FINITE LARMOR RADIUS EFFECT ON GRAVITATIONAL INSTABILITY OF A PLASMA

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The gravitational instability of an infinite homogeneous and infinitely conducting self-gravitating medium in the presence of a vertical magnetic field has been investigated by including the effects of finite Larmor radius (F.L.R.). It is found that Jeans' criterion of instability remains unaffected even if the F.L.R. effects are included. For waves propagated in the direction of the magnetic field, the effect of finite Larmor radius is to split the hydromagnetic waves into two waves travelling with different speeds, one greater and one less than 'Alfven velocity'. For waves propagated in the transverse direction of magnetic field, the F.L.R. effects add to the stability of the system. The critical wave numbers has been obtained.

1. Introduction. A detailed account of the gravitational instability of an infinite homogeneous self-gravitating medium has been given by Chandrasekhar (1961) and it has been found that a uniform magnetic field and rotation do not alter Jeans' criterion for instability.

Rosenbluth, Krall and Rostoker (1962), Roberts and Taylor (1962) and Jukes (1964) have demonstrated the stabilizing influence of finite Larmor radius, which exhibits itself in the form of a magnetic viscosity in the fluid equations, on plasma instabilities. The problem of Rosenbluth et al. (1962) has further been studied by Vandakurov (1964) with uniform cross-sectional temperatures of ions and electrons. The gravitational instability of an infinite homogeneous self-gravitating medium in the presence of a uniform horizontal magnetic field including the effects of F.L.R. has been studied by Ariel (1970).

The object of the present paper is to study the effects of finite Larmor radius on gravitational instability of a self-gravitating plasma. The plasma is considered to be infinite homogeneous and infinitely conducting.

2. Perturbation Equations. The linearized perturbation equations for an infinite, homogeneous and infinitely conducting self-gravitating medium in the presence of a uniform vertical magnetic field $\vec{H}(0,0,H)$ are

$$\rho \frac{\partial \vec{q}}{\partial t} = -\nabla \delta p - \nabla \overleftrightarrow{P} + \frac{1}{4\pi} (\nabla \times \vec{h}) \times \vec{H} + \rho \nabla \delta U , \tag{1}$$

$$\frac{\partial}{\partial t} \delta \rho = -\rho \nabla \cdot \vec{q} , \tag{2}$$

$$\delta p = c^2 \delta \rho , \tag{3}$$

$$\nabla^2 \delta U = -4\pi G \delta \rho , \tag{4}$$

$$\frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{q} \times \vec{H}) , \tag{5}$$

$$\nabla . \vec{h} = 0 . \tag{6}$$

In these equations $\vec{q}\,(u, v, \omega)$, $\vec{h}\,(h_x, h_y, h_z)$, δU, δp, $\delta\rho$ denote respectively the perturbations in velocity, magnetic field gravitational potential, scalar part of pressure and density. $c\,(= \sqrt{\nu p/\rho})$ denotes the velocity of sound in the medium.

For the vertical magnetic field along z-axis, the components of pressure tensor $\overleftrightarrow{P}$, taking into account the finite ion-gyration radius, are

$$P_{xx} = -\rho\nu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \; P_{xy} = P_{yx} = \rho\nu\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right),$$

$$P_{xz} = P_{zx} = -2\rho\nu\left(\frac{\partial v}{\partial z} + \frac{\partial \omega}{\partial y}\right), \; P_{yy} = \rho\nu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{7}$$

$$P_{yz} = P_{zy} = 2\rho\nu\left(\frac{\partial \omega}{\partial x} + \frac{\partial u}{\partial z}\right), \; P_{zz} = 0 .$$

In equations (7), $\rho\nu = \frac{NT}{4\omega_H}$, where ω_H is ion-gyration frequency, while N and T are, respectively, the number density and temperature of the ion.

3. Dispersion Relation and Discussion. Analyzing the disturbance into normal modes, we seek solutions of Eqs. (1)-(7) whose dependence on x, z and t is given by

$$\exp i\,(k_x x + k_z z + \sigma t) , \tag{8}$$

where σ is the frequency and k_x, k_z are the wave-numbers of the perturbations along the x- and z- axes.

From Eqs. (1)-(7), we have

$$\sigma u = -\left(\frac{k_x}{k^2}\right)\Omega_j^2 s + i\nu(k_x^2 + 2k_z^2)\,\mathrm{v} + \frac{H}{4\pi\rho}(k_z h_x - k_x h_z)\,, \tag{9}$$

$$\sigma \mathrm{v} = -i\nu\{(k_x^2 + 2k_z^2)u + 2k_x k_z \omega\} + \frac{H}{4\pi\rho}k_z h_y\,, \tag{10}$$

$$\sigma\omega = -\left(\frac{k_z}{k^2}\right)\Omega_j^2 s + 2i\nu k_x k_z \mathrm{v}\,, \tag{11}$$

$$\left.\begin{aligned} \sigma h_x &= H k_z u\,, \\ \sigma h_y &= H k_z \mathrm{v}\,, \\ \sigma h_z &= -H k_x u\,. \end{aligned}\right\} \tag{12}$$

