_{jk}^{\cdot\cdot i}) = \nabla_l (B_{mh\gamma}^{\cdots i} K_{jk}^{\cdot\cdot\gamma} - B_{mhj}^{\cdots\gamma} K_{\gamma k}^{\cdot\cdot i}$$

$$- B_{mhk}^{\cdots\gamma} K_{j\gamma}^{\cdot\cdot i}) + B_{lh\gamma}^{\cdots i} (\nabla_m K_{jk}^{\cdot\cdot\gamma}) - B_{lhm}^{\cdots\gamma} (\nabla_\gamma K_{jk}^{\cdot\cdot i})$$

$$- B_{lhj}^{\cdots\gamma} (\nabla_m K_{\gamma k}^{\cdot\cdot i}) - B_{lhk}^{\cdots\gamma} (\nabla_m K_{j\gamma}^{\cdot\cdot i}) . \qquad (2.22)$$

Proof. Applying the commutation formula (1.6) successively and noting (1.11), the equation (2.4) yields

$$\nabla_l \nabla_m R_{jkh}^{\cdots i} - a_{lm} R_{jkh}^{\cdots i} - \lambda_l \nabla_m R_{jkh}^{\cdots i} = (\nabla_h{}' a_{lm}) K_{jk}^{\cdot\cdot i}$$

$$+ \lambda_l (B_{mh\gamma}^{\cdots i} K_{jk}^{\cdot\cdot\gamma} - B_{mhj}^{\cdots\gamma} K_{\gamma k}^{\cdot\cdot i} - B_{mhk}^{\cdots\gamma} K_{j\gamma}^{\cdot\cdot i}$$

$$+ (\nabla_h{}' \lambda_l)(\nabla_m K_{jk}^{\cdot\cdot i}) - \nabla_l (B_{mh\gamma}^{\cdots i} K_{jk}^{\cdot\cdot\gamma} - B_{mhj}^{\cdots\gamma} K_{\gamma k}^{\cdot\cdot i}$$

$$- B_{mhk}^{\cdots\gamma} K_{j\gamma}^{\cdot\cdot i}) - B_{lh\gamma}^{\cdots i} (\nabla_m K_{jk}^{\cdot\cdot\gamma}) + B_{lhm}^{\cdots\gamma} (\nabla_\gamma K_{jk}^{\cdot\cdot i})$$

$$+ B_{lhj}^{\cdots\gamma} (\nabla_m K_{\gamma k}^{\cdot\cdot i}) + B_{lhk}^{\cdots\gamma} (\nabla_m K_{j\gamma}^{\cdot\cdot i}) . \tag{2.23}$$

If the space under consideration is generalized 2-recurrent the left-hand side of (2.23) vanishes and we have (2.22). Conversely, if (2.22) holds in our space, the right-hand side of (2.23) is zero and consequently the space is generalized 2-recurrent. Thus, we have the theorem (2.6).

Theorem (2.7). A necessary and sufficient condition for a special Kawaguchi space to admit (2.2) is that (2.3) holds.

Proof. Transvecting (2.23) by x'^h and noting (1.12), (1.4)[1] and (1.11), we have

$$\nabla_l \nabla_m K_{jk}^{\cdot\cdot i} - a_{lm} K_{jk}^{\cdot\cdot i} - \lambda_l \nabla_m K_{jk}^{\cdot\cdot i} = \{(\nabla_h{}' a_{lm}) K_{jk}^{\cdot\cdot i} +$$

$$+ (\nabla_h{}' \lambda_l)(\nabla_m K_{jk}^{\cdot\cdot i})\} x'^h , \tag{2.24}$$

which proves the theorem (2.7).

Note (2.1). It may be seen from (2.24) that the validity of (2.3) is not a sufficient condition for a special Kawaguchi space admitting (2.2) to become a generalized 2-recurrent special Kawaguchi space.

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ÖZET

Genelleştirilmiş bir çift indirgemeli özel Kawaguchi uzayı tanımlanmakta, bu uzaya özgü özellikler sunulmaktadır. Üzerinde genelleştirilmiş bir çift indirgemeli $K_{jk}^{\cdot\cdot i}$ tansör alanının tanımlanabildiği bir özel Kawaguchi uzayının genelleştirilmiş çift indirgemeli bir özel Kawaguchi uzayı olabilmesi için gerek ve yeter koşul verilmektedir.