Here $s\,(=\delta\rho/p)$ denotes the condensation in the medium,

$$\Omega_j^2 = c^2 k^2 - 4\pi G\rho \quad \text{and} \quad k^2 = k_x^2 + k_z^2.$$

From Eqs. (9), (10) and (12), we obtain

$$(\sigma^2 - k^2 V^2)u = -\sigma\left(\frac{k_x}{k^2}\right)\Omega_j^2 s + i\nu\sigma(k_x^2 + 2k_z^2)\,\mathrm{v}\,, \tag{13}$$

and

$$(\sigma^2 k_z^2 V^2)\,\mathrm{v} = -i\nu\sigma\{(k_x^2 + 2k_z^2)u + 2k_x k_z \omega\}\,. \tag{14}$$

Taking divergence of Eq. (1) and using Eqs. (2), (3), (4) and (7) we obtain

$$\sigma(\sigma^2 - \Omega_j^2)s = -k^2 V^2 k_x u - i\nu\sigma k_x(k_x^2 + 4k_z^2)\,\mathrm{v}\,, \tag{15}$$

where $V\left(=\sqrt{H^2/4\pi\rho}\right)$ is the Alfven velocity in the medium. The system of Eqs. (11) and (13)-(15) yields the dispersion relation

$$\begin{aligned} &\sigma^6 - [(k_x^2 + 2k_z^2)V^2 + \Omega_j^2 + 4\nu^2 k_x^2 k_z^2 + \{\nu(k_x^2 + 2k_z^2) - 2\Omega\}^2]\sigma^4 + \\ &+ \left[2k_z^2 V^2 \Omega_j^2 + k_z^2 k^2 V^4 + \Omega_j^2\left(\frac{k_z^2}{k^2}\right)\{\nu(2k_z^2 - k_x^2) - 2\Omega\}^2 + \right. \\ &\left. + 4\nu^2 k_x^2 k_z^2 k^2 V^2\right]\sigma^2 - k_z^4 V^4 \Omega_j^2 = 0\,. \end{aligned} \tag{16}$$

Equation (16) being cubic in σ^2, if σ_1, σ_2 and σ_3 denote the gyration frequencies of the three modes, then

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 = (k_x^2 + 2k_z^2)V^2 + \Omega_j^2 + 4\nu^2 k_x^2 k_z^2 + \{\nu(k_x^2 + 2k_z^2) - 2\Omega\}^2 \tag{17}$$

and

$$\sigma_1^2 \sigma_2^2 \sigma_3^2 = k_z^4 V^4 \Omega_j^2. \tag{18}$$

From Eq. (18) it follows that if $\Omega_j^2 < 0$, then one of the roots of σ^2 is negative, meaning thereby that one of the three modes is unstable. Thus Jeans' criterion for the gravitational instability of an infinite homogeneous medium is unaffected by the inclusion of F.L.R. effects.

Case I. When $k_x = 0$, $k_z = k$, i.e. for waves propagated in the direction of magnetic field, the dispersion relation (16) reduces to

$$(\sigma^2 - \Omega_j^2)\,[\sigma^4 - (2k^2 V^2 + 4\nu^2 k^4)\,\sigma^2 + k^4 V^4] = 0\,. \tag{19}$$

The factor

$$\sigma^2 - \Omega_j^2 = 0$$

gives the gravitational mode. It is independent of F.L.R. effect and its instability is given by Jeans' criterion i.e. if $\Omega_j^2 < 0$, σ^2 is negative meaning thereby that the system is gravitationally unstable.

The factor

$$\sigma^4 - (2\,k^2 V^2 + 4\nu^2\,k^4)\,\sigma^2 + k^4\,V^4 = 0$$

gives

$$\sigma = \pm\,[\,(k^2\,V^2 + \nu^2\,k^4)^{1/2} \pm \nu k^2]\,. \tag{20}$$

Eq. (20) shows that the F.L.R. effect is to split the hydromagnetic waves into two waves travelling with different wave speeds, one greater and one less than 'Alfven velocity'.

Case II. When $k_x = k$, $k_z = 0$, the dispersion Eq. (16) reduces to

$$\sigma^2 = k^2\,V^2 + \Omega_j^2 + \nu^2\,k^4\,. \tag{21}$$

From Eq. (21), it is clear that the effect of F.L.R. is to stabilize the system. The critical wave number k_* is given by

$$k_*^2 = \left\{-\frac{1}{2}(V^2 + c^2) \pm \frac{1}{2}\sqrt{(V^2 + c^2)^2 + 16\,\pi G\rho}\right\}/\nu^2\,. \tag{22}$$

The system is unstable for the wave number range $k < k_*$. This instability is of Jeans' type as the instability in this case occurs for sufficiently long wave lengths.

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ÖZET

Bu çalışmada bir düşey manyetik alan içinde, sonsuz homojen öz gravitasyonlu bir ortamın gravitasyon istikrarsızlığı, sonlu Larmor yarıçapı olayları da göz önüne alınarak incelenmektedir.